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A NEW APPROACH TO CHROMODYNAMICS*

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ABSTRACT

Chromodynamics is constructed in the framework of conformal symmetry. First we construct one action of the split octonion quarks, which is invariant under SU(2, 2). Then we construct the general relativistic theory of quarks by demanding both invariance under conformal and affine transformations. The model reveals that the split octonionic quark structure is realized on the space time topology as a spinor structure. We conclude that chromodynamics can be in essence constructed within the framework of a general relativistic theory.

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1. INTRODUCTION

We propose the possibility of constructing an exceptional realization of paraquark theory in the framework of space-time symmetry. The possibility that quarks are parafermion fields of order three has long been studied.[1] Recently Gürsey and Günaydin (GG) pioneered the exceptional realization of Klein transformation.[2] While the color gauging is generally forbidden in a theory with paraquark, it has been shown to be possible within the scheme of GG.[4] Casalbuoni, Domokos and Domokos (CDD) have extended the SU(3) group to compact G_2 and have considered the quantization problem of octonionic fields.[3] In general the automorphism group of the split octonions is the split $G_{2(2)}$ which is noncompact with signature 2. In this paper we use instead the SU(2, 2) group which is the maximal pseudounitary subgroup of SO(4, 4). SO(4, 4) is the multiplication group of the split octonions and preserves the octonionic norm.[5,6] Due to the product ansatz the color, space-time and internal symmetry (spins, flavors) of the exceptional paraquarks is just the direct product form. Here we consider the $SL(2,c) \otimes SU(2,2)$ and space-time transformations.

We identify the SU(2, 2) with the four-fold covering group of the connected conformal group. Hence we postulate that exceptional paraquarks are the realizations of the representation of the conformal group. If we consider the localized group transformation of $SL(2,c) \otimes SU(2,2)$, we will obtain the generalized version of the Einstein-Cartan-Weyl theory of graviatation by accommodating the general coordinate transformations.[8,9,13] The extended translation is dual to the intrinsic $SL(2,c) \otimes SU(2,2)$ transformations. Recently Ogievetsky has studied the group structure of these general coordinate transformations and has concluded that every higher order generators of the group are constructible from the joint configuration of generators of the affine and conformal groups.[7] Here we will construct our theory patterning after Ref. 7.

In Section 2 we define the paraquark field by the product ansatz with four color fermions and the split octonions and we write down the invariant action. In Section 3 we review the theory of Ref. 7, then we write down the invariant action of paraquarks, which encompasses the gravitational and strong interactions. Some concluding remarks are given in Section 4.

2. INVARIANT ACTION OF EXCEPTIONAL PARAQUARK FIELDS UNDER THE SU(2,2) GAUGE GROUP

We consider the split octonions at each point of space-time[6]

$$\begin{aligned} \omega_\alpha(x) &= (\omega_o, \omega_i) \\ \nu_\alpha(x) &= (\nu_o, \nu_i) \end{aligned} \tag{2.1}$$

and their octonionic conjugation $\tilde{\omega}_\alpha = \pi\,\omega_\alpha$ and $\tilde{\nu}_\alpha = \pi\,\nu_\alpha$

$$\begin{aligned} \tilde{\omega}_\alpha(x) &= (\nu_o, -\omega_i) \\ \tilde{\nu}_\alpha(x) &= (\omega_o, -\nu_i). \end{aligned} \tag{2.2}$$

Due to the product ansatz the exceptional paraquark field and its adjoint field are defined as

$$\begin{aligned} \Psi(x) &= \omega_\alpha(x)\psi_\alpha(x) + \nu_\alpha(x)\psi^c_\alpha(x) \\ \overline{\Psi}(x) &= \tilde{\omega}_\alpha(x)\overline{\psi}^c_\alpha(x) + \tilde{\nu}_\alpha(x)\overline{\psi}_\alpha(x) \end{aligned} \tag{2.3}$$

The free Lagrangian will be given formally as

$$\mathscr{L}_o = \frac{i}{8} \mathrm{Tr} \left[\overline{\Psi}(x), \gamma^{\mu} \partial_{\mu} \Psi(x)\right] \tag{2.4}$$

where Tr means the octonionic trace. This Lagrangian gives the color Lagrangian in a form similar to the usual one[10, 11]

$$\mathscr{L}_o = \frac{i}{2} \left\{ \overline{\psi}_{\alpha}(x) \gamma^{\mu} \partial_{\mu} \psi_{\alpha}(x) - \partial_{\mu} \overline{\psi}_{\alpha}(x) \gamma^{\mu} \psi_{\alpha}(x) \right\} \tag{2.5}$$

due to the octonion algebra with $\omega_o + \nu_o = 1$. If we assume the local octonion algebra[3] we must multiply the right hand side of Eq. (2.5) by a pathological infinite term.

Due to the fact that $\omega_{\alpha}(x)$ and its dual $\nu_{\alpha}(x)$ form the four-dimensional representation of SU(2,2) and its contragredient representation, we will assign the color quantum numbers to $\omega_{\alpha}(x)$ and $\nu_{\alpha}(x)$ as shown in Table 1.

The trace form of the split octonion is invariant under the SO(4,4) whose maximal rank pseudounitary subgroup is SU(2,2). Under the SU(2,2) group action the split octonions $s = \begin{bmatrix} \omega \\ \nu \end{bmatrix}$ transform as follows:

$$\begin{aligned} SU(2,2) &: \quad [s] \longrightarrow e^{iL} [s] , \\ L &= \begin{pmatrix} U_4 & O \\ O & -U_4^T \end{pmatrix} . \end{aligned} \tag{2.6}$$

where U_4 and $-U_4^T$ are respectively the four-dimensional gradient and contragredient representations of the SU(2,2) algebra. The L's include the 14 generators of the split G_2 with signature of Killing form 2 plus one diagonal generator made from the SO(4,4) algebra. The L gives the quantum number assignment of Table 1 under the classification of states under the SU(2,1) subgroup of SU(2,2).

Now we consider the local gauge transformation of the split octonionic quark field $\Psi(x)$:

$$\delta\Psi(x) = ig\,\epsilon(x) L\,\Psi(x)$$

with (2.7)

$$\delta x^{\mu} = 0.$$

Instead of constructing the usual gauge covariant derivative by introducing dynamically independent adjoint gauge fields, we consider the following fields composed from the split octonions: (2.8)

$$A_{\mu,\rho\sigma}(x) = g\left[\omega_{\alpha}(\partial_{\mu}\omega_{\beta}) - (\partial_{\mu}\omega_{\alpha})\omega_{\beta}\right](L_{\rho\sigma})_{\alpha\beta}\ ,$$

$$A^{c}_{\mu,\rho\sigma}(x) = g\left[\nu_{\alpha}(\partial_{\mu}\nu_{\beta}) - (\partial_{\mu}\nu_{\alpha})\nu_{\beta}\right](L_{\rho\sigma})_{\alpha\beta}$$

and

$$A_{\mu} = A_{\mu,\rho\sigma}(\omega_{\rho}\omega_{\sigma}),\quad A^{c}_{\mu} = A^{c}_{\mu,\rho\sigma}(\nu_{\rho}\nu_{\sigma}),$$

where $L_{\mu\nu}$ is the matrix basis of the six-dimensional representation of SU(2,2) and

$$(L_{\rho\sigma})_{\alpha\beta} = \delta_{\rho\alpha}\delta_{\sigma\beta} - \delta_{\rho\beta}\delta_{\sigma\alpha}\ . \qquad (2.9)$$

Then the covariant derivative of $\Psi(x)$ is given by

$$D_{\mu}\Psi(x) = \partial_{\mu}\Psi(x) + \left[A_{\mu}, \omega_{o}\Psi\right] + \left[A^{c}_{\mu}, \nu_{o}\Psi\right] \qquad (2.10)$$

Now we can give a formal invariant action of quark fields $\Psi(x)$:

$$I = \int d^{4}x\,\mathscr{L} \qquad (2.11)$$

where

$$\mathscr{L} = \mathscr{L}_{o} + \mathscr{L}_{I} \qquad (2.12)$$

$\mathscr{L}_o$ is given by Eq. (2.4) and

$$\mathscr{L}_I = \frac{i}{8} \mathrm{Tr} \left[\overline{\Psi}(x), \gamma^\mu \left\{ \left[A_\mu, \omega_o \Psi \right] + \left[A^c_\mu, \nu_o \Psi \right] \right\} \right] \tag{2.13}$$

The action I is readily shown to be invariant under the local SU(2,2) action. We could get the invariant action without introducing the independent gauge fields. Next we investigate the new situation by assuming that the SU(2,2) group should be the four-fold covering group of the connected conformal group C(3.1). Then we reach a new point of view on the exceptional paraquark fields. They are interpreted as objects with space-time quantum numbers which are transformed intrinsically as eight-dimensional representations of the full conformal group. Such fields have never been studied as quark fields. However, we observe that the mathematical quantities called "twistors" are analogous to our quarks here.[13]

By having identified the covering group C(3.1) with the SU(2,2) action on the exceptional paraquark fields we are in fact considering a nontrivial G bundle.[12] In this case it is natural to extend our formalism to the general relativistic framework. In gauge theories of gravitation, the general coordinate transformation (extended translation) and the SL(2,c) group structure play essential roles.[8] In our case the structure group must be in principle extended to SL(2,c) ⊗ SU(2,2).

The general coordinate transformation in space-time is the most general and a highly nonlinear one. Ogieveskey has shown that higher order generators of the transformation group are made up on the joint realization of the affine A(4) and conformal C(3.1) generators and the general relativistic theory of gravity has been constructed by the method of nonlinear realization of A(4) and C(3.1). In the next section we first recall this work on which we base our theory seen as a general relativistic theory of quarks.

3. JOINT REALIZATIONS OF AFFINE AND CONFORMAL SYMMETRIES

We now review briefly the work of Ref. 7. Consider the realizations of affine group $A(4)$ and the conformal group $C(3, 1)$ in the factor spaces $A(4)/L$ and $C(3, 1)/L$, where L is the Lorentz group. Then if $g \in A(4)$ and $\bar{g} \in C(3, 1)$, they are given as

$$g = e^{i\epsilon_\mu P_\mu} e^{\frac{i}{2}\epsilon_{\mu\nu} L_{\mu\nu}} e^{\frac{i}{2} h_{\mu\nu} R_{\mu\nu}} , \tag{3.1}$$

$$\bar{g} = e^{i\epsilon_\mu P_\mu} e^{\frac{i}{2}\epsilon_{\mu\nu} L_{\mu\nu}} e^{i\sigma D} e^{i c_\mu K_\mu} , \tag{3.2}$$

respectively. We introduce a symmetric tensor field $h_{\mu\nu}(x)$ and the Goldstone fields $C_\mu(x)$, $\sigma(x)$ and denote

$$R(x) = \exp(i\, x_\mu P_\mu) \exp\left(\frac{i}{2} h_{\mu\nu}(x) R_{\mu\nu}\right) , \tag{3.3}$$

$$C(x) = \exp(i\, x_\mu P_\mu) \exp(i\, C_\mu(x) K_\mu) \exp(i\sigma(x) D). \tag{3.4}$$

Define the action of an element of the group in accordance with

$$g : \; gR(x) = R'(x') \exp\left\{\frac{i}{2} U_{\mu\nu}(h, g) L_{\mu\nu}\right\} , \tag{3.5}$$

$$\bar{g} : \; \bar{g}\, C(x) = C'(x') \exp\left\{\frac{i}{2} U_{\mu\nu}(x, c_\mu, \sigma) L_{\mu\nu}\right\} . \tag{3.6}$$

The action of group $A(4)$ and $C(3, 1)$ on an arbitrary field $\Phi(x)$ is defined in infinitesimal form

$$\begin{aligned} \delta\Phi &= \Phi'(x') - \Phi(x) \\ &= \frac{i}{2} U_{\mu\nu}(h(x), g) L_{\mu\nu} \Phi(x) \end{aligned} \tag{3.7}$$

and

$$= \frac{i}{2} \bar{U}_{\mu\nu}(x, c_\mu(x), \sigma(x)) L_{\mu\nu} \Phi(x) . \tag{3.8}$$

In the vector representation of A(4), $R_{\mu\nu}$ is

$$(R_{\mu\nu})_{\alpha\beta} = -i\,(\delta_{\mu\alpha}\,\delta_{\nu\beta} + \delta_{\mu\beta}\,\delta_{\nu\alpha}) , \tag{3.9}$$

and we introduce the quantities

$$r_{\mu\nu}(x) = \left(\exp\left\{\frac{i}{2}\,h_{\alpha\beta}\;R_{\alpha\beta}\right\}\right)_{\mu\nu}$$

$$= (e^{h})_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu} + \frac{h_{\mu\alpha}\,h_{\alpha\nu}}{2} + \ldots \tag{3.10}$$

its inverse

$$r^{-1}_{\mu\nu}(x) = (e^{-h})_{\mu\nu} \quad , \tag{3.11}$$

and

$$g_{\mu\nu} = r_{\mu\alpha}(x)\; r_{\alpha\nu}(x) = (e^{2h})_{\mu\nu} \tag{3.12}$$

$$g^{\mu\nu} = r^{-1}_{\mu\alpha}(x)\; r^{-1}_{\alpha\nu}(x) = (e^{-2h})_{\mu\nu} \tag{3.13}$$

which are the contra- and co-variant metric tensor, with $r_{\mu\nu}$, $r^{-1}_{\mu\nu}$ corresponding to the tetrads familiar from gauge theories of gravitation. The covariant derivatives $D\Phi$ are

$$D\Phi(x) = \left(d + \frac{i}{2}\,\omega^{L}_{\mu\nu}\;L_{\mu\nu}\right)\Phi(x) , \tag{3.14}$$

so that

$$(D\,\Phi(x))' = \exp\left\{\frac{i}{2}\,U_{\mu\nu}\;L_{\mu\nu}\right\}\,(D\Phi) \tag{3.15}$$

Under the action of A(4) the covariant derivative of the $h_{\mu\nu}(x)$ and an arbitrary field $\Phi(x)$ take the forms

$$\nabla_\lambda h_{\mu\nu} = \frac{\omega^R_{\mu\nu}(d)}{\omega^P_\lambda(d)} = \frac{1}{2} r^{-1}_{\lambda\tau} \left\{ r^{-1}(x),\ \partial_\tau(x)\, r(x) \right\}_{\mu\nu} \tag{3.16}$$

and

$$\nabla_\lambda \Phi(x) = \frac{D\Phi}{\omega^P_\lambda(d)} = r^{-1}_{\lambda\tau}\partial_\tau \Phi + \frac{i}{2} v^{min}_{\mu\nu,\lambda} L_{\mu\nu} \Phi \tag{3.17}$$

where

$$v^{min}_{\mu\nu,\lambda}(x) = \frac{1}{2} r^{-1}_{\lambda\tau} \left[r^{-1}(x),\ \partial_\tau r(x) \right]_{\mu\nu} \tag{3.18}$$

There remain the same transformation properties of the covariant derivative $\nabla_\lambda \Phi$, when $v^{min}_{\mu\nu,\lambda}$ is replaced by (with arbitrary values of c_i)

$$\begin{aligned} V_{\mu\nu,\lambda} = v^{min}_{\mu\nu,\lambda} &+ c_1(\nabla_\mu h_{\nu\lambda} - \nabla_\nu h_{\mu\lambda}) + c_2(\delta_{\mu\lambda}\nabla_\nu h_{\sigma\sigma} \\ &- \delta_{\nu\lambda}\nabla_\mu h_{\sigma\sigma}) + c_3(\delta_{\mu\lambda}\nabla_\tau h_{\nu\tau} - \delta_{\nu\lambda}\nabla_\tau h_{\mu\tau}) \end{aligned} \tag{3.19}$$

Hence in general $\nabla_\lambda \Phi$ is:

$$\nabla_\lambda \Phi = r^{-1}_{\lambda\tau}\partial_\tau \Phi + \frac{i}{2} V_{\mu\nu,\lambda} L_{\mu\nu} \Phi \tag{3.20}$$

Under the C(3.1) action the covariant derivative is given as

$$\bar{\nabla}_\lambda \Phi = \frac{d\Phi + \frac{i}{2}\bar{\omega}^L_{\mu\nu} L_{\mu\nu}\Phi}{\omega^P_\lambda(d)} = e^{-\sigma(x)} (\partial_\lambda \Phi + i\,\partial_\nu \sigma\, L_{\lambda\nu}\Phi), \tag{3.21}$$

with the condition $C_\mu(x) = \frac{1}{2}\partial_\mu \sigma(x)$. For the tensor field $h_{\alpha\beta}(x)$ we have

$$\bar{\nabla}_\lambda h_{\alpha\beta}(x) = e^{-\sigma(x)}\Big\{\partial_\lambda h_{\alpha\beta} + \partial_\tau \sigma(x)(\delta_{\alpha\lambda} h_{\tau\beta} + \delta_{\beta\lambda} h_{\alpha\tau} - \delta_{\alpha\tau} h_{\lambda\beta} - \delta_{\beta\tau} h_{\alpha\lambda}\Big\} . \tag{3.22}$$

It is apparent that the above trace $R_{\mu\mu}$ of $R_{\mu\nu}$ is related to the dilatation generator D by $R_{\mu\mu} = 2D$. Hence $\sigma(x) = \frac{1}{4} h_{\mu\mu}(x)$ and set

$$h_{\mu\nu}(x) = \bar{h}_{\mu\nu}(x) + \delta_{\mu\nu}\,\sigma(x) \tag{3.23}$$

We can re-express $\nabla_\lambda h_{\mu\nu}$, $v^{min}_{\mu\nu,\lambda}$, and $r^{-1}_{\lambda\tau}\,\partial_\tau\,\Phi$ as

$$\begin{aligned}
\nabla_\lambda h_{\mu\nu} &= \frac{1}{2}(e^{-\bar{h}})_{\lambda\nu} e^{-\sigma}\Big\{e^{-\bar{h}},\ \partial_\nu\, e^{\bar{h}}\Big\}_{\mu\nu} \\
&\quad + e^{-\sigma}\,(e^{-\bar{h}})_{\lambda\tau}\,\partial_\tau \sigma \delta_{\mu\nu} \\
&= \frac{1}{2}(e^{-\bar{h}})_{\lambda\nu}\Big\{e^{-\bar{h}},\ \nabla_\nu e^{\bar{h}}\Big\}_{\mu\nu} \\
&\quad + \frac{\partial_\tau \sigma}{2}\, e^{-\sigma}\Big[(e^{-\bar{h}})_{\mu\tau}\,\delta_{\lambda\nu} - (e^{-2\bar{h}})_{\mu\lambda}\,(e^{\bar{h}})_{\tau\nu} \\
&\quad + (e^{-\bar{h}})_{\lambda\tau}\,\delta_{\mu\nu} + (\mu \longleftrightarrow \nu)\Big] ,
\end{aligned} \tag{3.24}$$

$$v^{\min}_{\mu\nu,\lambda} = \frac{1}{2} r^{-1}_{\lambda\tau} \left[r^{-1}, \partial_\tau r \right]_{\mu\nu}$$

$$= \frac{1}{2} e^{-\sigma} (e^{-\bar{h}})_{\lambda\tau} \left[e^{-\bar{h}}, \partial_\tau e^{-\bar{h}} \right]_{\mu\nu}$$

$$= \frac{1}{2} (e^{-\bar{h}})_{\lambda\tau} \left[e^{-\bar{h}}, \bar{\bar{\nabla}}_\tau e^{\bar{h}} \right]_{\mu\nu}$$

$$+ \frac{1}{2} \partial_\tau \sigma e^{-\sigma} \Big[(e^{-\bar{h}})_{\mu\tau} \delta_{\lambda\nu} + 2 (e^{-\bar{h}})_{\mu\lambda} \delta_{\tau\nu}$$

$$+ (e^{-2\bar{h}})_{\nu\lambda} (e^{\bar{h}})_{\mu\tau} - (\mu \longleftrightarrow \nu) \Big] , \tag{3.25}$$

$$r^{-1}_{\lambda\tau} \partial_\tau \Phi = (e^{-\bar{h}})_{\lambda\tau} \bar{\nabla}_\tau \bar{\Phi} - i \partial_\nu \sigma e^{-\sigma} (e^{-\bar{h}})_{\lambda\mu} L_{\mu\nu} \Phi . \tag{3.26}$$

Upon substitution in Eq. (3.9), and the requirement that $\nabla_\lambda \Phi$ depends on $\sigma(x)$ and $\partial_\nu \sigma(x)$ solely through the conformally covariant operations $\bar{\bar{\nabla}}$, we get $c_1 = -1$, $c_2 = c_3 = 0$.

Now the covariant derivative for both the affine and conformal symmetry of any field $\Phi(x)$ is

$$\nabla_\lambda \Phi = r^{-1}_{\lambda\tau} \partial_\tau \Phi + \frac{i}{2} V_{\mu\nu,\lambda} L_{\mu\nu} \Phi , \tag{3.27}$$

with the connection

$$V_{\mu\nu,\lambda} = \frac{1}{2} \Big\{ r^{-1}_{\lambda\gamma} \left[r^{-1}, \partial_\gamma r \right]_{\mu\nu} - r^{-1}_{\mu\gamma} \left\{ r^{-1}, \partial_\gamma r \right\}_{\lambda\nu}$$

$$+ r^{-1}_{\nu\gamma} \left\{ r^{-1}, \partial_\gamma r \right\}_{\lambda\mu} \Big\} . \tag{3.28}$$

In the joint realization of affine and conformal symmetries, both

$$\nabla_\lambda \sigma = 0 \tag{3.29}$$

and

$$\nabla_\lambda h_{\mu\nu} = 0 \tag{3.30}$$

Finally we consider the commutator of the covariant derivatives of any field Φ (x). We have

$$(\nabla_\lambda \nabla_\rho - \nabla_\rho \nabla_\lambda)\, \Phi = \frac{i}{2} R_{\mu\nu,\, \lambda\rho}\, L_{\mu\nu} \Phi \;, \tag{3.31}$$

where

$$R_{\mu\nu,\lambda\rho} = r^{-1}_{\lambda\gamma} \partial_\gamma V_{\mu\nu,\rho} + V_{\mu\nu,\gamma} V_{\rho\gamma,\lambda} + V_{\mu\gamma,\rho} V_{\nu\gamma,\lambda} - (\lambda \longleftrightarrow \rho) \;. \tag{3.32}$$

Its contraction

$$R = R_{\mu\nu,\mu\nu} = 2\, r^{-1}_{\mu\gamma} \partial_\gamma V_{\mu\nu,\nu} + V_{\mu\nu,\gamma} V_{\nu\gamma.\mu} - V_{\mu\gamma,\mu} V_{\nu\gamma,\nu} . \tag{3.33}$$

Then we obtain the invariant action under the affine and conformal transformation.

$$I = \int d^4x\, \det(r) \left[\mathscr{L}(\Phi,\; \nabla_\mu \Phi) + \frac{\Phi}{4f^2} R \right] \;. \tag{3.34}$$

The theory is identified with the theory of gravitational field by equating

$$\frac{1}{4\pi} f^2 = 6.67 \times 10^{-8}\, cm^3 \cdot g^{-1}\, sec^{-2} \;. \tag{3.35}$$

We are ready now to proceed with our theory. We will write down the action of the exceptional quark field is invariant under the A(4) · C (3, 1) ⊗ SL (2, c) ⊗

SU(2, 2) structure group:

$$I = \int d^4x \det(r) \left\{ \frac{i}{8} \mathrm{Tr} \left[\bar{\Psi}(x), \; \gamma^{\mu} \nabla_{\mu} \; \Psi(x) \right] + \frac{1}{4f^2} R \right\} , \tag{3.36}$$

where

$$\nabla_{\mu} \Psi(x) = r^{-1}_{\mu\tau} \, \partial_{\tau} \Psi - r^{-1}_{\mu\tau} \left\{ \left[A_{\tau}, \; \omega_0 \; \Psi \right] + \left[A^{c}_{\tau}, \nu_0 \Psi \right] \right\}$$

$$+ \frac{i}{4} V_{\rho\sigma,\mu} \; \sigma_{\rho\sigma} \Psi \tag{3.37}$$

It is apparent that the action (3.36) gives the gravitational theory for each color spinor due to the Eq. (2.5) with a condition that the split octonions are global. The detailed structures and the physical results of our system will be the objects of subsequent reports.

4. CONCLUDING REMARKS

The split octonions form the 8 dimensional representation of SO(4,4). Under the SU(2,2) subgroup of SO(4,4) the 8 octonions split into the 4 dimensional representation of SU(2,2) and its dual. We identified this SU(2,2) with the four-fold covering group of the conformal group C(3, 1). By virtue of this assumption we could consider the color symmetry as a realization of space-time symmetry. The situation is very similar to that of the spinors.[9] The spinor is transformed solely through spin generators (up to dilatation), but the split octonions are transformed by the full conformal group. Such an entity is a completely new physical object. Such a type of fundamental field has not been envisioned in the literature, though it shares certain features with Penrose's twistor,[13] nor have they been applied to quarks. Therefore it would be interesting to study any relations between our paraquark and the twistors.

We have shown the possibility of writing down a general relativistic theory of exceptional paraquarks. The point of view proposed here seems to provide an appealing formalism to bring gravitational and strong interactions within one single framework.[12] The detailed analysis of this formulation will be performed elsewhere.

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	I_3^c		Y_3^c		B	
ω_1	$\frac{1}{2}$		$\frac{1}{3}$		$\frac{1}{3}$	
ν_1		$-\frac{1}{2}$		$-\frac{1}{3}$		$-\frac{1}{3}$
ω_2	$-\frac{1}{2}$		$\frac{1}{3}$		$\frac{1}{3}$	
ν_2		$\frac{1}{2}$		$-\frac{1}{3}$		$-\frac{1}{3}$
ω_3	0		$-\frac{2}{3}$		$\frac{1}{3}$	
ν_3		0		$\frac{2}{3}$		$-\frac{1}{3}$
ω_0	0		0		-1	
ν_0		0		0		1

Table 1. SU(2, 2) assignment of quantum numbers to the split octonions. The sextet gluons transform as 3 and 3* under the SU(2, 1) subgroup and have number B = $\frac{2}{3}$, $-\frac{2}{3}$, respectively.