



T_{bbb} : A three B -meson bound state

H. Garcilazo^a, A. Valcarce^{b,*}

^a Escuela Superior de Física y Matemáticas, Instituto Politécnico Nacional, Edificio 9, 07738 México D.F., Mexico

^b Departamento de Física Fundamental and IUFFyM, Universidad de Salamanca, E-37008 Salamanca, Spain

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ABSTRACT

By solving exactly the Faddeev equations for the bound-state problem of three mesons, we demonstrate that current theoretical predictions pointing to the existence of a deeply-bound doubly bottom axial vector tetraquark lead to the existence of a unique bound state of three B mesons. We find that the $BB^*B^* - B^*B^*B^*$ state with quantum numbers $(I)J^P = (1/2)2^-$, T_{bbb} , is about 90 MeV below any possible three B -meson threshold for the reported binding of the doubly bottom axial vector tetraquark, T_{bb} .

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1. Introduction

There is a broad theoretical consensus about the existence of a deeply-bound doubly bottom tetraquark with quantum numbers $(i)j^P = (0)1^+$ strong- and electromagnetic-interaction stable [1–10]. In the pioneering work of Ref. [10] it was shown that $QQ\bar{q}\bar{q}$ four-quark configurations become more and more bound when the mass ratio M_Q/m_q increases, the critical value for binding being somewhat model dependent.

Lattice QCD calculations find unambiguous signals for a stable $j^P = 1^+$ bottom-light tetraquark [1]. Based on a diquark hypothesis, Ref. [2] uses the discovery of the Ξ_{cc}^{++} baryon [11] to calibrate the binding energy in a QQ diquark. Assuming that the same relation is true for the bb binding energy in a tetraquark, it concludes that the axial vector $bb\bar{u}\bar{d}$ state is stable. The Heavy-Quark Symmetry analysis of Ref. [3] predicts the existence of narrow doubly heavy tetraquarks. Using as input for the doubly bottom baryons, not yet experimentally measured, the diquark-model calculations of Ref. [2] also leads to a bound axial vector $bb\bar{u}\bar{d}$ tetraquark. Other approaches, using Wilson twisted mass lattice QCD [4], also find a bound state. Few-body calculations using quark–quark Cornell-like interactions [5,6], simple color magnetic models [7], QCD sum rule analysis [8], or phenomenological studies [9] come to similar conclusions.

The possible existence of deuteron-like hadronic molecular states made of vector–vector or pseudoscalar–vector two-meson systems was proposed in Ref. [12] in an exploratory study suggesting the *deusons*, two-meson states bound by the one-pion

exchange potential. This scenario of meson–meson stable states bound by some interacting potential, has later on been frequently used to draw conclusions about the existence of hadronic molecules [13–15] (see Refs. [16] for a recent compendium). The constituent quark and the meson–meson approaches to hadronic molecules must be equivalent [17], although, as will be discussed below, to get the results of the constituent quark approach would, in general, require a coupled-channel meson–meson study [18].

It is also worth to emphasize that when a two-body interaction is attractive, if the two-body system is merged with nuclear matter and the Pauli principle does not impose severe restrictions, the attraction may be reinforced. We find the simplest example of the effect of additional particles in the two-nucleon system. The deuteron, $(i)j^P = (0)1^+$, is bound by 2.225 MeV, while the triton, $(I)J^P = (1/2)1/2^+$, is bound by 8.480 MeV, and the α particle, $(I)J^P = (0)0^+$, is bound by 28.295 MeV. The binding per nucleon B/A increases as $1:3:7$. Thus, a challenging question is if the existence of a deeply bound two B -meson system¹ could give rise to bound states of a larger number of particles. As it was shown in Ref. [19] the answer is by no means trivial, because when the internal two-body thresholds of a three-body system are far away, they conspire against the stability of the three-body system.

2. Color dynamics

As it has been stated above, results based on meson–meson scattering or a constituent quark picture should be equivalent, provided that, in general, a coupled-channel meson–meson ap-

* Corresponding author.

E-mail addresses: humberto@esfm.ipn.mx (H. Garcilazo), valcarce@usal.es (A. Valcarce).

¹ The binding energy for the axial vector doubly bottom tetraquark reported in Refs. [1–10] ranges between 90 and 214 MeV.

Table 1
Different two-body channels (i, j) contributing to the $(I)J^P = (1/2)2^-$ $BB^*B^* - B^*B^*B^*$ system.

Interacting pair	(i, j)	Spectator
BB^*	(0, 1)	B^*
	(1, 1)	
B^*B^*	(0, 1)	B^*
	(1, 2)	
B^*B^*	(1, 2)	B

proach would be necessary to reproduce the constituent quark picture [17,18]. To be a little more specific, let us note that four-quark systems present a richer color structure than standard baryons or mesons. Although the color wave function for standard mesons and baryons leads to a single vector, working with four-quark states there are different vectors driving to a singlet color state out of colorless meson–meson (11) or colored two-body (88, 33, or 66) components. Thus, dealing with four-quark states an important question is whether one is in front of a colorless meson–meson molecule or a compact state (i.e., a system with two-body colored components). Note, however, that any hidden color vector can be expanded as an infinite sum of colorless singlet–singlet states [17]. This has been explicitly done for compact $QQ\bar{q}\bar{q}$ states in Ref. [18].

In the heavy-quark limit, the lowest lying tetraquark configuration resembles the helium atom [3], a factorized system with separate dynamics for the compact color $\bar{3} QQ$ nucleus and for the light quarks bound to the stationary color 3 state, to construct a $QQ\bar{q}\bar{q}$ color singlet. The validity of this argument has been mathematically proved and numerically checked in Ref. [18], see the probabilities shown in Table II for the axial vector $bb\bar{u}\bar{d}$ tetraquark. It has been recently revised in Ref. [6], showing in Fig. 8 how the probability of the 66 component in a compact $QQ\bar{q}\bar{q}$ tetraquark tends to zero for $M_Q \rightarrow \infty$. Therefore, heavy–light compact bound states would be almost a pure $\bar{3}3$ singlet color state and not a single colorless meson–meson 11 molecule. Such compact states with two-body colored components can be expanded as the mixture of several physical meson–meson channels [17], BB^* and B^*B^* for the axial vector $bb\bar{u}\bar{d}$ tetraquark (see Table II of Ref. [18]) and, thus, they can be also studied as an involved coupled-channel problem of physical meson–meson states [20,21].

Our aim in this work is to solve exactly the Faddeev equations for the three-meson bound state problem using as input the two-body t -matrices of Refs. [5,18–20], driving to the axial vector $bb\bar{u}\bar{d}$ bound state, T_{bb} , as an involved coupled-channel system made of pseudoscalar–vector and vector–vector two B -meson components. We show that for any of the recently reported values of the T_{bb} binding energy [1–10], the three-body system $BB^*B^* - B^*B^*B^*$ with quantum numbers $(I)J^P = (1/2)2^-$, T_{bbb} , is between 43 to 90 MeV below the lowest three B -meson threshold.

3. The three-body system

Out of the possible spin–isospin three-body channels $(I)J^P$ made of B and B^* mesons, we select those where, firstly, two-body subsystems containing two B -mesons are not allowed, because the BB interaction does not show an attractive character; and, secondly, they contain the axial vector $(i)j^P = (0)1^+$ doubly bottom tetraquark, T_{bb} . The three-body channel $(I)J^P = (1/2)2^-$ is the only one bringing together all these conditions to maximize the possible binding of the three-body system.² We indicate in Table 1

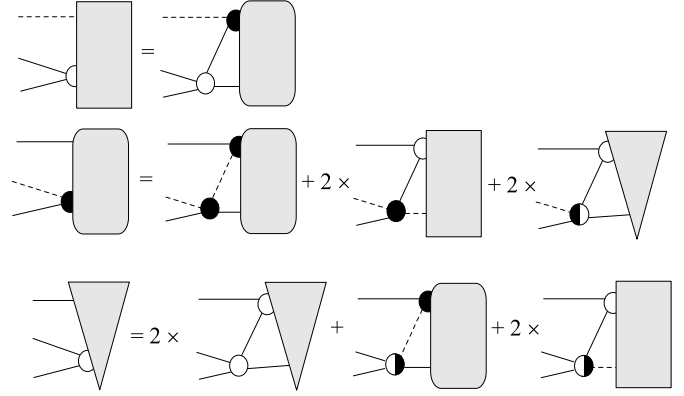


Fig. 1. Diagrammatic Faddeev equations for the three B -meson system.

the two-body channels contributing to this state that we examine in the following.

The Lippmann–Schwinger equation for the bound-state three-body problem is

$$T = (V_1 + V_2 + V_3)G_0T, \quad (1)$$

where V_i is the potential between particles j and k and G_0 is the propagator of three free particles. The Faddeev decomposition of Eq. (1),

$$T = T_1 + T_2 + T_3, \quad (2)$$

leads to the set of coupled equations,

$$T_i = V_iG_0T. \quad (3)$$

The Faddeev decomposition guarantees the uniqueness of the solution [22]. Eqs. (3) can be rewritten in the Faddeev form

$$T_i = t_iG_0(T_j + T_k), \quad (4)$$

with

$$t_i = V_i + V_iG_0t_i, \quad (5)$$

where t_i are the two-body t -matrices that already contain the coupling among all two-body channels contributing to a given three-body state, see Table 1. The two sets of equations (3) and (4) are completely equivalent for the bound-state problem. In the case of two three-body systems that are coupled together, like $BB^*B^* - B^*B^*B^*$, the amplitudes T_i become two-component vectors and the operators V_i , t_i , and G_0 become 2×2 matrices and lead to the equations depicted in Fig. 1. The solid lines represent the B^* mesons and the dashed lines the B meson. If in the second equation depicted in Fig. 1 one drops the last term in the r.h.s. then the first and second equations become the Faddeev equations of two identical bosons plus a third one that is different [19]. Similarly, if in the third equation depicted in Fig. 1 one drops the last two terms this equation becomes the Faddeev equation of a system of three identical bosons since in this case the three coupled Faddeev equations are all identical [19]. The additional terms in Fig. 1 are, of course, those responsible for the coupling between the BB^*B^* and $B^*B^*B^*$ components of the system.

² Note that the three-body channels with $J = 0$ or 1 would couple to two B -meson subsystems where no attraction has been reported [1–10], whereas the $J = 3$ would not contain a two-body subsystem with $j = 1$, the quantum numbers

of the deeply bound doubly-bottom tetraquark. The same reasoning excludes the $I = 3/2$ channels.

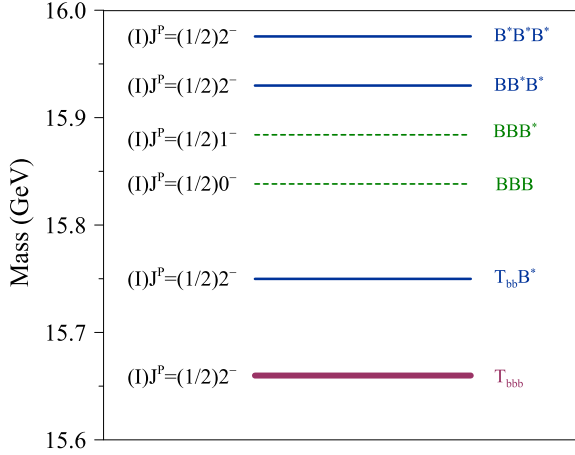


Fig. 2. Mass of the three-body $BB^*B^* - B^*B^*B^*$ bound-state $(I)J^P = (1/2)2^-$ T_{bbb} (purple thick line), compared to the different three B -meson strong (blue solid lines) and electromagnetic decay thresholds (green dashed lines). (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

4. Results

We show in Fig. 2 the results of our calculation. The blue solid lines stand for the different three B -meson strong decay thresholds of the $BB^*B^* - B^*B^*B^*$ system with quantum numbers $(I)J^P = (1/2)2^-$, that we have denoted by T_{bbb} . These thresholds are $B^*B^*B^*$, BB^*B^* and $T_{bb}B^*$, where T_{bb} represents the axial vector $(i)j^P = (0)1^+$ doubly bottom tetraquark. The green dashed lines stand for the possible three- B meson electromagnetic decay thresholds, BBB^* and BBB with quantum number $(I)J^P = (1/2)1^-$ and $(I)J^P = (1/2)0^-$, respectively. Finally, the purple thick line indicates the energy of the T_{bbb} state, that appears 90 MeV below the lowest threshold. The results shown in Fig. 2 correspond to the binding energy of the T_{bb} axial vector tetraquark obtained in Ref. [1].

There is also a baryon-antibaryon threshold $\Omega_{bbb} - \bar{p}$ clearly decoupled from the T_{bbb} , with a tetraquark-meson dominant component driving to the three B -meson bound state, due to the orthogonality of the color wave function. The decay of the T_{bbb} multiquark state $|\Psi_{T_{bbb}}\rangle$, with a dominant tetraquark-meson color component,³ into a baryon (B_1) plus and antibaryon (\bar{B}_2) is forbidden if the transition amplitude $\langle B_1\bar{B}_2|T|\Psi_{T_{bbb}}\rangle$ vanishes. In principle T is the transition matrix (or S matrix) which is roughly e^{iH} , but since $|\Psi_{T_{bbb}}\rangle$ is a true eigenstate of H , the transition amplitude vanishes if the overlap $\langle B_1\bar{B}_2|\Psi_{T_{bbb}}\rangle$ vanishes itself [24]. Since there are no experimental data for the Ω_{bbb} mass and there is a wide variety of theoretical estimations (see Table 1 of Ref. [25]) it has to be calculated within the same scheme. For the binding energy of the T_{bb} axial vector tetraquark obtained in Ref. [1], the Ω_{bbb} has a mass of 14.84 GeV. Thus, the $\Omega_{bbb} - \bar{p}$ threshold would lie at 15.78 GeV, above the T_{bbb} state. Let us note that even if the $\Omega_{bbb} - \bar{p}$ threshold would lie below the three B -meson energy, the T_{bbb} state will show up as a narrow resonance as recently discussed in Ref. [26], due to the negligible interaction between the Ω_{bbb} and the \bar{p} . The dynamics of this type of states would come controlled by the attraction in the three-body system and the channel made of almost non-interacting hadrons is mainly a tool for the detection. This is exactly the same situation observed in the case of the lower LHCb pentaquark $P_c^+(4380)$ [27] with a mass of

Table 2

Binding energy, in MeV, of the T_{bbb} $(I)J^P = (1/2)2^-$ $BB^*B^* - B^*B^*B^*$ three-body system as a function of the binding energy, in MeV, of the axial vector tetraquark T_{bb} . The T_{bbb} binding energy is calculated with respect to the lowest strong decay threshold: $m_B + 2m_{B^*} - B(T_{bb})$.

$B(T_{bb})$	$B(T_{bbb})$
180	90
144	77
117	57
87	43

$4380 \pm 8 \pm 29$ MeV, that it is seen to decay to the $J/\Psi - p$ channel with a width $\Gamma = 205 \pm 18 \pm 86$ MeV, while the phase space is of the order of 345 MeV.

We have checked that the T_{bbb} exotic state remains stable for the whole range of binding energies of the axial vector tetraquark T_{bb} reported in the different theoretical studies [1–10]. Thus, we have repeated the coupled-channel three-body calculation for different binding energies of the axial vector tetraquark T_{bb} , starting from the smallest binding of the order of 90 MeV obtained in Ref. [4]. The results are given in Table 2. It can be seen that the three-meson bound state T_{bbb} is comfortably stable for any of the binding energies of the axial vector tetraquark T_{bb} reported in the literature. If the binding energy of the T_{bb} state is reduced up to 50 MeV, the three-body system would have a binding of the order of 23 MeV that would lie already 19 MeV above the lowest BBB threshold, so that one does not expect any kind of Borromean binding in this system. The situation is even worse in the charm sector, because the vector-pseudoscalar meson mass difference changes from 45 MeV in the bottom sector to 141 MeV in the charm sector, so that the DDD and DDD^* thresholds would lie 282 MeV and 141 MeV below the DD^*D^* energy, respectively.

5. Summary

By solving exactly the Faddeev equations for the bound-state problem of three mesons, we demonstrate that the current theoretical predictions pointing to the existence of a deeply-bound doubly bottom axial vector tetraquark lead to the likelihood of a bound state of three B mesons. We find that the $BB^*B^* - B^*B^*B^*$ state with $(I)J^P = (1/2)2^-$, T_{bbb} , is about 90 MeV below any possible three B -meson threshold for the standard binding of the recently reported axial vector doubly bottom tetraquark, T_{bb} . It is important to note, as we have explained above, that this is the only three-body channel bringing together all necessary conditions about the two-body subsystems that allow to maximize the binding of the three-body system. In other words, this unconventional form of a three-body hadron is unique. The experimental search of these tetraquark, T_{bb} , and hexaquark, T_{bbb} , structures is a challenge well worth pursuing, because they are the first manifestly exotic hadrons stable under strong and electromagnetic interaction.

It is appealing that the stability of such hexaquark state with respect the lowest tetraquark-meson threshold was already anticipated in the exploratory study of Ref. [28] within a quark string model. Let us finally note that our discussion above could be extended to the charm sector, where the two-body bound state would lie close to threshold [21,29]. However, as we have noted above, going from the bottom to the charm sector there is a factor 3 in the mass difference between pseudoscalar and vector mesons, what makes the coupled-channel effect much less important in the charm case than in the bottom one. Thus, one does not expect binding in the three-meson charm sector.

³ This is in contrast to the analysis of Ref. [23] where baryon-antibaryon annihilation into three-mesons is studied by simple quark rearrangement.

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