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HIGH ENERGY LIMIT FOR THE REAL PART OF FORWARD COMPTON SCATTERING*

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We investigate the contribution of a pole at $\alpha = 0$ to forward Compton scattering using finite energy sum rules. We conclude that an experiment measuring the total photoabsorption cross section from $\sim$ 1 to 10-15 BeV with an accuracy of 5% will detect a contribution from such a pole whose magnitude is larger than 30% of the Thomson limit.

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Finite energy sum rules relate the low energy behavior of individual two particle transition amplitudes with their high energy behavior. Such a connection of high and low energy behaviors is accomplished as a result of the analyticity restrictions of Regge theory. In this paper we inquire into the relation between the exact low energy theorems for Compton scattering of a photon from protons and the high energy asymptotic limit. In particular, we concentrate on the forward scattering amplitude in order to introduce experimentally measurable total photoabsorption cross sections for the imaginary parts.

Similar analyses have been made for the pion nucleon amplitudes.[1] Because of the peculiar possibilities of fixed poles and/or singular residues, the Compton amplitude requires a separate discussion. First of all there is as yet no persuasive evidence for Regge behavior in amplitudes involving even one external photon line. Charged pion photoproduction over a broad range of momentum transfers, t, has an energy dependence from ≈ 2 GeV to 16 GeV mysteriously compatible with fixed $J = 0$ poles playing the dominant role in the t channel.[2] Furthermore, the dip in π^0 photoproduction at the "nonsense" zero in the ω exchange contribution recedes in prominence at the highest measured energies and no simple Regge parametrization can explain the observations within the framework of vector dominence.[3]

The pure simple Regge hypothesis also fails to fill the bill for the elastic Compton scattering from a proton as first discussed by Mur and subsequently in more detail by others.[4] In the forward direction we expect, in complete analogy with πp and pp elastic scattering, that we will find a forward diffraction peak corresponding to Pomeron exchange. However, a Pomeron leading to a constant total cross section, σ_T, at high energies must have a Regge trajectory intersecting at $\alpha_P(0) = 1$ and thus behaving under three dimensional rotations as a vector. In the forward direction the photon cannot flip helicity and an incident right circularly polarized γ (rhγ) must emerge as a rhγ simply by angular momentum conservation. Upon crossing to the t channel and the process $\gamma + \gamma \to p + \overline{p}$, the emerging rh$\gamma$ crosses to a ℓhγ incident and the two incoming γ's form a system with two units of helicity. This cannot however be deposited upon a Pomeron of unit spin if $\alpha_P(0) = 1$. If the Pomeron does not couple or if we must contrive to make $\alpha_P(0) < 1$, we do not predict a constant σ_T at high energies and we lose in an instant the motivating charm of the Pomeron trajectory in Reggeism. Originally it was designed to reproduce in hadron physics the classical diffraction picture in the classical problem of light scattering. To restore the constant cross section limit for very high energies we must either introduce a fixed pole at $J = 1$ or else resort to a singular residue to cancel the vanishing coupling described above when $\alpha_P(0) = 1$. This need not disturb us because the usual arguments against and about fixed poles are based on non-linear unitarity and thus are not applicable to the Compton amplitude when the photons are treated to lowest order in e^2.

Recognizing this we may ask whether the appearance of a pole at $\alpha = 0$ in the high energy behavior of the Compton amplitude might lead to experimentally identifiable contributions to the analysis with finite energy sum rules. We conclude that an answer can be found provided that we know the total photoabsorption cross-section to an accuracy of 5% for laboratory energies ranging up to 10-15 BeV.

The amplitude for forward Compton scattering from a proton, or arbitrary spin $\frac{1}{2}$ particle, is written[5]

$$f(\nu) = f_1(\nu)\, \underline{e}'^{*} \cdot \underline{e} + i\nu f_2(\nu)\, \underline{\sigma} \cdot \underline{e}'^{*} \times \underline{e} \tag{1}$$

where ν is the laboratory energy of the photon, and $\underline{e}$ and $\underline{e}'$ are the transverse polarization vectors of the incident and scattered photon. We shall be interested in the spin independent amplitude $f_1(\nu)$ which satisfies a low energy theorem (the Thomson limit)

$$f_1(0) = -\frac{e^2}{4\pi m} \tag{2}$$

where m is the proton mass, and which is related to the total spin averaged cross section through the optical theorem

$$\mathrm{Im}\, f_1(\nu) = \frac{\nu}{4\pi}\, \sigma_T(\nu) \tag{3}$$

We assume that the high energy behavior of $f_1(\nu)$ can be written as a Regge expansion of the form[6]

$$f_1(\nu) = \sum_{\alpha \leq 1} \frac{\beta_\alpha \nu^\alpha}{4\pi} \left(\frac{-1 - e^{-i\pi\alpha}}{\sin \pi\alpha} \right). \tag{4}$$

Eq. (4) determines the asymptotic form of the real and imaginary parts of the amplitude. When α is an even integer we demand that $\beta_\alpha / \sin \pi\alpha$ be finite.[7] Then the imaginary part vanishes and only the real part survives, giving a contribution from the even integers of the form

$$C + D/\nu^2 + \ldots .$$

Since we are concerned with high energy limiting behavior we shall consider only the possibility of $C \neq 0$ corresponding to a real term being added to Eq. (4) with $\alpha = 0$. For odd integer values of α the contribution to $\mathrm{Re} f_1(\nu)$ vanishes according to Eq. (4) and such poles contribute only in the imaginary part. In particular $\alpha = 1$ is the usual Pomeron or diffraction term. The specific question we want to get at is this: Is there in fact a real constant C to be added so that Eq. (4) becomes

$$f_1(\nu) = \sum_{\substack{\alpha \leq 1 \\ \alpha \neq 0}} \frac{\beta_\alpha \nu^\alpha}{4\pi} \left(\frac{-1 - e^{-i\pi\alpha}}{\sin \pi\alpha} \right) + C \tag{5}$$

and where C in a loose sense tells us how much of the Thomson limit, Eq. (2), survives as an $\alpha(0) = 0$ pole in the asymptotic behavior ?

To answer this we use the following two sum rules[8]

$$\int_\mu^N \sigma_T(\nu) d\nu = \sum_{\alpha > 0} \frac{\beta_\alpha N^\alpha}{\alpha} - \frac{e^2}{4\pi} \frac{2\pi^2}{m} - 2\pi^2 C + \sum_{\alpha < 0} \frac{\beta_\alpha N^\alpha}{\alpha} \tag{6}$$

$$\int_\mu^N \nu^2 \sigma_T(\nu) d\nu = \sum_{\alpha > 0} \frac{\beta_\alpha N^{\alpha+2}}{\alpha + 2} + \sum_{\alpha < 0} \frac{\beta_\alpha N^{\alpha+2}}{\alpha + 2} \tag{7}$$

In the analysis we include the P, P', and the A_2 trajectories along with the pole at $\alpha = 0$ giving rise to C. Furthermore, we assume one effective trajectory for both the P' and A_2 with $\overline{\alpha} \approx \frac{1}{2}$. By choosing N large enough we hope that poles with $\alpha < 0$ are negligible.

We are thus left with three parameters, β, the residue function for the Pomeron, $\beta_{\overline{\alpha}}$ for the effective P', A_2 trajectory, and the constant C. Eqs. (3) and (5) combine for $\nu \rightarrow \infty$ to tell us that $\beta_1 = \sigma_T(\infty)$ and the remaining two parameters are determined by Eqs. (6) and (7). We solve these using recent DESY data[9] up to 5.3 BeV obtaining

$$\sigma_T(\infty) = 110 \pm 30 \ \mu b$$

$$\beta_{\overline{\alpha}} = 10 \pm 50 \ \mu b \ (\mathrm{BeV})^{\frac{1}{2}}$$

$$C = -7 \pm 4 \ \mu b \ \mathrm{BeV}$$

In the absence of strong interactions and to lowest order in e^2, we expect only the Thomson term to survive so that

$$C = \frac{-e^2}{4\pi} \frac{1}{m} = -3.0 \ \mu b \ \mathrm{BeV}$$

but the above limits of error are too large to be significant. Indeed, if one allows the common intercept of the P' and A_2 trajectory to be treated as an independent parameter any value of $.3 \leq \overline{\alpha} \lesssim .7$ is compatible with the above values of C and $\sigma_T(\infty)$. The interesting question to consider is what accuracy of future experiments is required to put more stringent bounds on C. If the errors of the DESY experiment are reduced from ± 10% to ± 3% one could determine C to an accuracy of ~1 μb BeV. Extending the measurements to higher energies 10-15 BeV improves the results. It is then necessary only to do the measurements to a ± 5% accuracy with energy bins of 1 BeV or less in order to limit C to this accuracy of ~1 μb BeV. These conclusions necessarily rely on the assumption that our simple parametrization dominates the high energy region and provides, within the present knowledge, a working hypothesis in searching for fixed poles.

A recent paper by Costa, Savoy, and Shaw[10] deals with the determination of the Regge parameters through FESR, but they do not consider the possibility of a pole at $\alpha = 0$. Our analysis shows that such a pole could be large and greatly modify their results. An experiment of the accuracy described above will be important in determining if such a pole is necessary.

FOOTNOTES AND REFERENCES

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