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SELECTED TOPICS IN PION-NUCLEUS INTERACTIONS

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INTRODUCTION

These lectures will deal with the π nuclear interactions at low and intermediate energies ($\lesssim$ 500 MeV) only, since different hadrons show little individuality in their interaction at higher energies.

The first and obvious question is : why should we explore nuclear properties with pions when so many neutron and proton accelerators with excellent beams and high currents exist and when so many detailed nuclear properties already have been explored ? After all, nuclear physics is by now 35 years old.

There are two basic reasons for this :

1) presently there is only one strongly interacting elementary particle with which to explore the nucleus with ease, namely the nucleon (the neutron and the proton). Any other such particle with different properties and interaction is therefore most welcome and it is likely to view the nucleus in a different way than the nucleon. It is therefore important to begin by studying the trivial and elementary properties, which make the pion different from the nucleon;

2) the degrees of freedom of the pionic field in the nucleus, its modes of excitation, its coupling to its sources in the nucleus and its distribution over the nucleus are all fundamental questions of nuclear physics. Classical low energy nuclear physics virtually ignores the existence of pions in nuclei. Pionic effects mainly appear in the range of the nuclear forces due to meson exchange, but the mesons have been eliminated as explicit degrees of freedom. Among these mesons, the pion is exceptional by its very low mass which makes it dominant in the long part of the NN force. While explicit mesonic effects can occur in magnetic moments by exchange currents, as three-body interactions or as quantitative modifications of the weak interaction in nuclei no clear-cut direct evidence exists. The best experimental evidence for virtual pions in the nucleus is that for a pseudoscalar weak interaction in μ capture, but this already involves μ mesons. It should be much easier to study all the problems of the nuclear pion field directly using real pions.

1. GENERAL PROPERTIES OF THE PION

Even the trivial properties of the pion are important in its application to nuclei. We therefore rapidly review them below.

Units : $m_\pi = \hbar = c = 1$

length : $\hbar/(m_\pi c) = 1.413 \cdot 10^{-13}$ cm

energy : $m_\pi c^2 = 139.6$ MeV ($\pi^\pm$ only)

time : $\hbar/(m_\pi c^2) = 4.71 \cdot 10^{-24}$ s

The pion has three charge states π^+, π^- and π^0 reflecting that its isospin is 1.

The existence of a negative pion has the very important consequence that a π^- can be bound electromagnetically into a mesic atom similar to an electronic atom but for scale.

The pion is a boson. The process $N + \pi \rightleftarrows N$ is possible if the energy momentum balance is furnished. The required momentum for a pion at rest on a nucleon is $(2m_\pi M)^{\frac{1}{2}} \approx 500$ MeV/c which is usually not available for a single nucleon in a nucleus [Fermi momentum is about 250 MeV/c]. Nuclear pion absorption therefore typically involves several nucleons.

The boson nature of the pion allows the process $\pi + N \rightleftarrows N + \gamma$ with transfer of the full energy momentum balance into a high energy photon.

Since $\pi^0 \rightarrow 2\gamma$ is very fast [$1.2 \cdot 10^{-16}$ s] there are no π^0 beams. Expressed in energy, this corresponds to only 6.5 eV, which is much smaller than the characteristic nuclear transition energies [100 keV - few MeV]. The π^0 can therefore be considered to be stable in most nuclear phenomena. The charged pions are very stable on the same scale [lifetime $2.6 \cdot 10^{-8}$ s].

The πN interaction is quite weak at low energies. Qualitatively we can see this by comparing the πN and NN scattering lengths in units typical of inter-nucleon spacing, namely $1/m_\pi = 1.4$ fm :

scattering lengths πN : ~0.1 - 0.2

scattering lengths NN : ~10

The general properties of πN interaction below 500 MeV are heavily dominated by the isospin $\frac{3}{2}$, spin $\frac{3}{2}^{-}$, p wave πN resonance at 180 MeV. Even at much lower pion kinetic energies the p wave interaction is very strong. The πN scattering amplitude for s and p wave is :

$$f = b_0 + b_1(\vec{t}\cdot\vec{\tau}) + [c_0 + c_1(\vec{t}\cdot\vec{\tau})](\vec{k}\cdot\vec{k}') + i[d_0 + d_1(\vec{t}\cdot\vec{\tau})]\vec{\sigma}\cdot(\vec{k}\times\vec{k}'), \quad (1)$$

where $\vec{\tau}$ and $\vec{t}$ are isospin operators and $\vec{k}$, $\vec{k}'$ in- and outgoing pion momenta. The coefficients in general are energy dependent. In the low energy limit they become constants :

$$\begin{aligned} b_o &= -0.012 \pm 0.004 \quad ; \quad & b_1 &= 0.097 \pm 0.007 \\ c_o &= 0.208 \pm 0.008 \quad ; \quad & c_1 &= 0.180 \pm 0.005 \\ d_o &= -0.193 \pm 0.005 \quad ; \quad & d_1 &= -0.060 \pm 0.004 \end{aligned} \quad (1.a)$$

The coefficients are related to the πN s and p wave scattering lengths α_{2t} and α_{2t2j} :

$$b_0 = \frac{\alpha_1 + 2\alpha_3}{3} \quad ; \quad b_1 = \frac{1}{3}(\alpha_1 - \alpha_3).$$

$$c_0 = \frac{1}{3}(4\alpha_{33} + 2\alpha_{31} + 2\alpha_{13} + \alpha_{11}); \quad c_1 = \frac{1}{3}(2\alpha_{33} + \alpha_{31} - 2\alpha_{13} - \alpha_{11}). \quad (1.b)$$

$$d_0 = \frac{1}{3}(2\alpha_{31} - 2\alpha_{33} + \alpha_{11} - \alpha_{13}) \; ; \; d_1 = \frac{1}{3}(\alpha_{31} - \alpha_{33} - \alpha_{11} + \alpha_{13}).$$

Note that both $b_o = -0.012$ and $(c_o - c_1) = 0.028$ are extremely small. Therefore :

1) the s wave $\pi^- n$ scattering length $(b_o + b_1)$ is nearly exactly equal to the $\pi^- p$ scattering length $(b_o - b_1)$ but with opposite sign;

2) the π^-p p-wave scattering length (c_0-c_1) is nearly vanishing; it is ~14 times smaller than the π^-n p-wave scattering length (c_0+c_1). There is thus practically no π^-p interaction in this state at low energy.

Problems

1) The reaction $D+D \rightarrow \pi^0+{}^4He$ is $\lesssim 10^{-7}$ of the total D+D cross-section at 460 MeV deuteron energy. What does this indicate for the π isotopic spin ? [Note : the process giving $\pi^++\pi^-+{}^4He$ has been observed with much larger cross-section when energetically possible.]

2) The ratio

$$\frac{k_p^2\,\sigma(p+D \rightarrow \pi^+ + {}^3He)}{k_\pi^2\,\sigma(\pi^- + {}^3He \rightarrow n + D)} = \frac{1}{3}$$

for the same energy. What do you conclude for the pion spin ? (Use detailed balance.)

2. SCALE FACTORS AND OBSERVABLES IN THE MESIC ATOM

A mesic atom is the bound states of a negative meson in the nuclear Coulomb field $Z\alpha/r$. In the absence of strong interactions and for a point nucleus this is an ordinary Bohr atom with a boson. In the absence of any other scale factor lengths and energies are simply scaled by the meson mass and the charge Z :

$$\text{Bohr radius} \qquad B = \frac{1}{m_\pi Z\alpha} \approx \frac{200}{Z}\ \mathrm{F}$$

$$\text{Bohr energy} \qquad = \frac{m_\pi (Z\alpha)^2}{2} \approx 3.7\ Z^2\ \mathrm{keV}$$

For an infinitely heavy nucleus, the Klein-Gordon equation gives the energy $E_{n\ell}$ for a state of principal quantum number n and orbital momentum ℓ.

$$E_{n\ell} = -\frac{m_\pi (Z\alpha)^2}{2n^2}\left\{1 + \frac{(Z\alpha)^2}{n^2}\left(\frac{n}{\ell+\frac{1}{2}} - \frac{3}{4}\right) - \cdots\right\}$$

(higher electromagnetic corrections are not included). For circular orbits, i.e., $\ell = \ell_{max} = n-1$, the radius of the orbit is $B_n = n^2 B$.

The initial capture of the pion into a high orbit is a complicated problem. It is believed to occur for $n \approx (m_\pi/m)^{\frac{1}{2}} \approx 16$, since the electron K orbit $(m_e Z\alpha)^{-1}$ equals the pion orbit $n^2(m_\pi Z\alpha)^{-1}$ there. The atom de-excites by Auger and X-ray emission. There is a strong preference for populating orbits of $n = \ell+1$ (i.e., circular orbits) during this process.

The observable effects produced by nuclear size and strong interactions are mainly $\varepsilon_{n\ell}$, $\Gamma_{n\ell}$ and $Y_{n\ell;n'\ell'}$; the deviation of an energy from the point energy, the absorption broadening and the yield of the transition $(n'\ell')$ to $(n\ell)$. More detailed effects like hyperfine splittings, nuclear γ rays associated with absorption, etc., are also accessible for study.

The yield in a transition $(n'\ell') \to (n\ell)$ is related to the level broadening in the originating level $\Gamma_{n'\ell'}$ using the branching ratio between radiative and absorptive processes :

$$Y_{n\ell;n'\ell'} = P_{n'\ell'} \frac{\Gamma_{rad\ n'\ell';n\ell}}{\Gamma_{rad\ n'\ell'} + \Gamma_{abs\ n'\ell'}} \quad . \qquad (2)$$

Here $P_{n'\ell'}$ is the original population of the level $(n'\ell')$, while $\Gamma_{rad\ n'\ell';n\ell}$ and $\Gamma_{rad\ n'\ell'}$ are the well-known partial and total radiative widths for the level $n'\ell'$ (including Auger emission). The absorption width to all non-pionic states is $\Gamma_{abs\ n'\ell'}$. The indirect method of width determination becomes important when $\Gamma_{abs\ n'\ell'} \gtrsim \Gamma_{rad\ n'\ell'}$ and allows the determination of extremely small widths in some cases.

The pionic Bohr atom is in practice always large compared to the nuclear size with a small probability of finding the pion inside the nucleus. As an example : the 1s level of ^{16}O has an important strong interaction shift which is 10% of the 2p-1s transition energy. The radius is 25 F which is much larger than the nuclear radius of 3.5 F. There is only 0.3% probability of finding the pion inside ^{16}O in the 1s state. States with much higher probability seem not observable at present due to the very high absorption rate of pions in nuclear matter ($\sim 5\times 10^{-22}$ s).

Problem

For which nucleus is the unperturbed 1s and 2p Bohr orbit equal to the nuclear radius for μ^-, π^-, K^- and $\bar{p}$?

3. GROSS BEHAVIOUR OF SHIFTS AND WIDTHS WITH Z

A major part of the systematic variation of $\mathcal{E}_{n\ell}$ and $\Gamma_{n\ell}$ with the nuclear charge Z comes from the change of the size of the atom with Z. We will here isolate this Z dependence by simple arguments.

The normal scale of energies in the Bohr atom is $E_n \sim Z^2$: binding and transition energies vary relatively slowly with Z.

The radiative transition rates are dipole transitions. Thus $\Gamma_{\mathrm{rad}\ n\ell}$ is dimensionally $[(\mathrm{length})^2 \times (\mathrm{energy})^3]$, so that

$$\Gamma_{\mathrm{rad}} \propto B^2 (E_2 - E_1)^3 \propto Z^4 ,$$

since $B \propto Z^{-1}$ and $(E_1 - E_2) \propto Z^2$.

The strong interaction effects have usually a much stronger Z dependence. Assume for simplicity that the strong interaction can be described by a square well V_0 up to the nuclear radius R and that perturbation theory at least crudely applies.

Close to $r = 0$ the wave function $\phi_{n\ell}$ is dominated by the centrifugal barrier. For dimensional reasons, it must then have the form

$$\phi_{n\ell} \propto B^{-3/2} \left(\frac{r}{B}\right)^{\ell} .$$

The complex energy shift $\Delta E_{n\ell} = \mathcal{E}_{n\ell} - i\,\Gamma_{n\ell}$ is

$$\Delta E_{n\ell} \simeq \int V(r)\, \phi_{n\ell}^2\, d\vec{r} \propto \frac{\int^{\infty} r^{2\ell+2}\, V(r)\, dr}{B^{2\ell+3}}$$

$$\propto V_0\, B^{-(2\ell+3)} \int_0^R r^{2\ell+2}\, dr \propto Z^{(2\ell+3)}\, R^{2\ell+3} .$$

Since the nuclear radius $R \propto A^{1/3}$, we have

$$\Delta E_{n\ell} \propto (Z A^{1/3})^{2\ell+3} \propto Z^{\frac{4}{3}(2\ell+3)} ,$$

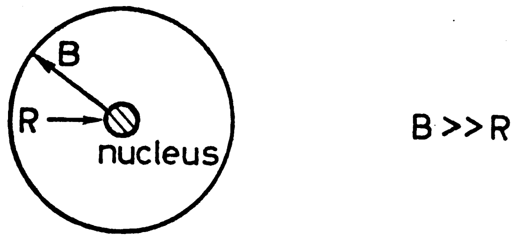
where the last expression results by taking $A \propto Z$. The powers of Z in different states are (note that in practice $\ell = n-1$) :

angular momentum	power of Z
0	4
1	6 2/3
2	9 1/3
3	12

The consequence of the exceedingly strong behaviour with Z in higher states is that for these the shifts and widths are very hard to observe for low Z. When they become observable they quickly become quite large as a function of Z.

4. A RELATION BETWEEN (COMPLEX) ENERGY SHIFTS AND SCATTERING LENGTHS

The pion is weakly bound in the nuclear Coulomb field. The Bohr orbits are usually very large compared to nuclear dimensions (see Fig.)



The pion is in a state of well-defined angular momentum ℓ with respect to the nucleus (we assume central interactions) and it has a slightly negative energy $E_{n\ell}$. It is physically obvious that the interaction of the pion with the nucleus is elastic scattering in such a state. In so far as nuclear scattering properties vary slowly with energy, it should thus be possible to relate the energy shift and width to low energy elastic scattering in the same angular momentum state. The π mesic atom should be a natural analysis for scattering phase shifts.

Accurate and model independent expressions for this connection can be derived using S matrix theory [1),2)]. We give below a simple derivation of the leading term which is sufficient in many cases. The relation between energy shifts and elastic scattering for atoms is more general than for pions. The results are, for example, applicable to e^-, μ^-, K^- and $\bar{p}$ atoms as well.

The scattering amplitude for a spinless particle is :

$$f(\theta) = \sum_{\ell=0}^{\infty} \frac{(e^{2i\delta_\ell}-1)}{2ik}(2\ell+1)P_\ell(\cos\theta) \tag{3}$$

with $(\vec{k}\cdot\vec{k}') = k^2\cos\theta$ where $\vec{k}$ is incident, $\vec{k}'$ outgoing momentum. As $k\to 0$, every partial wave has an effective range formula

$$k^{2\ell+1}\cot\delta_\ell = \frac{1}{A_\ell} + \frac{r_\ell}{2}k^2 + \cdots \tag{4}$$

or, in the limit of $k \to 0$

$$\lim_{k \to 0} \left(\delta_\ell / k^{2\ell+1} \right) = A_\ell \,. \tag{5}$$

Here A_ℓ is a generalized "scattering length" [indeed it is of dimension $(\text{length})^{2\ell+1}$, so it is not a length generally]. Therefore, if we retain only the leading term for small k for each partial wave, the contribution of the ℓ^{th} wave to $f(\theta)$ is :

$$f_\ell(\theta) = (2\ell+1) A_\ell \, k^{2\ell} P_\ell(\cos\theta). \tag{6}$$

The trick to relate A_ℓ to the energy $\Delta E_{n\ell}$ is to exploit that the pion orbits are very much larger than the nuclear radius. We introduce a short ranged <u>pseudopotential</u> $V(\vec{r})$, for which the amplitude is

$$f(\theta) = -\frac{2m_\pi}{4\pi} \int e^{-i\vec{k}\cdot\vec{r}} \, V(r) \, e^{i\vec{k}'\cdot\vec{r}} \, d\vec{r} \tag{7}$$

and the energy shift

$$\Delta E_{n\ell} = \int V(r) \, |\Psi_{n\ell}|^2 d\vec{r}. \tag{8}$$

Since we use a pseudopotential, we are permitted to use the Born approximation in both cases, so that $\Psi_{n\ell}$ refers to a hydrogenic, unperturbed wave function.

Let us apply this to $\ell = 0$. In this case we obtain $f_0(\theta)$ simply by letting $k \to 0$, so that

$$f_0(\theta) = A_0 = -\frac{2m_\pi}{4\pi} \int V(r) \, d\vec{r} = -2m_\pi \int_0^\infty r^2 V(r) \, dr \tag{9}$$

Similarly since $\psi_{n\ell} = Y_{\ell m}\,\phi_{n\ell}$

$$\Delta E_{n\,\ell=0} = \int_0^\infty |\phi_{n0}|^2 V(r)\, r^2 dr = |\phi_{n0}(0)|^2 \int_0^\infty r^2 V(r)\, dr \,. \tag{10}$$

Hence

$$\Delta E_{n\,\ell=0} = -\,|\phi_{n0}(0)|^2 \,\frac{A_0}{2m_\pi} \,.$$

This can be simplified further into a more usual form by observing that the hydrogenic wave function

$$\phi_{n\,\ell=0}(0) = 2\left(\frac{m_\pi Z\alpha}{n}\right)^{3/2} ,$$

so that

$$\Delta E_{n\,\ell=0} = -2\,\frac{m_\pi^2 Z^3 \alpha^3}{n^3}\,A_0 = -E_{n0}\,\frac{4}{n}\,\frac{A_0}{B} \,. \tag{11}$$

We can easily generalize this result to an arbitrary partial wave. By projecting out the leading contribution of the ℓ^{th} partial wave from Eq. (7)

$$f_\ell(\theta) = -\,\frac{2m_\pi}{4\pi}\,(2\ell+1)\left[\int j_\ell^2(kr)\, V(r)\, d\vec{r}\right] P_\ell(\cos\theta) \,, \tag{12}$$

where $j_\ell(kr) \simeq (k^\ell r^\ell)/(2\ell+1)!! + \ldots$ is a spherical Bessel function. Since $f_\ell(\theta) \rightarrow k^{2\ell} A_\ell P_\ell(\cos\theta)$ as $k \rightarrow 0$, we have

$$A_\ell = -\,\frac{2m_\pi}{4\pi} \int \frac{r^{2\ell}}{[(2\ell+1)!!]^2}\, V(r)\, d\vec{r} \,. \tag{13}$$

On the other hand the bound state wave function

$$\Psi_{n\ell}(\vec{r}) = \phi_{n\ell}(r) Y_{\ell m} \simeq \frac{1}{(2\ell+1)!}\left[\frac{(n+\ell)!}{(n-\ell-1)!\,2n}\right]^{1/2}\left(\frac{2}{n}\frac{1}{B}\right)^{3/2}\left(\frac{2r}{nB}\right)^{\ell} + \cdots \tag{14}$$

Therefore Eq. (8) gives

$$\Delta E_{n\ell} = \frac{1}{4\pi}\int \phi_{n\ell}^2 V d\vec{r} = a_{n\ell} B^{-2\ell-3} \frac{1}{4\pi}\int r^{2\ell} V d\vec{r} \quad , \tag{15}$$

where we have introduced a constant $a_{n\ell}$ to describe the numerical factors in n and ℓ. There is clearly a common dynamical factor $\int r^{2\ell} V(r) d\vec{r}$ in both expressions which can be eliminated. The general relation is therefore

$$\frac{\Delta E_{n\ell}}{E_n} = -\frac{2}{n}\frac{A_\ell}{(\frac{1}{2}nB)^{2\ell+1}}\frac{1}{(2\ell!!)^2}\frac{(n+\ell)!}{(n-\ell-1)!} \; . \tag{16}$$

In the special cases of $\ell = 0$, 1, 2 and 3

$$\frac{\Delta E_{n0}}{E_n} = -\frac{4A_0}{nB} \quad , \tag{17.a}$$

$$\frac{\Delta E_{n1}}{E_n} = -\left(1-\frac{1}{n^2}\right)\frac{4}{n}\frac{A_1}{B^3} \; , \tag{17.b}$$

$$\frac{\Delta E_{n2}}{E_n} = -\frac{1}{n}\left(1-\frac{1}{n^2}\right)\left(1-\frac{4}{n^2}\right)\frac{A_2}{B^5} \; , \tag{17.c}$$

$$\frac{\Delta E_{n3}}{E_n} = -\frac{1}{9n}\left(1-\frac{1}{n^2}\right)\left(1-\frac{4}{n^2}\right)\left(1-\frac{9}{n^2}\right)\frac{A_3}{B^7} \; . \tag{17.d}$$

These relations are true for $\Delta E_{n\ell}$ complex, i.e., $\mathcal{E}_{n\ell} - i\,\Gamma_{n\ell}/2$ since we nowhere used that V was a real pseudopotential.

5. EXPECTED BEHAVIOUR OF SHIFTS

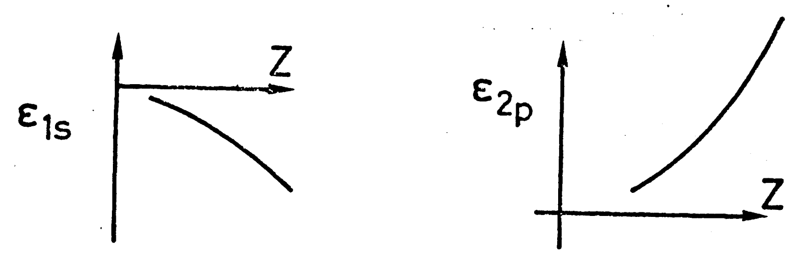
If we take a very naïve and qualitative view of likely behaviour of strong interaction shifts, we would probably argue as follows : the πN interaction is dominantly attractive, as we easily see from nuclear binding. Hence $V(r)$ is attractive. If we have a reasonable potential which follows the mass distribution of the nucleus, then :

$$\int_0^\infty r^n V(r)\, dr$$

the n^{th} moment of $V(r)$, should have the sign of $V(r)$, so that s,p,d,... levels of the atom should all have shifts of the same sign. This is true for example for the finite size shifts in μ mesic atoms due to the smeared out charge (all repulsive).

6. OBSERVED BEHAVIOUR AND MAGNITUDE OF SHIFTS AND WIDTHS

The energy shifts deviate strongly from this picture (see Fig.), Because of the strong variation of the probability of finding a pion inside the nucleus as a function of the Bohr orbit, the strong interaction shifts and widths are in practice determined to high precision by the lower state in an X-ray transition.



repulsive attractive

It is observed that the binding in the attractive Coulomb field is decreased for s wave pions, i.e., the interaction is effectively repulsive, while it is increased in the 2p, 3d, etc., levels, i.e., in higher orbits the interaction is effectively attractive. This indicates that a major feature of the interaction structure is not up to this point included.

A second interesting feature is that the 1s states have energy shifts shifts which depend very strongly on neutron excess. So, for example, for the two boron isotopes ^{10}B and ^{11}B for which the a priori probability of finding a pion inside the nucleus is nearly identical, the energy shift changes from -3.18 ± 0.18 keV to -4.23 ± 0.18 keV, i.e., by over 30%. The addition of a neutron produces a more repulsive interaction than before.

Below we give the typical ranges of nuclei for which strong interaction shifts and broadenings have been directly observed as well as typical values of energies and widths. The experimental technique used is usually solid state germanium detectors.

In addition, indirect measurements of the Γ_{2p} and Γ_{3d} by intensity attenuations exist for a series of nuclei. They are consistent with and extrapolate smoothly to the directly measured widths.

For further details on the experimental data, the reader is referred to the lecture by Professor H. Daniel [3].

2p-1s transitions (observed from ^{4}He to ^{23}Na)

	E_{2p-1s} (keV)	ε_{1s} (keV)	Γ_{1s} (keV)
$^{10}_{5}B$	65.95±0.18	− 3.18±0.18	1.27±0.25
$^{23}_{41}Na$	276.2 ±1.0	−64.74±1.0	10.3 ±4.0

3d-2p transitions (^{27}Al to ^{30}Zn)

	E_{3d-2p} (keV)	ε_{2p} (keV)	Γ_{2p} (keV)
$^{27}_{13}Al$	87.40±0.10	0.20±0.10	0.36±0.15
$^{59}_{27}Co$	356.43±0.30	3.81±0.35	7.37±0.70

4f-3d transitions (^{89}Y to ^{141}Pr)

	E_{4f-3d} (keV)	ε_{3d} (keV)	Γ_{3d} (keV)
$^{115}_{49}In$	442.9 ±0.5	2.6 ±0.5	2.6 ±0.6
$^{133}_{55}Cs$	560.5 ±1.1	5.4 ±1.5	3.3 ±1.5

5g-4f transitions (^{181}Ta to ^{239}Pu)

	E_{5g-4f} (keV)	ε_{4f} (keV)	Γ_{4f} (keV)
$^{238}_{92}U$	731.4 ±1.1	6.0 ±1.1	6.1 ±1.0

7. PION MASS MEASUREMENT

Since we know empirically the strength of the pion-nucleon interaction in a given angular momentum state as well as its scale with Z it is possible to choose the strong interaction effects arbitrarily small simply by choosing a nucleus of sufficiently low Z. The energies in the Bohr orbits are then scaled by the pion mass m_π which is the only scale energy (excluding vacuum polarization and electron screening). Hence π mesic atoms can be used to measure the pion mass to high accuracy with strong effects eliminated.

By a crystal spectrometer, Crowe et al. [4] have obtained :

$$m_\pi = (139.580 \pm 0.015)\ \mathrm{MeV}$$

from the 4f-3d transitions in Ca and Ti.

Typical energy is :

$$E_{4f-3d}(\mathrm{Ca}) = (72.388 \pm 0.009)\ \mathrm{keV}.$$

8. QUALITATIVE ESTIMATE OF LEVEL SHIFTS FROM THE π N INTERACTION

The close relation between energy shifts and scattering lengths in the same angular momentum state was emphasized in Section 4. It can be used as follows.

Imagine the nucleus to be a collection of free nucleons and neglect effects of pion absorption. If an incident pion has very long wavelength we would expect the s wave π nuclear interaction to come predominantly from the s wave π N interaction. A first approximation for the scattering length $A_{\ell=0}$ would then be the coherent sum of π N scattering lengths (clearly this neglects rescattering which is quite bad for heavier nuclei). Further, if the nucleus is sufficiently small and the pion is in a state of angular momentum ℓ with respect to the nucleus, it will be predominantly in the same angular momentum state also with respect to the nucleons (again this becomes bad as the size of the nucleus increases). We can then also try to obtain $A_{\ell\neq0}$ as the coherent sum π N scattering lengths for that ℓ state :

$$A_\ell \simeq Z a^\ell_{\pi^- p} + N a^\ell_{\pi^- n} \,. \tag{18}$$

From the numerical values in Section 1 [Eqs. (1.a) and (1.b)], we have since

$$a^{\ell=0}_{\pi^- n} = \alpha_3 \quad ; \quad a^{\ell=0}_{\pi^- p} = \frac{2\alpha_1 + \alpha_3}{3} \,,$$

that

$$A_{\ell=0} \simeq b_0 A - b_1 (Z-N) = \left[-0.012 A - 0.097 (N-Z)\right] m_\pi^{-1} . \tag{19}$$

From this expression, we see from the sign of b_0 that the interaction is repulsive in a nucleus of $N = Z$; however, b_0 results as a small number due to important cancellations, so that this cannot be based on this term. We will later discuss the interesting origin of the repulsive interaction in the s state. We see further that the addition of neutrons produces a more negative, that is more repulsive, $A_{\ell=0}$. This is in agreement with observations on energy shifts although the effect is about half of this estimate.

For the analogous estimate for p waves and higher waves, we average over spin directions. Since according to Eqs. (1.a) and (1.b)

$$a^{\ell=1}_{\pi^- n} = \frac{1}{3}\left(2\alpha_{33}+\alpha_{31}\right) = \frac{1}{3}\left(c_0+c_1\right); \quad a^{\ell=1}_{\pi^- p} = \frac{1}{9}\left(2\alpha_{33}+\alpha_{31}+4\alpha_{13}+2\alpha_{11}\right) = \frac{c_0-c_1}{3}$$

we have

$$A_{\ell=1} \simeq \left(0.129\,N + 0.09\,Z\right) m_\pi^{-3}. \tag{20}$$

We note the very small contribution from the π^-p interaction and that the scattering length in this state is positive, i.e., attractive, so that the effective interaction in this state is attractive. A comparison of $\mathcal{E}_{2p}/E_{n=2}$ for ${}^{27}_{13}A$, as given in Section 6, to the value predicted by Eq. (20), using Eq. (17.b) to connect $A_{\ell=1}$ to $\mathcal{E}_{2p}$ gives

$${}^{27}_{13}Al: \left(\frac{\mathcal{E}_{2p}}{E_{n=2}}\right)_{exp} = 1.3\cdot 10^{-3}; \quad \left(\frac{\mathcal{E}_{2p}}{E_{n=2}}\right)_{coherent} = 2.5\cdot 10^{-3}.$$

In view of the crude approximation for $A_{\ell=1}$ the agreement is close.

Similar estimates for $A_{\ell=2}$ and $A_{\ell=3}$ with the measured d and f wave πN scattering lengths give

$$\begin{aligned} A_{\ell=2} &\simeq \left[-0.4\,A - 6(N-Z)\right]\cdot 10^{-4}\, m_\pi^{-5} \\ A_{\ell=3} &\simeq 4A\, 10^{-4}\, m_\pi^{-7}. \end{aligned} \tag{21}$$

We compare these estimates to the energy shifts for ${}^{133}_{55}Cs$ and ${}^{238}_{92}U$ using the connections (17.c) and (17.d). This will indicate if such estimates are good enough for such large systems

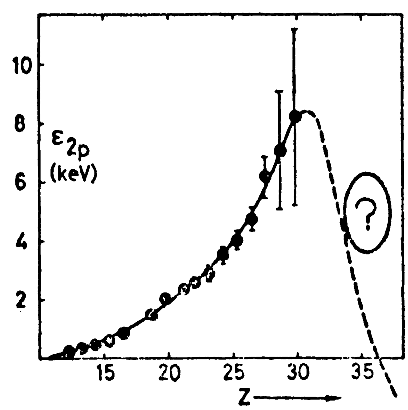
$${}^{133}_{55}Cs: \left(\frac{\mathcal{E}_{3d}}{E_{n=3}}\right)_{exp} \simeq 43\cdot 10^{-4}; \quad \left(\frac{\mathcal{E}_{3d}}{E_{n=3}}\right)_{coherent} \simeq -0.35\cdot 10^{-4}.$$

$${}^{238}_{92}U: \left(\frac{\mathcal{E}_{4f}}{E_{n=4}}\right)_{exp} \simeq 26\cdot 10^{-4}; \quad \left(\frac{\mathcal{E}_{4f}}{E_{n=4}}\right)_{coherent} \simeq 0.8\cdot 10^{-4}.$$

Clearly the coherent sum over the scattering lengths in these angular momentum states is utterly insufficient to explain the observed shifts and they give only a small contribution to them [2]. The reason is that the interaction in these states is dominated by s and p wave π N interaction on individual nucleons. This will be discussed in detail in the following Sections.

9. AN ANOMALY IN THE π NUCLEUS INTERACTION ?

The π nuclear interaction is repulsive in the 1s state, attractive in the 2p, 3d, 4f,... states for nuclei experimentally studied. The repulsion in the 1s state originates in s wave π N scattering, the attraction in higher states from the attractive p wave π N interaction. Higher waves were shown to be unimportant in heavy elements in Section 8.



The observed interaction shift for the pionic 2p state is shown above as a function of Z. It is a growing function of Z up to $Z = 30$, the heaviest nucleus for which observations exist. An argument will now be given that the proper extrapolation of this curve is a dramatic downward trend with a change of sign for $Z \approx 36$. The observed attraction should therefore reverse into repulsion for a slight further increase of nuclear radius [5]. The origin of this predicted effect is the following : for a sufficiently small nucleus, a pion in a relative p state with respect to the nucleus will also be very predominantly in a relative p state with respect to the individual nucleons. For a larger nuclear radius, the pion can increasingly be in a relative s state with respect to the nucleons, and thus feel the repulsive interaction of this state. There can therefore be a critical nuclear radius for which the attraction and repulsion exactly balance (with no net interaction). A very simple but in essence correct estimate of the critical nuclear size can be made as follows : the π nuclear interaction strength is given approximately by the expectation value of π N scattering amplitudes on the individual nucleons in the nucleus. In practice only s and p wave π N scattering is of importance, as mentioned above. For a single nucleon and a plane wave

$$\exp\{i\vec{k}\cdot\vec{x}\} = 1 + i\vec{k}\cdot\vec{x} + \cdots \quad .$$

The interaction strength is then

$$I_1 \simeq a_s + 3a_p k^2 + \cdots \quad , \tag{22}$$

where a_s and a_p are πN s and p wave scattering lengths. For the bound wave function $\phi(\vec{r})$ we have to take proper s and p state contributions for the individual nucleons. In the neighbourhood of a point $\vec{r}$ this is immediately obtained by comparison of the plane wave expansion above with the Taylor expansion of the wave function,

$$\phi(\vec{r}+\vec{x}) \simeq \phi(\vec{r}) + \vec{x}\cdot(\vec{\nabla}\phi)_{\vec{r}} \; .$$

With a nuclear density $\rho(\vec{r})$ and the average scattering lengths $\bar{a}_s$ and $\bar{a}_p$, the A particle interaction strength is

$$I_A \simeq \int [\bar{a}_s \phi^2(\vec{r}) + 3\bar{a}_p (\vec{\nabla}\phi)^2] \rho(\vec{r}) \, d\vec{r} \, . \tag{23}$$

The parameters $\bar{a}_s$ and $\bar{a}_p$ are to be interpreted as the effective πN scattering lengths in the nucleus. It is obvious from the expression for the interaction strength I_A that the net interaction strength may be repulsive <u>or</u> attractive, depending on the relative size of the integrals containing ϕ^2 and $(\vec{\nabla}\phi)^2$, since $\bar{a}_s$ and $\bar{a}_p$ have opposite signs. These integrals depend strongly on angular momentum and nuclear radius.

An insight into this dependence is obtained by taking the matter distribution to be uniform to a radius R and the pion wave function $\phi(r) \propto r^{\ell} Y_{\ell m}$ over nuclear dimensions. These are gross oversimplifications, of course, of the real situation. We now determine the critical radius R_ℓ for which attraction and repulsion exactly balance $[I_A=0]$ by an elementary integration :

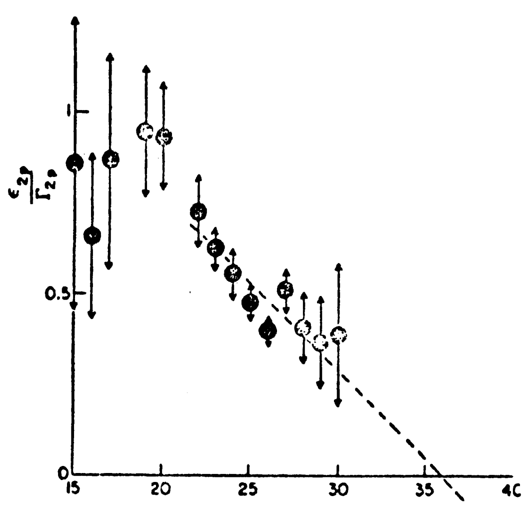
$$I_A \simeq 4\pi A \rho_0 [\bar{a}_s \frac{R^{2\ell+3}}{(2\ell+3)} + 3\bar{a}_p \ell R^{2\ell+1}] = 0 \tag{24}$$

so that

$$R_\ell^2 = -\frac{3\bar{a}_p}{\bar{a}_s}\,\ell(2\ell+3). \tag{25}$$

The critical radius is a strong function of the angular momentum. It is obvious that the higher the angular momentum, the larger the critical radius. With $\bar{a}_s$ and $\bar{a}_p$ empirically determined, this leads to the following predicted critical radii : $R_1 = 6$ f, $R_2 = 11$ f and $R_3 = 15$ f, which correspond to $A \sim 100$, $A \sim 250$-350 and $A \sim 1000$. These indicate that for real nuclei the effect should be observable for $\ell = 1$, possibly for $\ell = 2$, while effects in $\ell \geq 3$ seem impossible to detect at present. A more detailed numerical analysis on a more sophisticated basis indicates that the effect should occur around $Z \approx 36$, i.e., for $A \approx 80$.

Since the experimental interaction shifts do not apparently show any effect at $Z = 30$, we decided to plot $\varepsilon_{2p}/\Gamma_{2p}$, the ratio of shifts to widths versus Z. The rationale for this was that this should eliminate the probability of the presence of the pion in the nucleus, which scales strongly with Z as the Bohr orbit shrinks according to section 4. The result in shown below, which strongly indicates that our prediction may be true. We did not rely on any parameter dependent calculation to obtain this curve. It is simply the ratio of two experimental quantities versus Z.



It would therefore be of considerable interest to extend present measurements of 2p shifts to Z values only a little bit higher than presently observed to establish whether the effect exists or not. This is extremely difficult since absorption attenuates the 3d-2p line to less than 1% of its normal intensity, but it merits a special effort.

An interaction that changes from attraction to repulsion as a function of nuclear size is unusual in nuclear physics. Since the amplitude changes sign even in the Born approximation, the nature of the effect is completely different from the well-known change of sign of neutron scattering lengths at a nuclear size resonance (anomalous dispersion).

An interesting corollary of this little calculation is worth mentioning. It is clear that the energy shifts in 3d and 4f states depend almost entirely on the p wave π N interaction, since the dominance of the s wave π N interaction occurs for much heavier nuclei than those presently studied. Since $a_p^{\pi^- n} \simeq 14 a_p^{\pi^- p}$ when averaged over spin (Section 8), the interaction in the 3d and 4f states are almost entirely with the neutrons in the nucleus and hardly at all with the protons. Any effect in these states which can be linked to the nuclear size therefore measures the nuclear <u>neutron</u> size, which normally is an elusive quantity.

10. THE EFFECTIVE FIELD OR WHY IS THE s WAVE INTERACTION REPULSIVE ?

The π^- nuclear interaction will now be discussed in a more sophisticated way using multiple scattering theory [*]. In this picture the individual nucleons have scattering properties which fundamentally are unchanged by the presence of neighbouring nucleons. The scattered waves are followed in the repeated scatterings on different nucleons as well on the scattering back to a nucleon which already has scattered the particle previously.

There are two fundamentally different regimes for multiple scattering : 1) the high energy Glauber limit and 2) the low energy limit.

The high energy limit considers the nucleus as the equivalent of a box of transparent glass beads (the nucleons) through which light rays pass (the hadron). The phase changes for the individual glass beads through which the ray passes at a given impact parameter are simply summed up. The validity of this picture requires that the scatterers are extended compared to the incident wavelength, so that many partial waves contribute in each scattering. This ensures the applicability of ray optics. More exactly it requires that the distance between successive scatterings $L < b^2/\lambda$ where b is the nucleon size. It is characteristic of this high energy approximation that no scatterer is struck more than once.

Low energy multiple scattering is largely the antithesis of the Glauber picture. The scatterers are small compared to wavelength, so that only one or a few partial waves contribute in each scattering. The scattered waves do not obey ray optics from one scattering to the next, but are spherical outgoing waves. It is now possible to scatter more than once on the same particle. There is also a characteristic modification of the <u>local field</u>, due to the nuclear polarization at low energy. Such effects, which are associated with the discrete structure of the nucleus (or correlations), can be looked at in terms of virtual nuclear excitations. The local effective field is a very important effect at low energy, in contrast to the high energy case.

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*) A pedagogical and systematic discussion of approximations is given in Ref. [6]. A more complete account is found in Ref. [7].

The most striking effect of the granularity of the nuclear medium appears in the 1s state of the mesic atom. The physics is obtained from the following oversimplified model. The π N scattering is taken to be point-like with a strength parameter a given by the πN s-wave scattering length, since the 1s state is dominated by the elementary s-wave π N interaction.

If we now consider the total wave $\phi(\vec{r})$ produced by an incident wave $\exp i\,\vec{k}\cdot\vec{r}$ on a system of such nucleons this wave is the sum of the incident wave and the scattered waves from all the individual nucleons. The latter in turn are each simply (effective wave ϕ^{eff} at a scatterer)× ×(scattering strength a)×(spherical outgoing wave). Thus

$$\phi(\vec{r}) = \exp\{i\vec{k}\cdot\vec{r}\} + \sum_i \frac{\exp\{ik|\vec{r}-\vec{r}_i|\}}{|\vec{r}-\vec{r}_i|}\, a\, \phi^{\text{eff}}_{(\vec{r}_i)}$$

$$\equiv \exp\{i\vec{k}\cdot\vec{r}\} + \int \frac{e^{ik|\vec{r}-\vec{r}_1|}}{|\vec{r}-\vec{r}_1|}\, a\, \rho(\vec{r}_1)\, \phi^{\text{eff}}_{(\vec{r}_1)}\, d\vec{r}_1, \tag{26}$$

where we introduced the mass density of the nucleus. The equation above solves no problems since it simply expresses the desired wave function $\phi(\vec{r})$ in terms of the effective wave $\phi^{\text{eff}}(\vec{r})$. If we operate on Eq. (26) by $(\vec{\nabla}^2+k^2)$ and use the famous relations :

$$\begin{cases} (\vec{\nabla}^2+k^2)\, \dfrac{e^{ikx}}{x} = -4\pi\, \delta(\vec{x}) \\ (\vec{\nabla}^2+k^2)\exp\{i\vec{k}\cdot\vec{x}\} = 0 \end{cases} \tag{27}$$

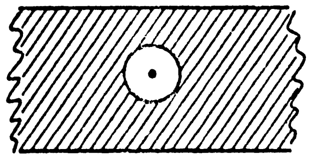
then

$$(\vec{\nabla}^2+k^2)\phi = -4\pi a\, \rho(\vec{r})\, \phi^{\text{eff}}_{(\vec{r})}\ . \tag{28}$$

If the nucleus were a piece of homogeneous and uniform medium with no discrete structure whatsoever there would be no difference between ϕ and ϕ^{eff}. In this case there would be an equivalent potential $V(\vec{r})$ for the elastic scattering

$$2 m_\pi V = -4\pi a\, \rho(\vec{r}). \tag{29}$$

A nucleus is <u>not</u> a piece of amorphous matter. A pion within the nucleus clearly experiences the discrete distribution of matter within. In the neighbourhood of a particular nucleon, the pion sees that the mass of the nucleon is concentrated into a point with a corresponding absence of medium (a hole in the medium) surrounding this nucleon (see Fig.).



Since a structure of this kind can only be seen by virtually exciting the system, this effect is exactly equivalent to calculations of nuclear polarization using excited states of the nucleus. More exactly, this hole is described by the nuclear pair correlation function as we will now show by giving the equation for ϕ^{eff}.

In complete analogy to Eq. (26) the effective field at a scatterer i is given by the incident wave and the scattered wave from all other particles <u>but itself</u>

$$\phi^{eff}(\vec{r}_i) = \exp\{i\vec{k}\cdot\vec{r}_i\} + \sum_{j\neq i} \frac{e^{ik|\vec{r}_i-\vec{r}_j|}}{|\vec{r}_i-\vec{r}_j|}\, a\, \phi^{eff}(\vec{r}_j) \tag{30}$$

or identically introducing the nuclear pair correlation function $C(\vec{r}_1,\vec{r}_2)$ defined by the expectation value $\langle 0|\sum_{i\neq j} \delta(\vec{r}_1-\vec{r}_i)\,\delta(\vec{r}_2-\vec{r}_j)|0\rangle \equiv \rho(\vec{r}_1)\rho(\vec{r}_2)\times [1+C(\vec{r}_1,\vec{r}_2)]$.

$$\phi^{eff}(\vec{r}_1) = \exp\{i\vec{k}\cdot\vec{r}_1\} + \int \frac{e^{ik|\vec{r}_1-\vec{r}_2|}}{|\vec{r}_1-\vec{r}_2|}\,\rho(\vec{r}_2)[1+C(\vec{r}_1,\vec{r}_2)]\,a\,\phi^{eff}(\vec{r}_2)\,d\vec{r}_2$$

$$\equiv \phi(\vec{r}_1) - \int \frac{e^{ik|\vec{r}_1-\vec{r}_2|}}{|\vec{r}_1-\vec{r}_2|}\,\rho(\vec{r}_2)\,\ell(\vec{r}_1,\vec{r}_2)\,a\,\phi^{eff}(\vec{r}_2)\,d\vec{r}_2\,. \tag{31}$$

The effect of $C(\vec{r}_1,\vec{r}_2)$ is to describe how the nucleons are distributed close to $\vec{r}_1$. The fact that there is a scatterer in this position influences the local density of other nucleons. In particular, since the average density stays fixed, there is on the average one particle removed from the rest of the system, so that

$$\int \rho(\vec{r}_2)\, C(\vec{r}_1,\vec{r}_2)\, d\vec{r}_2 = -1 . \tag{32}$$

We may think of the nucleon as having "created a hole around itself".

The important effect of the hole is now this : in the uniform medium there is an average wave function $\phi(\vec{r})$. The wave incident on a scatterer in the hole is not $\phi(\vec{r})$, but an effective wave $\phi^{eff}(\vec{r})$. In the long wavelength limit, these are related by [see Eq. (31)]

$$\phi^{eff} = \phi - a \left\langle \tfrac{1}{r} \right\rangle_{hole} \phi^{eff} . \tag{33}$$

The effective field in the middle of the hole has thus two contributions : i) the average field $\phi(\vec{r})$ that would result from the background medium itself; ii) the local modification by the hole, which is a removal of uniform matter close to $\vec{r}$ in accordance with our ansatz. At the centre of the hole this contribution is proportional to the exciting field ϕ^{eff} over the hole, the strength of scattering a for one particle, and the average of the inverse distance of the centre to the various parts of the hole (from the propagation of a spherical wave). The sign is negative since these contributions have been overcounted in $\phi(\vec{r})$. Hence

$$\phi^{eff} = \frac{1}{\left(1 + a \left\langle \frac{1}{r} \right\rangle_{hole}\right)}\, \phi . \tag{34}$$

Since the derivation of the potential for a uniform medium supposed that the exciting wave function was ϕ and not ϕ^{eff}, the potential is modified due to the correlations :

$$2 m_n V = -4\pi \frac{a}{\left(1 + a \left\langle \frac{1}{r} \right\rangle_{hole}\right)}\, \rho(\vec{r}) . \tag{35}$$

Therefore, the scattering length a can simply be considered to be replaced by an effective scattering length

$$a \to a_{\mathrm{eff}} = \frac{a}{\left(1 + a\langle 1/r\rangle_{hole}\right)} \simeq \left[a - a\langle \tfrac{1}{r}\rangle_{hole} a + \dots\right] \tag{36}$$

for $\langle \frac{a}{r}\rangle_{\mathrm{hole}} \ll 1$. This correction is always <u>repulsive</u> in this order since it depends on a^2.

We now generalize this to a nucleus which has both neutrons and protons and for which virtual charge exchange also can occur. We identify $\langle\frac{1}{r}\rangle_{\mathrm{hole}}$ with $\langle \frac{1}{r}\rangle_{\mathrm{corr}}$, the expectation value of $1/r$ over the normally defined pair correlation function. For a $T=0$ nucleus, the leading order terms are

$$2m_\pi V = -4\pi\left[\frac{a_n + a_p}{2} - \frac{2(a_n^2 + a_p^2) + (a_n - a_p)^2}{4}\langle\frac{1}{r}\rangle_{corr}\right]\rho(\vec{r}). \tag{37}$$

The remarkable thing for pions is now that $(a_n + a_p) \approx 0$. In the absence of the higher order term, nuclei would be nearly transparent to s wave pions. The elastic scattering occurs predominantly (to about 70%) by the second order term. The important correlations are the Pauli correlations, which have the largest range. <u>The repulsive interaction is thus caused by the granular structure of the nucleus</u> [8),9),6)].

The importance of the second term in Eq. (37) can be understood as follows. The exceptionally small value of the pion mass has led to the study of pion interactions in the limit $m_\pi \to 0$. It emerges that for most models [10)] one finds $a_n \simeq -a_p = \mathrm{const}\cdot m_\pi$ + [terms in m_π^2], so that $(a_n + a_p) \propto m_\pi^2$. However, since a_n^2, a_p^2 and $(a_n + a_p)^2$ are all proportional to m_π^2 also, we find that all terms to this order are systematically included in the potential

$$V \propto m_\pi .$$

It is a dynamical feature that the second term is the dominant one numerically.

11. THE GRADIENT POTENTIAL

The π N scattering contains a very important dipole component from p wave scattering. The previous results can be generalized to include this effect.

Consider as before point scatterers which now can interact both in s and p states with the pion. The π N amplitude is then schematically

$$\left[a + c(\vec{k}\cdot\vec{k}')\right]\delta(\vec{r}-\vec{r}_i) . \tag{38}$$

We use the same picture of the nucleus as previously. The effective wave falling in on a scatterer is ϕ^{eff} in the s state and $\vec{E}^{eff}$ in the p state.

For the dipole scattering in the nucleus a scattered wave from a nucleon is [exciting field $\vec{E}^{eff}$] $\times$ [strength of p wave scattering c] $\times$ $\times$ [outgoing p wave $-\vec{\nabla}(\exp ik|\vec{r}-\vec{r}_i|/|\vec{r}-\vec{r}_i|)$] :

outgoing p wave from a scatterer =

$$= -c\left(\vec{\nabla}\cdot\frac{e^{ik|\vec{r}-\vec{r}_i|}}{|\vec{r}-\vec{r}_i|}\right)\vec{E}^{eff}(\vec{r}_i) . \tag{39}$$

It is therefore immediate to generalize Eq. (26) :

$$\phi(\vec{r}) = \exp\{i\vec{k}\cdot\vec{r}\} + \int\frac{e^{ik|\vec{r}-\vec{r}_1|}}{|\vec{r}-\vec{r}_1|}\,a\,\rho(\vec{r}_1)\,\phi^{eff}(\vec{r}_1)\,d\vec{r}_1 - \int\left(\vec{\nabla}\frac{e^{ik|\vec{r}-\vec{r}_1|}}{|\vec{r}-\vec{r}_1|}\right)c\,\rho(\vec{r}_1)\,\vec{E}^{eff}(\vec{r}_1)\,d\vec{r}_1 . \tag{40}$$

In the last integral we can switch gradient from $\vec{r}$ to $\vec{r}_1$, so that a partial integration gives the simpler form :

$$\phi = \exp\{i\vec{k}\cdot\vec{r}\} + \int\frac{e^{ik|\vec{r}-\vec{r}_1|}}{|\vec{r}-\vec{r}_1|}\left[a\rho\phi^{eff} - c\left(\vec{\nabla}_1\cdot\rho\quad\vec{E}^{eff}\right)\right]d\vec{r}_1 \tag{41}$$

We can therefore immediately write the equivalent relation to Eq. (28) :

$$(\vec{\nabla}^2+k^2)\phi = -4\pi[a\rho\phi^{eff} - c(\vec{\nabla}\cdot\rho\vec{E}^{eff})]. \tag{42}$$

In the first and oversimplified approximation of a homogeneous and uniform medium $\phi^{eff}=\phi$ and $\vec{E}^{eff}=\vec{\nabla}\ \phi$, since the average and effective fields are the same. This gives a velocity dependent potential equivalent to Eq. (42) :

$$2m_\pi V = -4\pi[a\rho-(\vec{\nabla}\cdot c\rho\vec{\nabla})]. \tag{43}$$

This equation is of course to be expected since we could have obtained it simply by replacing $\vec{k}\to -i\vec{\nabla}$ in Eq. (42) using a π N amplitude with both s and p wave scattering.

The velocity dependent potential is a special case of the more general potential

$$2m_\pi V = g(\vec{r})+(\vec{\nabla}\cdot\alpha(\vec{r})\vec{\nabla}). \tag{44}$$

Since the principal effect of nuclear polarization is a renormalization of the exciting wave, its effect is to transform Eq. (43) into a case of Eq. (44). We will now investigate this in more detail.

12. THE NUCLEAR LORENTZ-LORENZ EFFECT

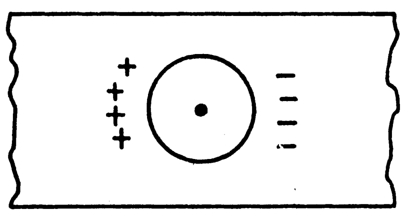
We made the assumption in deriving Eq. (43) that the p wave effective field is simply $\vec{\nabla}\phi$, where ϕ is the average wave in the medium, which gave an equation of type (44) :

$$\left[\vec{\nabla}\cdot(1-\alpha(\vec{r}))\vec{\nabla}\right]\phi + k^2\phi = q(\vec{r})\,\phi \tag{45}$$

with

$$q = -4\pi a\,\rho(\vec{r})$$
$$\alpha = 4\pi c\,\rho(\vec{r}).$$

While this assumption is good for a very dilute medium ($\rho(r)$ small), we must look at it with some suspicion in general; it is well known from the theory of propagation of a (dipole) wave in a classical dielectric that the effective field seen by a scatterer is modified by induced dipole charges on the surface of a small hole cut in the medium (Lorentz-Lorenz, 1871).



This modifies the refractive index for dense media. A similar effect occurs for the dipole scattering of the (pseudo) scalar pion field ϕ.

To obtain the necessary modification (we only discuss the effect for p waves and higher order terms are neglected !), we consider the scattering from a small homogeneous sphere of radius R with a scattering strength α as in Eq. (45). The wave function is continuous on the surface :

$$\phi_-(R) = \phi_+(R).$$

The condition on the derivatives are obtained by integrating the equation along a short line perpendicular through the surface :

$$(1-\alpha)\left(\frac{d}{dr}\phi_-\right)_R = \left(\frac{d}{dr}\phi_+\right)_R . \tag{46}$$

If the radius is small, the inside and outside solutions are

$$\phi_- = A r^\ell Y_{\ell m}$$

$$\phi_+ = B(r^\ell + b r^{-\ell-1}) Y_{\ell m} \tag{47}$$

so that the logarithmic derivative gives (note no derivation on $Y_{\ell m}$!) :

$$(1-\alpha)\frac{\ell}{R} = \frac{\ell - b(\ell+1)R^{-2\ell-1}}{R(1+bR^{-2\ell-1})}$$

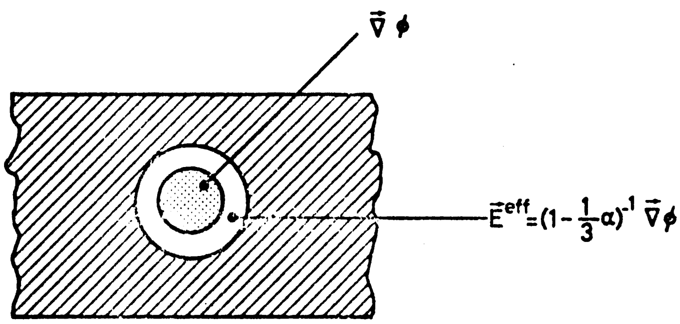
or :

$$A/B = (1+bR^{-2\ell-1}) = \frac{1}{(1-\frac{\ell}{2\ell+1}\alpha)} . \tag{48}$$

This expression was first obtained by M. Ericson [11]. Therefore if we excite such a small system by an initial wave Br^ℓ, the effective wave reaching the centre is a factor $1/(1-\alpha\ell/(2\ell+1))$ different [11]. In particular for dipole waves ($\ell=1$), this is a factor $1/(1-\alpha/3)$.

The effective field for p wave scattering in the nuclear medium is now easily obtained.

In the homogeneous medium the p wave field at a point is $\vec{\nabla}\phi$. Imagine now that the homogeneous medium is made up of a small, spherical hole around this point with a corresponding active "plug" which fills the hole. Imagine further that the plug is slightly smaller than the hole (see Fig.). According to Eq. (48), the incident field proportional to r^ℓ in the slot is a factor $(1-(\ell\alpha/2\ell+1))$ stronger than the field inside the plug, so that for dipoles ($\ell=1$) we have



$$\vec{E}^{\,\mathrm{eff}} = \left(1 - \frac{\alpha}{3}\right) \vec{\nabla}\phi \tag{49}$$

in the limit that the size of the plug goes to zero *).

If we now inspect Eq. (42) which contains the p wave scattering contribution and compare this to Eq. (44) we see that by Eq. (49)

$$\alpha(\vec{r})\,\vec{\nabla}\phi = 4\pi c\rho\,\vec{E}^{\,\mathrm{eff}} = 4\pi c\rho\,(1 - \alpha/3)\,\vec{\nabla}\phi \tag{50}$$

Therefore if we introduce the notation $\alpha_0 \equiv 4\pi c\rho$ we have

$$\alpha = \alpha_0\,(1 - \alpha/3)$$

or (51)

$$\alpha = \frac{\alpha_0}{\left(1 + \frac{\alpha_0}{3}\right)} \; .$$

An alternative way of writing Eq. (49) for the effective field which connects more directly to the π N scattering strength c is therefore

$$\vec{E}^{\,\mathrm{eff}} = \frac{\vec{\nabla}\phi}{\left(1 + \frac{\alpha_0}{3}\right)} \; . \tag{52}$$

Substitution of Eq. (51) into the potential equation (44) gives

$$2m_p V = q + \left(\vec{\nabla}\cdot \frac{\alpha_0}{\left(1 + \frac{\alpha_0}{3}\right)}\,\vec{\nabla}\right) \tag{53}$$

The s wave part of this equation, $q(\vec{r})$, is simply Eq. (38).

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*) Equation (49), or its equivalent, can be derived in several ways, of which the more rigorous ones start from the pair correlations function [7),8)]. The crucial point above is that matter has to be left at the centre of the hole because of the singular contribution from the centre.

The approximations made here in the derivation of ϕ^{eff} and $\vec{E}^{eff}$ seem presently sufficient for pionic problems. However, there are in general modifications of the effective fields with decreasing wavelength both in the medium and between scatterings. These cause in general a mixing of s and p wave contributions to ϕ^{eff} and $\vec{E}^{eff}$. Some of the approximations are discussed more in detail in Ref. 7).

Problem

If a wave equation has the form

$$[\vec{\nabla}\cdot(1-\alpha(r))\vec{\nabla}]\,\phi + q(r)\,\phi = 0$$

and if

$$\alpha = \alpha_1 \qquad \text{for} \quad r < r_0$$
$$\alpha = \alpha_2 \qquad \text{for} \quad r > r_0$$

show that

$$(1-\alpha_1)\left(\frac{d\phi}{dr}\right)_{r_0^{(-)}} = (1-\alpha_2)\left(\frac{d\phi}{dr}\right)_{r_0^{(+)}}$$

with $r_0^{(-)}$, $r_0^{(+)}$ the values $\vec{r}$ just below and above the limit layer. (Integrate radially from $r_0^{(-)}$ to $r_0^{(+)}$ and let the difference go to zero !)

Compare to the behaviour of the field $\vec{E}=-\vec{\nabla}\phi$ in electromagnetic theory ! What is $\vec{D}=\varepsilon\vec{E}$ for a pion ?

Problem

$$V(\vec{r}) = V_0(r) + \frac{1}{2m}\left(\vec{\nabla}\cdot 4\pi c\rho\,\vec{\nabla}\right)$$

is a short range interaction with $\rho(r)$ uniform inside a small sphere of radius R and zero outside,

$$\phi_\ell(r), \qquad R_\ell(r) \to Y_{\ell m} r^\ell B_\ell$$

at the origin for unperturbed wave functions with B_ℓ a constant. The energy shift

$$\Delta E_\ell = \int \phi_\ell(\vec{r})\, V(\vec{r})\, \psi_\ell(\vec{r})\, d\vec{r}$$

Calculate the perturbed energy as $R \to 0$.

[Use that at the origin $\psi_\ell = Y_{\ell m} r^\ell A_\ell$ inside R and $Y_{\ell m}(r^\ell + a_\ell r^{-\ell-1})B_\ell$ outside R (show this with A_ℓ and a_ℓ constant and proceed as in Eqs. (46)-(49) !].

Will the Born approximation hold or not in the limit $R \to 0$?

13. NUCLEAR PION ABSORPTION (SHORT REMARKS)

One of the most characteristic processes of a pion in contact with nuclear matter is its rapid strong absorption. This important phenomenon cannot occur for a free nucleon because of energy momentum unbalance. The lightest nucleus for which it occurs is the deuteron by

$$\pi + D \rightleftarrows N + N.$$

The energy momentum balance is then satisfied using the relative momenta of the two outgoing nucleons. It has been observed experimentally since the earliest days of pion physics that π absorption in more complex nuclei predominantly leads to the emission of two nucleons anticorrelated in their direction. This led very early to the quasi-deuteron model for nuclear pion absorption [12], elaborated later by Eckstein [13] and by M. Ericson and collaborators [14],[8].

From the point of view of strict multiple scattering theory the whole problem of pion absorption is, to some extent, an embarrassment since it falls out of the framework of multiple scattering. We will here only make a few remarks in the spirit of the two-nucleon absorption model in spite of the abundant experimental and in particular abundant theoretical literature on this topic [see Ref. [15] for further references].

The most economical and simplest assumption is that nucleon pion absorption is the same process in nuclei and in the deuteron but for scale factors [8],[9],[14]. This assumption amounts to saying that very complex, many-particle processes are not vital to pion absorption. It further assumes that the short range structure of the (np) wave function in the deuteron and in the nucleus are closely similar but for a scale factor. These assumptions are not necessarily correct, but we will try to get away with them.

If we consider the absorption of an s wave pion on the deuteron, the threshold absorption rate is

$$\beta_{11} \left| \phi_D(0) \right|^2 ,$$

where $\phi_D(\vec{r})$ is the deuteron wave function [i.e., rate = (absorption strength β_{11}) $\times$ (probability of finding the two nucleons at the origin $|\phi_D(0)|^2$)].

Within our assumptions above the <u>nuclear</u> local absorption rate is proportional to the same β_{11} and the probability of a chance encounter of a neutron and a proton at a particular point :

$$\text{local absorption rate} \propto \beta_{11} \frac{3}{4} \rho_n \rho_p . \qquad (54)$$

The factor 3/4 is simply the weight factor for the three deuteron spin states out of the four possible spin states of two randomly oriented nucleons.

To the expression (54) one should also add the absorption on a nucleon pair in relative singlet state as well as absorption of pions which are in a relative p state with respect to the two nucleons. The singlet absorption can be scaled from π production experiments of the type $n+p \rightarrow p+p+\pi^0$ and they are found to give relatively small contributions. This small empirical value has an important (and observed) consequence : in π^- absorption on nuclei the (nn) pairs emitted are much more frequent than the (np) pairs, since the latter only come from singlet absorption [14].

We can therefore from this view point describe nuclear π absorption by essentially two parameters : the s and p wave absorption strength on the deuteron, which have been experimentally determined. If we make the approximate further assumption that $\rho_n \approx \rho_p \approx \rho/2$, we can simply generalize Eq. (53) to have absorptive components of $q(\vec{r})$ and $\alpha_0(\vec{r})$. If we denote their strength by the constants $\mathrm{Im}\,B_0$ for s waves and $\mathrm{Im}\,C_0$ for p waves (these are of course proportional to β_{11}, etc.), we have

$$\begin{aligned} \mathrm{Im}\, q(r) &= -16\pi\, \mathrm{Im}\, B_0\, \rho_n \rho_p \simeq -4\pi\, \mathrm{Im}\, B_0\, \rho^2 \\ \mathrm{Im}\, \alpha_0(r) &= -16\pi\, \mathrm{Im}\, C_0\, \rho_n \rho_p \simeq -4\pi\, \mathrm{Im}\, C_0\, \rho^2 \end{aligned} \qquad (55)$$

From Eq. (55) we note that this looks formally as if "deuteronic particles" were floating around in the nucleus with a given absorption strength, and these "particles" are simply added into the multiple scattering equations as a new, additional type of scatterers.

As will be seen shortly, this simple, semi-phenomenological, description of the absorption is surprisingly good. It is in fact much more quantitative in the description of experimental data than more ambitious, detailed approaches which attempt explicit, dynamical descriptions of the absorptive process. One should realize, however, that the price for accuracy is that only total absorption rates and the ratio of emitted (nn)/(np) pairs can be described.

14. ANALYSIS OF DATA ON PIONIC ATOMS

Early experimental data on pionic atoms were analyzed using perturbative expansions in the interaction as well as nuclear matter distributions of the square box type [11),8)]. The technical problem of analyzing π mesic atoms with high precision using realistic matter distribution has recently been solved [9)] and a detailed analysis can now be made with ease.

It is useful to perform the analysis in two steps :

1) the determination of the effective parameters in an effective potential of the form (53);

2) the comparison of the effective parameters to multiple scattering predictions.

The non-local potential (53) depends on the mass densities ρ_n, ρ_p and $\rho = (\rho_n + \rho_p)$ as well as on constants b_o, b_1, etc., related to s and p wave π N scattering lengths [Eq. (1.a)] and production amplitudes. The leading terms neglecting s wave effective field effects are :

$$q = -4\pi \left[b_0 \rho + b_1 (\rho_n - \rho_p) + i \, \mathrm{Im} B_0 \rho^2 \right] + \text{corrections}$$

$$\alpha_0 = 4\pi \left[c_0 \rho + c_1 (\rho_n - \rho_p) + i \, \mathrm{Im} C_0' \rho^2 \right] + \text{corrections.} \qquad (56)$$

The parameters are considered to be "effective" parameters. Simple kinematical binding corrections are included in "corrections".

The pionic atom is described by the Klein-Gordon equation

$$\nabla^2 + \left[(E - V_c(r))^2 - m_\pi^2 \right] = 2 m_\pi V(r) \qquad (57)$$

where $V_c(r)$ is the Coulomb potential for the extended nuclear charge. (Vacuum polarization effects in the presence of the strong interaction have, of course, to be included.)

The analysis proceeded as follows [9] : we first took 1s and 2p shifts and widths for pure isotopes with reasonably well measured charge distributions (18 elements in the range ^{10}B to ^{59}Co fulfilled this criterion). The proton distributions were parametrized by the Saxon-Woods shape

$$\rho = \frac{N}{\left[1 + \exp\left\{4 \ln 3 \frac{r-c}{t}\right\}\right]} \tag{58}$$

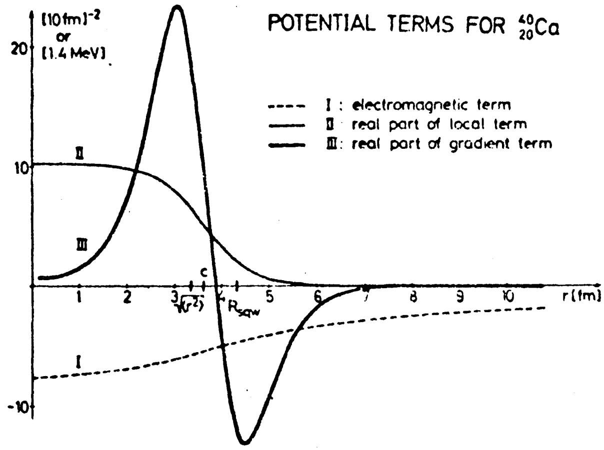
with c the radial parameter and t the surface thickness (90% to 10% variation of ρ) and N normalizing to the correct number of particles.

We then <u>assumed</u> the neutron and proton matter distributions to coincide : this is a very innocent assumption for shifts (possible exceptions near ^{59}Co) and for widths in $T = 0$ nuclei. It may not be quite so good for widths in $T \neq 0$ and could introduce there minor systematic deviations. The constants of Eq. (56) can then be directly determined and they are in fact strongly overdetermined. The results are given in the Table :

	Exp.	Theor.
b_o	-0.030	$-0.025 + a^{(+)}$
b_1	-0.08	-0.09
c_o	0.22	0.19
c_1	[0.18]	0.017
$\mathrm{Im}\, B_o$	0.040	0.017
$\mathrm{Im}\, C_o$	0.08	0.07

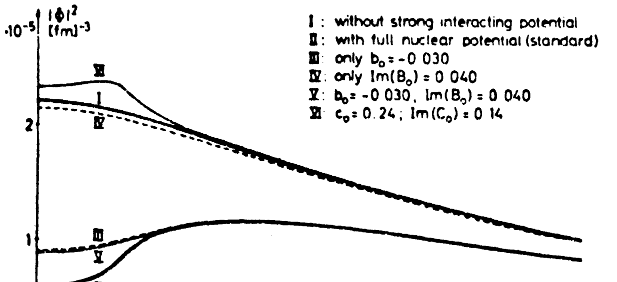
The dependence on the constant c_1 is unimportant so its value was introduced theoretically. The parameters agree surprisingly well with previous analyses of less sophistication.

To obtain a feeling for the properties of the non-local potential (53) with parameters (56), we transformed into a local representation [see problem at the end of this Section]. The corresponding local potential for ^{40}Ca is seen in the Figure. The Coulomb interaction is included for comparison. The local repulsive potential q is of the magnitude of the Coulomb potential over the nuclear region, but with opposite sign.



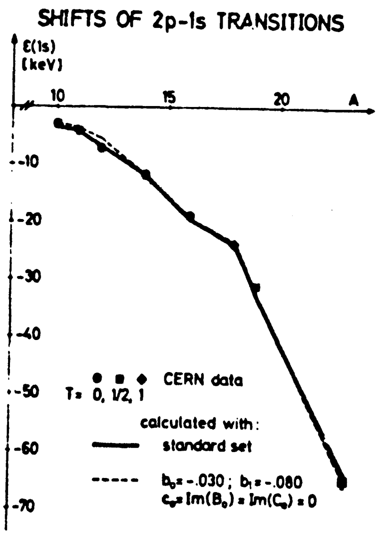
The gradient interaction corresponds to a dipole layer with attraction in the nuclear surface and repulsion inside. (Note that the maximal attraction is even <u>outside</u> the radius of an equivalent, uniform matter distribution !) Since the centrifugal barrier makes wave functions of high ℓ small in the nuclear interior, bigger further out, they feel the outside attraction of the dipole layer predominantly and give attractive energy shifts.

It is interesting for many applications to know how the wave function is changed by the strong interaction. The Figure below shows the probability density of a pion in the 1s state of ^{16}O for various assumptions about the interaction. It shows reduction by a factor of three in the nuclear interior, but this reduction is nearly model independent. Outside the nucleus where there is no interaction the wave



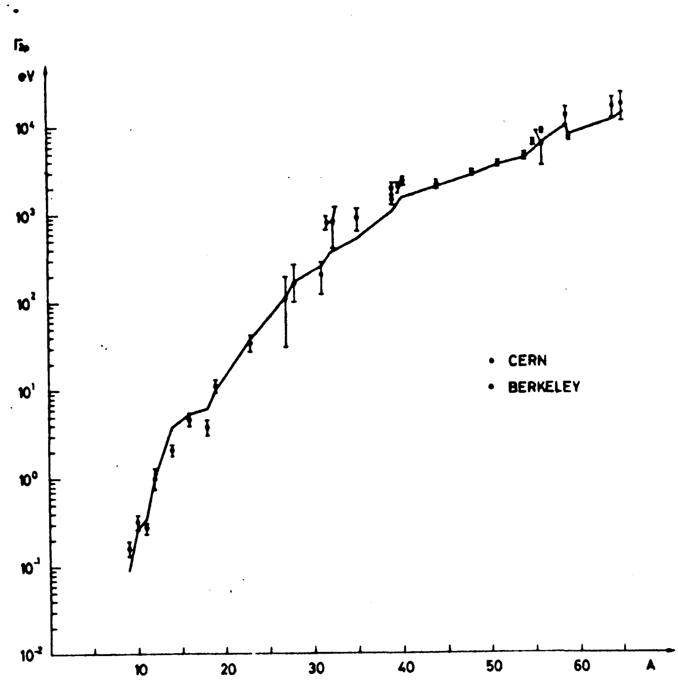
function is exactly determined by the measured energy, and it is seen to deviate strongly from its unperturbed value very far outside the nucleus. Inside the nucleus any interaction giving the correct energy shift gives similar values for the probability density. It is mainly the repulsion in the 1s state that decreases the wave function, while the absorption plays no noticeable role. The gradient interaction enhances the repulsion in the nuclear interior somewhat by decreasing the mean free path, but it has no effect outside the nucleus.

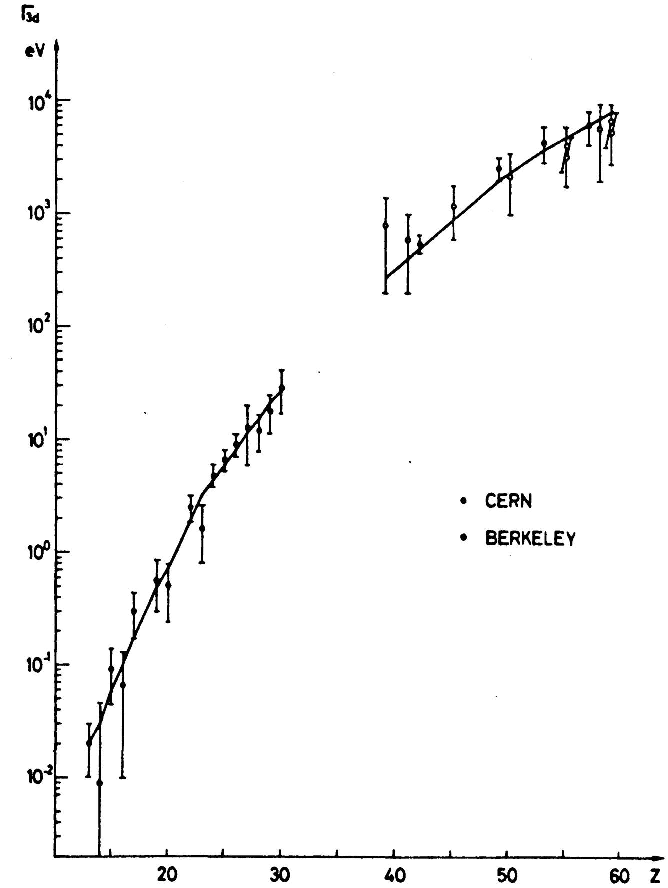
The parameters are not much correlated. An extreme example is seen for the 1s states in the next Figure, where the energy shifts have been fitted using an interaction without absorption and without gradient terms (which is a wild change). The fit is nearly identical with nearly identical values for b_o and b_1. These are the only parameters of importance for describing 1s shifts.



The absorption parameters depend importantly on the strength of the real part of the potential. The major part of this dependence is of trivial origin, however, since it only reflects that the probability of presence in the nucleus is changed : this is nearly model independent as just discussed. The absorptive parameters are therefore also very stable.

The extent to which the absorption can be described is seen in the widths of the 2p and 3d states presented below. These span both over more than five magnitudes each and are described mainly by $\mathrm{Im}\,C_o$ over this range.


Γ_{2p}
eV
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10
20
30
40
50
60
A
CERN
BERKELEY


Γ_{3d}
eV
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
20
30
40
50
60
Z
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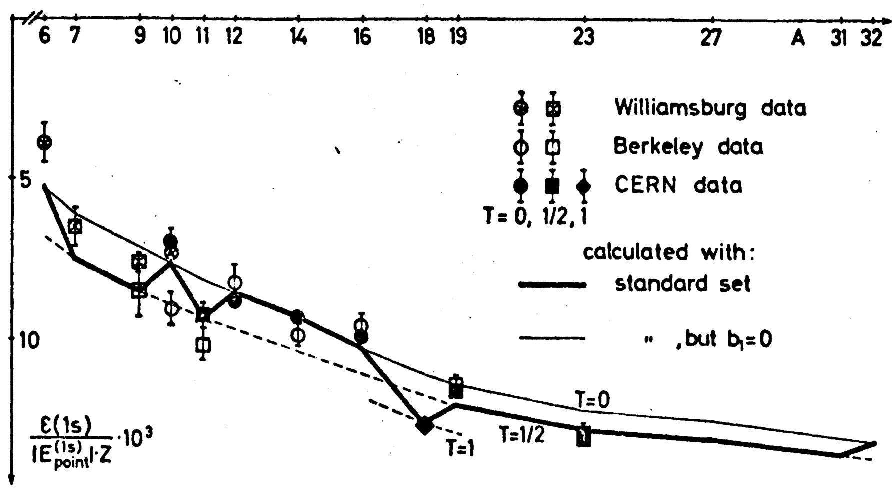
BERKELEY

The precision of the best measured X-ray lines, those of the 2p-1s transition, is so great that it is necessary to use a reduced scale to reveal the details of the strong interaction shifts. It is clearly seen in this scale that the 1s shifts have a strong dependence on neutron excess, particularly apparent in $^{11}B-^{10}B$ and $^{18}O-^{16}O$.



Present analyses have reached the precision that some information on neutron distributions can be obtained. So, for example, Jenkins et al. [16] have measured the difference of the 2p levels for the $^{60}Ni-^{58}Ni$ pair and find them identical within experimental errors, although two neutrons were added. They interpret this to mean that the neutron radius with high precision is the same for these two elements.

<u>Problem</u>

Consider the wave equation

$$\left[\vec{\nabla}\cdot(1-\alpha(\vec{r})\vec{\nabla}\right]\phi + k^2\phi = q(\vec{r})\,\phi$$

corresponding to a non-local potential

$$2m_n V = q(\vec{r}) + \left[\vec{\nabla}\cdot\alpha(\vec{r})\vec{\nabla}\right].$$

Show that the substitution $\phi(\vec{r}) = (1-\alpha(\vec{r}))^n\tilde{\phi}(\vec{r})$ gives a wave equation for $\tilde{\phi}$ which exactly corresponds to a <u>local</u> equivalent potential for a suitable choice of n [see Ref. 9]. Which is the value of n for which this happens ? Show that the equivalent potential is

$$2m_n\tilde{V} = (1-\alpha)^{-1}\left[q - \alpha k^2 - \frac{1}{2}(\nabla^2\alpha) - \frac{1}{4}\frac{(\vec{\nabla}\alpha)^2}{1-\alpha}\right].$$

Note that $\tilde{V}$ is <u>energy dependent</u> since k^2 appears explicitly !

15. COMPARISON OF EFFECTIVE AND PREDICTED PARAMETERS

The theoretical predictions of the experimental parameters (given in the Table of p.40) are in general in good agreement with predictions.

The limitation on the predicted value for

$$b_0 = \frac{(a_n+a_p)}{2} - \frac{2(a_n^2+a_p^2)+(a_n-a_p)^2}{4}\left\langle\frac{1}{r}\right\rangle_{corr}$$

[see Eq. (38)] is the very badly known value of $(a_n+a_p)/2$ for the πN scattering lengths, of which even the sign is disputed. About 70% of b_0 is expected to come from the effective field correction. We may turn the argument around and use the mesic atoms to give a value for $(a_n+a_p)/2$ and final $-0.010\ m_\pi^{-1} < a^{(+)} = (a_n+a_p)/2 < 0$.

Beyond the multiple scattering contributions to b_0 dispersive corrections are expected from absorption. These corrections are generally believed to be small [of order $(\Gamma_{1s}/\mathcal{E}_{1s})b_0$], but the question is still open. A recent estimate for deuterium [17] indicates that they may be considerably larger than previously believed.

The only parameter falling out of line is $\mathrm{Im}\,B_0$, the absorptive parameter for s states, which is predicted a factor of two weaker than actually observed. In a sense, this is a very good result in view of the simple quasi-deuteron picture used, in particular since the p absorption $(\mathrm{Im}\,C_0)$ describes widths extremely well for wide ranges of nuclei.

This is, however, the very real possibility that this discrepancy reflects a modification of the nuclear s wave pion absorption by the presence of other nucleons, and this should be further explored.

It should finally be remarked that the parameter b_1 can be compared to predictions of generalized soft pion theorems. For nuclei these predict the Born approximation of the potential universally [18] and there is agreement up to about 15%.

* * * *

In the final Sections, we will briefly comment on some features of the π nuclear interaction which are directly connected to the structure of the nuclear pion field.

* * * * * *

16. LOW ENERGY PHOTOPIONS AND RADIATIVE π ABSORPTION

In addition to the strong absorption of pions in nuclei, electromagnetic absorption (photoproduction) occurs with a typical strength of several per cent of the main absorptive process.

There is a strong analogy between radiative pion absorption and μ capture processes both kinematically and dynamically. This can be seen as follows : assume these processes to occur on individual nucleons in the nucleus. The basic process is then

$$\pi + N \rightleftarrows N + \gamma + 139\ \mathrm{MeV}$$
$$\mu + N \rightleftarrows N + \nu_\mu + 106\ \mathrm{MeV}.$$

In both casses a massless particle $[\gamma$ or $\nu_\mu]$ is involved and the momentum transfer is also similar.

At low energies of the pion, the $(\pi\gamma)$ process can, to a very good approximation, be described by an interaction

$$H^{\alpha}_{\pi\gamma} = \frac{eg}{4M_n}\,(\vec{\sigma}_i\cdot\vec{\varepsilon})\,\tau_i^{\alpha}\,\delta(\vec{x}-\vec{x}_i) \tag{59}$$

valid for s wave pions. Here $e^2/4\pi = \alpha$ and $g^2/4\pi\,(m_\pi/2M_n)^2 = f^2 = 0.08$ is the πN coupling constant. The photon polarization vector is $\vec{\varepsilon}$.

The weak (μ, ν_μ) capture is described by a (current×current) interaction between the lepton current and the nucleon current. The effective μ capture Hamiltonian has an <u>axial</u> part which is in the non-relativistic limit proportional to

$$H^{\alpha}_{\mu\nu_\mu} \propto \left(j^{\nu}_{lepton}\cdot J^{A}_{\nu}\right) \propto \vec{\sigma}_i\,\tau_i^{\alpha}\,\delta(\vec{x}-\vec{x}_i). \tag{60}$$

In this approximation the nucleon part of Eq. (59) is therefore extremely similar.

Assume now that these processes occur in the same way on nucleons in the nucleus. Then the <u>nuclear</u> effective Hamiltonians are

$$H^{\alpha}_{\pi\gamma} = \frac{eg}{4M_n} \sum_i (\vec{\sigma}_i \cdot \vec{\varepsilon})\, \tau_i^{\alpha}\, \delta(\vec{x} - \vec{x}_i) \qquad (59')$$

and

$$H^{\alpha}_{\mu\nu_\mu} \propto \sum_i \vec{\sigma}_i\, \tau_i^{\alpha}\, \delta(\vec{x} - \vec{x}_i) \equiv \vec{\mathcal{O}}^{\alpha}(\vec{x}) . \qquad (60')$$

In this approximation the matrix elements for the two processes contain the same operator $\vec{\mathcal{O}}^{\alpha}(\vec{x})$. The conclusion is therefore that the nuclear states which are excited by the two reactions under otherwise similar conditions (for example if the initial angular momentum state is the same) must be very similar. In particular, one concludes that the ordinary μ capture theory from atomic orbits can be immediately transposed to radiative π capture [19),20)].

Equations (59') and (60') refer to the effective Hamiltonian. In the case of the pion, the wave function is modified importantly in the neighbourhood of the nucleus by the strong interaction (see Section 14). The probability of presence for an s wave pion in ^{16}O is changed by a factor ≈ 3. It is therefore necessary to use correct pion wave functions in numerical applications of the $(\pi\gamma)$ effective Hamiltonian.

The similarity between the weak μ capture and the threshold $(\pi\gamma)$ process extends beyond the impulse approximation. It can be shown [21),22)] that the threshold $(\pi\gamma)$ amplitude for soft pions (i.e., hypothetical pions of mass $m_\pi \to 0$) has an effective Hamiltonian

$$H^{\alpha}_{\pi\gamma}(m_\pi \to 0) \Longrightarrow \frac{e}{2f_\pi} (\vec{J}^{\alpha}_{A} \cdot \vec{\varepsilon}) \qquad (61)$$

where J^{μ}_{A} is the nuclear axial current and f_π the pion decay constant for the $\pi \to \mu\nu$ reaction.

This is therefore the same axial current which occurs in μ capture. Equation (61) is the nuclear analogy to the so-called Kroll-Ruderman theorem for the nucleon.

It has further been proved [23] by dispersive techniques, that the correct extension of the zero pion results to actual pions is mainly to include the distortion effects of the π nuclear interaction, so that Eq. (60) can be interpreted as the effective Hamiltonian also for real pions.

17. THE PION-NUCLEAR SOURCE FUNCTION

The nucleus is the source of the nuclear pion field $\phi^{\alpha}(\vec{x})$. The field is related to its source strength by the Klein-Gordon equation

$$(\Box - m_\pi^2)\,\phi^{\alpha}(\vec{x}) = j_\pi^{\alpha}(\vec{x}). \tag{62}$$

Since the virtual emission of a pion in general transfers the nucleus into an excited state, both field and source functions are <u>operators</u> in the nuclear variables.

The virtual emission of a pion gives a momentum transfer of order m_π to the nucleus. The characteristic nuclear excitation energy is then of order $m_\pi^2/2M_n \ll m_\pi$. The nucleus is therefore approximately static and the excitation energy can be neglected.

Every point of the source gives rise to a Yukawa-like virtual pion field $e^{-m_\pi x}/x$, so that

$$\phi^{\alpha}(\vec{x}) \simeq -\int j_\pi^{\alpha}(\vec{x}\,')\,\frac{1}{4\pi}\,\frac{e^{-m_\pi|\vec{x}-\vec{x}\,'|}}{|\vec{x}-\vec{x}\,'|}\,d\vec{x}\,'. \tag{63}$$

Assume the nuclear source to be the individual nucleons considered to be static and pointlike. Then

$$j_\pi^{\alpha}(\vec{x}) = \sum_i (4\pi)^{1/2}\,\frac{f}{m_\pi}\,(\vec{\sigma}_i\cdot\vec{q})\,\tau_i^{\alpha}\,\delta(\vec{x}-\vec{x}_i), \tag{64}$$

where $\vec{q} = -i\vec{\nabla}$ is the pion momentum. Substitute this source into Eq. (63) and consider the asymptotic region outside the nucleus, for which $|\vec{x}| \gg |\vec{x}_i|$:

$$\begin{aligned}
\phi^{\alpha}(\vec{x}) &= i\,(4\pi)^{-1/2}\,\frac{f}{m_\pi}\sum_i(\vec{\sigma}_i\cdot\vec{\nabla})\,\frac{e^{-m_\pi|\vec{x}-\vec{x}_i|}}{|\vec{x}-\vec{x}_i|}\,\tau_i^{\alpha} \rightarrow \\
&\rightarrow i\,(4\pi)^{-1/2}\,\frac{f}{m_\pi}\left(\vec{\nabla}\,\frac{e^{-m_\pi x}}{x}\right)\cdot\sum_i\vec{\sigma}_i\,\tau_i^{\alpha}\,e^{\vec{m}_\pi\cdot\vec{x}_i} \\
&\equiv i\,(4\pi)^{-1}\,\frac{f}{m_\pi}\,(\vec{a}^{\alpha}\cdot\vec{\nabla})\left(\frac{e^{-m_\pi x}}{x}\right),
\end{aligned} \tag{65}$$

where the direction $\vec{m}_\pi$ is the direction of $\vec{x}$. The operator

$$\vec{a}^{\alpha} \equiv \sum_i \vec{\sigma}_i \tau_i^{\alpha} e^{\vec{m}_\pi \cdot \vec{x}_i} \tag{66}$$

introduces transitions between different nuclear states because of the virtual pion emission. The asymptotic region outside the nucleus contains therefore a superposition of nuclear excited states and it is a dynamical structure. In principle, $\vec{a}^{\alpha}$ can be determined by measuring $\langle i|\vec{a}^{\alpha}|f\rangle$ between nuclear states.

Which are the transient nuclear states in the region outside the nucleus ?

Take the specific example of a nucleus with a saturated spin-isospin structure like ^4He, ^{16}O, etc. These are nuclei with equal probability for any direction of spin and isospin of the nucleons. For such a nucleus

$$\sum_i \vec{\sigma}_i \tau_i^{\alpha} |0\rangle = 0 . \tag{67}$$

The exponent $m_\pi x_i$ is a relatively small number and an expansion can be made in this quantity. If the leading term of the multipoles is retained, one finds :

1) $\ell = 0$. Since $\sum_i \vec{\sigma}_i \tau_i^{\alpha}|0\rangle = 0$ there are no such positive parity states to this order.

2) $\ell = 1$. The states are $\sum_i \vec{\sigma}_i \tau_i^{\alpha} (\vec{m}_\pi \cdot \vec{x}_i)|0\rangle$ which have negative parity, isospin 1 and angular momentum 0, 1 or 2. These states are analogues to the nuclear giant dipole excitations $\sum_i \tau_i^{\alpha} z_i|0\rangle$ but correspond to spin-isospin waves in the nucleus, instead of isospin waves. These are the dominant contributions in lighter elements.

3) Higher multipoles are relatively of less importance.

The nuclear states which occur in the asymptotic pion field are very similar to those which are associated with radiative pion capture at rest [cf., Eq. (59')]. The reason is that the spin-isospin structure is the same and the momentum transfer is nearly the same.

17. THE EFFECTIVE π NUCLEAR COUPLING CONSTANT

A massive particle acquires no kinetic energy by a change of its momentum. A pion of energy $\omega = (\vec{q}^{\,2}+m_\pi^2)^{\frac{1}{2}} = 0$ can therefore be freely emitted and absorbed by a massive system whenever quantum numbers allow this. This reflects itself both by the singularity $(\vec{q}^{\,2}+m_\pi^2)^{-1} = \omega^{-2}$ in the Fourier transform of the asymptotic field and in a pole at $\omega = 0$ in the pion dispersion relations.

Even the nucleon is nearly massive in this sense. For a massive nucleon with a π N forward scattering amplitude $f(\omega) = f^{(+)}(\omega) - (\vec{t}\cdot\vec{\tau})f^{(-)}(\omega)$ [normalized so that $\langle |f|^2 \rangle = (d\sigma/d\Omega)_{0\omega}$], the antisymmetric amplitude $f^{(-)}(\omega)$ obeys the unsubtracted forward dispersion relation :

$$\mathrm{Re}\, f^{(-)}(\omega) = \frac{2f^2}{\omega} + \frac{2}{\pi}\,\omega\, P\int_{m_\pi}^{\infty} \frac{\mathrm{Im}\, f^{(-)}(\omega')\,d\omega'}{(\omega'^2-\omega^2)} \,. \tag{68}$$

This equation can be used to determine the π N coupling constant $f^2 \simeq 0.08$ from measured real amplitude and $\mathrm{Im}\, f^{(-)}(\omega) = \frac{k}{4\pi} \times \frac{1}{2}[\sigma_{\pi^-p} - \sigma_{\pi^+p}]$.

The complex nucleus also emits virtual pions, in general associated with virtual excitation of the nucleus. In the static limit in which excitation energies are small compared to the pion mass (Section 16), there will be an <u>effective</u> pion pole at $\omega = 0$ with an <u>effective π nuclear coupling constant</u> $(f^2)_{\mathrm{eff}}$ [24]. In the particular case of an isospin $\frac{1}{2}$ nucleus, there will be an antisymmetric amplitude analogous to Eq. (68)

$$\mathrm{Re}\, f^{(-)}(\omega) = \frac{2(f^2)_{\mathrm{eff}}}{\omega} + \frac{2}{\pi}\,\omega\, P\int_{m_\pi}^{\infty} \frac{\mathrm{Im}\, f^{(-)}(\omega')\,d\omega'}{(\omega'^2-\omega^2)} \,. \tag{69}$$

Two questions are natural :

1) what is the experimental value of $(f^2)_{\mathrm{eff}}$?

2) how is $(f^2)_{\mathrm{eff}}$ related to the pion field and what is its relation to f^2 ? (This question is discussed in Section 18.)

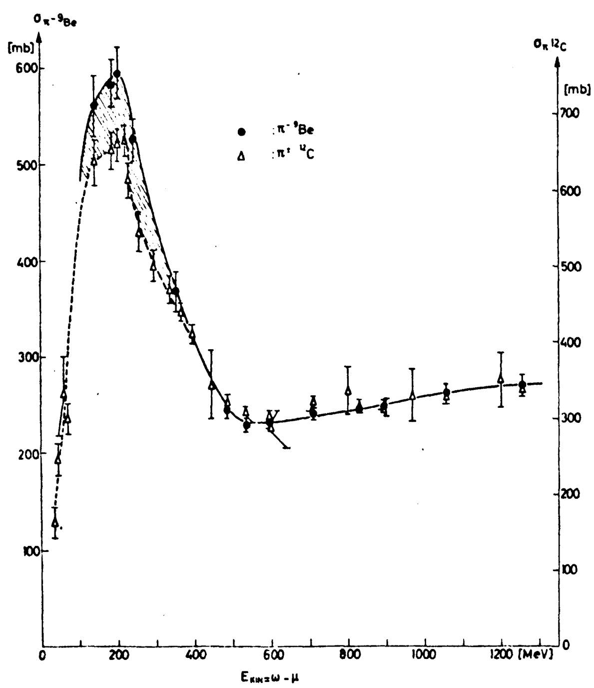
The only nucleus of $T=\frac{1}{2}$ for which even marginally useful data on $\mathrm{Re}\, f^{(-)}(\omega)$. and $\mathrm{Im}\, f^{(-)}(\omega)$ exist is ^{9}Be which has $J^{\pi} = \frac{3}{2}^-$. Relatively good measurements exist for the total cross-section for π^- and for the π^- ^{9}Be scattering length as deduced from π^- mesic atoms. The input values of Eq. (69) can therefore be obtained from $\mathrm{Im}\, f^{(-)}(\omega) = \frac{k}{4\pi}\, \sigma^{(-)}(\omega)$, where

$$\sigma^{(-)}(\omega) = \frac{1}{2}\left\{ \sigma_{\pi^+ {}^9Be}(\omega) - \sigma_{\pi^- {}^9Be}(\omega) \right\} = \frac{1}{2}\left(\sigma_{\pi^+ {}^9Be} + \sigma_{\pi^- {}^9Be}\right) - \sigma_{\pi^- {}^9Be} .$$

There is no measurement of $\sigma_{\pi^+ \, {}^9Be}(\omega)$, but the total cross-section on neighbouring $T=0$ elements can be used as a crude interpolated value for $\frac{1}{2}\left[\sigma_{\pi^+ \, {}^9Be} + \sigma_{\pi^- \, {}^9Be}\right]$, since

$$\sigma_{\pi^-}^{I=0} = \sigma_{\pi^+}^{I=0} = \frac{1}{2}\left(\sigma_{\pi^+}^{I=0} + \sigma_{\pi^-}^{I=0}\right).$$

The extent to which such an interpolation can be done is shown below.



A similar procedure gives the threshold amplitude $\mathrm{Re}\, f^{(-)}(\omega = m_\pi)$. With these values, Eq. (69) gives

$$\left(f^2\right)_{eff} = (0.06 \pm 0.03) \tag{70}$$

for ^{9}Be. This value is very close to the nucleon value.

A simple way of obtaining the nucleon value is to approximate the nucleon pole by a coherent sum of nucleon poles. This gives universally

$$(f^2)_{eff} = f^2 . \tag{71}$$

This approximation is too simple, however, since it ignores that $(f^2)_{eff}$ consists of contributions from many nuclear excited states. If a relation like Eq. (70) holds in the nucleus, it must necessarily result as a sum rule over these states.

Problem

A pion of energy $\omega = \sqrt{\vec{q}^2+m_\pi^2}$ is incident on a nucleon at rest on which it is absorbed. For which energy ω_0 is the process allowed ? Compare this energy to the pion rest mass. What is the nucleon recoil kinetic energy ?

18. $(f^2)_{eff}$ AND THE ASYMPTOTIC FIELD

Since forward scattering has no momentum transfer, one would expect it to depend mostly on the peripheral nature of the interaction. This is indeed the case. The poles of forward dispersion relations have residues directly dependent on the Yukawa strength outside the interaction region. So for example, if a pion described by isospin index α is scattered and emerges as β on the nucleon, there is a one-nucleon pole contribution (65) for $\vec{q}^{\,2} = -m_\pi^2$:

$$f_{pole} \propto \left[f(\vec{\sigma}_i \cdot \vec{q})\tau_i^\beta \, f(\vec{\sigma}_i \cdot \vec{q})\tau_i^\alpha - f(\vec{\sigma}_i \cdot \vec{q})\tau_i^\alpha \, f(\vec{\sigma}_i \cdot \vec{q})\tau_i^\beta \right]$$

$$= f^2 m_\pi^2 \left[\tau_i^\alpha, \tau_i^\beta\right]. \qquad (72)$$

The two orderings of the isospin indices α and β as well as the two signs originate from the two possibilities :

a) absorption of α and emission of β which has sign (+);

b) emission of β in the presence of α with subsequent absorption of α which has sign (-).

The situation is exactly analogous in the nuclear case. The effective pole strength has a similar form like in Eq. (72) with $\vec{\sigma}_i \tau_i^\alpha$ replaced by the operator $\vec{a}^{\,\alpha}(\vec{m}_\pi)$ which describes the asymptotic field and the virtual nuclear excitations. More exactly

$$f_{pole} \propto \left\{ f(\vec{q}\cdot \vec{a}^{\,\beta}(\vec{m}_n))\, f(\vec{q}\cdot \vec{a}^{\,\alpha}(-\vec{m}_n)) - f(\vec{q}\cdot \vec{a}^{\,\alpha}(-\vec{m}_n))\, f(\vec{q}\cdot \vec{a}^{\,\beta}(\vec{m}_n)) \right\}$$

$$\equiv f^2 \left[(\vec{q}\cdot \vec{a}^{\,\beta}(\vec{m}_n)), (\vec{q}\cdot \vec{a}^{\,\alpha}(-\vec{m}_n)) \right] \qquad (73)$$

$$= f^2 \sum_{i,j} (\vec{q}\cdot\vec{\sigma}_i)(\vec{q}\cdot\vec{\sigma}_j)\, e^{\vec{m}_n (\vec{x}_i - \vec{x}_j)} \left[\tau_j^\beta, \tau_i^\alpha\right],$$

where we simply substituted Eq. (66) by $\vec{m}_\pi = \hat{q}m_\pi$. Since now, $[\tau_j^\beta, \tau_i^\alpha] = \delta_{ij}[\tau_i^\beta, \tau_i^\alpha]$, i.e., it vanishes for $i \neq j$ and since $(\vec{\sigma}_i \cdot \vec{q})^2 = q^2 = -m_\pi^2$, we have

$$f_{pole} \propto f^2 m_\pi^2 \sum_{i,j} \delta_{ij} [\tau_i^\alpha, \tau_i^\beta] = f^2 m_\pi^2 \sum_i [\tau_i^\alpha, \tau_i^\beta]. \quad (74)$$

A comparison between Eq. (72) and (74) shows then immediately that $(f^2)_{eff}$ is simply the coherent sum of nucleon poles (72) in this approximation. This can be made even more vivid by introducing $[\tau^\alpha, \tau^\beta] = \varepsilon^{\alpha\beta}_{\gamma} \tau^\gamma$, the third isospin component. We can then express Eq. (74) as :

$$f_{pole} \propto f^2 m_\pi^2 \, \varepsilon^{\alpha\beta}_{\gamma} \sum_i \tau_i^\gamma \propto (f^2)_{eff} T^\gamma \quad (75)$$

i.e., $(f^2)_{eff}$ is a universal constant f^2 and the pole strength is proportional to the total isospin component T^γ independent of the structure of the nucleus and its size [25].

We re-emphasize the assumptions used :

1) single nucleon sources with no meson terms and no multi-nucleon terms;

2) a static nucleus.

Although these restrictions are important, the result (75) is far from trivial. Individual nuclear states do not exhaust the sum rule in general. It is therefore of considerable interest to establish more exactly from experiment to what extent a <u>quantitative</u> universality is valid or not.

At present, the crude estimate above for ^{9}Be as well as the good description of the (π,γ) capture in ^{3}He from the Panofsky ratio [23] constitute the only two direct determinations of the asymptotic pion field (65) in the first case and the pion-nuclear source strength in the second case.

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