

## **Personal and scientific recollections of Aurelio Grillo**

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### **Abstract**

In this talk I will present some personal and scientific recollections of my dear friend Aurelio Grillo. These recollections will be mainly devoted to his early part of his career, because at that time we had frequent scientific exchanges and we were working on similar field (often together).

### **1 Introduction**

Aurelio was born in 1945: he was three years older than me. We did not see each other too much while he was studying at the university. However we started to see each other very frequently after he left the university: we had a very good common friend, Massimo Testa. When I got a fellowship at the National Frascati Laboratories (January 1971), he had already a permanent position there.

At the Frascati Laboratories there was the largest  $e^+e^-$  colliding beam in the world: it was constructed under the scientific leadership of Bruno Touschek. Unfortunately, the project-energy was 1.5+1.5 GeV, not 1.6 GeV+1.6 GeV, so Frascati was unable to discover the  $\psi$ <sup>1</sup>.

When arrived at Frascati I found a small, but wonderful, theory group: beyond Aurelio there were Gianni de Franceschi, Paolo di Vecchia, Antonino Drago, Etim Etim, Sergio Ferrara, and Mario Greco. We were working at a few meters one from the others and we were going to eat together at the excellent cantine of the laboratories. At lunch time we were discussing everything, maybe the most popular arguments were connected to the experiments that were done in Frascati at that particular moment.

Indeed it was an exciting time. The first results from the experiments were coming out and they presented evidence for a completed unexpected production of many pions with a quite high cross section. This was in variance with the most fashionable theories that predicted that there should be only two or three mesons production and that the total cross section for hadron production should be much smaller than the observed one. An explanation was in the parton model, but this interpretation of the data was not so compelling as it is nowadays. We were very interested to understand how sound were the experimental results, we were pondering the information that was coming out from informal discussions with the experimentalist and we were trying to figure out which was the best interpretation of the data.

People would hardly visually recognize Aurelio: at that time he had not his beard we are accustomed now. We were part of a company of very good friends: we met very frequently outside working time, eating and drinking beer in Roman pubs, playing poker, etc. Aurelio had a strange luck at poker. I remember that once in 8 consecutive deals he had 7 very good deals: one quad, one flush, three full houses, two straights. It was a very impressive sequence: I cannot remember anything similar. Unfortunately for him and fortunately for us, he lost all the deals.

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<sup>1</sup>The  $\psi$  was observed by increasing the electron and positron energy to a value that was of about %3 higher than the project-energy. Fortunately it turned out that this increase was within the tolerance of the magnets and everything was fine.

## 2 Aurelio, the conformist

Aurelio started to work (with di Vecchia and Drago in Frascati and other people from outside) on the modifications of the Veneziano model produced by the effect of unitarity<sup>1, 2)</sup>. It was a very hot problem at that time, that faded with time with the introduction of the systematic loop expansion and with the awareness that the Veneziano model could not be a viable starting point for strong interactions.

His scientific interest changed when Raul Gatto came to Rome (1971). He strongly pushed Aurelio and Sergio Ferrara to work on the conformal group. He was already working on this field while in Padova and he wanted to continue his investigations in Rome. The collaboration was extremely successful and 15 papers were produced: most of them were written between 1971 and 1974<sup>3, 4, 5, 6, 7, 8)</sup>. I joined the group 1972 and I cosigned 5 of them<sup>9, 10)</sup>.

If a quantum field theory is scaling invariant, the trace of the energy-momentum tensor must be zero, and usually, the theory is also conformal invariant (at least for gauge invariant quantities). Conformal invariance is thus a natural extension of scale invariance. Indeed the four-dimensional conformal group (i.e.  $O(4, 2)$ ) is an extension of the direct product of Lorentz group and of dilatations.

Why at the time people were deeply interested in studying the conformal group? Bjorken scaling was suggesting that the strong interaction theory was scaling invariant at high energies. Extending scaling invariance to conformal invariance could give some extra clues to understanding the physics of the (at that time) mysterious scaling invariance.

In a very nice review<sup>11)</sup> Aurelio explains very clearly the motivations for these studies: *It is an important idea, due to Wilson, that the renormalization procedure of any sensible field theory could eventually give an anomalous part to the dimensions of the fields: this comes from the infinite strength renormalization and it is a parameter which is determined by the interaction and in some sense characterizes the dynamics. (...) Since the discovery of scaling behavior in deep inelastic electroproduction, many theoretical investigations have been devoted to the study of the origin of this phenomenon.*

*Various models, such as the parton model, have been invented, that give a partially satisfactory (or unsatisfactory) explanation of experimental results, but the most important achievement which emerges is the emphasis that has*

been put on fundamental properties of field theory such as dilatation and conformal invariance.

At that time, in the relatively small circles of people that believed that strong interactions should be understood via a renormalizable quantum field theory, the standard folklore was:

- At high energy (more precisely at short distances) strong interactions were supposed to be a strongly coupled theory: indeed in all the known theories, the interaction was increasing with the energy (decreasing the distance).
- Renormalization group equations could be written and the scaling invariance of the theory implied that the physical coupling constant  $g$  at large energy is a fixed point of the renormalization group, i.e.  $\beta(g) = 0$ .
- Asymptotic free theories were not known: moreover they would be considered to be not natural. For example, in QCD there are two scales of masses:  $\Lambda_{QCD}$  and the quark masses. The following scenario would be more elegant: strong interaction theory are a perfectly scaling invariant at high energies where the scaling invariance is broken only by a mass term. However as suggested by Einstein in the preface of his 1916 book *Relativity* we should *adhere to the precept of that brilliant theoretical physicist L. Boltzmann, according to whom matter of elegance should be left to the tailor and to the cobbler*.
- The strong interaction theory was not known at that time. As far as the renormalization group fixed point was supposed to be in the strong coupling regime, perturbative techniques were useless. The only hopes were based on the possibility of using symmetry arguments (like the conformal group) in order to get predictions *interra incognita*.

Let me summarize some of the most important Aurelio's results in this period.

- The construction of a manifestly conformal covariant operator-product expansion <sup>3)</sup>. This paper was the mathematical and physical base of the following papers on the conformal group. The group theoretical aspects were deepened in a subsequent paper on the tensor representations of

conformal algebra and on their contributions to conformally covariant operator product expansion <sup>7)</sup>.

- A crucial problem was the computation of the contribution of terms arising from the covariant operator-product expansion to the conformal four-point function (the so-called conformal blocks). This was done mainly in two papers <sup>9, 10)</sup> where we introduced the shadow operator formalism for operator-product expansion and we applied it to the computation of vacuum expectation values, mainly for the four-point function.
- In a remarkable paper Aurelio studied deeply the conformal algebra in two space-time dimensions and it applies the insight he obtains to the Thirring model, that is a non-trivial interacting model with many interesting features <sup>5)</sup>. This paper is very interesting as far the two-dimensional conformal group is much larger: it is an infinite dimensional Lie group, strongly related to the Virasoro algebra.
- In a short but deep paper Aurelio showed that the constraint of positivity (i.e. the Hilbert space of physical states should have a positive norm) gave strong restrictions on the values of anomalous dimensions. <sup>8)</sup>

The collaboration faded around in 1973-1974 for many reasons. Logistic difficulties were important: Sergio Ferrara moved away to CERN and I moved to Columbia University. However scientific reasons were the most important. Asymptotic freedom for strong interactions was discovered and it was becoming increasingly popular in our community. At the end of the day, the scaling invariant strong interaction theory was free-field theory and it could be understood in a naive way. The only delicate point was the computation of scaling corrections: they can be evaluated in perturbation theory.

The conformal group turned out to be useless in high energy physics and the interest of conformal theories for strong interactions disappeared for some time and it resurrected much later in string theory.

### **3 The conformal bootstrap**

It is well known that second-order phase transitions in two and tree Euclidean dimensions provide non-trivial strongly coupled scaling invariant theories in the

infrared region when the correlation length goes to infinity<sup>2</sup>. Critical exponents (that are measured both in experiments and in numerical simulations) are related to anomalous dimensions.

The computation of the critical exponents was an evident field of application of the conformal group. This led to the proposal by Ferrara, Gatto, and Grillo of the conformal bootstrap idea. The attempt to implement this project was my main motivation to start the very complex computations of [9, 10].

The idea at the basis of the conformal bootstrap is quite simple. Conformal invariant Wilson expansion gives:

$$\langle \phi(x)\phi(y)\phi(z)\phi(t) \rangle = \sum_{\mathcal{O}} \int dw \langle \phi(x)\phi(y)\mathcal{O}(w) \rangle \langle \tilde{\mathcal{O}}(w)\phi(z)\phi(t) \rangle, \quad (1)$$

where the sum is done over the primary conformal fields  $\mathcal{O}$ , i.e. those fields that transform a simple way under the action of the conformal group and  $\tilde{\mathcal{O}}$  is the shadow operator corresponding to the operator  $\mathcal{O}$ . This is essentially a  $s$ -channel decomposition in conformal partial waves.

Each term is not symmetric (exchange  $x$  with  $z$ ). The hope was that imposing this symmetry we should get the dimensions of the operators  $\phi$  and  $\mathcal{O}$ 's. In other terms, we were asking that the sum of  $s$ -channel poles should be equal to the sum of  $t$ -channel poles. Obviously, there are no simple solutions with a few terms. We needed an infinite number of terms and we did not find a simple way to deal with the problem. It was a natural generalization of the duality arguments that lead to the Veneziano formula.

We abandoned the problem because we were stuck. However, as we learned much later, we were on the right track!

We have already seen that the  $D = 2$  conformal group is contained in a much larger group. Each representation of the larger conformal group contains an infinite number of representations of the smaller conformal group (i.e. a Virasoro tower).

In 1984 Belavin, Polyakov and Zamolodchikov [12] computed the  $2D$  equivalent of equation (1) using the invariance under the larger conformal group (the one that was studied in [5]). Only for a discrete set of values, one could get

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<sup>2</sup>In Euclidean field theory the correlation length is the inverse of the inverse of the relativistic mass and it goes to zero at the critical point.

perfect symmetry with only a few terms. This magnificent computation leads to the exact evaluation of the critical exponents in  $D = 2$  for many different models. It provides an invaluable tool for studying two-dimensional physics.

I always regret that we never tried to do the simplest attempt to compute the  $2D$  equivalent of equation (1). I do not know why: maybe the effort to compute one term in  $D = 3$  was so high that we believed that the  $D = 2$  computation with an infinite number of terms was too difficult (ironically the computation was much simpler).

In  $D = 3$  the story was quite different. In 2012 appeared an other magnificent paper <sup>13)</sup>, where the conformal bootstrap program was used to put a strict bound on the value of the critical exponents for the three dimensional Ising model. Further developments of these techniques allow a high precision computation of the critical exponents. The paper is based on clever new ideas: moreover one can consider the contributions of more than 100 terms: I firmly believe that such a computation could not be done with the computers of 45 years ago. A side effect of these computations is that the citations of Aurelio's old work on the conformal invariance nearly doubled in these recent years.

#### 4 Looking around

In 1974 Aurelio started to look around for other fields and he started to study new topics.

He started to perform some computations that were useful to interpret the experimental data coming from the colliding beam experiments: for example, he analyzed the radiative asymmetry in  $e^+e^- \rightarrow \mu^+\mu^-$  near a narrow resonance in the case where the beams were polarized <sup>14)</sup>.

He moved later to study other problems of astrophysical relevance as the production of cosmological black holes in the framework of grand unified theories <sup>15)</sup> and the production of Fermions during monopole-antimonopole annihilation <sup>16)</sup>.

At the end of the eighties, he started a very interesting program of investigations of lattice quantum electrodynamics, trying to arrive to clear-cut results and to a deep understanding of the problems: these achievements were possible given the simplicity of QED with respect to QCD. Some of the most interesting results were:

- The study of the stability of quantum vortices on the lattice in the case of the quantum electrodynamics ( $U(1)$  gauge group) in the presence of a charged Higgs-like field in the phase where the Higgs field were breaking the  $U(1)$  symmetry, as in the standard model <sup>17)</sup>.
- In the framework of compact pure gauge lattice QED a careful study of the phase transition that separates the confined phase from the unconfined phases. The delicate point of the order of the phase transition was successfully addressed <sup>18)</sup>.
- In long series of papers Aurelio addressed the problem of introducing dynamical Fermions in lattice gauge theory and analyzed many of theoretical aspects mainly in the case of QED. In one of these very interesting papers, he introduced a new method for simulating lattice gauge theories with dynamical Fermions based on microcanonical Fermionic average <sup>19)</sup>. Many of his later papers are based on the results he obtained with this method.

## 5 Transforming himself into a refined experimentalist

A complete change of interests happens in Aurelio when he starts the MACRO adventure and he became a refined experimentalist. Everything started with the proposal of MACRO: the title was *Proposal for a large area detector dedicated to monopole search, astrophysics, and cosmic ray physics at the Gran Sasso Lab*: it was signed by 73 physicists, among them Aurelio, who used his deep theoretical knowledge of many areas of physics to give a great contribution to the planning of future experiments, especially in understanding the physical relevance of the future results.

MACRO was located underground Gran Sasso Laboratories and it started the data taking with a part of the apparatus in 1989; it has completed in early 1995 and was running in its final configuration until the end of 2000. It was large of the order of  $10^4 m^3$  : it produced 50 scientific papers that were signed also by Aurelio, who was a crucial part of the collaboration.

It would be difficult for me to summarize MACRO's results: the interested reader can look at the very nice review <sup>20)</sup>. I will only mention a selection some of Aurelio's papers just to stress the diversity of his activities:



- An analysis of the performance of the MACRO streamer tube system that was used in the search for magnetic monopoles <sup>21)</sup>. Magnetic monopoles were indeed one of his recurrent theoretical interest. Moreover, the magnetic monopoles search was a very important goal of MACRO.
- A theoretical analysis where the high-energy neutrino emission from binary X-ray sources was estimated using the data for very high energy  $\gamma$  rays. The implications of these findings for the future MACRO experiments were clearly spelled out <sup>22)</sup>.
- An analysis of the multiple muons event in MACRO: it was possible to extract from these data very interesting information on the ultrahigh-energy primary-cosmic-ray composition <sup>23)</sup>: the data exhibited a preference towards the light composition model.
- In another paper, he analyzes the atmospheric neutrino interactions using the data on the induced upgoing muon flux <sup>24)</sup>.

With the turn of the millennium, he started a new adventure: the Pierre Auger Observatory. The Pierre Auger Observatory has some similarities with MACRO, however, the scaling of the involved energies of the primaries and the physical extensions of the experiments are quite different: it was a great leap forward.

Aurelio was a crucial component of the team from the beginning in of the project: he wrote a paper on the properties and performance of the prototype instrument for the Pierre Auger Observatory <sup>25)</sup>: this study was a crucial analysis that played an important role for the success of the project.

He was involved in technical papers concerning the crucial study of the composition of the atmosphere above the experimental area: as an example, I would like to recall the two papers on the study of the performance of the LIDAR system <sup>26)</sup> and of the related measurement of aerosols <sup>27)</sup>.

He also gave crucial contributions to the much more interesting papers where the main experimental results were presented. I will recall only two the so many papers

- The measurement of the energy spectrum of cosmic rays above  $10^{18}$  eV. This was one of the main motivation for the construction of the Pierre

Auger Observatory <sup>28)</sup>: the results are really impressive and they have very deep theoretical implications.

- The high-resolution studies of the anisotropy of ultrahigh-energy cosmic rays. These studies give us information that is crucial to identify the source of these ultrahigh-energy particles <sup>29)</sup> and to understand the physical mechanism that produces them.

## 6 Conclusions

Aurelio suddenly died on the February of 2017: we all lost a wonderful colleague and a dear friend. At the moment that this paper is written, it is was one and half years ago. It is difficult to convince oneself that we shall not see again, that we will not able to follow his deep advice that he was able to give in his characteristic outspoken way. The only relief may come from the consideration that Aurelio managed to dedicate his life to his two great passions, to music (in this helped by his beloved daughter Stefania) and to physics. I remember that when we were both in Frascati he used to tell to me "Being a physicist is a hard job, but it's always better than working".

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