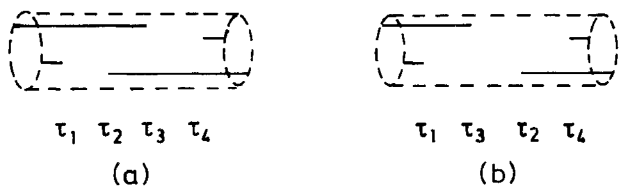

Fig. 2 Non-planar loop and the Pomeron

the non-planar loop (a) and the Pomeron (b); the strings have been drawn as arcs of circles moving along a cylinder. The topological equivalence between the non-planar loop and the Pomeron diagram appears in a new light in the string picture. The fact that the Pomeron pole factorizes with the Virasoro-Shapiro spectrum is now manifest.

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QUANTUM GRAVITY*

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Our report is divided into three distinct sections; the first deals with tree diagrams, and is devoted to a derivation of general relativity as the unique generator (at low frequency) of the tree expansion, given only the observed properties of gravitational forces and the principles of S-matrix theory. The second section deals with the ultraviolet behaviour of general relativity considered as a quantum field theory. In the last section we discuss some prospects for improving the catastrophic situation which arises there.

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I. Trees

We summarize here the work of D. Boulware and the author (submitted to Ann. Phys.) which carries out a program first proposed by Weinberg (1964) and by Feynman (1962). There exist static, macroscopic, attractive, long range forces; by S-matrix theory these must be mediated by exchange of physical (non-ghost) particles. A well-known argument then shows that the forces cannot be due to quanta of spin other than 2 (gravitons). Lorentz invariance requires these gravitons to couple to a conserved symmetric matrix element, $T_{\mu\nu}(p,p')$. As a low energy theorem, $T_{\mu\nu}$ is determined uniquely to first order in emitted graviton momentum. It has a universal strength and form for all systems (which is known as the equivalence principle) and is thus derived to be the usual stress tensor of the source in question. More important for us is that gravitons, if emitted by any system, must also self-couple through a three-graviton vertex V_3. Its structure is determined by the Ward identities derived from Lorentz invariance of the graviton polarisation tensor, at least to quadratic order in the momenta (which we stick to throughout). There is an infinite series of higher-point vertices, whose sum also satisfies a (more general) Ward identity. Many solutions of the identity exist but all yield the same amplitudes. We need only exhibit one solution, which is the Einstein action: all tree graphs are generated by the Einstein functional $I = \int\sqrt{g}R$, plus the usual minimally coupled matter action. But since the classical limit of a theory is determined by its tree generator, this shows that any theory of gravitation which respects the stated principles will necessarily have general relativity as its classical limit. The above quantum derivation is formally similar to, but considerably stronger than, the usual classical arguments based on self-coupling or gauge invariance.

II. Closed Loops and Nonrenormalizability

Let us now consider general relativity as a local quantum field theory and examine its ultraviolet behaviour, at the one-loop level. What sort of counter-terms will be required, i.e. is it renormalizable? The recent powerful methods of covariant quantization in nonabelian gauge theory provided the incentive to attack the full quantized gravity-matter system. The pioneer work here is by 't Hooft and Veltman (Ann. Inst. H. Poincare 1974), who calculated the one-loop devergences of pure gravity and of gravity plus scalar field. The form of the counter terms in pure gravity is easily established without calculation, using the background field method and dimensional regularization. The counter-Lagrangian ΔL must be invariant in the background metric and have dimension 4; it is then necessarily a linear combination of the three quadratic curvature invariants:

$$\Delta L = (1/\epsilon)\left[\alpha R^2_{\mu\nu\alpha\beta} + \beta R^2_{\mu\nu} + \gamma R^2\right] \sqrt{g} \qquad (1)$$

For renormalizability ΔL must either vanish on mass shell ($R_{\mu\nu} = 0 = R$), or be proportional to L itself. But although $R_{\mu\nu\alpha\beta}$ does not vanish, a peculiar identity in 4 dimensions says its square is a linear combination of the other two invariants, and ΔL is "accidentally" zero at one loop. As soon as a scalar field is coupled, however, ΔL no longer vanishes on the scalar-graviton mass shell.

In the hope that more physical matter systems do not share this fate, P van Nieuwenhuizen and I (Phys. Rev. D15 , July 1974) investigated the coupling to photons and to fermions, van Nieuwenhuizen, Tsao and I (Phys. Letters, June 1974) calculated coupled Einstein-Yang-Mills loops. We concluded that general relativity is nonrenormalizable when coupled to spin 0, ½ or 1 (it would require an infinity of miraculous cancellations for higher order counter terms to stop at a finite member). This disaster is linked to the presence of the dimensional coupling constant κ. The Brans-Dicke theory is also nonrenormalizable (being equivalent to the Einstein-Scalar system).

III Prospects

It is of course possible that general relativity is renormalizable (or even finite) when treated nonperturbatively, but this is difficult to check, as is the possibility that some magic set of coupled matter fields will cancel each other's infinities when coupled to gravitation. The renormalizable model, first suggested by De Witt and Utiyama (1962) and also advocated by Weinberg at this Conference (a sum of Einstein and Weyl ($\alpha R_{\mu\nu}^2 + \beta R^2$) terms), unfortunately, has propagator ghost difficulties. Perhaps the supergauge theories, either as matter sources, or in the spin 2 plus 3/2 gauge fields they generate (see Zumino's lecture) will behave better. Whatever the correct road, it is essential to resolve this conflict between gravitation and quantum theory; the nonrenormalizability of graviton-matter interactions pollutes, in principle, all calculations of the properties of matter.

WBK METHOD IN FIELD THEORY

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There have been many reports at this conference on classical solutions of interacting field theories, or in general of systems with an infinite number of degrees of freedom, which are believed to be relevant to particle physics. This is a short report on the general method to deal with such systems in a quantum mechanical fashion. Details and examples are presented in ref. 1.

The idea is to extract as much information as possible from the classical system, to obtain results beyond perturbation theory. This is typically fruitful for bound state problems.

The problem is to generalize in some sense the Bohr-Sommerfeld quantization rules to systems with many degrees of freedom in the non separable case. It turns out that the difficult problem is to go from one to two degrees of freedom. The analysis can then be extended fairly easily to field theories.

Typically, one knows one or several families of classical solutions to the interacting field theory which have finite energy and are periodic in time. The period is a continuous classical variable, together with the energy, and other constants of motion (if any) for which one has to find quantization conditions.

The general method for WKB quantization of non separable systems is explained in ref. 2 and 3. It is illustrated by the following considerations: For two separable degrees of freedom, where the hamiltonian is

$$H = \tfrac{1}{2} P_1^2 + \tfrac{1}{2} P_2^2 + V(q_1) + V(q_2),$$

the energy is separately conserved in each mode:

$$E_1 = \tfrac{1}{2} P_1^2 + V(q_1) = \text{constant}$$

$$E_2 = \tfrac{1}{2} P_2^2 + V(q_2) = \text{constant}$$

These two equations define a manifold in classical (p_i, q_i) phase space, which is called an invariant torus. The important point is that such manifolds also exist in the non separable case (see ref. 4). Maslov's quantization conditions are then

$$\int_{C_i} p\, dq = 2\pi\hbar\, (n_i + \ell_i) + O(\hbar^2),\ n_i \text{ integer},$$