UNSTABLE PARTICLES IN GENERAL FIELD THEORY (†)

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We consider here (*) the problem of the definition of unstable particles in quantum field theory. The difficulty met with in framing such a definition lies in the fact that the asymptotic ' in ' and ' out ' states containing such particles do not exist and so the usual methods of field theory based on these asymptotic states do not apply. We follow here the suggestion of Peierls [1] that unstable particles may be associated with poles appearing in the unphysical sheets of the analytic continuation of the particle propagator in momentum space. The work of Lévy [2] and others shows that the Lee model provides a satisfactory demonstration of this suggestion. We attempt to extend the methods developed by Lévy to a more general field theory. However, it is not yet possible to give a satisfactory definition of the local field to be associated with an unstable particle in a general field theory satisfying the usual axioms of causality, Lorentz invariance and an asymptotic condition. In consequence, we consider mainly the interpretation of poles appearing in the scattering amplitudes and then show that the same poles should occur in the propagator if it exists. We do not consider here the decay properties of unstable particles as a function of time but only how they manifest themselves as resonances in scattering or production processes.

We consider first the scattering amplitude for two neutral scalar particles of masses m and μ. We assume that the partial wave amplitude $M_l(W^2)$ for total energy W^2 and angular momentum l is analytic in the cut W^2 plane, with branch points at $W^2=(m+n\mu)^2$ for $n=1, 2, \ldots$; we will not consider explicitly the cuts in the half plane Re $W^2<0$. Similar results also follow for a suitable smaller region of analyticity. The value M_l' of M_l on its first unphysical sheet is $M_l' = M_l+2iA_{l_2}$ where A_{l_2} is the continuation of the contribution to the absorptive part of the amplitude arising from two particle intermediate states. For real $W^2 \geq (m+\mu)^2$ unitarity gives $A_{l_2} = M_l^* M_l$ and we continue this relation into the lower half of the cut energy plane as $A_{l_2} = M_l' M_l$. Hence $M_l' = M_l/(1-2iM_l)$, so that the only singularities of M_l' other than those coming from M_l will be isolated poles. We conjecture that such a pole corresponds to an ' unstable particle ' or ' resonance ' state of spin l. In terms of a phase shift δ_l we have the expected formula

$$M_l' = M_l e^{2i\delta_l}\,.$$

Our interpretation of the poles in M_l' is supported by the influence the poles have on the physical scattering amplitude. For, we may deform the contour of the dispersion relation integral for $M_l(W^2)$ to obtain

$$M_l(W^2) = \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{A_l^{(r)}(x)dx}{(x-W^2)}+\frac{1}{\pi}\int_{-\infty}^{(m+\mu)^2}\frac{A_{l_2}(x)dx}{(x-W^2)}+ \\ +\frac{1}{\pi}\int_{(m+\mu)^2}^{-\infty}\frac{A_{l_2}^{(c)}(x)dx}{(x-W^2)}+2i\sum_i\frac{R(z_i)}{(z_i-W^2)} \qquad (1)$$

where $A_l^{(r)} = A_l - A_{l_2}$, $A_{l_2}^{(c)}$ is the continuation of A_{l_2} from the cut $(m+\mu)^2$ to ∞ and $R(z_i)$ is the residue of $A_{l_2}^{(c)}$ at the pole z_i. For a pole z_i close to the real axis the most rapidly varying term in M_l for W^2 near Re z_i will be from the last term in Eq. (1), and will

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give a resonance-type contribution to scattering cross sections.

If we suppose that the Mandelstam representation holds for the complete amplitude on the physical sheet, we may obtain the region of analyticity of $M(W^2, \cos\theta)$ in $\cos\theta$, where θ is the centre of mass scattering angle, for fixed W^2 on the first unphysical sheet. If branch points in the $\cos\theta$ plane are at $\pm\alpha$ for W^2 on the physical sheet, then the corresponding branch points in $\cos\theta$ for W^2 on the first unphysical sheet are at the points $\alpha_n (n = 0, 1, 2, \ldots \infty)$ with $\alpha_{n+1} = \pm(\alpha\alpha_n + \sqrt{(\alpha^2-1)(\alpha_n^2-1)})$, $\alpha_0 = \alpha$. The Mandelstam representation is thus not valid for W^2 on the first unphysical sheet.

If we assume that a local field $\phi_\mu(x)$ and current $J_\mu(x)$ can be constructed to describe an unstable particle of spin l, then with suitable conditions on the mass spectrum the vertex function $\Gamma_\mu(W^2) = \langle pk|J_\mu(0)|0\rangle$ is analytic in the cut W^2 plane cut from $(m+\mu)^2$ to ∞ ($|pk\rangle$ is a state containing) of one particle of mass m and one of mass μ, $(p+k)^2 = W^2$). The continuation Γ'_μ of Γ_μ to the first unphysical sheet is $\Gamma'_\mu = \Gamma_\mu + 2iG_\mu$ where $G_\mu = M_l^* \Gamma_\mu$ may be continued into the lower half energy plane to give $G_\mu = M_l \Gamma'_\mu$ so $\Gamma'_\mu = \Gamma_\mu/(1-2iM_l)$. Thus Γ'_μ has the same singularities in the first unphysical sheet as the two particle scattering amplitude. The two particle propagator has the same property, since the two particle contribution to the Lehmann weight function $\rho_2(W^2)$ can be expressed as a product of two functions which on continuation to the lower half W^2 plane pick up the extra factor $(1-2iM_l)^{-1}$.

The problem of continuing the two particle scattering amplitude into higher sheets differs from the problem of the first unphysical sheet in that unitarity involves higher order amplitudes. So we go to the general problem of the continuation of all possible amplitudes into physical sheets. We assume at least analyticity of all amplitudes in the cut physical plane of the centre of mass energy variable W^2. The value $M^p(q'_1, \ldots, q'_n|q_1, \ldots, q_m)$ of the amplitude for m in and n out particles on the p^{th} unphysical sheet is

$$M^p(q'_1,\ldots,q'_n|q_1,\ldots,q_m) = M^{p-1}(q'_1,\ldots,q'_n|q_1,\ldots,q_m) + \\ + 2iA_{p+n}(q'_1,\ldots,q'_n|q_1,\ldots,q_m)$$

where

$$A_{p+1} = \int d^3k_1 \ldots d^3k_{p+1}\delta^4(\Sigma q_i - \Sigma k_j) \times \\ \times M(q'_1,\ldots,q'_n|k_1,\ldots,k_{p+1})M^p(k_1,\ldots,k_{p+1}|q_1,\ldots,q_m)$$

A_{p+1} could be immediately continued into the lower half plane of $W^2 = (\sum_1^m q_i)^2$ if we knew the continuation $M^p(k_1,\ldots,k_{p+1}|q_1,\ldots,q_m)$ since $M^*(k_1,\ldots,k_{p+1}|q'_1,\ldots,q^*_n)$ has an immediate continuation from below the real W^2 axis. The continuation of M^p can be solved by induction on p since if we write

$$M^p(k_1,\ldots,k_{p+1}|q_1,\ldots,q_m) = \\ = M^{p-1}(k_1,\ldots,k_{p+1}|q_1,\ldots,q_m) + \\ + 2i\int d^3r_1,\ldots,d^3r_{p+1}\delta(\Sigma q_i - \Sigma r_j) \times \\ \times M^*(r_1,\ldots,r_{p+1}|k_1,\ldots,k_{p+1})M^p(r_1\ldots r_{p+1}|q_1\ldots q_m)$$

this equation can be reduced to a Fredholm integral equation for M^p, regarded as a function of the possible Lorentz invariants, of the form

$$M^p(W^2abc) = M^{p-1}(W^2abc) + \\ + 2i\int da'db' K(W^2, a, b, a'b')M^p(W^2a'b'c)$$

where the kernel K has suitable continuity properties in a, b, a', b', and is analytic in W^2 in a cut plane. (a, b, c each denotes a set of invariants, suitably chosen.) Then

$$M^p(W^2abc) = M^{p-1}(W^2abc) + \\ + \Delta^{-1}(W^2)\int da'db'D(W^2aba'b')M^{p-1}(W^2a'b'c)$$

where Δ, D are the Fredholm determinant and first minor respectively. From induction on the $(p-1)^{th}$ sheet D is analytic in the cut W^2 plane, so M^p is analytic there, except for possible poles in Δ^{-1} arising from zeros of Δ. We interpret these poles as ' unstable particle ' or ' resonance ' intermediate states.

LIST OF REFERENCES AND NOTES

1. Peierls, R. E. Proc. Glasgow Conf. on Nucl. and Meson Phys., p. 296 (Pergamon Press, Inc., London, 1954).
2. Lévy, M. Nuovo Cimento **13**, p. 115 (1959). This paper contains further references.