

Gravitational waves versus cosmic strings

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Abstract. The equation which governs the temporal evolution of a gravitational wave (GW) in curved space-time can be treated as the Schrödinger equation for a particle moving in the presence of an *effective potential*. When GWs propagate in an expanding Universe with *constant* effective potential, there is a *critical value* (k_c) of the comoving wave-number which discriminates the metric perturbations into *oscillating* ($k > k_c$) and *non-oscillating* ($k < k_c$) modes. The effective potential is reduced to a non-vanishing constant in a cosmological model which is driven by a *two-component fluid*, consisting of *radiation* (dominant) and *cosmic strings* (subdominant). However, the cosmological evolution (gradually) results in the *scaling* of any long-cosmic-string network and, therefore, after some time ($\Delta\tau$) the Universe enters in the *pure-radiation* epoch. The evolution of the non-oscillatory GW modes during $\Delta\tau$, results in the *distortion* of the *low-frequency part* of the *stochastic GW power-spectrum*, which, therefore, departs from *scale invariance* (anticipated in the pure-radiation case). As regards the corresponding *high-frequency part* (which is determined by the evolution of the oscillating modes), we find that the presence of cosmic strings gives rise to the quantum-gravitational creation of *gravitons*, leading to the *amplification* of the GW signal by (almost) two orders of magnitude.

1. Introduction

The so-called *cosmological gravitational waves* (CGWs) represent small-scale perturbations to the Universe metric tensor [1]. Since gravity is the weakest of the four known forces, these metric corrections decouple from the rest of the Universe at very early times, presumably at the Planck epoch [2]. Their subsequent propagation is governed by the space-time curvature, encapsulating in the field equations the inherent coupling between relic GWs and the Universe matter-content; the latter being responsible for the background gravitational field [3].

In this context, we consider the interaction between CGWs and *cosmic strings*. They are one-dimensional objects that can be formed as linear defects at a symmetry-breaking phase transition [4], [5]. If they exist, they may help us to explain some of the large-scale structures seen in the Universe today, such as gravitational lenses [6]. They may also serve as *seeds* for density perturbations [7], [8], as well as potential sources of relic gravitational radiation [9], [10].

In this article we present some recent results concerning another possibility: The presence of cosmic strings in a radiation-dominated Universe being responsible for the constancy of the *effective potential* which drives the temporal evolution of a CGW in curved space-time.

In particular, we shall demonstrate that, a (non-vanishing) constant effective potential is associated to a *critical value* (k_c) of the comoving GW wave-number, which discriminates the metric perturbations into *oscillating* ($k > k_c$) and *non-oscillating* ($k < k_c$) modes, something

that is reflected in the CGWs' *power-spectrum* [11]. In this context, the presence of cosmic strings at the early stages of the Universal evolution will modify the expected *profile* of any *relic-GW signal* that can be observed today. In what follows, we shall attempt to illustrate how, in a realistic setting.

We begin with the fact that the cosmological evolution (gradually) results in the *scaling* of any long-cosmic-string network [12] and, therefore, the Universe, if ever experienced a short *radiation-plus-strings stage*, eventually, will enter in the *pure-radiation* epoch. Accordingly, following the evolution of the non-oscillatory GW modes during the radiation-plus-strings stage, we find that the *low-frequency part* of a *relic-GW signal* will depart from *scale-invariance*, in contrast to what is anticipated in the pure-radiation case [13]. On the other hand, as regards the oscillatory GW modes, their evolution from an *early* radiation epoch to a *late* radiation era through the radiation-plus-strings stage leads to the quantum-gravitational creation of *gravitons*, thus resulting in the *amplification* of the *high-frequency part* of the relic-GW signal by (almost) two orders of magnitude.

2. CGWs in a Friedmann model

In the system of units where c (the velocity of light) equals to unity, a weak CGW ($|h_{ij}| \ll 1$), propagating in a spatially flat Friedmann - Robertson - Walker (FRW) cosmological model (as *inflation* advocates), is defined by [9]

$$ds^2 = dt^2 - R^2(t)(\delta_{ij} + h_{ij})dx^i dx^j, \quad (1)$$

and satisfies the Klein-Gordon equation [14]

$$h_{ij};\alpha = 0, \quad (2)$$

In Eqs (1) and (2), Greek indices refer to the four-dimensional space-time (in connection, Latin indices refer to the three-dimensional spatial section), the semicolon denotes covariant derivative, δ_{ij} is the Kronecker symbol and the dimensionless scale factor $R(t)$ is a solution to the Friedmann equations with matter-content in the form of a *perfect fluid*. In terms of the so-called *conformal-time* coordinate, $\tau = \int dt/R(t)$, the solution to Eq (2) is a *linear superposition of plane-wave modes*

$$h_{ij}(\tau, x^r) = \phi_k(\tau) \varepsilon_{ij} e^{ik_r x^r}, \quad (3)$$

where, $\phi_k(\tau)$ is a (complex) function of time and ε_{ij} is the polarization tensor, depending only on the direction of the *comoving* wave-vector k_r . For a fixed wave-number $k^2 = \sum k_r^2$, the time-dependent part of the corresponding GW mode satisfies the second-order differential equation (e.g. see [15])

$$\phi_k'' + 2\frac{R'}{R} \phi_k' + k^2 \phi_k = 0, \quad (4)$$

where, the prime denotes differentiation with respect to τ . Under the further decomposition

$$h_k(\tau) = \phi_k(\tau) R(\tau), \quad (5)$$

the equation which governs the temporal evolution of a CGW in a FRW model, is written in the form

$$h_k'' + (k^2 - \frac{R''}{R}) h_k = 0. \quad (6)$$

Eq (6) can be treated as the *Schrödinger equation* for a particle moving in the presence of the *effective potential*

$$V_{eff} = \frac{R''}{R}. \quad (7)$$

To solve this equation, we need an evolution formula for the cosmological model under consideration.

In terms of the conformal time, the spatially flat FRW model is a solution to the Friedmann equation

$$\left(\frac{R'}{R^2}\right)^2 = \frac{8\pi G}{3} \rho(\tau), \quad (8)$$

(where, G is Newton's constant) with matter-content in the form of a perfect fluid, $T_{\mu\nu} = \text{diag}(\rho, -p, -p, -p)$, which obeys the conservation law

$$\rho' + 3\frac{R'}{R}(\rho + p) = 0, \quad (9)$$

and the equation of state

$$p = \left(\frac{m}{3} - 1\right) \rho, \quad (10)$$

where, $\rho(\tau)$ and $p(\tau)$ represent the mass-density and the pressure, respectively.

The linear equation of state (10) covers most of the matter-components considered to drive the evolution of the Universe, such as quantum vacuum ($m = 0$), a network of domain walls ($m = 1$), a gas of cosmic strings ($m = 2$), dust ($m = 3$), radiation ($m = 4$) and Zel'dovich ultra-stiff matter ($m = 6$). For each component, the continuity equation (9) yields

$$\rho = \frac{M_m}{R^m}, \quad (11)$$

where, M_m is an integration constant. Provided that the various components *do not interact* with each other, a mixture of them obeys [16]

$$\rho = \sum_m \frac{M_m}{R^m}, \quad (12)$$

where, now, Eq (9) holds for each matter-constituent separately.

3. Constancy of the effective potential

There is a case of particular interest involved in the temporal evolution of a primordial GW, in which the effective potential is *constant* for every τ , namely

$$\frac{R''}{R} = \text{const} = \frac{8\pi G}{3} M, \quad (13)$$

where, M is a non-negative constant of units gr/cm^3 . In this case, Eq (6) is written in the form

$$h_k'' + \left(k^2 - \frac{8\pi G}{3} M\right) h_k = 0. \quad (14)$$

According to Eq (14), a *critical value* of the comoving wave-number arises, through the condition

$$k > k_c = \sqrt{\frac{8\pi G}{3} M}. \quad (15)$$

This critical value discriminates the primordial GWs in modes, with $k > k_c$, which *oscillate* for every τ ,

$$h_{k>k_c}(\tau) \sim e^{i\sqrt{k^2 - k_c^2} \tau}, \quad (16)$$

and modes, with $k < k_c$, which *grow exponentially* for every τ ,

$$h_{k < k_c}(\tau) \sim e^{\sqrt{k_c^2 - k^2} \tau}, \quad (17)$$

(the corresponding *exponentially-decaying* solutions are neglected).

Now, the question arises on whether there exists a spatially flat FRW cosmological model in which the effective potential is constant. To answer this question, we set $\mathcal{H} = \frac{R'}{R}$. Accordingly, V_{eff} is written in the form

$$\frac{R''}{R} = \mathcal{H}' + \mathcal{H}^2. \quad (18)$$

Upon consideration of Eqs (8) and (18), Eq (13) results in the ordinary differential equation

$$\rho' + 4\rho \frac{R'}{R} = 2M \frac{R'}{R^3}, \quad (19)$$

which admits the solution

$$\rho(\tau) = \frac{M}{R^2} + \frac{C}{R^4}, \quad (20)$$

where, C is an integration constant of units gr/cm^3 . For $C = 0$, Eq (20) results in $\rho \sim R^{-2}$, which, by virtue of Eqs (10) and (11), leads to $p = -\frac{1}{3}\rho$. This case, now known as the *string-dominated Universe* [17], was considered by Ford and Parker [18] in their effort for a natural *renormalization* of the energy-momentum tensor of the *gravitational-perturbation field*. Upon consideration of Eqs (11) and (12), we distinguish two cases [in Eq (20)] with respect to M : (i) $M = 0$ and (ii) $M \neq 0$. We consider each one of them separately.

3.1. CGWs in a radiation-dominated Universe

For $M = 0$, in correspondence to Eq (11), Eq (20) reads

$$\rho(\tau) = \frac{M_4}{R^4}. \quad (21)$$

This is the matter-content of a cosmological model filled with relativistic particles - radiation. Now, the effective potential vanishes and Eq (14) is written in the form

$$h_k'' + k^2 h_k = 0, \quad (22)$$

i.e. it is a simple *harmonic-oscillator* equation. In this case, a *conformally-invariant field* (the metric perturbation) propagates in a *conformally-flat* space-time and there is no particle production [19]. Therefore, the choice of a solution to Eq (22) corresponds to the choice of an *initial (or final) quantum state* for the gravitational-perturbation field. Eq (22) admits the solution

$$h_k^{in}(\tau) = \sqrt{\frac{2}{\pi k}} e^{-ik\tau}, \quad (23)$$

and the gravitational-perturbation field is in the well-defined *adiabatic-vacuum* state [20].

3.2. CGWs in the presence of cosmic strings

For $M \neq 0$, in correspondence to Eq (12), Eq (20) reads

$$\rho(\tau) = \frac{M_2}{R^2} + \frac{M_4}{R^4}. \quad (24)$$

This type of matter-content corresponds to a radiation model *contaminated* by cosmic strings, in which the two constituents (relativistic particles and linear defects) do not interact with each other, as it should be the case shortly after the *dynamic friction* between them [4] became unimportant [5]. It is worth noting that, the constant M appearing in Eq (13) is related to the initial amount of the linear defects in the mixture (M_2). Now, Eq (14) results in

$$h_k'' + (k^2 - k_c^2) h_k = 0, \quad (25)$$

and, as regards the oscillating ($k > k_c$) modes, it can be put in the more convenient form

$$h_k'' + \gamma^2 k^2 h_k = 0, \quad (26)$$

where, we have introduced the parameter

$$\gamma = \sqrt{1 - \left(\frac{k_c}{k}\right)^2}, \quad (27)$$

which measures the departure from the *pure-radiation* case. For every $k_c < k < \infty$ ($0 < \gamma < 1$), the general solution to Eq (26) is written in the form

$$h_k(\tau) = \sqrt{\frac{2}{\pi\gamma k}} \left[c_1 e^{-i\gamma k\tau} + c_2 e^{+i\gamma k\tau} \right], \quad (28)$$

where, c_1 and c_2 are complex numbers, satisfying the Wronskian condition

$$|c_1|^2 - |c_2|^2 = 1. \quad (29)$$

We conclude that, in the presence of cosmic strings the effective potential is reduced to a non-vanishing constant and, therefore, oscillation of the metric perturbations is possible only if their comoving wave-number is larger than a critical value, depending on the mass-density of the linear defects

$$k > k_c = \sqrt{\frac{8\pi G}{3}} M_2. \quad (30)$$

In other words, a cosmic-string network discriminates the primordial GWs predicted by inflation into oscillating and non-oscillating modes, something that should be reflected in the power-spectrum of the stochastic GW background. We shall attempt to illustrate how, in a realistic setting.

4. A tale of cosmic strings

The presence of cosmic strings in the Universe is purely a question of *topology* [21]. In particular, after inflation (and reheating) the Universe enters in an *early-radiation* (ER) epoch [22], during which the temperature of the Universe drops monotonically ($T \sim R^{-1}$). This cooling process may have resulted in the breaking of a fundamental U(1) local gauge symmetry, which, in turn, led to the formation of *linear topological defects* (for a detailed analysis see [4] and/or [5]). Within this era, the cosmological scale factor behaves as

$$R_{ER}(t) = \left(\frac{t}{t_{cr}}\right)^{1/2} \Rightarrow R_{ER}(\tau) = \frac{\tau}{\tau_{cr}}, \quad (31)$$

where $\tau_{cr} = 2t_{cr}$ marks the time at which the cosmic strings are formed and we have normalized $R(t_{cr})$ to unity.

By the time these linear defects are formed ($t_{cr} \sim 10^{-31}$ sec for GUT-scale strings), they are moving in a very dense environment and, hence, their motion is heavily damped due to string-particle scattering [23] - [25]. This *friction* becomes subdominant to expansion damping [23] at

$$t_* = \frac{1}{G\mu} t_{cr} \Rightarrow \tau_* = \frac{1}{\sqrt{G\mu}} \tau_{cr}, \quad (32)$$

where μ is the mass per unit-length of the linear defect. For $\tau \geq \tau_*$, the motion of the long cosmic strings can be considered essentially independent of anything else in the Universe and soon they acquire relativistic velocities [4]. Now, the gas of cosmic strings *does not interact* with the fluid of relativistic particles and, hence, Eq (24) holds. Therefore, we may consider that, for $\tau \geq \tau_*$, the evolution of the curved space-time is driven by a two-component fluid, consisting of radiation (dominant) and cosmic strings (subdominant). Accordingly, τ_* marks the beginning of a *radiation-plus-strings* stage. During this stage, the Friedmann equation (8) admits the solution

$$R(\tau) = \sqrt{\frac{M_4}{M_2}} \sinh \sqrt{\frac{8\pi G}{3} M_2} \tau. \quad (33)$$

Nevertheless, the scale factor (33) can drive the Universe expansion only for a short period of time after τ_* , since cosmic strings should (at any time) be a small proportion of the Universe energy-content. This means that the equation of state (24) has validity only for a limited time-period, otherwise, cosmic strings would eventually dominate the energy-density (see [17]).

In fact, the radiation-plus-strings stage (if ever existed) does not last very long. Numerical simulations of the early 90's [26] - [29], as well as their present-time counterparts [30] - [32], suggest that, after the friction becomes unimportant, the production of loops smaller than the Hubble radius gradually results in the *scaling* of the long-string network. In other words, the linear defects form a *self-similar configuration*, the density of which, eventually, behaves as R^{-4} . In this way, at some (physical) time $t_{sc} > t_*$ the Universe re-enters in the (*late*) radiation (LR) era

$$R_{LR}(t) = R_{sc} \left(\frac{t}{t_{sc}}\right)^{1/2} \Rightarrow R_{LR}(\tau) = R_{sc} \frac{\tau}{\tau_{sc}}, \quad (34)$$

before it can become string-dominated. The duration ($\Delta\tau = \tau_{sc} - \tau_*$) of the radiation-plus-strings stage is quite uncertain, mostly due to the fact that numerical simulations (which revealed the scaling of the long-strings network) can be run for relatively limited times. For example, the earliest treatments [28], [29] suggested that $t_{sc} \simeq 30 t_*$ ($\tau_{sc} \simeq 5.5 \tau_*$), while, the most recent ones [31] raise this value to $t_{sc} \simeq 300 t_*$ ($\tau_{sc} \simeq 17 \tau_*$).

5. Implications on the propagation of metric perturbations

5.1. Evolution of the non-oscillatory modes

5.1.1. *GW modes outside of the horizon:* CGWs are produced by quantum fluctuations during inflation (e.g. see [13]). Some of them escape from the visible Universe, once their reduced physical wavelength [$\lambda_{ph} = \frac{\lambda}{2\pi} R(\tau)$] becomes larger than the (constant) inflationary horizon [$\ell_H = H_{dS}^{-1}$, H_{dS} being the Hubble parameter of the de Sitter space]. Eventually, every CGW with $k \leq k_{max} = H_{dS} R_{dS}$ is *exiled* from the Hubble sphere and *freezes out*, acquiring a constant amplitude [33], [34]

$$\alpha_k^2 = \left[\frac{h_k(\tau)}{R(\tau)} \right]^2 = \frac{16}{\pi} \left(\frac{H_{dS}}{m_{Pl}} \right)^2 k^{-3}, \quad (35)$$

where $m_{Pl} = G^{-1/2}$ is the Planck mass. After inflation, i.e. within the subsequent radiation epoch, analytic solutions for $\alpha_k(\tau)$ can be expressed in terms of the Bessel function $J_{\frac{1}{2}}(k\tau)$ [11]

$$\alpha_k(\tau) \sim \alpha_k \frac{\sqrt{\tau}}{R(\tau)} J_{\frac{1}{2}}(k\tau) \sim \alpha_k \frac{\sin k\tau}{k\tau}. \quad (36)$$

Accordingly, when $k\tau \ll 1$, the perturbation's amplitude evolves slowly and is approximately constant. Once $k\tau \approx 1$, the amplitude decays away rapidly before entering an oscillatory phase with slowly decreasing amplitude, when $k\tau \gg 1$. Physically, this corresponds to a mode that is (almost) frozen beyond the horizon, until its physical wavelength becomes comparable to the Hubble radius, at which point it enters in the visible Universe (e.g. see [35]).

In other words, as the Universe expands, a fraction of the modes that lie beyond the horizon re-enters inside the Hubble sphere. At the time of re-entry their amplitude is given by Eq (34), while, afterwards, they begin oscillating. The k -dependence of their amplitude implies a *scale-invariant* power-spectrum [13].

However, if the cosmological evolution includes a radiation-plus-strings stage, then, during this stage, the effective potential (in the equation which governs the temporal evolution of a CGW) is a non-vanishing constant. In other words, $k_c \neq 0$ and the modes with $k < k_c$ do not oscillate. As a consequence (in contrast to what previously stated), even if they lie outside the horizon, *they do not freeze out*.

At the beginning of the radiation-plus-strings stage, the GW modes that *fit* inside the visible Universe obey the condition

$$\lambda_{ph}(\tau_*) \leq \ell_H(\tau_*) \Rightarrow k \geq H(\tau_*)R(\tau_*) = \frac{1}{\tau_*}, \quad (37)$$

while, modes of $k < k_* = \tau_*^{-1}$ lie outside the horizon. In order to examine whether $k_c > k_*$ we need to determine the initial mass-density of the linear defects, since, by definition,

$$M_2 = \rho_{str}(\tau_*)R^2(\tau_*). \quad (38)$$

Let us consider a network of cosmic strings characterized by a *correlation length* $\xi(\tau)$. This may be defined as the length such that the mass within a typical volume ξ^3 , is $\mu\xi$ [4]. In this case, at $\tau = \tau_*$, the cosmic strings contribute to the Universe matter-content a mean density

$$\rho_{str}(\tau_*) = \frac{\mu}{\xi^2(\tau_*)} = \gamma_*^2 \frac{\mu}{\ell_H^2(\tau_*)}, \quad (39)$$

where γ_* is a numerical constant of the order of unity (in fact, $\gamma_* \simeq 4-11$, e.g. see [4], reflecting the differences among the various methods of numerical integration in [26] - [29]), representing the number of correlation lengths inside the horizon at τ_* . Accordingly,

$$M_2 = \mu \left(\frac{\gamma_*}{\tau_*} \right)^2, \quad (40)$$

and hence

$$k_c = \sqrt{\frac{8\pi}{3}} \sqrt{G\mu} \frac{\gamma_*}{\tau_*}. \quad (41)$$

The dimensionless quantity $(G\mu)$ characterizes the *strength* of the gravitational interactions of strings. The observations give an upper limit on the value of this parameter. In particular, for GUT-scale strings, the current CMB bound is [36] $G\mu \leq 1.3 \times 10^{-6}$, while, more recent studies on a cosmic-string contribution in the WMAP data [37] have yielded the tighter bound $G\mu \leq 3.3 \times 10^{-7}$, something that is confirmed also by gravitational lensing observations: $G\mu \geq 4 \times 10^{-7}$ [38], [39]. Admitting that $G\mu \sim 10^{-6}$ and $\gamma_* \simeq 7$, Eq (41) results in

$$k_c \simeq 2 \times 10^{-2} k_* \ll k_*. \quad (42)$$

In other words, at the beginning of the radiation-plus-strings stage, the GW modes of $k < k_c$ *do not fit* inside the horizon.

On the other hand, for $\tau_* < \tau \leq \tau_{sc}$, the condition of fitting inside the Hubble sphere is written in the form

$$\frac{k}{k_c} \geq \coth \sqrt{\frac{8\pi G}{3}} M_2 \tau. \quad (43)$$

Since the *hyperbolic cotangent* is larger than unity for every τ , Eq (43) suggests that during the whole radiation-plus-strings stage the GW modes of comoving wave-numbers $k < k_c$ remain outside the horizon.

Nevertheless, due to the constancy of the effective potential along this stage, these modes (although being outside the horizon) do not freeze out. Indeed, the combination of Eqs (17) and (35) suggests that, for $\tau_* < \tau \leq \tau_{sc}$, their amplitude evolves as

$$\alpha_{k < k_c}(\tau) = \frac{4}{\sqrt{\pi}} \left(\frac{H_{dS}}{m_{Pl}} \right) \frac{1}{k^{3/2}} \left[\frac{R(\tau_*)}{R(\tau)} \right] e^{\sqrt{k_c^2 - k^2} (\tau - \tau_*)}. \quad (44)$$

This behavior ends at τ_{sc} , when the scaling of the long-string network is completed and the Universe re-enters in the (late) radiation era. For $\tau > \tau_{sc}$ the GW modes of $k < k_c$ are no longer influenced by the radiation-plus-strings stage and therefore, just like the rest of the metric perturbations outside the horizon, (*re*)freeze out. As a consequence, their amplitude acquires the constant value

$$\alpha_{k < k_c} = \frac{4}{\sqrt{\pi}} \left(\frac{H_{dS}}{m_{Pl}} \right) \frac{1}{k^{3/2}} \left[\frac{R(\tau_*)}{R(\tau_{sc})} \right] e^{\sqrt{k_c^2 - k^2} \Delta\tau}. \quad (45)$$

5.1.2. The distorted power-spectrum: Within the late-radiation era these modes remain frozen until the time τ_c . At that time, the mode k_c enters inside the visible Universe, since its physical wavelength ($\lambda_{cph} \sim \tau_c$) becomes smaller than the corresponding Hubble radius ($\ell_H \sim \tau_c^2$). In accordance, for $\tau > \tau_c$, GW modes of $k < k_c$ also enter inside the Hubble sphere. After entering inside the horizon, the GW modes under consideration begin oscillating, thus producing a part of the power-spectrum we observe today (or at some time in the future). However, since they have experienced the influence of the radiation-plus-strings (*rps*) stage, their amplitude is no longer given by Eq (35), but by Eq (45), thus resulting in the *distortion* of the GW power-spectrum ($\mathcal{P}_k \sim k^3 \alpha_k^2$), from what it is anticipated in a *pure-radiation (rad)* model, at comoving wave-numbers $k < k_c$

$$\begin{aligned} \mathcal{P}_{k < k_c}^{rps} &= \frac{16}{\pi} \left(\frac{H_{dS}}{m_{Pl}} \right)^2 \left[\frac{R(\tau_*)}{R(\tau_{sc})} \right]^2 e^{2\sqrt{k_c^2 - k^2} \Delta\tau} \Rightarrow \\ \mathcal{P}_{k < k_c}^{rps} &= \mathcal{P}_{k < k_c}^{rad} \left[\frac{R(\tau_*)}{R(\tau_{sc})} \right]^2 e^{2\sqrt{k_c^2 - k^2} \Delta\tau}. \end{aligned} \quad (46)$$

Upon consideration of Eq (33), we obtain

$$\frac{R(\tau_*)}{R(\tau_{sc})} = \frac{1}{\cosh(k_c \Delta\tau) + \coth(k_c \tau_*) \sinh(k_c \Delta\tau)}, \quad (47)$$

and therefore

$$\frac{\mathcal{P}_{k < k_c}^{rps}}{\mathcal{P}_{k < k_c}^{rad}} = \frac{4 e^{2(1 + \sqrt{1 - x^2}) k_c \Delta\tau}}{\{[\coth(k_c \tau_*) + 1] e^{2k_c \Delta\tau} - [\coth(k_c \tau_*) - 1]\}^2}, \quad (48)$$

where, we have set

$$0 < \frac{k}{k_c} = x = \frac{f}{f_c} < 1, \quad (49)$$

and f is the frequency attributed to the GW mode denoted by k . By virtue of Eq (42), $\coth(k_c \tau_*) \simeq 5$ and, therefore, Eq (48) results in

$$\frac{\mathcal{P}_{k < k_c}^{rps}}{\mathcal{P}_{k < k_c}^{rad}} = \frac{e^{2(1+\sqrt{1-x^2}) k_c \Delta\tau}}{(3 e^{2k_c \Delta\tau} - 2)^2}. \quad (50)$$

Clearly, for $\Delta\tau = 0$ (i.e. in the absence of the radiation-plus-strings stage) $\mathcal{P}_{k < k_c}^{rps} = \mathcal{P}_{k < k_c}^{rad}$, while, for $\Delta\tau \neq 0$ the inflationary-GW power-spectrum is no longer *scale-invariant*.

The *spectral function* Ω_{gw} , appropriate to describe the *intensity* of a stochastic GW background [40], is related to the power-spectrum as $\Omega_{gw} \sim k \mathcal{P}_k$ (e.g. see [2], [9], [41]), so that, Eq (50) is written in the form

$$\frac{\Omega_{gw}^{rps}(f < f_c)}{\Omega_{gw}^{rad}(f < f_c)} = \frac{e^{2(1+\sqrt{1-x^2}) k_c \Delta\tau}}{(3 e^{2k_c \Delta\tau} - 2)^2}. \quad (51)$$

Notice that, for every $0 \leq x \leq 1$, Eq (51) yields $\Omega_{gw}^{rps}(f < f_c) \leq \Omega_{gw}^{rad}(f < f_c)$ with the equality being valid only for $\Delta\tau = 0$. In other words, the involvement of a radiation-plus-strings stage in the evolution of the Universe reduces the stochastic GW intensity at lower levels than those expected in a pure-radiation model. To give some numbers, we take into account the numerical results of [28], as well as those of [29]. Accordingly, a reasonable estimate on the duration of radiation-plus-strings stage would be $\tau_{sc} = 5.5 \tau_*$ and therefore, $k_c \Delta\tau \simeq 9 \times 10^{-2}$. In this case, Eq (51) is written in the form

$$\frac{\Omega_{gw}^{rps}(f < f_c)}{\Omega_{gw}^{rad}(f < f_c)} \simeq 0.47 \times e^{0.18\sqrt{1-x^2}}, \quad (52)$$

from which, it becomes evident that, for $f < f_c$, the value of Ω_{gw} is no longer 8×10^{-14} , as it is predicted by pure-radiation (e.g. see [2], [9]), but rather

$$\Omega_{gw}^{rps} \simeq 0.5 \Omega_{gw}^{rad} \sim 4 \times 10^{-14}, \quad (53)$$

(Fig. 1). On the other hand, according to [31], which deals with the scaling of a cosmic-string network in an updated numerical fashion, the duration of a potential radiation-plus-strings stage in terms of the conformal time (the *dynamical range*, as it is referred to) is $\tau_{sc} = 17\tau_*$ (corresponding to a factor of 300 in terms of t). Adoption of this result, would lead to a more evident distortion of the inflationary GW spectrum, modifying Eq (53) to

$$\Omega_{gw}^{rps} \simeq 0.2 \Omega_{gw}^{rad} \sim 1.6 \times 10^{-14}. \quad (54)$$

Such a distortion reflects a change in the distribution of the GW energy-density among the various frequency intervals, probably due to the coupling between metric perturbations and cosmic strings.

The question that arises now is, whether these results are observable by the detectors currently available. To answer this question, we should determine explicitly both f_c (the critical frequency) and t_c (the physical time at which modes of $f < f_c$ begin entering inside the horizon). In what follows, $c \neq 1$.

During the early-radiation epoch, the *physical time* is defined as

$$t = \int R(\tau) d\tau \Rightarrow t = \frac{\tau^2}{2\tau_{cr}}. \quad (55)$$

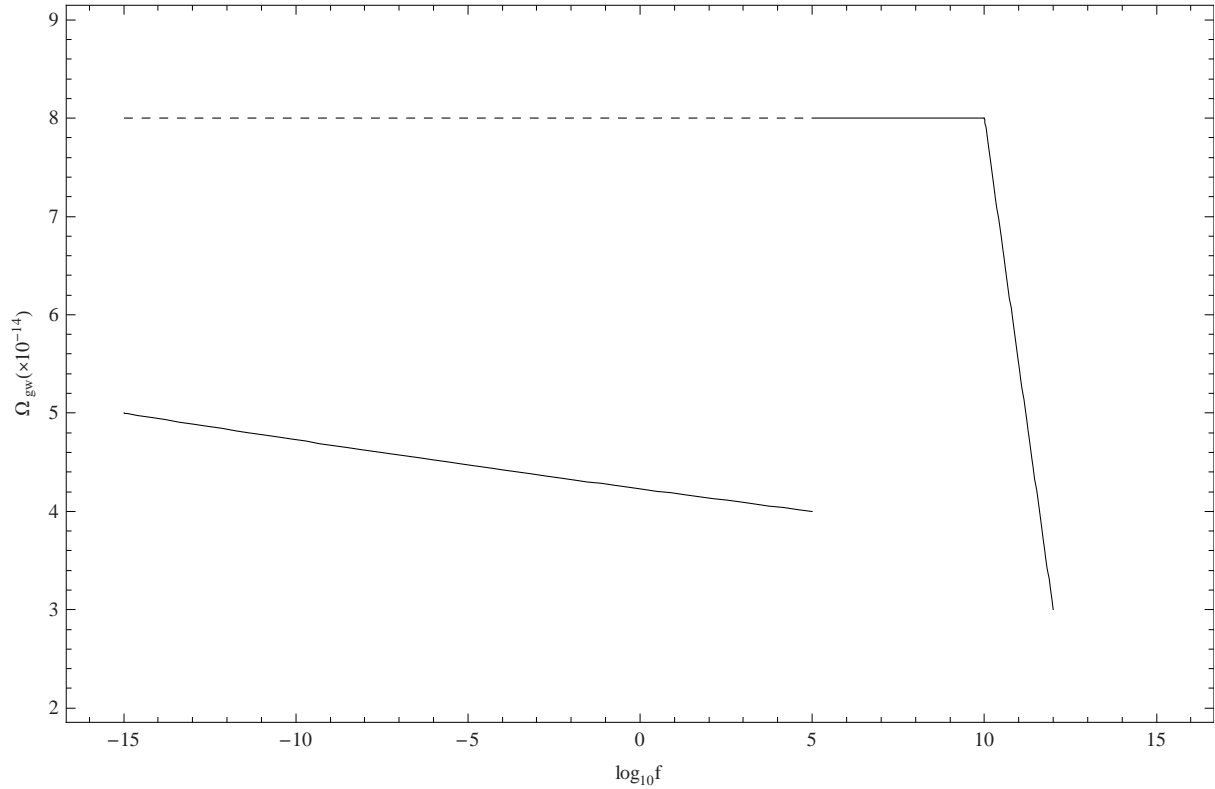


Figure 1. The stochastic GW background from inflation at frequencies within the radiation era, $f \geq 10^{-16} \text{ Hz}$ (dashed line), under the influence of a radiation-plus-strings stage produced by GUT-scale cosmic strings (solid line).

With the aid of Eqs (32) and (55), Eq (41) is written in the form

$$k_c = \sqrt{\frac{2\pi}{3}} \left(\frac{G\mu}{c^2} \right) \frac{\gamma_*}{c t_{cr}}, \quad (56)$$

and therefore

$$f_c = \frac{1}{\sqrt{6\pi}} \left(\frac{G\mu}{c^2} \right) \frac{\gamma_*}{t_{cr}}. \quad (57)$$

The first of the GW modes under consideration which enters inside the visible Universe, is the one with the shortest comoving wavelength (λ_c), i.e. the one with the largest frequency, f_c . In terms of the physical time, this process begins at t_c at which $\lambda_{c_{ph}}(t_c) \leq \ell_H(t_c)$.

Within the late-radiation era, the physical time is defined as

$$t = \int R(\tau) d\tau \Rightarrow t = R_{sc} \frac{\tau^2}{2\tau_{sc}}, \quad (58)$$

and therefore

$$t_c \geq \frac{3\pi}{11\gamma_*^2} \left(\frac{G\mu}{c^2} \right)^{-1/2} R_{sc} t_*, \quad (59)$$

where we have used both Eqs (55) and (58), together with Eq (32) and the fact that $\tau_{sc} = 5.5\tau_*$. In an expanding Universe

$$R_{sc} > R(t_*) = \left(\frac{G\mu}{c^2} \right)^{-1/2}, \quad (60)$$

and hence

$$t_c > \frac{3\pi}{11\gamma_*^2} \left(\frac{G\mu}{c^2} \right)^{-1} t_* = \frac{3\pi}{11\gamma_*^2} \left(\frac{G\mu}{c^2} \right)^{-2} t_{cr}. \quad (61)$$

Within the Hubble sphere the GW modes of $\lambda > \lambda_c$ correspond to CGWs of frequencies $f < f_c$. Extrapolation of this result into the present epoch ($t_{pr} \simeq 13.7 \times 10^9$ y), suggests that at frequencies

$$\begin{aligned} f^{pr} \leq f_c^{pr} &= f_c \left(\frac{t_c}{t_{rec}} \right)^{1/2} \left(\frac{t_{rec}}{t_{pr}} \right)^{2/3} \Rightarrow \\ f^{pr} \leq f_c^{pr} &= \frac{1}{\sqrt{22}} \left(\frac{t_{cr}}{t_{rec}} \right)^{1/2} \left(\frac{t_{rec}}{t_{pr}} \right)^{2/3} \frac{1}{t_{cr}}, \end{aligned} \quad (62)$$

(where $t_{rec} = 1.2 \times 10^{13}$ sec is the *recombination time*) the inflationary-GW power-spectrum is distorted, departing from scale-invariance.

We note that f_c^{pr} depends only on the (physical) time at which the cosmic strings are formed. These linear defects may have been formed at a grand unification (GUT) transition or, conceivably, much later, at the electro-weak transition or somewhere in between [4]. For GUT-scale strings, $t_{cr} \sim 10^{-31}$ sec [5] and therefore $f_c^{pr} \simeq 1.5 \times 10^5$ Hz. Clearly, this value is far outside of the range where both the ground-based and the space-based laser interferometers may operate. A GW of this frequency could be detected only by a system of *coupled super-conducting microwave cavities* [42], [43].

However, one should have in mind that, this is only the *upper bound* of the distorted GW power-spectrum. In fact, if cosmic strings have contributed to the evolution of the Universe, the GW power-spectrum (that we hope we observe some day) will decline from what it is anticipated by a pure-radiation model at every present-time frequency in the range 10^{-16} Hz $< f \leq f_c^{pr}$ (see Fig. 1). The lower bound of this range arises from the GWs that began entering inside the horizon after the Universe has become matter-dominated [39]. Clearly, a potential detection of CGWs would give us valuable information on the epoch (and therefore on the physical mechanism, as well) at which the cosmic strings were formed.

5.2. Evolution of the oscillatory modes

The quantum-gravitational creation of gravitons in an expanding FRW Universe was first demonstrated by Grishchuk [14] and Starobinsky [44]. They showed that, in the *linear approximation*, the behavior of a CGW propagating in curved space-time is identical to that of a massless, minimally-coupled scalar field. In particular, each of the two polarization states of the metric perturbation satisfy the Klein-Gordon equation (2). The quantization of primordial GWs and that of the minimally-coupled, massless scalar fields also proceeds along identical lines, as it was demonstrated by Ford and Parker [18], [45].

In what follows, we will show that, if the Universal evolution includes a radiation-plus-strings stage, then, although it could last only for a short period of time, its presence would have resulted also in a measurable effect on the *high-frequency* CGWs' power-spectrum.

Indeed, around τ_* and τ_{sc} (the boundaries of the radiation-plus-strings stage) both $R(\tau)$ and $R'(\tau)$ can be matched to be continuous, but $R''(\tau)$ is essentially discontinuous, acquiring a non-zero value through Eq (13). Consequently, the *scalar curvature* of the space-time changes discontinuously both at τ_* and at τ_{sc} . A discontinuous change in the scalar curvature produces *gravitons* [19], [46].

As a consequence, the evolution of the oscillating GW modes in the transition of the Universe from an early-radiation epoch to the late-radiation era through a radiation-plus-strings stage, is modified as follows:

- For $\tau \leq \tau_*$ the effective potential (13) vanishes. The temporal evolution of the metric perturbations is governed by Eq (22), admitting the solution (23) and the gravitational-perturbation field is in an adiabatic-vacuum state.
- For $\tau_* < \tau < \tau_{sc}$ the effective potential reduces to a non-vanishing constant ($V_{eff} = k_c^2 c^2$). The propagation of the metric perturbations is now driven by Eq (25), for $k > k_c$. As a result of the discontinuous change of the scalar curvature at τ_* , during the whole radiation-plus-strings stage the gravitational-perturbation field is no longer in its vacuum-state and the general solution to Eq (25) is the linear superposition of positive- and negative-frequency solutions (28).
- Finally, for $\tau \geq \tau_{sc}$, $V_{eff} = 0$ and the temporal evolution of a metric perturbation in curved space-time is (once again) governed by Eq (22). However, due to the discontinuity of the scalar curvature at τ_{sc} , this time the general solution to Eq (22) is written in the form ($c \neq 1$)

$$h_k^{out}(\tau) = \sqrt{\frac{2}{\pi k c}} \left[\alpha_k e^{-i k c \tau} + \beta_k e^{+i k c \tau} \right], \quad (63)$$

where, the constant (Bogoliubov) coefficients α_k and β_k should satisfy the Wronskian condition

$$|\alpha_k|^2 - |\beta_k|^2 = 1. \quad (64)$$

For $k > k_c$, Eq (63) determines the *final* quantum state of the gravitational-perturbation field. As a result of conformal invariance, this state is also an adiabatic vacuum, but, as long as $\beta_k \neq 0$, it differs from the corresponding state of the early-radiation epoch. Clearly, the occurrence of a non-zero β_k in the late-radiation era would signal that $h_k^{out}(\tau)$ is not a pure *positive-frequency* solution, but contains also a *negative-frequency* component. This means that, if the *in-state* of the gravitational-perturbation field is taken to be vacuum, then, particles are found in the *out-state* [20].

In what follows, we discuss the evolution of the modes (23) with $k > k_c$, in the transition of the Universe from the early-radiation epoch to the late-radiation era through a radiation-plus-strings stage. The coefficients c_1 , c_2 , α_k and β_k can be determined by the requirement that $h_k(\tau)$ and its first derivative $h'_k(\tau)$ are continuous across the boundaries τ_* and τ_{sc} .

5.2.1. Graviton production by long cosmic strings: Matching the modes (23) and (28), as well as their first derivatives, at $\tau = \tau_*$, we obtain

$$c_1 = \frac{\gamma + 1}{2\sqrt{\gamma}} e^{i k c (\gamma - 1) \tau_*}, \quad (65)$$

and

$$c_2 = \frac{\gamma - 1}{2\sqrt{\gamma}} e^{-i k c (\gamma + 1) \tau_*}, \quad (66)$$

for which the Wronskian condition (29) holds. Similarly, matching the modes (28) and (63), as well as their first derivatives, at $\tau = \tau_{sc}$, we obtain

$$\alpha_k = \frac{1}{4\gamma} \left[(\gamma + 1)^2 e^{-i(\gamma - 1) k c \Delta \tau} - (\gamma - 1)^2 e^{+i(\gamma + 1) k c \Delta \tau} \right], \quad (67)$$

and

$$\beta_k = i \frac{\gamma^2 - 1}{2\gamma} \sin(\gamma k c \Delta \tau) e^{-i k c (\tau_* + \tau_{sc})}, \quad (68)$$

where we have taken into account Eqs (65) and (66). Using Eq (27), the number of gravitons [20] created in the mode denoted by k , is written in the form

$$N_k = |\beta_k|^2 = \frac{k_c^4}{4 k^2} \left[\frac{\sin(c\Delta\tau \sqrt{k^2 - k_c^2})}{\sqrt{k^2 - k_c^2}} \right]^2, \quad (69)$$

and the normalization condition $|\alpha_k|^2 - |\beta_k|^2 = 1$, holds. The observable quantity N_k possesses a series of very interesting properties.

Obviously, for $k_c = 0$, N_k vanishes, i.e. no gravitons are created in the absence of cosmic strings. On the other hand, for $k \rightarrow k_c$ and/or $\Delta\tau \rightarrow 0$, Eq (69) results in

$$N_k \rightarrow \frac{k_c^4}{4 k^2} c^2 \Delta\tau^2. \quad (70)$$

We distinguish the following cases:

- (i) $\Delta\tau = 0$: In this case, $N_k = 0$. Although this may look like an unexpected result, it is not. Usually, the instantaneous transition from one epoch to another is a pathological feature of quantum field theory, leading to an infinite number of created particles (e.g. see [20]). However, in our case, as long as $\Delta\tau = 0$ no transition ever takes place. The Universe remains in the radiation era, where no gravitons are produced.
- (ii) $k = k_c$: In this case, N_k acquires the constant value

$$N_{k_c} = \frac{k_c^2}{4} c^2 \Delta\tau^2. \quad (71)$$

This is a very important result, suggesting that the number of gravitons in the *lowest-allowed* wave-number is well-defined and finite. Therefore, there should be no *infrared divergences*.

Finally, for $k \gg k_c$ and $k \rightarrow \infty$, $N_k \rightarrow 0$ and particle production ceases. Consequently, there should be no *ultraviolet divergences*, as well.

On the other hand, according to Eq (69), the *expectation value* of the *number-operator* varies periodically with k . Hence:

- (iii) For comoving wave-numbers satisfying the condition

$$c \Delta\tau \sqrt{k^2 - k_c^2} = m\pi \quad (m = 1, 2, \dots), \quad (72)$$

N_k admits its absolute minimum value, i.e. $N_k = 0$. The case $m = 0$ is excluded due to the fact that $N_{k_c} \neq 0$. As a consequence, creation of gravitons with comoving wave-numbers

$$k_m^2 = k_c^2 + m^2 \frac{\pi^2}{(c\Delta\tau)^2} \quad (m = 1, 2, \dots), \quad (73)$$

never occurs by this mechanism. In quantum physics, this case is referred to as *perfect-transmission* [47]. Notice that, the spectrum of these states is *discrete* and their position on the k -axis is determined solely by the choice of $\Delta\tau \neq 0$.

5.2.2. *The power-spectrum of these relic gravitons:* During the (late) radiation stage of the expansion, the *physical* frequency of a primordial GW is defined as $\omega = ck/R(\tau)$ and the number of states with frequencies in the interval ω and $\omega + d\omega$ is written in the form $dn_\omega = \omega^2 d\omega / 2\pi^2 c^3$. Accordingly, the distribution of the gravitational energy-density, summed over the two polarization states of the gravitons present in the late-radiation epoch, is given by

$$d\epsilon_{gw} = \mathcal{P}(\omega)d\omega = 2\hbar\omega \frac{\omega^2}{2\pi^2 c^3} |\beta_k|^2 d\omega, \quad (74)$$

and, in terms of k , is reduced to

$$d\epsilon_{gw} = \mathcal{P}(k)dk = \frac{1}{R^4(\tau)} \frac{\hbar c}{4\pi^2} k_c^4 \left[\frac{\sin \left(c\Delta\tau \sqrt{k^2 - k_c^2} \right)}{\sqrt{k^2 - k_c^2}} \right]^2 k dk. \quad (75)$$

Therefore, the power-spectrum of the created gravitons [15], [19], [46] reads

$$\begin{aligned} \mathcal{P}(k) &= \frac{d\epsilon_{gw}}{dk} = \\ &= \frac{\hbar c}{4\pi^2} k_{c_{ph}}^4 \left[\frac{\sin \left(c\Delta\tau \sqrt{k^2 - k_c^2} \right)}{\sqrt{k^2 - k_c^2}} \right]^2 k, \end{aligned} \quad (76)$$

where, we have introduced the (physical) critical wave-number at every τ as

$$k_{c_{ph}}(\tau) = k_c/R(\tau). \quad (77)$$

In terms of the dimensionless parameter

$$x = \frac{k}{k_c} \geq 1, \quad (78)$$

which, in this case, measures the comoving wave-number in units of the corresponding *lowest-allowed* k_c , Eq (76) is written in the form

$$\begin{aligned} \mathcal{P}(x) &= \frac{d\epsilon_{gw}}{dx} = k_c \frac{d\epsilon_{gw}}{dk} = \\ &= (k_c c \Delta\tau)^2 \frac{\hbar c}{4\pi^2} k_{c_{ph}}^4 \left[\frac{\sin \left(k_c c \Delta\tau \sqrt{x^2 - 1} \right)}{\left(k_c c \Delta\tau \sqrt{x^2 - 1} \right)} \right]^2 x, \end{aligned} \quad (79)$$

or else

$$\begin{aligned} \mathcal{P}(x) &= 4\pi \frac{1}{4} (k_c c \Delta\tau)^2 2\hbar c k_{c_{ph}} \frac{1}{\lambda_{c_{ph}}^3} \\ &\times \left[\frac{\sin \left(k_c c \Delta\tau \sqrt{x^2 - 1} \right)}{\left(k_c c \Delta\tau \sqrt{x^2 - 1} \right)} \right]^2 x, \end{aligned} \quad (80)$$

where, $\lambda_{c_{ph}} = 2\pi/k_{c_{ph}}$ is the *correlation (wave)length*, associated to $k_{c_{ph}}$.

At a fixed $\tau \geq \tau_{sc}$, the gravitational energy contained within the volume $\lambda_{c_{ph}}^3$, is $2\hbar ck_{c_{ph}}$, i.e. the energy of a graviton with physical wave-number $k_{c_{ph}}$ summed over the two polarization states. Therefore, the quantity

$$\epsilon_c(\tau) = N_{k_c} 2\hbar c k_{c_{ph}} \frac{1}{\lambda_{c_{ph}}^3}, \quad (81)$$

[where N_{k_c} is given by Eq (71)] represents the overall energy-density of the gravitons created in the state denoted by k_c , at every $\tau_{sc} \leq \tau \leq \tau_c$, where τ_c is the time at which the longest metric perturbation created by this mechanism (i.e. the one with $k = k_c$), enters inside the horizon. At that time its *physical wavelength* [$\lambda_{ph}(\tau_c) = \lambda_c R(\tau_c)$] becomes smaller than the corresponding *Hubble radius* [$\ell_H(\tau_c) = c/H(\tau_c)$, H being the Hubble parameter]. According to Eq (61),

$$\tau_c \geq \left(\frac{G\mu}{c^2}\right)^{-1/2} \tau_*, \quad (82)$$

and, therefore, $\tau_c \sim 10^3 \tau_* \gg \tau_{sc} \sim 17 \tau_*$. Since k_c is the lowest-allowed comoving wave-number, after τ_c there are no gravitons (created by this mechanism) to enter inside the Hubble sphere.

Upon consideration of Eqs (71) and (81), the power-spectrum (80) results in

$$\mathcal{P}(x) = 4\pi \epsilon_c(\tau) \left[\frac{\sin(k_c c \Delta \tau \sqrt{x^2 - 1})}{(k_c c \Delta \tau \sqrt{x^2 - 1})} \right]^2 x. \quad (83)$$

For every $\tau_{sc} \leq \tau \leq \tau_c$, $\mathcal{P}(1)$ is the energy-density of the gravitons created in the state $k = k_c$ (within the solid angle of a sphere - 4π), while, as we depart from the lowest-allowed comoving wave-number, the gravitational power is distributed among the various k -intervals according to Eq (83). In particular, for k very close to k_c ($x \simeq 1$), $\mathcal{P}(k) \sim k$, while, in the large- k limit ($x \rightarrow \infty$) it results in a damped oscillation

$$\mathcal{P}(k) \sim \frac{1}{k} \sin^2(kc\Delta\tau). \quad (84)$$

According to Eq (84), graviton production is suppressed at high frequencies.

On the other hand, the *logarithmic spectrum* is written in the form

$$\begin{aligned} \frac{d\epsilon_{gw}}{d(\ln k)} &= k \mathcal{P}(k) = x \mathcal{P}(x) = \\ &4\pi \epsilon_c(\tau) \left[\frac{\sin(k_c c \Delta \tau \sqrt{x^2 - 1})}{(k_c c \Delta \tau \sqrt{x^2 - 1})} \right]^2 x^2. \end{aligned} \quad (85)$$

For $k \simeq k_c$ ($x \rightarrow 1$), the logarithmic spectrum is quadratic in k , while, for $k \gg k_c$ ($x \rightarrow \infty$), it reduces to a periodic function of the comoving wave-number with constant amplitude (Fig. 2)

$$\frac{d\epsilon_{gw}}{d(\ln k)} \sim \sin^2(kc\Delta\tau). \quad (86)$$

The importance of this result rests in the fact that the logarithmic spectrum is a re-scaling of the *spectral intensity*

$$\frac{1}{\epsilon_{cr}} \frac{d\epsilon_{gw}}{d(\ln k)} = \Omega_{gw} = \frac{1}{\epsilon_{cr}} \frac{d\epsilon_{gw}}{d(\ln f)}, \quad (87)$$

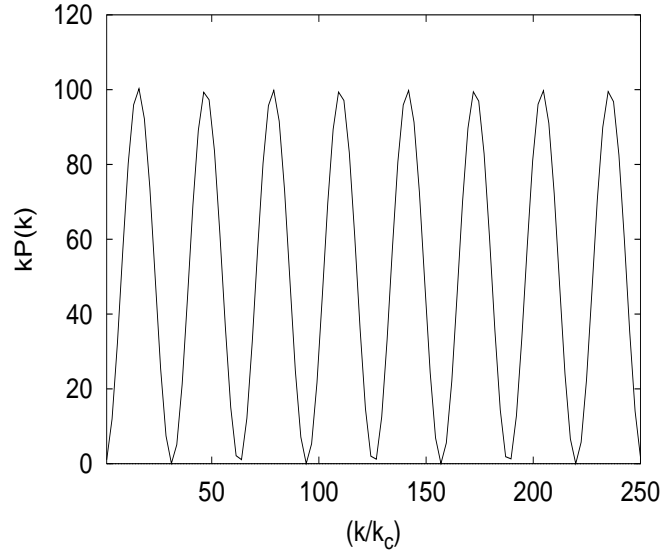


Figure 2. The logarithmic spectrum $k \mathcal{P}(k)$ of the metric perturbations created during the scaling of a long-cosmic-string network (normalized over $4\pi \epsilon_c$), versus k in units of k_c , for $k_c c \Delta\tau = 0.32$ (corresponding to $\tau_{sc} = 17\tau_*$ [31]).

where ϵ_{cr} is the *energy-density for closing the Universe*.

Accordingly, if the Universal evolution has undergone through a radiation-plus-strings stage, then, at present-time frequencies *larger* than the critical value (62) (corresponding to $k_{c_{ph}}$ at the present epoch), the relic GW spectrum would no longer be scale invariant (as it is predicted by pure-radiation), but it should have been reduced to a *periodic function* of the frequency (Fig. 2). For $k = k_c$ ($x = 1$), Eq (85) yields

$$k_c \mathcal{P}(k_c) = 4\pi \epsilon_c(\tau). \quad (88)$$

In this case, from Fig. 2 we observe that, the *amplitude* of the logarithmic spectrum (and therefore of Ω_{gw} , as well) reaches up to 100 times the (initial) value of this function for $k = k_c$. This is a very important result!

It suggests that, after being *filtered* by a long-cosmic-string network, the high-frequency part of the relic GW signal will be *amplified* by (almost) *two orders of magnitude*. To the best of our knowledge, *this is the first time* (since Grishchuk [14]) that an amplification mechanism of a stochastic GW signal is proposed and discussed. In this context, an indirect interaction between relic GWs and cosmic strings could have a major impact on the detection of these waves.

Since the only interference to the propagation of the relic GWs from an early-radiation epoch to the late-radiation era comes from cosmic strings (encapsulated in the cosmological model), it is natural to assume that any *energization* of the gravitational-perturbation field is due to a corresponding *energy-loss* of the linear-defect network. This assumption is compatible to the absence of *high-frequency harmonics* on long cosmic strings [48].

In particular, cosmic-string simulations indicate that, on small scales, the strings are extremely *wiggly*. In other words, there is a substantial amount of small-scale structure on a string network [4], [5]. The small-amplitude perturbations (or waves) on an *infinite* string can become the source of gravitational radiation with the same frequency [49], [50]. However, it has been found that the production of high-frequency gravitational waves from a long cosmic string is negligible, suggesting that the corresponding higher harmonics are strongly suppressed [48].

Our model could provide a physical interpretation for this result, in the sense that, the higher harmonics of the long linear defects might have been *consumed* in the excitation of high-frequency relic GWs (those produced by inflation at earlier epochs and the wavelength of which became less than the horizon during the radiation-plus-strings stage) and not in the production of *new* CGWs. The corresponding physical mechanism probably involves a *resonant interaction* between inflationary GWs and long cosmic strings, which becomes prominent at high frequencies. Such a mechanism is yet unknown and its exploration will be the scope of a future work.

6. Conclusions

The equation which governs the temporal evolution of a CGW in a Friedmann model can be treated as the Schrödinger equation for a particle moving in the presence of the effective potential $V_{eff} = R''/R$. In the present article we show that, if there is a period where the effective potential is constant, this would lead to a critical value (k_c) in the comoving wave-number of the metric fluctuations, discriminating them into oscillating ($k > k_c$) and non-oscillating ($k < k_c$) modes. As a consequence, when the non-oscillatory modes lie outside the horizon do not freeze out, something that should be reflected in the relic-GW power-spectrum.

This property is met in a radiation model contaminated by a fraction of cosmic strings. Therefore, if the cosmological evolution includes a radiation-plus-strings stage, some of the long-wavelength GW modes (although being outside the Hubble sphere) continue to evolve. However, this stage (if ever existed) does not last very long, since, the cosmological evolution gradually results in the scaling of the cosmic-string network and, after some time ($\Delta\tau$), the Universe enters in the late-radiation era.

In a radiation-dominated Universe the metric perturbations of $k < k_c$ can enter the horizon, which now expands faster than their physical wavelength. However, the evolution of the non-oscillatory GW modes during $\Delta\tau$ (while they were outside the horizon) has modified their amplitude and, therefore, oscillation of these modes within the Hubble sphere results in the distortion of the scale-invariant GW power-spectrum at present-time frequencies in the range $10^{-16} Hz < f \leq 10^5 Hz$.

On the other hand, if there was a period in the early Universe in which the matter-content can be modelled by a two-component fluid consisting of radiation and cosmic strings, this would have resulted in a measurable effect on the high-frequency CGWs' power-spectrum, as well. In fact, along the transition from an early-radiation epoch to the late-radiation era through a radiation-plus-strings stage, gravitons with a highly recognizable profile could have been produced. As a consequence, at high (present-time) frequencies ($f \geq 10^5 Hz$), the GW spectrum would also depart from scale-invariance, becoming a *periodic* function of the frequency, the period of which depends solely on the *duration* of the radiation-plus-strings stage.

But, what's most important for the detection of these waves, is that: The semiclassical interaction between metric perturbations and cosmic strings would lead to the *amplification of the relic GW signal* by (almost) *two orders of magnitude*. To the best of our knowledge, this is the first time since Grishchuk's work, that an amplification mechanism of a stochastic GW signal is proposed and discussed.

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