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HELICAL UNDULATORS OF MAGNETIZED HELICES AND RING SECTORS

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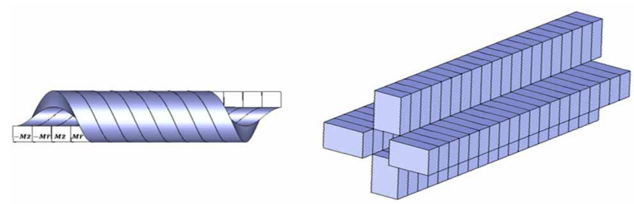
Abstract

The periodic system of a spiral array of magnetized ring sectors (quasi-helix) creates a helical field, which is close in structure and magnitude to the field in the system of helical magnets. Such a system of a relatively small number of readily accessible magnets can be easier to manufacture and assemble than a system containing magnetized helices made from single pieces. In this paper, we study the dependence of the helical field on the number of sectors per undulator period. Short prototypes consisting of longitudinally and radially magnetized sectors were experimentally studied. The maximum value of the helical field on the axis of a quasi-helix of longitudinally and radially magnetized ring NdFeB sectors with a period of 2 cm and a relatively large inner diameter of 8 mm is 0.28 T and 0.34 T, respectively. This corresponds to a field of 0.74 T in the case of four alternately longitudinally and radially magnetized Halbach-like quasi-helices of pre-magnetized sectors and a hybrid system assembled from longitudinally pre-magnetized sectors and preliminarily non-magnetized steel sectors. Such undulators can provide a high oscillatory electron velocity and seem promising for increasing the efficiency of FELs and inverse FELs in various frequency ranges.

NEW VARIANTS OF PERMANENT HELICAL UNDULATORS

According to calculations, the helical undulators from magnetized rare earth helices proposed in [1] make it possible to obtain stronger fields than planar undulators. Thus, a Halbach-like set of four alternately longitudinally and radially magnetized helices (Fig. 1 a) can provide about a factor of $\sqrt{2}$ greater field than the commonly used set of two perpendicular and quarter-period-shifted Halbach undulators with linear field polarization (Fig. 1 b). When implementing undulators of the type under discussion, two obvious problems arise, namely, the manufacture of rare earth helices and the assembly of a structure from such strongly interacting and brittle parts. The first problem can be successfully solved by using the Wire Electrical Discharge Machining [2, 3], which makes it possible to fabricate NdFeB helices with very short periods down to about 1 mm or even less (see also [4]). In order to simplify fabrication and assembly, we propose a trade-off here, which is to move away from the concept of helices made from one piece and replace some or all of them with a quasi-helical set of simple and widely available magnetized ring sectors (Fig. 2). As in the case of a Halbach-type array of helices (Fig. 1 a), a hybrid design using two quasi-helices from longitudinally magnetized rare earth sectors and two quasi-helices from preliminary non-magnetized steel (or vanadium permendur) sectors (Fig. 3) turns out to be simpler and even more efficient.



a) b)

Figure 1: Permanent helical undulators: a) a Halbach-like helical undulator comprising four identical rare earth helices with wave-like alternating axial and radial magnetizations, b) commonly used two shifted planar Halbach arrays.

a) b) c)

Figure 2: Rare earth helix and quasi-helix manufactured from: a) one piece and b) set of ring sectors; c) photo of quasi-helix from sectors.

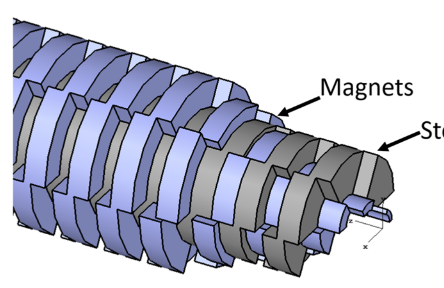


Figure 3: Hybrid helical undulator comprising two spirally arranged and oppositely longitudinally magnetized sets of ring sectors and two non-pre-magnetized spirally arranged steel ring sectors (right shows a two-period adiabatic entry).

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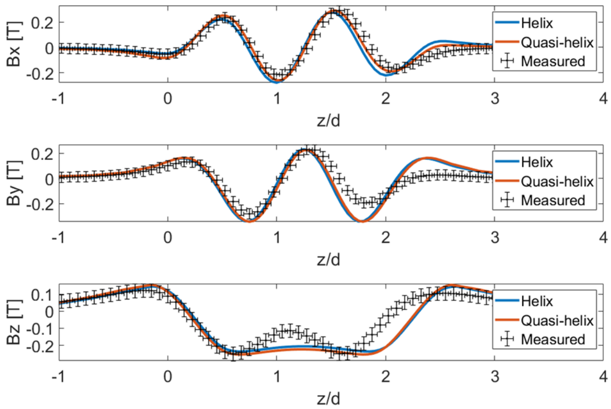


Figure 4: CST simulations for fields on the axes of a two-period axially magnetized solid helix and a spirally arranged set of 24 ring sectors per period in comparison with the experimentally measured field of 24 ring sectors per period (see Fig. 5).

FIELDS OF A HELIX FROM ONE PIECE AND A QUASI-HELIX FROM RING SECTORS

The fields of infinitely long helices of rectangular cross section and axial size of a quarter of the period *d* with longitudinal and radial magnetizations, respectively, are described by the expressions [1, 4].

$$\vec{B}_{u_l} = \frac{B}{\pi} \sin\eta \int_{\xi_1}^{\xi_2} \xi K_1(\xi) d\xi \, (\hat{x}\sin\zeta + \hat{y}\cos\zeta) \quad (1)$$

$$\vec{B}_{u_r} = \frac{B}{\pi} \sin\eta \int_{\xi_1}^{\xi_2} \xi K_1{}'(\xi) d\xi \, (\hat{x}\sin\zeta - \hat{y}\cos\zeta) \quad (2)$$

Here, $B = \mu_0 M, M$ is the remanent magnetization of the ferromagnetic, μ_0 is the magnetic permeability of vacuum, $\xi_{1,2} = hR_{1,2}$, $\zeta = hz, h = 2\pi/d,$ $R_{1,2}$ are the inner and outer radii of the helix, $\eta = ha/2,$ where a is the width of the magnet , K_1 and $K_1{}'$ are a first-order Macdonald function and its derivative. Equations (1) and (2) give a good approximation even for a small number of helical periods (5-10) and can be used for estimation even with a shorter length. For example, the results of CST [5] calculations for two periods differ from the values found from Eq. (1) and Eq. (2) less than 5%. In turn, calculations for a spirally arranged set containing 24 magnetized circular sectors per period. (Fig. 4 b and Fig. 4 c) and for a solid helix (Fig. 1a) are close to each other and to the experimentally measured values (Fig. 5 and Fig. 7) for 24 sectors per period (Fig. 4 and Fig. 6). It is important that the field magnitude depends only weakly on the number of sectors *N* per period. For example, when *N* decreases from 24 to 6, the field decreases by only 3% (Fig. 8).

For FELs and IFELs, in which the electron beam moves near the undulator axis, the homogeneity of the transverse undulator field is very important. In a Halbach-type undulator formed by a set of sectors, the field is fairly uniform even with *N*=6 sectors per period (Fig. 9).

a) b)

Figure 5: Assembly (a) and axial magnetization (b) of a two-period helix consisting of a spirally arranged set of 24 ring sectors per period.

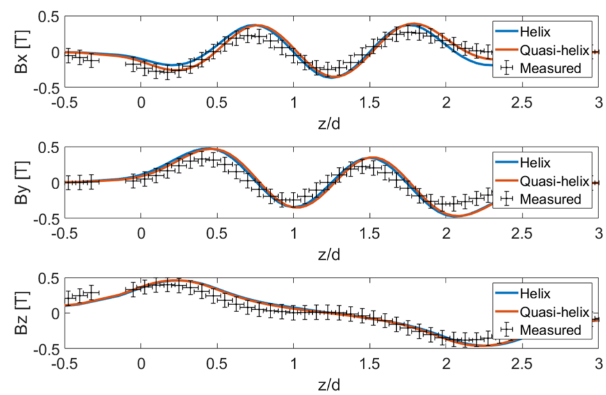
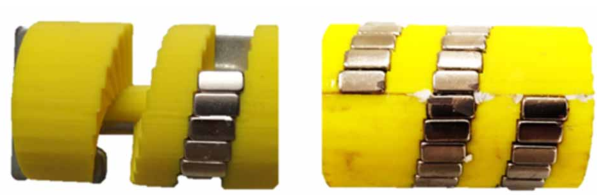


Figure 6: CST simulations for fields on the axes of a two-period radially magnetized solid helix and a spirally arranged set of 24 ring sectors per period in comparison with the experimentally measured field of 24 ring sectors per period (see Fig. 7).



a) b)

Figure 7: Assembly and axial magnetization of a two-period quasi-helix consisting of a spirally arranged set of 24 radially magnetized ring sectors per period. Magnetized sectors are glued into a mold placed on steel to weaken the forces of interaction between the magnets: a) the beginning and b) the end of the process.

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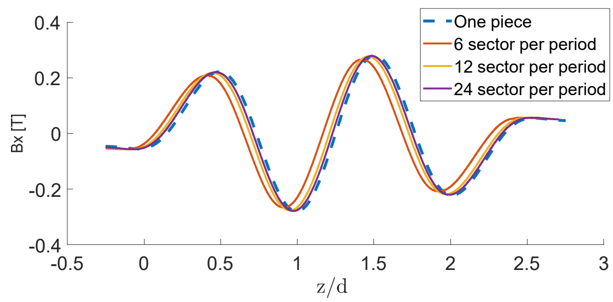


Figure 8: CST simulation results for the field on the axis by the number of sectors per period.

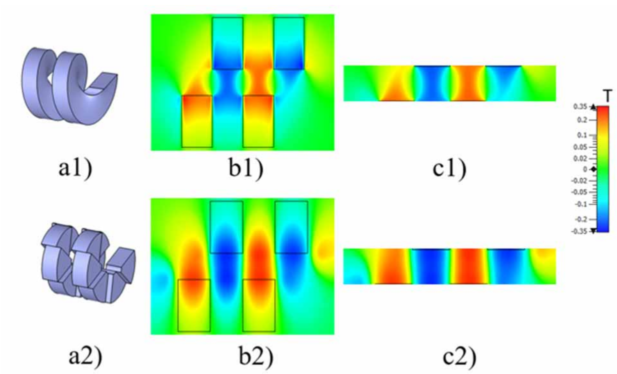
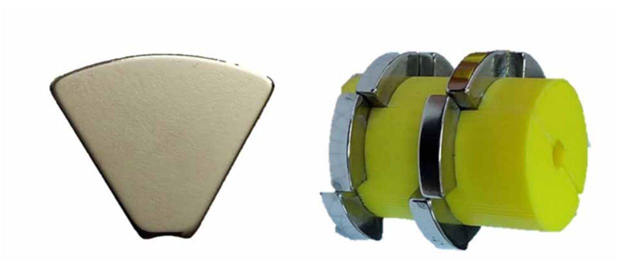


Figure 9: CST simulations for a two-period solid helix (top row) and a quasi-helix of 6 ring sectors per period (bottom row): a) structures; b) field intensity in a transverse cross section, c) field intensity inside the helix and quasi-helix.



a) b)

Figure 10: Photo of a) one 60° NdFeB ring sector and b) a two-period quasi-helix formed by N=6 longitudinally magnetized ring sectors per period (d=20 mm, R_1=4 mm, and R_2=20 mm).

In the experiments, we measured the fields of short two-period quasi-helices formed by longitudinally and radially magnetized circular sectors, with a relatively large internal hole having a relatively large radius R_1=4 mm (Fig. 10 and Fig. 11), which is related to the size of the magnetic probe. The measured magnitudes of the field are close to the calculated ones and are only 7% less than the field of infinite helices. For a Halbach-like system of quasi-helices, the CST calculation gives a field of 0.74 T, and when the inner radius is decreased to R_1=1 mm, the field increases to 1.93 T, which corresponds to the undulator parameter K=3.6.

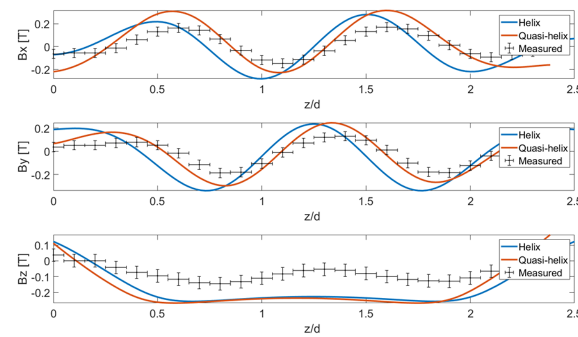


Figure 11: Experimentally measured field of a two-period quasi-helix with N=6 longitudinally magnetized ring sectors per period.

CONCLUSIONS

Permanent undulators consisting of several helices, each of which is made of one piece of a ferromagnet, seem to be the most promising in terms of achieving a strong field with high magnetization uniformity and a small spread of field parameters. The possibility of fabricating such helices was demonstrated for other purposes in [2, 3], and specifically for undulators in [4]. As a next step, we plan to assemble a hybrid set of rather fragile magnetized rare earth and preliminarily non-magnetized steel helices.

In this paper, we have also considered much simpler and cheaper systems of quasi-helices formed by spirally arranged sets of easily accessible ring sectors. The experimentally measured fields of short quasi-helices with a small number of longitudinally and radially magnetized sectors per period are in good agreement with the calculated values and differ from the fields of the helices by less than 10%. In assembling the quasi-helices, we have overcome the difficulties associated with the significant forces acting between the individual magnets.

Both types of undulators, consisting of helices and quasi-helices, can create strong magnetic fields and seem promising for FELs and inverse FELs in various frequency ranges.

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