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Light Meson Decay with Gaussian Confinement in a JKJ Decay Model

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A form to describe hadron strong decays is in terms of quark and gluon degrees of freedom in microscopic decay models. Initially we assume that strong decays are driven by the same inter-quark Hamiltonian which determines the spectrum, and that it incorporates gaussian confinement. An $A \to BC$ decay matrix element of the JKJ Hamiltonian involves a pair-production current matrix elements times a scattering matrix element. Diagrammatically this corresponds to an interaction between an initial line and produced pair. In this work we apply the model to the light meson sector and calculate the decay rate, comparing with the experimental values.

Keywords: Meson, Decay, Confinement.

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1. Introduction

Historically, the quark model calculations were used to describe the hadron spectrum. In particular in 80's this method was used by Oka and Yazaki for the calculation of the NN scattering with quark interchange. In the 90's the quark interchange techniques to meson-meson and baryon-baryon scattering were extended by Barnes and Swanson (Ref. [1]) and Hadjimichef *et al* (Refs. [2−4]). Recently these techniques were applied to meson decay, glueballs and other exotics.

In the present work we shall use the Fock-Tani formalism to obtain a decay amplitude for vector mesons ρ.

2. The Meson in the Fock-Tani Formalism

In the Fock-Tani formalism we can write the meson creation operators in the following form $M_\alpha^\dagger = \Phi_\alpha^{\mu\nu} q_\mu^\dagger \bar{q}_\nu^\dagger$, where $\Phi_\alpha^{\mu\nu}$ is the bound-state wave-function for two-quarks. The quark and anti-quark operators obey the following anti-commutation relations

$$\{q_\mu, q_\nu\} = \{\bar{q}_\mu, \bar{q}_\nu\} = \{q_\mu, \bar{q}_\nu\} = \{q_\mu, \bar{q}_\nu^\dagger\} = 0\,,$$
$$\{q_\mu, q_\nu^\dagger\} = \{\bar{q}_\mu, \bar{q}_\nu^\dagger\} = \delta_{\mu\nu}\,. \tag{1}$$

The composite meson operators satisfy non-canonical commutation relations

$$[M_\alpha, M_\beta] = 0\,, \tag{2}$$

and

$$[M_\alpha, M_\beta^\dagger] = \delta_{\alpha\beta} - \Delta_{\alpha\beta}\,, \tag{3}$$

where

$$\Delta_{\alpha\beta} = \Phi_\alpha^{\star\mu\gamma}\Phi_\beta^{\gamma\rho} q_\rho^\dagger q_\mu + \Phi_\alpha^{\star\mu\gamma}\Phi_\beta^{\gamma\rho} \bar{q}_\rho^\dagger \bar{q}_\mu\,. \tag{4}$$

The idea of the Fock-Tani formalism is to make a representation change, where the composite particles operators are described by operators that satisfy canonical commutation relations, i. e., which obey canonical relations

$$[m_\alpha, m_\beta] = 0\,, \tag{5}$$

and

$$[m_\alpha, m_\beta^\dagger] = \delta_{\alpha\beta}\,, \tag{6}$$

where $m_\alpha^\dagger$ is the operators of the "ideal particles" creation. This way one can transform the composite state $|\alpha\rangle$ into an ideal state $|\alpha)$. The transformations are given of the following form

$$|\alpha\rangle \to |\alpha) = U^{-1}|\alpha\rangle\,, \tag{7}$$

and

$$O \to O_{FT} = U^{-1} O\, U\,, \tag{8}$$

where the U operator must be unitary so that

$$\langle\alpha|\alpha\rangle = (\alpha|\alpha)\,, \tag{9}$$

and

$$\langle\alpha\,|O|\,\alpha\rangle = (\alpha\,|O_{FT}|\,\alpha)\,. \tag{10}$$

In the meson case for example we have

$$U^{-1}\, M^\dagger_\alpha|0) = m^\dagger_\alpha|0) \equiv |\alpha)\,, \tag{11}$$

where $U = exp\,(tF)$ and F is the generator of the meson transformation given by

$$F = m^\dagger_\alpha \tilde{M}_\alpha - \tilde{M}^\dagger_\alpha m_\alpha\,, \tag{12}$$

with

$$\tilde{M}_\alpha(t) = \sum_{i=0}^{\infty} \tilde{M}^{(i)}_\alpha(t)\,. \tag{13}$$

3. The Microscopic Hamiltonian

The JKJ Hamiltonian is given by

$$H = \frac{1}{2}\int d^3x d^3y J^\mu_a(\vec{x}) K^{ab}_{\mu\nu}(\vec{x}-\vec{y}) J^\nu_b(\vec{y})\,, \tag{14}$$

with

$$J^\mu_a(\vec{x}) = {\Psi_\alpha}^\dagger(\vec{x})(\Gamma^\mu_a)_{\alpha\delta}\Psi_\delta(\vec{x})\,, \tag{15}$$

where

$$\Gamma^\mu_a = A^\mu \otimes C_a\,, \tag{16}$$

and A^μ is related to the γ^μ matrixes (Ref. [1]). The expression of the Dirac quarks field is

$$\Psi_\alpha(\vec{x}) = \frac{1}{(2\pi)^{3/2}}\int d^3k[u_{s\alpha}(\vec{k})q_s(\vec{k}) + v_{s\alpha}(\vec{k})\bar{q}^\dagger_s(-\vec{k})]e^{i\vec{k}\cdot\vec{x}}\,. \tag{17}$$

After some algebraic manipulations we found

$$H_I = V_{qq} q^\dagger_1 \bar{q}^\dagger_2 q^\dagger_3 q_4 - V_{\bar{q}\bar{q}} q^\dagger_1 \bar{q}^\dagger_2 \bar{q}^\dagger_3 \bar{q}_4\,, \tag{18}$$

where

$$\begin{aligned} V_{qq} = {} & \frac{1}{2(2\pi)^3}\int d^3k_1\, d^3k_2\, d^3k_3\, d^3k_4 \delta(\vec{k_3}-\vec{k_4}+\vec{k_1}+\vec{k_2}) \\ & \times\left[D^{ba}_{\nu\mu}(\vec{k_3}-\vec{k_4}) + D^{ab}_{\mu\nu}(\vec{k_1}+\vec{k_2})\right]\left\{[u^\dagger_3(\Gamma^\nu_b)_{3\,4}u_4][u^\dagger_1(\Gamma^\mu_a)_{2\,1}v_2]\right\}, \quad (19) \\ V_{\bar{q}\bar{q}} = {} & \frac{1}{2(2\pi)^3}\int d^3k_1\, d^3k_2\, d^3k_3\, d^3k_4 \delta(\vec{k_3}-\vec{k_4}+\vec{k_1}+\vec{k_2}) \\ & \times\left[D^{ba}_{\nu\mu}(\vec{k_3}-\vec{k_4}) + D^{ab}_{\mu\nu}(\vec{k_1}+\vec{k_2})\right]\left\{[v^\dagger_4(\Gamma^\nu_b)_{4\,3}v_3][u^\dagger_1(\Gamma^\mu_a)_{2\,1}v_2]\right\}. \quad (20) \end{aligned}$$

We are interested in the calculation of h_{fi}. Applying the Fock-Tani transformation on the interaction Hamiltonian H_I we have

$$\langle f \mid H_{FT} \mid i\rangle = \delta(P_\alpha + P_\beta - P_\gamma) h_{fi}\,, \tag{21}$$

where,

$$\mid f\rangle = m^\dagger_\beta m^\dagger_\alpha \mid 0\rangle\,, \tag{22}$$

and

$$\mid i\rangle = m^\dagger_\gamma \mid 0\rangle\,. \tag{23}$$

Calculating the transition matrix we find

$$\begin{aligned}\langle f \mid H_{FT} \mid i\rangle = V_{qq}\,(\mu\nu;\sigma\rho)\left(\Phi^{\mu\nu}_\alpha\Phi^{\rho\epsilon}_\beta + \Phi^{\mu\nu}_\beta\Phi^{\rho\epsilon}_\alpha - \Phi^{\mu\epsilon}_\alpha\Phi^{\rho\nu}_\beta - \Phi^{\mu\epsilon}_\beta\Phi^{\rho\nu}_\alpha\right)\Phi^{\sigma\epsilon}_\gamma \\ + V_{\bar{q}\bar{q}}\,(\mu\nu;\sigma\rho)\left(\Phi^{\mu\nu}_\alpha\Phi^{\eta\rho}_\beta + \Phi^{\mu\nu}_\beta\Phi^{\eta\rho}_\alpha - \Phi^{\mu\rho}_\alpha\Phi^{\eta\nu}_\beta - \Phi^{\mu\rho}_\beta\Phi^{\eta\nu}_\alpha\right)\Phi^{\eta\sigma}_\gamma\,,\end{aligned} \tag{24}$$

where the ground state wavefunction can be expressed as

$$\Phi^{\mu\nu}_\alpha = \delta(P_\alpha - k_\mu - k_\nu)\varphi(k_\mu, k_\nu)\chi^{\mu\nu}_\alpha \xi^{\mu\nu}_\alpha \mathcal{C}^{\mu\nu}_\alpha\,, \tag{25}$$

with $\varphi(k_\mu, k_\nu)$, $\chi^{\mu\nu}_\alpha$, $\xi^{\mu\nu}_\alpha$ and $\mathcal{C}^{\mu\nu}_\alpha$, being respectively space, spin, flavor and color wavefunction. For this work we consider the following process $\rho \to \pi^+\pi^0$ of the light sector, with ρ meson at rest. Then $P_\gamma = 0$, $P_\beta = -P_\alpha$ and only the confinement term will be considered ($(\Gamma^{\mu_1}_a)_{2\,1} = T^a\gamma^0$).

4. The Confinement, Flavor and Color Factors

In this work we consider that the confinement is gaussian

$$D^{ab}_{\mu_1\nu_1}(\vec{k_1} + \vec{k_2}) = Ae^{-B(\vec{k_1}+\vec{k_2})^2}\,, \tag{26}$$

where A and B are the gaussian parameters. The space wave function is already a gaussian

$$\varphi(k_1, k_2) = \left(\frac{1}{\pi b^2}\right)^{3/4} e^{-\frac{1}{8\beta^2}(k_1-k_2)^2}\,. \tag{27}$$

The flavor factors are $f_1 = f_4 = -1/\sqrt{2}$, $f_2 = f_3 = 1/\sqrt{2}$ and the color factor is $\mathcal{C} = 2^2/3^{3/2}$. Then

$$h_{fi} = \mathcal{C}\left(f_1 h_{fi_1} + f_2 h_{fi_2} + f_3 h_{fi_3} + f_4 h_{fi_4}\right)\,. \tag{28}$$

The full expression for h_{fi} is

$$h_{fi} = \mathcal{C}\left\{ \frac{4}{\sqrt{2}} \frac{1}{(2\pi)^3} A T^a T^b \frac{1}{2m_1} \left(\frac{1}{\pi\beta^2}\right)^{9/4} \left(\frac{\pi}{\frac{1}{\beta^2}+B}\right)^{3/2} \left(\frac{\pi}{\left(\frac{1}{2\beta^2}+B-\frac{B^2\beta^2}{1+B\beta^2}\right)}\right)^{3/2} \right. \\ \left. \times \left[\frac{(5+16B\beta^2)(P_x+iP_y)}{4+12B\beta^2}\right] e^{-\frac{(1+4B\beta^2)P^2}{16\beta^2(1+3B\beta^2)}} \right\}. \quad (29)$$

The decay amplitude for this process is given by

$$\frac{d\Gamma_{\rho\to\pi^0\pi^+}}{d\Omega} = 2\pi P \frac{E_{\pi^0}E_{\pi^+}}{M_\rho} \mid h_{fi} \mid^2 , \quad (30)$$

of where finally we obtain

$$\Gamma_{\rho\to\pi^0\pi^+} = 2\pi^2 P^3 \frac{E_{\pi^0}E_{\pi^+}}{M_\rho} A^2 \frac{4}{3^4} \frac{1}{m_1^2} \left(\frac{1}{\pi\beta^2}\right)^{9/2} \left(\frac{1}{\frac{1}{\beta^2}+B}\right)^3 \\ \times \left(\frac{1}{\left(\frac{1}{2\beta^2}+B-\frac{B^2\beta^2}{1+B\beta^2}\right)}\right)^3 \left[\frac{(5+16B\beta^2)}{4+12B\beta^2}\right]^2 e^{-\frac{2(1+4B\beta^2)P^2}{16\beta^2(1+3B\beta^2)}} . \quad (31)$$

Adjusting the theoretical result to the experimental data of this process, we find $\Gamma = 149.4MeV$ (Ref. [5]), being used for the parameters the following values: $A = 450$ GeV, $B = 5.55$ GeV^{-2}, $m = 0.33$ GeV, $M_\pi = 0.138$ GeV, $M_\rho = 0.77$ GeV and $\beta \approx 0.3 - 0.4$ GeV.

5. Conclusions

The Fock Tani formalism is proven appropriate not only for hadron scattering but for decay. The next step will be to apply the model for all the light sector and compare with other models and experimental data.

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