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RESONANCES: A QUARK VIEW OF HADRON SPECTROSCOPY AND TRANSITIONS*

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I. INTRODUCTION

At this time, when so much of our concern is already focused on what is happening in the TeV energy region for strong interactions, why be interested in resonances? Of course, one might answer that the enumeration of what hadronic states exist and their quantum numbers is one of many subject areas within particle physics, and an incomplete one at that, which should be studied like any other as part of our understanding of physical phenomena. But this increasingly neglected area is still of great interest and importance for reasons other than that of completing a catalogue of states and their properties.

First, the arrangement of states tells us about the symmetries of strong interactions. As we all know, internal symmetry groups like isospin and SU(3), when realized in the normal manner, imply that single particle states fall into multiplets which correspond to irreducible representations of the appropriate group. Further, the existence of such a symmetry group implies relations among amplitudes, e.g., among three point functions appropriate to resonance decays.

Second, if hadrons are "made" of still simpler constituents like quarks,[1,2] this structure may be reflected in a recognizable way in the spectrum of states. In fact, on the basis of a non-relativistic picture of building hadrons out of quarks, an $SU(6) \times O(3)$ description of the hadronic spectrum has arisen. Furthermore, one may employ this to calculate relations among transition amplitudes, although to do so it is necessary to understand both the structure of the states involved and the nature of the operators that induce the appropriate transitions. In such a picture the existence of additional "charmed, quarks should result in "charmed" hadronic states, which remain to be found.[3]

Third, resonances and their properties can tell one about the dynamics of strong interactions at many levels. At a fundamental level, those aspects of the spectrum and amplitudes which point toward an underlying quark basis for strong interactions require one to consider the question of quark confinement. Some very interesting recent approaches to this problem involve either the use of "infrared slavery" arising in asymptotically free gauge theories[4] or the "bag" model.[5] Particularly in the latter case, a number of properties of the low-lying hadron states come simply from the confinement of the quarks.

At a less fundamental level, the multiplet structure, the ordering of states, and the mass splittings between multiplets give us information on the "forces" involved between constituents. A popular model for many years has been that of quarks in a harmonic oscillator potential.[6,7]

Given a hadronic spectrum and two body decay amplitudes, then one may proceed to the next level of dynamics by building up the (non-diffractive) four-point function. According to duality one may obtain the imaginary part of such a non-diffractive amplitude either in terms of a complete sum of resonant states in the direct or crossed channel. In fact, many of the major successes of the duality approach arise from considering cases where one channel is exotic, i.e., where there are no resonances possible according to the quark model. Thus, results of imposing duality such as exchange degeneracy follow from the resonance spectrum, and more particularly, from the absence of exotic states. Finally, the hadron states may be used as "Born terms" in the t-channel to calculate elastic and inelastic two-body and multiparticle scattering amplitudes. As such, what happens at very low energies and how the hadronic resonance spectrum and couplings are organized has a very direct effect on what happens even in the TeV region.

II. SYMMETRIES AND NON-SYMMETRIES

Throughout the following we shall assume that the SU(2) group of isotopic spin transformations is an exact symmetry of strong interactions. Evid for this comes both from the observation of nearly degenerate isospin multipl and from amplitude relations. Whatever breaking of the mass or amplitude rel tions occurs is of order α and therefore seemingly attributable to electromagnetic effects, although these have not actually been calculated since thei magnitude depends on the details of strong interactions themselves.

The larger symmetry group[8] of SU(3) is clearly broken at the 10 to 2C level in masses, but it still gives rise to many clearly identifiable SU(3) multiplets among mesons and baryons. With more than a dozen identified baryo multiplets being slowly filled with states, no one seriously doubts the appli cability and usefulness of SU(3) as a strong interaction symmetry.

SU(3) has also had some success when applied to amplitudes. In particular, application to matrix elements of currents, yielding relations among baryon magnetic moments and among the vector and axial-vector couplings in we decays,[9] are in striking agreement with experiment. Results for decay amplitudes,[10] say for baryon$' \to$ meson + baryon, are in fair agreement with experiment, although there is often some leeway in the choice of barrier factors and mixing parameters when comparison is made with experiment. More striking are the relative signs of amplitudes in reactions like $\bar{K}N \to \pi\Lambda$ and $\bar{K}N \to \pi\Sigma$ which agree very well with SU(3). For four point functions there are major discrepancies when a naive comparison is made with SU(3), but this is well understood in terms of kinematic effects induced by SU(3) breaking on the mas of exchanged particles or on thresholds and barrier factors.[11]

Combining the SU(2) of quark spin with SU(3) gives one an SU(6)--usua called $SU(6)_S$ where S stands for spin. In a non-relativistic picture of quarks bound in a spin and SU(3) independent potential, with total orbital angular momentum L, one could classify the bound states in terms of

$SU(6) \times O(3)$. Much investigation a decade ago showed that $SU(6)$ <u>cannot</u> be a true symmetry in a relativistic theory.[12] Nevertheless, it has increasingly proven to be a very useful algebra with which to classify the hadron states we observe. Although other theoretical approaches (e.g. the bootstrap) presumably have some applicability to hadron spectroscopy, and may well be complementary, it is the quark model which up to now has shown the most promise of a general and basic understanding of the subject. In the following, we shall examine in some detail the consequences of such a quark viewpoint as a basis of both hadron spectroscopy and transitions.

The quark model rules for constructing states go as follows. Mesons are constructed out of a $q\bar{q}$ pair. Adding internal orbital angular momentum, $\vec{L}$, to the quark spin, $\vec{S}$, gives the total $\vec{J}$ of the state. As quarks have spin 1/2, S can only take the values 0 and 1. Given that quarks are in the basic 6 representation of $SU(6)$, and antiquarks in a $\bar{6}$, all meson states are then in the representations contained in $6 \times \bar{6} = 35 + 1$. These have the $SU(3)$ and quark spin content:

$$\begin{aligned} 35:\ & 8 \text{ of } SU(3) \text{ with } S = 1 \\ & 8 \text{ of } SU(3) \text{ with } S = 0 \\ & 1 \text{ of } SU(3) \text{ with } S = 1 \\ 1:\ & 1 \text{ of } SU(3) \text{ with } S = 0\ . \end{aligned} \tag{1}$$

Since fermion and antifermion have opposite intrinsic parity, one has $P = (-1)^{L+1}$ for the $q\bar{q}$ state. For neutral, non-strange mesons, $C = (-1)^{L+S}$ and $G = (-1)^{L+S+I}$.

With these rules there are two kinds of meson states which cannot be formed, i.e., exotic meson states:

1. $SU(3)$ exotics--only 1 and 8 representations of $SU(3)$ are allowed. The 10, $\overline{10}$ and 27 representations, for example, are exotic.
2. CP exotics--only $CP = +1$ natural spin parity ($J^P = 0^+, 1^-, 2^+, \ldots$) states are allowed. There is no state with $J^{PC} = 0^{--}$.

Up to this point, neither type of exotic meson has been found.[13]

For baryons one constructs states from three quarks with a wave function which has overall symmetry in SU(3), L, and S. The antisymmetry expected for fermions may be avoided by postulating quarks to be parafermions[6] (of rank 3), or more simply, by introducing the new quantum number of color.[14] Insisting that all hadrons are singlets under the SU(3)' of color forces the three fermion quarks in a baryon into an antisymmetric 1 representation of color, leaving the remaining part of the wave function to be symmetric.

From the standpoint of SU(6) one would then expect baryons to fall in the representations spanned by $6 \times 6 \times 6 = 56 + 70 + 70 + 20$, whose SU(3) and total quark spin S content are given by:

56: (symmetric)	: 8 of SU(3) with S = 1/2	
	10 of SU(3) with S = 3/2	
70: (mixed symmetry)	: 1 of SU(3) with S = 1/2	
	8 of SU(3) with S = 1/2	
	10 of SU(3) with S = 1/2	(2)
	8 of SU(3) with S = 3/2	
20: (antisymmetric)	8 of SU(3) with S = 1/2	
	1 of SU(3) with S = 3/2	

The parity will be simply given by $P = (-1)^L$. Since only the 1, 8, and 10 representations of SU(3) are contained above, any other representation (e.g. $\overline{10}$, 27) is exotic for baryons. Evidence for exotic baryons is not conclusive.[15]

III. MESON STATES

Let us then proceed to see how the known meson states compare with the SU(6) × O(3) picture outlined above. We do so with the vague idea that any reasonable "potential" will have states with small values of L lying lowest.

For $L = 0$ we expect $35 + 1$ states, all with parity $P = -1$, including $8 + 1$ vector and $8 + 1$ pseudoscalar mesons. The observed lowest mass mesons exactly fill this multiplet structure as shown[16] in Table I. As is well known, the physical ω and φ are mixtures of the octet and singlet states with a "magic" mixing angle $\theta = \cos^{-1}(2/3)$. The η may also be slightly mixed with the η' or higher mass states. Another candidate for the slot occupied by the η' (or mixed with it) is the E (1422). Although it is usually forgotten, it is entirely non-trivial that the lowest mass mesons have negative parity and that exactly those states required by the quark model are found, and no more!

TABLE I

Meson States with $L = 0$

SU(6)	SU(3)	S	J^{PC}	States[17]
35	8	0	0^{--}	π (140) K (495) η (550)
35	8 + 1	1	1^{--}	ρ (770) K^* (890) ω (784) φ (1020)
1	1	0	0^{--}	probably mainly η' (958)

The next principal set of states we expect are those with $L = 1$, all of which have positive parity. We first examine those with quark spin $S = 1$, shown in Table II. The B is now well established[18] from massive $\pi^+ p$ bubble chamber experiments and has $\pi\omega$ as a main decay mode. None of the other states is established, with the K^* state being lost in the non-resonant

"Q-bump" in $K^{\pm}N$ reactions. However, SU(3) tells us that the rest of the octet had better be there. Nondiffractive processes are the obvious place to look. The H and/or H' may be very broad (decaying into $\pi\rho$), and correspondingly difficult to find.[19]

TABLE II

Meson States[17] with $L = 1$ and $S = 0$

SU(6)	SU(3)	S	J^{PC}	States
35	8	0	1^{+-}	B (1235) K^* (1320?)
1	1	0	1^{+-}	H ? H' ? } may be mixed

The $S = 1$ states with $L = 1$ are even more problematic, as shown in Table III.

TABLE III

Meson States[17] with $L = 1$ and $S = 1$

SU(6)	SU(3)	S	J^{PC}	States
35	8 + 1	1	2^{++}	A_2 (1310) K^* (1420) f (1270) f' (1514)
35	8 + 1	1	1^{++}	A_1 (1100?) K^* (1240?) D (1285) D' ?
35	8 + 1	1	0^{++}	δ (970) K^* (1300?) ϵ (700) ? S^* (997) ϵ' (1300) ?

Here the $J^{PC} = 2^{++}$ states are all found, with the f and f' again being mixed octet and singlet states like the ω and φ. The 1^{++} states are an embarassment with only the D now well established. The famous A_1 and its K^* SU(3) partner are not found as resonances in the dominantly diffractive reactions $\pi p \rightarrow (3\pi)p$ or $Kp \rightarrow (K\pi\pi)p$. While there are hints of a $K\pi\pi$ state at ~ 1240 MeV in $\bar{p}p$ annihilations and in $\pi^- p \rightarrow (K\pi\pi)\Lambda$, these results are not conclusive by any means. A much more intensive look at nondiffractive channels is needed to search for these states, and in the process we should not be prejudiced by the "masses" for the corresponding diffractively produced non-resonant bumps.

The scalar mesons[18] are in fair shape now that the δ (970) is established, with $\pi\eta$ as principle decay mode. With the s-wave $K\pi$ phase shift rising through 90° at ~ 1300 MeV, it seems likely there is an appropriate K^* near that mass. At present there are too many $I = 0$ candidates, although both the ϵ and ϵ' may not survive as resonant states. One possibility, explored by Morgan,[20] is to form the scalar octet plus singlet out of δ, K^*, S^* and ϵ', with the S^* and ϵ' mixed.

Candidates to fill out the $L = 2$ multiplets are lacking in most cases. As seen in Table IV, only the 3^{--} states have been mostly established. The K^* (1800) has only recently been established by a SLAC group investigating $K\pi$ scattering.[21] The assignment, and even existence, of the F_1, ρ', and A_3 is somewhat speculative.

There are hints of a few other multiplets for mesons.[22] One possibility is a radial excitation of the ground state $35 + 1$ with $L = 0$. Candidates for this include the E (1420) and a proposed ρ' (1250) or the ρ' (1600). Note that given the quark model, not only does the proposal of a new state require in general the remainder of its SU(3) multiplet be found, but all of its SU(6) partners. Here one needs to see a π', an ω', a φ', etc.--a nontrivial requirement which should make one somewhat skeptical on the existence of all these unseen states.

TABLE IV

Meson States[17] with $L = 2$

SU(6)	SU(3)	S	J^{PC}	States
35	8 + 1	1	3^{--}	g (1680) K^* (~ 1800) ω_3 (1675) φ_3 (?)
35	8 + 1	1	2^{--}	F_1 (1540)? ?
35	8 + 1	1	1^{--}	ρ' (1680) ? ?
35	8	0	2^{-+}	A_3 (1640) ? ?
1	1	0	2^{-+}	?

At still higher mass there is now evidence[18] from $\pi\pi \rightarrow K\bar{K}$ for the first of the 4^{++} states expected for $L = 3$. And then there are indications from $\bar{p}p$ reactions for bumps in the T (2190) and U (2360) regions. The particularly interesting possibility of towers of states has been raised from a recent analysis[23] of $\bar{p}p \leftrightarrow \pi\pi$, although evidence could already be deduced[24] for this from the spectrum of states at lower mass. How and if the quark model states coexist with a pattern of towers or of Regge daughter states is one of many unsettled questions concerning the spectrum of hadron states.

IV. BARYON STATES

Because of extensive phase shift analyses, baryon spectroscopy is a much richer experimental area with which to compare our theoretical expectations. Even so, only the nucleon resonances below about 2 GeV in mass can be said to have been investigated with any claim of completeness. As such, we shall only list N^* candidates for each SU(3) multiplet, with the exception of the ground state. The Y^*'s are still only in fair shape, while the status of Ξ^*'s can only be described as poor.

The full set of states in the ground state 56 with $L = 0$ was completed ten years ago with the discovery of the Ω^-. They appear[16] in Table V.

TABLE V

Baryons in the 56 L = 0 Ground State

SU(6)	SU(3)	S	J^P	States[17]
56	8	1/2	$1/2^+$	N (940)
				Λ (1115)
				Σ (1193)
				Ξ (1317)
56	10	3/2	$3/2^+$	Δ (1232)
				Σ^* (1385)
				Ξ^* (1530)
				Ω^- (1672)

The next highest mass states observed all have negative parity, as befits $L = 1$, and they fit nicely into a 70 of SU(6). As Table VI shows, the established negative parity N^*'s below 2 GeV provide all the candidates for the SU(3) and J^P multiplets in a 70 L = 1 with no omissions or additions. Mixing of the two $J^P = 1/2^-$ N^*'s, $3/2^-$ N^*'s, and three $1/2^-$ Λ^*'s, $3/2^-$ Σ^*'s, etc., can, and presumably does, take place. A recent discussion of candidates for the Y^* states (most of which are now known) and the possible mixings can be found in Cashmore et al.[25]

Also essentially complete in having candidates for all the nonstrange states is a 56 with $L = 2$ and $P = +1$, as shown in Table VII.

TABLE VI

Non-Strange Baryons in the 70 L = 1

SU(6)	SU(3)	S	J^P	States
70	1	1/2	$1/2^-$	[Λ^* (1405)]
			$3/2^-$	[Λ^* (1520)]
	8	1/2	$1/2^-$	S_{11} (1535)...
			$3/2^-$	D_{13} (1520)...
	10	1/2	$1/2^-$	S_{13} (1650)...
			$3/2^-$	D_{33} (1670)...
	8	3/2	$1/2^-$	S_{11} (1700)
			$3/2^-$	D_{13} (1700)...
			$5/2^-$	D_{15} (1670)...

TABLE VII

Baryons in the 56 with L = 2

SU(6)	SU(3)	S	J^P	States[17]
56	8	1/2	$5/2^+$	F_{15} (1688)...
			$3/2^+$	P_{13} (1810)...
	10	3/2	$7/2^+$	F_{37} (1950)...
			$5/2^+$	F_{35} (1890)...
			$3/2^+$	P_{33} (~1900 ?)...
			$1/2^+$	P_{31} (1910)...

The remaining P_{33} state below 2 GeV we classify with the Roper resonance as forming a radially excited 56 with L = 0 (Table VIII).

TABLE VIII

Baryons in a radially excited 56 with L = 0

SU(6)	SU(3)	S	J^P	States[17]
56	8	1/2	$1/2^+$	P_{11} (1470)...
	10	3/2	$3/2^+$	P_{33} (1690)...

We do this both for reasons of mass and because of inelastic amplitude signs, to be discussed later.

Some other possible multiplets can be proposed on the basis of picking through the relatively few nonstrange baryon states remaining in the tables.[17] First, the $P_{11}(1780)$ probably belongs in a second radially excited 56 L = 0. This requires finding yet another P_{33} state, presumably around 2100 MeV, to be its non-strange companion in a 56.

There are several negative parity states in the 2000-2200 MeV range which are good candidates for members of a 70 L = 3 multiplet. In particular the G_{17} (2190) and D_{15} (2100) states fit the S = 1/2 octet slots in such a multiplet, while the D_{35} (1960) and an undiscovered G_{37} state could be the decuplet S = 1/2 members. That leaves D_{13}, D_{15}, G_{17}, and G_{19} states to be found, presumably several hundred MeV higher in mass, to fit into the required S = 3/2 octets.

There are also several candidates for a radially excited 70 L = 1 multiplet in the same region. The D_{13} (2040) and S_{11} (2100) fit in as the octet S = 1/2 states. The S_{31} (1900) and a D_{33} would be in the S = 1/2 decuplet, leaving S_{11}, D_{13}, and D_{15} states to be found at ~ 2200 MeV to fill the octets with S = 3/2.

As for higher mass positive parity states, there is the beginning of a 56 L = 4 multiplet containing the $H_{19}(2200)$ and $F_{17}(1990)$ as octet S = 1/2 members, and the $H_{3,11}(2420)$ as the highest spin Δ^*, with H_{39}, F_{37}, and F_{35} states yet to be found for the remaining $S = 3/2$ slots. Finally another F_{15} state at ~ 2000 MeV would be the beginning of a radially excited 56 L = 2 multiplet.

On looking back over the above classification of baryons into multiplets there is an obvious pattern:[15,22] 56 representations have even L, 70 representations odd L. While one could classify the observed states in a way which breaks this "rule," they fit it well and there is no compelling reason to do so. Note that this "rule" and the baryon spectrum are then not consistent with the states expected from a three dimensional harmonic oscillator potential where, for example, one expects[6] a 70 L = 2, 70 L = 0, 56 L = 0, and 20 L = 1 in the same mass region as the 56 L = 2. If the spectrum of baryons is as simple as it now seems to be, one hopes there would be a deeper reason for that simplicity. Another interesting way of looking at the $I = 1/2$ N^* states we have been discussing is shown in Fig. 1. Is it possible we have a tower structure developing? And if so, as for mesons, what is its relation to the quark model picture we have been discussing?

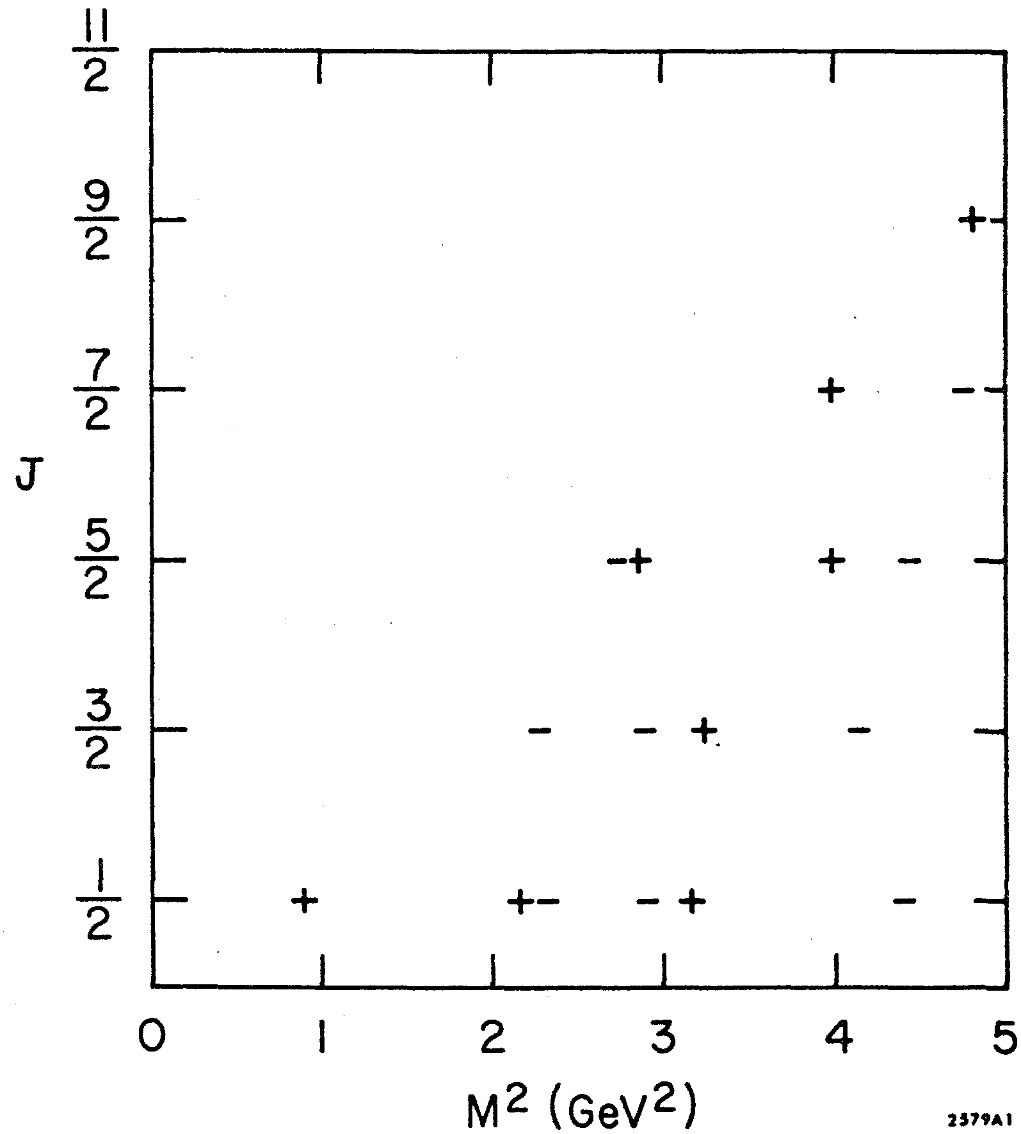

Figure 1

Spins vs. mass squared for the known $I = 1/2$ N^* resonances. Positive (negative) parity states are denoted by + (-).

V. TRANSITIONS AMONG HADRONS

Given the spectroscopy of hadrons in terms of quark constituents which we have built up in the preceeding sections, we now turn to transitions between these states. We restrict ourselves to matrix elements of currents at $q^2 = 0$. For the vector current, such matrix elements are directly related to the amplitudes for one photon decay or excitation. The axial-vector current presents more of a problem in that few weak axial-vector transitions are measured. But via the PCAC hypothesis,[26] one may relate such matrix elements to pion amplitudes, which are the mainstay of strong interaction decays.

However, to be able to carry out a calculation of such matrix elements we must actually solve two problems at once. First, we must understand the currents, their symmetry properties, and commutation relations. Second, we must understand hadron spectroscopy, how different hadron states are related, and how these currents "flow" inside them. These two problems in fact have been partially solved in recent times by relating them, i.e. by finding a transformation between the quarks seen by currents and those which we used earlier as the building blocks of hadrons.

The result is an approximate theory of photon and pion transition matrix elements within the context of the quark model. The theory yields many relations among decay widths and predicts with great success the relative ampli tude signs in inelastic processes like $\pi N \to N^* \to \pi\Delta$ and $\gamma N \to N^* \to \pi N$. The agreement with experiment that is found leads one to have further confidence in the quark model for spectroscopy, particularly if the assignment of observed resonances to the states in the model, and lends support as well to the theory of current-induced transitions.

VI. CURRENTS AND QUARKS

In order to formulate a theory of current-induced-transitions among hadrons composed of quarks we need a group theoretic frame work for labeling the states and operators involved. For this purpose it is natural to turn to an algebra of charges formed by integrating weak and electromagentic current densities over all space.

To start with, consider vector and axial-vector charges:

$$Q^{\alpha}(t) = \int d^3x\ V_0^{\alpha}(\vec{x},t) \tag{3a}$$

$$Q_5^{\alpha}(t) = \int d^3x\ A_0^{\alpha}(\vec{x},t) \tag{3b}$$

where α is an SU(3) index which runs from 1 to 8 and $V_{\mu}^{\alpha}(\vec{x},t)$ and $A_{\mu}^{\alpha}(\vec{x},t)$ are the local vector and axial-vector current densities with measurable matrix elements. The vector charges are just the generators of SU(3). These integrals over the time components of the current densities are assumed to satisfy the equal-time commutation relations proposed by Gell-Mann[8]

$$\begin{aligned} [Q^{\alpha}(t),\ Q^{\beta}(t)] &= if^{\alpha\beta\gamma}\ Q^{\gamma}(t) \\ [Q^{\alpha}(t),\ Q_5^{\beta}(t)] &= if^{\alpha\beta\gamma}\ Q_5^{\gamma}(t) \\ [Q_5^{\alpha}(t),\ Q_5^{\beta}(t)] &= if^{\alpha\beta\gamma}\ Q^{\gamma}(t)\ , \end{aligned} \tag{4}$$

where $f^{\alpha\beta\gamma}$ are the structure constants of SU(3). Sandwiched between nucleon states at infinite momentum, the last of Eqs. (4) gives rise to the Adler-Weisberger sum rule.[27] From this point on, we shall always be considering matrix elements to be taken between hadron states[28] with $p_z \to \infty$.

For the purposes at hand we need a somewhat larger algebraic system then that provided by the measurable vector and axial-vector charges in Eqs. (3), which form the algebra of SU(3) $\times$ SU(3) according to Eqs. (4). To obtain the larger algebra we adjoin to the integrals over all space of[29] $V_0^\alpha(\vec{x},t)$ and $A_z^\alpha(\vec{x},t)$, those of the tensor current densities $T_{yz}^\alpha(\vec{x},t)$ and $T_{zx}^\alpha(\vec{x},t)$. In the free quark model these charges have the form:

$$\begin{aligned}
\int d^3x\ V_0^\alpha(\vec{x},t) &\sim \int d^3x\ \psi^+(x)\ (\tfrac{\lambda^\alpha}{2})\ \mathbb{1}\ \psi(x) \\
\int d^3x\ A_z^\alpha(\vec{x},t) &\sim \int d^3x\ \psi^+(x)\ (\tfrac{\lambda^\alpha}{2})\ \sigma_z\ \psi(x) \\
\int d^3x\ T_{yz}^\alpha(\vec{x},t) &\sim \int d^3x\ \psi^+(x)\ (\tfrac{\lambda^\alpha}{2})\ \beta\sigma_x\ \psi(x) \\
\int d^3x\ T_{zx}^\alpha(\vec{x},t) &\sim \int d^3x\ \psi^+(x)\ (\tfrac{\lambda^\alpha}{2})\ \beta\sigma_y\ \psi(x)
\end{aligned} \tag{5}$$

where $\psi(x)$ is the Dirac (and SU(3)) spinor representing the quark field. When commuted using the free quark field commutation relations, these charges act algebraically like the product of SU(3) and Dirac matrices $(\lambda^\alpha/2)\mathbb{1}$ $(\lambda^\alpha/2)\sigma_z$, $(\lambda^\alpha/2)\beta\sigma_x$, and $(\lambda^\alpha/2)\beta\sigma_y$ respectively.[30] The Dirac matrices $\beta\sigma_x$, $\beta\sigma_y$, and σ_z form the so-called W-spin.[31] They are invariant under boosts in the z-direction and the corresponding charges are "good," in the sense that they have finite (generally non-vanishing) matrix elements between states as $p_z \to \infty$. This makes them the correct set of charges to use to label states in terms of their internal quark spin components. If we let $\alpha = 0$ correspond to the SU(3) singlet representation (and λ^0 be a multiple of the unit matrix), then Eqs. (5) consists of 36 charges which close under commutation. They act like an identity operator plus 35 other generators of an SU(6) algebra. We call this algebra the $SU(6)_W$ of currents[30] because of its origin. Q^α and Q_5^α then essentially[29] form a chiral $SU(3) \times SU(3)$ subalgebra of this larger algebra.

Given such an algebra, we define the smallest representations of it (other than the singlet), the 6 and $\bar{6}$ representations, as the current quark (q) and current antiquark ($\bar{q}$) respectively. We may build up all the larger representations of $SU(6)_W$ out of these basic ones.

Can then real baryons be written as three current quarks, qqq, and real mesons as current quark and antiquark, $q\bar{q}$, with internal angular momentum L, as in the constituent quark model used for hadron spectroscopy? While possible in principle, it is a disaster when compared with experiment. For it leads to $g_A = 5/3$, zero anomalous magnetic moment of the nucleon, no electromagnetic transition from the nucleon to the 3-3 resonance (Δ), no decay of ω to $\gamma\pi$, etc. It would also yield results for masses like $M_N = M_\Delta$, $M_\pi = M_\rho$, etc. The hadron states we see cannot be simple in terms of current quarks. They must lie in mixed representations of the $SU(6)_W$ of currents. Work in past years has shown directly that hadron states are quite complicated when viewed in terms of current algebra.[32]

We may restate this complication in terms of the definition of an operator V for any hadron:

$$\begin{aligned} |\text{Hadron}\rangle &\equiv V|\text{simple } qqq \text{ or } q\bar{q} \text{ state of current quarks}\rangle \\ &= |\text{simple } qqq \text{ or } q\bar{q} \text{ state of constituent quarks}\rangle \qquad (6) \end{aligned}$$

All the complication of real hadrons under the $SU(6)_W$ of currents (i.e., in terms of current quarks) has been swept into the operator V. On the other algebraic properties of the most general combination of single quark operators consistent with SU(3) and Lorentz invariance.

Thus, while Eq. (5) shows that Q_5^α itself behaves under the $SU(6)_W$ of currents as simply

$$\int d^3x \; \psi^+(x) \; (\frac{\lambda^\alpha}{2}) \; \sigma_z \; \psi(x) \; ,$$

a direct calculation in the free quark model shows that algebraically $V^{-1} Q_5^\alpha V$ behaves as a sum of two terms.[40]

$$V^{-1} \, Q_5^\alpha \, V$$

$$\sim (\frac{\lambda^\alpha}{2}) \; \sigma_z + (\frac{\lambda^\alpha}{2}) \; [(\beta\sigma_x + i\beta\sigma_y)(v_x - iv_y) - (\beta\sigma_x - i\beta_y)(v_x + iv_y)] \; , \tag{8}$$

where the products of Dirac and SU(3) matrices are understood to be taken between quark spinors (and integrated over all space). Here v_μ is a vector in configuration space, so that $v_x \pm iv_y$ raises (lowers) the z component of angular momentum (L_z) by one unit. The particular combination of Dirac matrices and vector indices in the two terms in Eq. (8) is dictated by the demands that the total $J_z = 0$ and the parity be odd for the axial-vector charge, Q_5^α, and for $V^{-1} \, Q_5^\alpha V$.

For the vector charge, Q^α, we must have

$$V^{-1} Q^\alpha V = Q^\alpha \; , \tag{9}$$

since we want these charges to be the generators of SU(3), both before and after the transformation. However, the first moment of the charge density,[41]

$$D_+^\alpha = i \int d^3x \left(\frac{-x-iy}{\sqrt{2}} \right) \; V_0^\alpha(\vec{x},t) \; , \tag{10}$$

is not a generator and is transformed non-trivially by V. One finds in the free quark model that in algebraic properties $V^{-1} \, D_+^\alpha V$ behaves as a sum of four terms under the $SU(6)_W$ of currents:[42]

hand, real hadrons are supposed to be simple in terms of the "constituent quarks" used for spectroscopy purposes, as indicated by the second equality in Eq. (6). In other words, the transformation V connects the two simple descriptions in terms of current quarks and constituent quarks.[33] It is for this reason that it is sometimes called the "transformation from current to constituent quarks."[34,35]

Up to this point we have only managed to restate the problem via Eq. (6). But as often happens, phrasing the problem right is a major way toward the solution. For what we are after in the end are matrix elements of various current operators, $\mathcal{O}$. Using Eq. (6) and assuming V is unitary we may write

$$\langle \text{Hadron}' | \mathcal{O} | \text{Hadron} \rangle$$

$$= \langle (\text{simple current quark state})' | V^{-1} \mathcal{O} V \, | (\text{simple current quark state}) \rangle \tag{7}$$

This has two important advantages. First, we may study the properties of $V^{-1}\mathcal{O}V$ in isolation, and then apply what we learn to the matrix elements of $\mathcal{O}$ between any two hadron states. Second, even though V itself is very complicated and contains (by definition) all information on the current quark composition of each hadron, it is possible that the object $V^{-1}\mathcal{O}V$ for some operators $\mathcal{O}$ may be relatively simple in its algebraic transformation properties.

This last possibility is of course exactly what we shall assume on the basis of calculations done in the free quark model. In that model, Melosh[36] and others[37,38,39] have been able to formulate and explicitly calculate the transformation V. While one would not take the details of the transformation found there as correctly reflecting the real world, one might try to abstract the algebraic properties of some transformed operators $V^{-1}\mathcal{O}V$, from such a calculation. In cases of interest, this turns out to be equivalent to assuming that the transformed operators $V^{-1}\mathcal{O}V$ have the

$$V^{-1}D_+^\alpha V$$

$$\sim (\tfrac{\lambda^\alpha}{2})\, 1\!\!1(v_x + iv_y) + (\tfrac{\lambda^\alpha}{2})\,(\beta\sigma_x + i\beta\sigma_y) + (\tfrac{\lambda^\alpha}{2})\,\sigma_z(v_x + iv_y)$$

$$+ (\tfrac{\lambda^\alpha}{2})\,(\beta\sigma_x - i\beta\sigma_y)(v_x + iv_y)(v_x + iv_y)\ , \tag{11}$$

where again the Dirac and SU(3) matrices are understood to be taken between quark spinors.

We abstract the algebraic properties of $V^{-1}Q_5^\alpha V$ and $V^{-1}D_+^\alpha V$ given in Eqs. (8) and (11) from the free quark model and assume them to hold in the real world. We are then able to treat matrix elements of Q_5^α and D_+^α between hadron states as follows:

(1) We identify the hadrons with qqq or $q\bar{q}$ states of the constituent quark model where the total quark spin S is coupled to the internal angular momentum L to form the total J of the hadron. The states so constructed fall into $SU(6)_W \times O(3)$ multiplets.

(2) Since very few weak axial-vector transitions are measured, given a matrix element of Q_5^α, we use PCAC to relate it to a measured pion transition amplitude. Application of the golden rule then yields:

$$\Gamma(H' \to \pi^- H) = \frac{1}{4\pi f_\pi^2}\,\frac{p_\pi}{2J'+1}\,\frac{(M'^2 - M^2)^2}{M'^2}\ \sum_\lambda \left|\langle H',\lambda|\ (1/\sqrt{2})\ (Q_5^1 - iQ_5^2)|\ H,\lambda\rangle\right|^2\ , \tag{12}$$

where $f_\pi \simeq 135$ MeV. The factors in Eq. (12) are forced on us by PCAC and kinematics--there are no arbitrary phase space factors.

For real photon transitions, matrix elements of $D_+^3 + (1/\sqrt{3})D_+^8$ are directly proportional to the corresponding Feynman amplitudes. The width for $H' \to \gamma H$ is given by[41]

$$\Gamma(H' \to \gamma H) = \frac{e^2}{\pi}\,\frac{p_\gamma^3}{2J'+1}\sum_\lambda \left|\langle H',\ \lambda|\ D_+^3 + (1/\sqrt{3})\ D_+^8|H,\ \lambda-1\rangle\right|^2\ . \tag{13}$$

(3) Given a matrix element of Q_5^α or D_+^α between hadron states which is related to measurements by either Eq. (12) or (13), we transform using V from simple constituent to simple current quark states. The particular matrix element is thus rewritten in terms of $V^{-1}Q_5V$ or $V^{-1}D_+V$, and simple current quark states. We know the algebraic properties of all these quantities under the $SU(6)_W$ of currents via abstraction of Eqs. (8) and (11) from the free quark model and our identification of hadrons with quark model states. We may then apply the Wigner-Eckart theorem to each term to express it as a Clebsch-Gordan coefficient[43] (of $SU(6)_W$) times a reduced matrix element. Since the same reduced-matrix element occurs in many different transitions, relations among the corresponding transition amplitudes follow.

VII. CONSEQUENCES FOR TRANSITION AMPLITUDES

The experimental consequences of the theory outlined in the last section have been considered by a number of authors.[36,44-53] These consequences fall into the following three categories:

(1) Selection Rules. For transitions by pion or photon emission from states (either mesons or baryons) with internal angular momentum L' to those with L, one finds[46,47]

$$\left||L' - L| - 1\right| \leq \ell_\pi \leq L + L' + 1 \tag{14a}$$

$$\left||L' - L| - 1\right| \leq j_\gamma \leq L + L' + 1 , \tag{14b}$$

where ℓ_π and j_γ are the total angular momentum carried off by the pion and photon in the overall transition.

For example, ℓ_π can be 0 or 2 ($\ell_\pi = 1$ is forbidden by parity), but not 4 for a pion decay from $L' = 1$ to $L = 0$. Thus the decay of the $D_{15}(1670)$, the $J^P = 5/2^- N^*$ resonance with $L' = 1$, into $\pi\Delta$ is forbidden in g-wave ($\ell_\pi = 4$), although otherwise allowed by kinematical considerations. Similarly, only $j_\gamma = 1$ is allowed for $L' = 0$ to $L = 0$ photon transitions, although

$j_\gamma = 2$ (and even $j_\gamma = 3$ for $\Delta' \to \gamma\Delta$) is generally permitted by kinematics. This particular rule is well-known for $\Delta \to \gamma N$, where it is just the successful quark model result[54] that the transition is purely magnetic dipole in character, i.e. the possible electric quadrupole amplitude is forbidden. The inequalities in Eqs. (14) might be regarded as the generalization of these particular results to all L and L' in the present theoretical context.

Note that for $|L - L'| \geq 3$ the lower limit of the inequalities becomes operative in a non-trivial way, forbidding low values of ℓ_π or j_γ which would otherwise have been favored kinematically. Unfortunately, the relevant hadron states which would provide an interesting test of this have not yet been found.

Selection rules of a different sort govern the number of independent reduced matrix elements. For pion transitions from a hadron multiplet with internal angular momentum L' down to the ground state hadrons with $L = 0$, there are at most two independent reduced matrix elements, corresponding to the two terms in Eq. (8). For real photon transitions between the same two multiplets there are at most four independent reduced matrix elements, corresponding to Eq. (11).

In general structure, the theory described above includes various concrete quark model calculations, both non-relativistic[55] and relativistic.[56] In fact, a one-to-one correspondence exists between the quantities calculated in such models and the reduced matrix elements in the present theory. However, such models are usually much more specific, with parameters like the strength of the "potential," quark masses, etc. fixed. Since the quantities corresponding to reduced matrix elements are expressed explicitly in terms of such parameters, they are computable numerically and the scale of the reduced matrix elements is determined.

Also included in the general structure of the theory are the results following from assuming strong interaction $SU(6)_W$ conservation.[31] For pion transitions, this corresponds in the present theory to retaining only the st term in $V^{-1}Q_5^\alpha V$. Since this hypothesis fails experimentally, various hoc schemes for breaking $SU(6)_W$ have been proposed.[57] Such schemes still l within the general structure of amplitudes presented above,[58] and they similar in giving relations between amplitudes while not setting their solute scale.[59] However, as we shall see below, they are generally more strictive in that they tie together pion and rho decay amplitudes.

(2) Decay Widths. The simplest such set of relations are those for on transitions from $L' = 0$ to $L = 0$ mesons. Here there is only one reced matrix element (the second term in Eq. (6) has $\Delta L_z = \pm 1$ and so cannot ntribute when $L' = L = 0$), so that the amplitudes for $\rho \to \pi\pi$, $K^*(890) \to \pi K$, d $\omega \to \pi\rho$ are all proportional. The ratio of the amplitudes for the first o processes may be obtained from $\Gamma(\rho \to \pi\pi)/(K^* \to \pi K)$, while the amplitude r the latter is obtainable from $\omega \to 3\pi$ and rho dominance. Within errors, e ratio of the three amplitudes is that predicted by the theory.[60]

For pion transitions from mesons with internal angular momentum $L' = 1$ those with $L = 0$, both terms in Eq. (8) are possible and there are conseuently two independent reduced matrix elements which describe all such decays. ather than performing a fit to all the data, we choose two measured widths s input and thereby determine all the other decay rates. For this purpose e take $\Gamma(A_2 \to \pi\rho) = 71.5$ MeV, from the latest particle data tables,[17] and $\Gamma_{\lambda=0}(B \to \pi\omega) = 0$. This latter condition, the vanishing of the helicity zero (longitudinal) decay of $B \to \pi\omega$, is suggested by high statistics experiments[61] which find the transverse decay to be strongly dominant. While probably not exactly zero, we take this as a very reasonable first approximation to the data. Exact vanishing of $\Gamma_{\lambda=0}(B \to \pi\omega)$ corresponds to only the second term in $V^{-1}Q_5^\alpha V$, with the algebraic properties of $(\lambda^\alpha/2)[(\beta\sigma_x + i\beta\sigma_y)(v_x - iv_y) - (\beta\sigma_x - i\beta\sigma_y)(v_x + iv_y)]$, having a non-zero reduced matrix element. This well illustrates the experimental necessity of a non-trivial transformation V; for if $V = \mathbb{1}$, only the term behaving as $(\lambda^\alpha/2)\sigma_z$ would be present and the predicted helicity structure for $B \to \pi\omega$ would be completely opposite that observed.

The results[62] can be seen in Table IX. The correct values for $\Gamma(A_2 \to \pi\rho)/\Gamma(K^*(1420) \to \pi K^*)$ and $\Gamma(f \to \pi\pi)/\Gamma(K^*(1420) \to \pi K)$ may be regarded as testing the SU(3) component of the theory, while, for example, the value of $\Gamma(A_2 \to \pi\rho)$ or $\Gamma(K^*(1420) \to \pi K^*)$ relative to $\Gamma(f \to \pi\pi)$, $\Gamma(K^*(1420) \to \pi K)$ or $\Gamma(A_2 \to \pi\eta)$ tests the full theory, including the phase space factors in Eq. (12), since one is relating d-wave pion decays into pseudoscalar vs. vector mesons. As for the other decays in the table, we note that: (a) other strong interaction decay modes of the B meson very likely exist, although $\pi\omega$ is certainly dominant; (b) the "real" A_1 resonance still remains to be found for comparison with the theory; (c) the now established $I = 1$ scalar meson, δ, only has $\pi\eta$ as a possible strong decay channel, so the total width should almost coincide with that into $\pi\eta$; (d) we have chosen 1300 MeV, the mass where the s-wave πK phase shift goes through 90°, as the mass of the strange, $J^P = 0^+$ meson.[63] The overall agreement found in Table IX between theory and experiment is quite good, with the exception of $\Gamma(A_2 \to \pi\eta')$. While mixing of the pseudo-scalar mesons is such as to alleviate this discrepancy, reasonable mixing angles do not change the width appreciably from the value in Table IX. A more likely source of trouble lies in the theoretical assignment of the η' to be dominantly that SU(3) singlet pseudoscalar meson associated with the octet containing the pion and eta. In any case, an actual measurement of the $A_2 \to \pi\eta'$ decay width, rather than an upper limit, would be an interesting quantity to determine experimentally.

For $L' = 2$ mesons decaying by pion emission to the $L = 0$ states, there are again two independent reduced matrix elements. About the only decay width determined with any certainty is $g \to \pi\pi$. The meagre information available on other decays is consistent with the theory within the large experimental errors.[47]

For photon decays of mesons the data are even more sparse, although there are plenty of theoretical predictions.[52] In fact, only a few decays

TABLE IX

Decays of $L' = 1$ Mesons to $L = 0$ Mesons by Pion Emission.[62]

Decay	Γ(predicted) (MeV)	Γ(experimental)[17] (MeV)
$A_2(1310) \to \pi\rho$	71.5 (input)	71.5 ± 8
$K^*(1420) \to \pi K^*$	27	29.5 ± 4
$f\ (1270) \to \pi\pi$	112	141 ± 26
$K^*(1420) \to \pi K$	55	55 ± 6
$A_2(1310) \to \pi\eta$	16	15 ± 2
$A_2(1310) \to \pi\eta'$	5	< 1
$B\ (1235) \to \pi\omega,\ \lambda = 0$	0 (input)	$\Gamma_{total} = 120 \pm 20$
$\lambda = 1$	75	$\pi\omega$, with $\lambda = 1$ strongly dominant, only mode seen
$A_1(1100) \to \pi\rho,\ \lambda = 0$	63	??
$\lambda = 1$	31	
$\delta\ (970) \to \pi\eta$	41	50 ± 20
$\kappa(1300) \to \pi K$	380	?, broad

among $L' = 0$ mesons are actually measured, where there is just one possible reduced matrix element. Fixing this from $\Gamma(\omega \to \gamma\pi)$, the predictions[64] are collected in Table X. What widths have been measured are consistent with the predictions of the theory, although at the limits of the error bars in several cases.

TABLE X

Decays of $L' = 0$ Mesons to Other $L = 0$ Mesons by Photon Emission

	Γ(predicted) no mixing (KeV)	Γ(predicted) $\theta_p = -10.5°$ (KeV)	$\Gamma(\text{experimental})^{17}$ (KeV)
$\omega \to \gamma\pi$	870 (input)	870 (input)	870 ± 60
$\rho \to \gamma\pi$	92	92	$30 \pm 10 \leq \Gamma \leq 80 \pm 10$ (Ref. 65)
$\varphi \to \gamma\pi$	0	0	< 14
$\rho \to \gamma\eta$	36	56	< 160
$\omega \to \gamma\eta$	5	7	< 50
$\varphi \to \gamma\eta$	220	170	126 ± 46
$\eta' \to \gamma\rho$	160	120	$0.27\Gamma(\eta' \to \text{all})$
$\eta' \to \gamma\omega$	15	11	
$\varphi \to \gamma\eta'$	0.5	0.6	

There are a large number of pion and photon transitions among baryons which are predicted by the theory. They are compared with experiment elsewhere.[46,47,52,25] Overall there is fair agreement between theory and experiment, with a number of predicted pion widths "right on the nose," but others off by factors of 2 to 3. In many of these cases there are large experimental uncertainties, as well as the theoretical uncertainty inherent in using the narrow resonance approximation to compute decays of one broad resonance into another.

(3) Relative Signs. In the process $\pi N \to N^* \to \pi\Delta$, the couplings to both πN and $\pi\Delta$ of all the N^*'s with a given value of L are related by $(SU(6)_W)$ Clebsch-Gordan coefficients to the same reduced matrix element(s). The signs of the amplitudes for passing through the various N^*'s in $\pi N \to \pi\Delta$ are then computable group theoretically. The correctness of these sign predictions is crucial, for while, for example, one may be willing to

to envisage a small amount of mixing of the constituent quark states, and corresponding corrections of say, 20%, to amplitudes (and 40% to widths), this will not change their signs. A wrong sign prediction could well spell the end of the theory!

This in fact seemed to be the case last year[66] when a comparison of the theoretical predictions[46,67] was made with the amplitude signs observed in an earlier phase shift solution of $\pi N \to \pi\Delta$ by the LBL-SLAC collaboration.[68] Since then a newer solution[69,70] with much better χ^2 has been found--in fact, the new solution is the only one left once additional data in the previous energy "gap" between 1540 and 1650 MeV is used as a constraint.[71]

The present situation with regard to amplitude signs for intermediate N^*'s with $L = 1$ in $\pi N \to N^* \to \pi\Delta$ is shown[72] in Table X. Aside from an overall phase (chosen so as to give agreement with the sign of the $DD_{15}(1670)$ amplitude), there is one other free quantity. This is the relative size of the reduced matrix elements of the two terms in $V^{-1}Q_5^\alpha V$ or, what turns out to be equivalent, the sign of an s-wave relative to a d-wave transition amplitude. In Table XI we have fixed this by using the sign of the $SD_{31}(1640)$ amplitude. All other signs for N^*'s in the 70 L = 1 multiplet are then predicted theoretically. The seven other signs determined experimentally agree with these predictions. The sign of the s-wave relative to d-wave amplitude is such as to show that the reduced matrix element of the second term in $V^{-1}Q_5^\alpha V$ with the algebraic properties of $(\lambda^\alpha/2)\ [(\beta\sigma_x + i\beta\sigma_y)(v_x - iv_y) - (\beta\sigma_x - i\beta\sigma_y)(v_x + iv_y)]$, is dominant for $L' = 1$ to $L = 0$ pion transitions of baryons, just as it is for $L' = 1$ to $L = 0$ pion transitions of mesons.

TABLE XI

Signs of Resonant Amplitudes[72] in $\pi N \to N^* \to \pi\Delta$ for N^*'s in the 70 L = 1 multiplet of $SU(6)_W \times O(3)$. Amplitudes are labeled by $(\ell_{\pi N}\ell_{\pi\Delta})_{2I,2J}$ and the resonance mass in MeV.

Resonant Amplitude	Theoretical Sign	Experimental Sign[70]
$DS_{13}(1520)$	-	-
$DD_{13}(1520)$	-	-
$SD_{11}(1550)$	+	?
$SD_{31}(1640)$	+ (input)	+
$DS_{33}(1690)$	-	-
$DD_{33}(1690)$	-	-
$DD_{15}(1670)$	+ (input)	+
$DS_{13}(1700)$	-	-
$DD_{13}(1700)$	+	+
$SD_{11}(1715)$	+	+

For N^*'s with $L = 2$, many of the amplitudes have not been seen experimentally. As the overall phase is already fixed, there is just one parameter free. Again this is the relative size of the two possible reduced matrix elements, only now it corresponds to the sign of a p-wave relative to an f-wave pion decay amplitude. We use the FP_{15} (1688) amplitude in Table XII to fix this sign[72] it corresponds to the reduced matrix element of the first term in $V^{-1}Q_5^\alpha V$ behaving algebraically as $(\lambda^\alpha/2)\sigma_z$, being dominant. All other signs (3) which are measured in Table XII agree with the theory.

TABLE XII

Signs of Resonant Amplitudes[72] in $\pi N \to N^{*} \to \pi\Delta$ for N^{*}'s in the 56 L = 2 Multiplet of $SU(6)_W \times O(3)$. Amplitudes are labeled as in Table XI.

Resonant Amplitude	Theoretical Sign	Experimental Sign[70]
$FP_{15}(1688)$	- (input)	-
$FF_{15}(1688)$	+	+
$PP_{13}(1860)$	-	?
$PF_{13}(1860)$	+	?
$FF_{37}(1950)$	-	-
$FP_{35}(1880)$	-	?
$FF_{35}(1880)$	-	-
$PP_{33}(\quad)$	+	?
$PF_{33}(\quad)$	+	?
$PP_{31}(1960)$	+	?

The signs of amplitudes for resonances in the radially excited 56 L = C are given in Table XIII. The sign of the $PP_{33}(1690)$ amplitude is in fact the principal reason for its previous assignment as the partner of the Roper resonance, since the alternative assignment to a 56 L = 2 leads to an opposite sign prediction.

TABLE XIII

Signs of Resonant Amplitudes[72] in $\pi N \to N^{*} \to \pi\Delta$ for N^{*}'s in a Radially Excited 56 L = 0 Multiplet of $SU(6)_W \times O(3)$. Amplitudes are Labeled as in Table XI.

Resonant Amplitude	Theoretical Sign	Experimental Sign[70]
$P_{11}(1470)$	+	+
$P_{33}(1690)$	-	-

Another reaction where relative signs are predicted is $\gamma N \to N^* \to \pi N$. This involves the theory at both the $\gamma N N^*$ and $\pi N N^*$ vertices. Although the situation is more complicated, there are also more amplitudes determined experimentally. An analysis[50,52] of the situation shows that not only are there 15 or so signs correctly predicted, but the information on the $\pi N N^*$ vertex so obtained agrees with that from $\pi N \to N^* \to \pi\Delta$ as to which term in $V^{-1} Q_5^\alpha V$ has the dominant reduced matrix element.

What emerges from all this is another possible systematics: for pion amplitudes, both meson and baryon show that the term transforming as $(\lambda^\alpha/2)\sigma_z$ is dominant in known L' even $\to L = 0$ transitions, while $(\lambda^\alpha/2)[\beta\sigma_+ v_- - \beta\sigma_- v_+]$ is dominant for $L' = 1 \to L = 0$ transitions. This might generalize to all L' even and L' odd decays. If it does, we will have yet another simple regularity to explain.

VIII. CONCLUSION

The theory of pion and photon transitions which we have outlined has had great success in predicting the signs of amplitudes--more than 25 relative signs are correctly predicted in the reactions $\pi N \to N^* \to \pi\Delta$ and $\gamma N \to N^* \to \pi N$. There is also at least fair success in predicting the relative magnitude of decay amplitudes, particularly for mesons.

This success lends support both to the theory of current-induced-transitions we have presented and to the assignment of hadron states to constituent quark model multiplets. In particular, the amplitude signs found to be in agreement with experiment mean that, at least in a rough sense, the relationship between the wave functions of different hadrons is that of the quark model. At $q^2 = 0$ one sees evidence for a quark picture of hadrons which is just as compelling as that obtained in a very different way as $q^2 \to \infty$ in deep inelastic scattering.

Aside from pushing further on questions like masses, the extension[53] to $q^2 \neq 0$ current induced transitions, the relationship[73] of V and PCAC, etc., what is most needed is a deeper understanding of why we can get away with such simple assumptions--why can we abstract anything relevant about transformed current operators from the free quark model? Even given that, why can we recognize so clearly the hadrons corresponding to the constituent quark model states? Why aren't the multiplets more badly split in mass and mixed? Most of all, to answer these and other questions we need at least part of the dynamics at which point we might be able to calculate magnitudes of the matrix elements as well. Then we truly will have a quark picture of hadron structure, spectroscopy, and amplitudes.

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40. Under the chiral SU(3) × SU(3) subalgebra of the $SU(6)_W$ of currents, the two terms in Eq. (8) transform as $(8,1)-(1,8)$ and $(3,\bar{3})-(\bar{3},3)$, respectively.

41. The operators D_+^α have $J_z = +1$. The corresponding operators D_-^α with $J_z = -1$, and all their matrix elements, are related (up to a sign) by a parity transformation. Hence we need only consider the properties of D_+^α.

42. Under the chiral SU(3) × SU(3) subalgebra of the $SU(6)_W$ of currents the four terms in Eq. (11) transform as $(8,1) + (1,8)$, $(3,\bar{3})$, $(8,1) - (1,8)$, and $(\bar{3},3)$ repsectively.

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57. Such schemes center around adding an L = 1 "spurion" in a 35: see J.C. Carter and M.E.M. Head, Phys. Rev. 176, 1808 (1968); D. Horn and Y. Ne'eman, Phys. Rev. D1, 2710 (1970); R. Carlitz and M. Kislinger, Phys. Rev. D2, 336 (1970). Specific broken $SU(6)_W$ calculational schemes with the same general algebraic structure as the theory considered here have been developed by L. Micu, Nucl. Phys. B10, 521 (1969); E.W. Colglazier and J.L. Rosner, Nucl. Phys. B27, 349 (1971); W.P. Petersen and J.L. Rosner, Phys. Rev. D6, 820 (1972); W.P. Petersen and J.L. Rosner, Phys. Rev. D7, 747 (1963). See also D. Faiman and D. Plane, Nucl. Phys. B50, 379 (1972).

58. The algebraic structure of broken $SU(6)_W$ schemes and their relation to the present theory are discussed in Ref. 48.

59. The relation between various quark model, $SU(6)_W$, broken $SU(6)_W$, and constituent to current quark transformation calculations of pion decay amplitudes is discussed by H.J. Lipkin, NAL preprint NAL-PUB-73/62-THY, 1973 (unpublished).

60. See the discussion in Ref. 44, particularly footnote 13.

61. S.U. Chung et al., Brookhaven preprint BNL 18340, 1973 (unpublished); V. Chaloupka et al., CERN preprint, 1974 (unpublished); U. Karshon et al., Weizmann Institute preprint WIS-73/44-Ph, 1973 (unpublished) and references to previous work therein.

62. This is essentially an updated version of results found in F.J. Gilman, M. Kugler, and S. Meshkov, Refs. 46 and 47, and also in J.L. Rosner, Phys. Reports (to be published) and Stanford Linear Accelerator Center Report No. SLAC-PUB-1391, 1974 (unpublished). The decay widths are calculated from the expressions in Table I of ref. 47, with quark model assignments as described there.

63. We have not treated the decay of the $I = 0$ scalar mesons into $\pi\pi$ in Table III because of the unclear situation in assigning the observed states to the quark model multiplet. For recent assessments see J.L. Rosner, Ref. 62 and D. Morgan, Ref. 20.

64. This is an updated version of Table II of F.J. Gilman and I. Karliner, Ref. 52. The decay widths can be calculated from Table I of Ref. 52, with the quark model assignments as described there.

65. See J. Rosen, invited talk at the IV International Conference on Experimental Meson Spectroscopy, Northeastern University, Boston, 1974 (unpublished).

66. F.J. Gilman, in Baryon Resonances-73, E.C. Fowler, Ed. (Purdue University, West Lafayette, 1973), p. 441.

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72. Tables XI and XII update the comparison of theory and experiment contained in Table VIII of Ref. 47 (see also Ref. 67). The ordering in angular momentum and isospin Clebsch-Gordan coefficients is the same as in Ref. 47 (see particularly footnote 59) even though there is a changed isospin convention in the new experimental papers, Ref. 70.

73. The connection of chiral representation mixing and the Nambu-Goldstone mechanism of chiral symmetry realization is explored by R. Carlitz and W.K. Tung, University of Chicago preprint EFI 73/19, 1974 (unpublished).