SEEING BEAUTY

IN THE SIMPLE AND THE COMPLEX :

CHANDRASEKHAR AND GENERAL RELATIVITY

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Abstract

Some aspects of S.Chandrasekhar's contribution to General Relativity are reviewed. These cover the areas of post-Newtonian approximation and its application to radiation reaction, black hole theory, colliding gravitational waves and non-radial oscillations of a star. Some examples of his perception of beauty in these areas are given, as also the way symmetries seem to speak to him like to no one else. His attempt to find counterparts to space components of the metric in Newtonian theory in the context of non-radial oscillations is also presented.

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Sometimes in the course of human interests things happen that truly lift the spirit. For me your winning the Nobel Prize is such an occasion.

I have always regarded my relationship with you as one of special inspiration for me. Your kindness, graciousness, absolutely uncompromising dedication to science, culture and integrity have really had a profound impact on me.

Every now and then the Nobel Committee does something truly great. This is one of those times. I cannot adequately express how happy I am for you and this is a feeling shared by all those who have been privileged to know you.

— MURPH GOLDBERGER.

His development is motivated by deep insight into the forms and symmetries of the differential equations with which he struggles. The equations speak to him in a tongue they speak to no one else leading him inexorably forward to new results. This is particularly so when the mathematics becomes horrendously complex as in the development of the 2PN approximation to general relativity and the analysis of pulsations of Kerr black holes.

In Mathematical Theory of Black Holes (MTBH) he concentrates on the last idea and the last word: He cleans up and completes in a thorough manner the body of incomplete theory that his younger colleagues left behind. And he does it without entering into controversy with them- at least not on the surface. However, if one knows something of the literature and reads beneath the surface one sees Chandrasekhar riding smoothshod over the works of his younger colleagues- smoothshod on an elegant steed with velvet covered hooves'

'Other insights tied up in incomplete non-Chandrasekhar versions of black hole mathematics may be lost to researchers..Chandrasekhar's method has been canonised and in ten or twenty years Chandra may truely have the last word..In return for losing other viewpoints we get from Chandrasekhar's book a monumental and almost complete body of mathematical theory presented in a totally coherent and aesthetically pleasing way. We are struck by the splendour of the theory, by the intricacies of its interconnections, by the mysterious amenability of black holes to totally analytical analysis.

One of those exceedingly rare books that will have a useful lifetime of fifty years. A book filled with new approaches to old subjects, old approaches to new subjects; completes unfinished researches of other physicists and maddeningly for the first time in Chandra'a career leaves unfinished his researches. It is filled with nuggets of mathematical insight.

— KIP THORNE

1 Introduction

Subrahmanyan Chandrasekhar started his work in General Relativity in the early 1960s. He was then past 50 years of age. In retrospect it seems historically inevitable that he should have moved over into the study of general relativity especially relativistic astrophysics. Discussing the maximum mass of white dwarfs that he had discovered in 1931, Chandrasekhar had said in 1934 'A star of large mass can not pass into the white dwarf stage and one is left speculating on other possibilities.' These other possibilities, it turned out later, were collapse to a neutron star or to a black hole. These were discussed by Oppenheimer and Volkov and Oppenheimer and Snyder in 1939. Chandrasekhar's studies of these possibilities, neutron stars and black holes thus seem a natural culmination of the ideas that led to the Chandrasekhar limit. But the way he entered the subject was unique and bears his own distinctive stamp.

In 1963 came his first major discovery in this area. He showed that general relativistic gravity creates a radial instability to gravitational collapse in stars with adiabatic index a little larger than 4/3. For the next twenty years he was one of the leaders in the field of relativistic astrophysics research. Research in this area, according to Kip Thorne [1], focussed on the structure, pulsations and stability of stars, star clusters and black holes, the gravitational collapse to form black holes and the generation of gravitational waves and the back reaction of waves on their sources. Chandrasekhar contributed to all these fields except star clusters and gravitational collapse.

The methods of research used for these studies are: Global Methods (Differential Topology), Discovery and Study of Exact Solutions (like Kerr metric), Perturbations of Exact Solutions and Post - Newtonian approximation. Chandrasekhar was a major figure in the study of perturbations and created the post- Newtonian approximation and was its master.

The major areas of his interest during the last 35 years of his life can be roughly chronologically listed.

1. 1961 -1975: Post Newtonian Approximation and its applications to general relativistic instability, radiation reaction and radiation reaction induced instability.
   (a) 1975-1982: Black Hole theory, perturbation and stability, separation of variables, transformation theory, reflection and transmission of waves.
   (b) 1983-1990: Colliding gravitational waves, relation to Kerr metric.
2. 1990 - 1992: Non - radial oscillations of neutron stars.
3. 1990-1995: Newton's Principia (outside the scope of the present lecture)

2 Post Newtonian Approximation

We first present the general relativistic instability discovered by Chandrasekhar using post - Newtonian approximation and then go on to present the general formalism of the approximation scheme.

In Newtonian theory the total energy of a star (sum of kinetic and potential energy) as a function of central density is given by

$$E = aKM{\rho_c}^{\Gamma-1} - bGM^{5/3}\rho_c^{1/3}$$

where M is the mass of the star, ρ_c is the central density and Γ is the adiabatic index. a and b are constants. Setting

$$\frac{dE}{d\rho_c} = 0,$$

keeping M constant, we find

$$M \propto \rho_c^{(\Gamma-\frac{4}{3})\frac{3}{2}}$$

$$\frac{dM}{d\rho_c} \propto (\Gamma - 4/3).$$

So $\Gamma > 4/3$ implies stabiliy.

In the case of general relativity we find the extra energy is given by (using post-Newtonian approximation)

$$\Delta E_{GTR} = -0.9 M^{7/3} \rho_c^{2/3}.$$

This leads to

$$\Gamma > 4/3 + k\frac{GM}{Rc^2}$$

$$\rho_c = 2.6\ 10^{10} \left(\frac{\mu_c}{2}\right)^2\ g/cc$$

where R is the radius of the star.

In many cases, before this density is reached the constituents become neutrons. If the critical density ρ_c given above is smaller than the density at which neutronisation takes place then general relativity decides the density limit for stability.

Chandrasekhar was led to this discovery after his mathematical analysis based on post-Newtonian approximation. A physical insight following a mathematical analysis ! Let us compare this with the way Feynman and Fowler came to the same conclusion at Caltech around the same time.

Fowler was giving a seminar at Caltech and Feynman was in the audience. When supermassive stars were mentioned Feynman seems to have remarked that there may be an instability as gravity is stronger (in general relativity) and so collapse must be easier. Fowler calculated the effect and discovered the instability. Physical insight followed by mathematical confirmation! Did Chandra have no physical insight before calculating? I think there is some oversimplification here. Without insight one can not have the will or patience to do such complicated calculations. It was probably the desire for verification before announcement that played a role in Chandra's case.

2.1 Formalism: Conservation laws in Newtonian hydrodynamics [2]

The Energy-Momentum tensor is defined as

$$T^{00} = \rho c^2, T^{0\alpha} = \rho c v_\alpha$$

$$T^{\alpha\beta} = \rho v_\alpha v_\beta + p\delta_{\alpha\beta}$$

where $\alpha, \beta = 1, 2, 3.$ and $i, j = 0, 1, 2, 3.$

$$T^{ij}{}_{,j} \equiv \frac{d}{dx_j} T^{ij} = 0.$$

The momentum

$$p^i = \int_V T^{0i}\, d\overline{x} = \text{constant}$$

So is the angular momentum

$$L_\gamma = \epsilon_{\alpha\beta\gamma} \int_V \rho x_\alpha v_\beta d\overline{x} = \text{constant}$$

If we assume preservation of entropy by every fluid element we get another Energy integral. Change in thermodynamic energy (Π) per unit mass is equal to work done by the pressure in changing volume

$$\frac{d\Pi}{dt} = -p\frac{d}{dt}(1/\rho)$$

Then

$$E = \int_V \rho\left(\frac{1}{2}v^2 + \Pi\right) d\overline{x} = \text{constant}.$$

So far no external forces, not even its own gravitation is included.

When we include gravitation, if U is the gravitational potential given by distribution of ρ then

$$\nabla^2 U = -4\pi G \rho$$

and

$$T^{\alpha j}{}_{,j} - \rho \frac{\partial U}{\partial x_\alpha} = 0.$$

Using the symmetric tensor defined by

$$t^{00} = 0, t^{0\alpha} = 0, t^{\alpha\beta} = \frac{1}{16\pi G}[4\frac{\partial U}{\partial x_\alpha}\frac{\partial U}{\partial x_\beta} - 2\delta_{\alpha\beta}(\frac{\partial U}{\partial x_\mu})^2]$$

and letting $\Theta^{ij} = T^{ij} + t^{ij}$ we can write

$$\Theta^{ij}{}_{,j} = 0$$

which again leads to conservation laws. We also have

$$E = \int_V \rho(\frac{1}{2}v^2 + \Pi - \frac{1}{2}U)d\overline{x} = \text{constant}$$

2.2 General Relativity

In General Relativity the physical character of the system is completely specified by choice of T^{ij}. For a perfect fluid

$$T^{ij} = \rho(c^2 + \Pi + \frac{p}{\rho})u^i u^j - pg^{ij}$$

where $u^i = \frac{dx^i}{ds}$. Note that the metric is yet ***unspecified.***

When T^{ij} is inserted in the field equation viz.

$$G^{ij} \equiv R^{ij} - \frac{1}{2}Rg^{ij} = -\frac{8\pi G}{c^4}T^{ij}$$

we do not have the choices we had in Newtonian Theory. $T^{ij}_{;j} = 0$ necessarily includes the effect of gravitational field on fluid motions. The covariant derivative has in addition to the ordinary derivative the effect of gravitational fields included in it. In Newtonian limit the additional term is the same as that encountered earlier, that is $-\rho\frac{\partial U}{\partial x_\alpha}$.

As T^{ij} has no dissipative mechanism the flow must preserve its entropy. We have

$$(\rho u^j)_{;j}(c^2 + \Pi + \frac{p}{\rho}) + \rho u^j[\Pi_{,j} + p(\frac{1}{\rho})_{,j}] = 0.$$

So conservation of mass $(\rho u^j)_{;j} = 0$ is compatible with the equation of motion only if

$$u^j[\Pi_{,j} + p(\frac{1}{\rho})_{,j}] = 0$$

which is the requirement that motion is ***isentropic.*** Thus in general relativity conservation of mass and conservation of entropy are not independent.

To get a conservation law we need $\Theta^{ik}{}_{,k} = 0$, where we have an ordinary derivative. For this we have to add a 'pseudo-tensor' t^{ik} to the energy- momentum tensor T^{ik} and t^{ik} is symmetric but not a tensor. Defining the 'Energy-Momentum' complex $\Theta^{ik} = (-g)(T^{ik} + t^{ik})$ we have $\Theta^{ik}{}_{,k} = 0$. This leads to the conservation law

$$E = \int_V (\Theta^{00} - c^2\rho u^0\sqrt{-g})d\overline{x} = \text{constant}$$

2.3 Basic Problem

In Newtonian framework given a well defined physical system (such as n - mass points under their mutual gravitational attraction or a mass of perfect fluid subject to internal stresses and its own gravitation) we can write down a set of ***equations of motion*** which govern all possible motions that can occur in the system.

The question arises : Can we write a similar set of equations in the framework of general relativity ? Chandrasekhar's response is : It appears that in general we can not do so. We are then led to a more modest inquiry : Can we write down an explicit set of equations of motion which govern departures from Newtonian motion due to effects of general relativity in a well defined scheme of successive post- Newtonian approximations. Further can we specify conserved quantities which are generalisations of the corresponding Newtonian quantities and which are constants of the post-Newtonian equations of motion ?

Around 1938 Einstein, Infeld and Hoffman (EIH) did pioneering investigations on the n-body problem. They wrote down the Lagrangian which differs from the Newtonian Lagrangian by terms of the $\sim \frac{v^2}{c^2}, \frac{U}{c^2}$ and comparable ones. The formalism gives the Keplarian orbit of two finite mass points about one another. However extensions to higher orders has not been possible. Chandrasekhar preferred the use of perfect fluid instead of mass points. He writes "mass points are not concepts that are strictly consistent with the spirit of general relativity. Hermann Bondi says 'General Relativity is a peculiarly complete theory and may not give sensible solutions for situations too far removed from what is physically reasonable'". Thus is Einstein suitably admonished ! According to Chandrasekhar, 'the concept of perfect fluid does not suffer from such limitations. In any event we confine ourselves to relativistic hydrodynamics of a perfect fluid.'

One starts with

$$T^{ij} = \rho(c^2 + \Pi + \frac{p}{\rho})u^i u^j - pg^{ij}$$

and the equation for conservation of rest mass $c^2(\rho u^i\sqrt{-g})_{,i} = 0$. The field equation

$$G^{ij} = -\frac{8\pi G}{c^4}T^{ij}$$

completes the set of equations.

The basic question is how is the field equation to be solved for g^{ij} so that the equation $T^{ij}_{;j} = 0$ when written out explicitly will provide the equation of motion as a power series in a suitable parameter.

2.4 Scheme of Approximation

First we identify the small parameters. The physical quantities of interest are the kinetic energy $K.E. = \frac{1}{2}\rho v^2$, the potential energy $P.E. = -\frac{1}{2}\rho U$, internal energy $= \rho\Pi$ and energy of molecular motion $= \frac{p}{\rho}$. The rest mass $= \rho c^2$ dominates over all these quantities. So $\frac{v^2}{c^2}, \frac{U}{c^2}, \frac{\Pi}{c^2}$ and $\frac{p}{\rho c^2}$ are the small parameters in what is usually called 'slow motion approximation'.

Secondly Equivalence principle, in its weak form, implies the following relation between rate of clock and gravitational potential.

$$g_{00} = 1 - \frac{2U}{c^2} + O(c^4)$$

So for Newtonian theory $g_{00} = 1 - \frac{2U}{c^2}, g_{0\alpha} = 0, g_{\alpha\beta} = -\delta_{\alpha\beta}$. These two considerations suffice to develop an entirely consistent scheme of successive post Newtonian approximation.

The first post-Newtonian is of order $O(c^{-2})$, the second is of order $O(c^{-4})$. These are orders of the equation of motion. For a given order of the equation of motion the different $g_{ij}s$ have to be known to different orders. If $T_{00} = \rho c^2 + O(1)$ then (0,0) component of $R_{ij} = -\frac{8\pi G}{c^4}(T_{ij} - \frac{1}{2}Tg_{ij})$ combined with $g_{00} = 1 - \frac{2U}{c^2} + O(c^{-4})$ reduces to Poisson's equation

$$\nabla^2 U = -4\pi G\rho$$

confirming that Newtonian equations are indeed 'zero order' solutions to Einstein's field equations.

To get to the first post-Newtonian approximation we proceed farther to the (α, β) component of the field equation (with a suitable coordinate condition) and find $g_{\alpha\beta} = -\delta_{\alpha\beta}(1 + \frac{2U}{c^2})$. Curvature of space implied by this is what is at the base of deflection of light by a gravitational field in general relativity. We next modify T^{ij}. We take

$$T^{00} = \rho c^2[1 + \frac{1}{c^2}(v^2 + 2U + \Pi)] + O(c^{-2})$$

$$T^{0\alpha} = \text{..............} + O(c-3)$$

$$T^{\alpha\beta} = \text{.............} + O(c^{-4}).$$

Used in the field equation they give

$$g_{00} = 1 - \frac{2U}{c^2} + \frac{2}{c^4}(U^2 - 2\phi)$$

$$g_{0\alpha} = \frac{P_\alpha}{c^3}$$

We can then write $T^{ij}_{;j} = 0$ to order $O(c^{-2})$ giving first order post Newtonian approximation.

The post- Newtonian series is even. The non trivial odd step is required in the imposition of the outgoing wave boundary condition. As $\frac{\partial}{\partial r}$ and $\frac{\partial}{c\partial t}$ are in different orders this is not possible. We are restricted to the near zone (r, ct). The way to match in the far zone was given by Trautman in 1958 who, unfortunately, used, wrongly, T^{ij} instead of Θ^{ij} for the Newtonian expression. When Chandra corrected this it gives the right result. Chandrasekhar and Esposito [3] found that $T^{\alpha j}_{;j}$ gives (in 2.5 post-Newtonian)

$$< \frac{d}{dt} \int E_{2.5} dx >= -\frac{G}{45c^5} < (\frac{d^3 D_{\alpha\beta}}{dt^3})^2 >$$

in exact agreement with rate of emission of gravitational radiation predicted by linear theory. Here $D_{\alpha\beta} = 3I_{\alpha\beta} - \delta_{\alpha\beta} I_{\sigma\sigma}$ is the quadrupole moment of the system.

It took 15 years (till mid 1980) for this result to be accepted by the scientific community [1]. There was controversy over handling of some divergent integrals and over the method of imposing the out going wave boundary condition.One should not be too far ahead of one's time, perhaps! Infeld, who was a collaborator of Einstein and Hoffman in the pioneering work mentioned earlier has written a book called 'Quest'. He discusses in that book the excitement and disappointment felt by them when they were trying to decide whether gravitational radiation is theoretically predicted in general relativity. Chandra's result finally showed the consistency of the formalism of gravitational radiation and provided for its theoretical acceptance. However it should be mentioned that work by Bondi and his collaborators, in the 1960s, had also convinced the community of the consistency of gravitational radiation though not in such a transparent manner.

The discussion of the radiation reaction led to the discovery of 'Radiation Reaction Induced Instability' in 1970. In the years from 1965 to 1968 Chandra was working on an area which seemed archaic to many of us, then, at Chicago. These were presented in a prestigious lecture series at Yale in 1968 and was published in a book form under the title 'Ellipsoidal figures of equilibrium'[4]. Chandra talks about the Maclaurin spheroid, the Jacobi ellipsoid and the Dedekind ellipsoid and how a rotating star can pass through these phases and meet bifurcation points when one form separates from the other. All these archaic concepts suddenly became very important in the context of the stability of rotating stars especially in the case of neutron stars which were discovered as pulsars in the year 1968. When dissipation is included, Chandrasekhar's study, in 1970, revealed the existence of an instability caused by radiation reaction due to emission of large amount of radiation. This has been confirmed in later studies.

5 Black Holes

[illegible] study of black holes occupied Chandrasekhar for the next ten to fifteen years. [illegible] Theory of Black Holes [5] was published in 1982. The analysis of stability was one of the [illegible] considerations in the study. One perturbed a system and then studied whether the [illegible] was damped returning the system to its original configuration or the perturbation [illegible] grow. To do this separation of variables was one of the important considerations.

The black hole metric has the well known form

$$ds^2 = (1 - \frac{2M}{r})dt^2 - (1 - \frac{2M}{r})^{-1} dr^2 + r^2[d\theta^2 + sin^2\theta d\phi^2].$$

When perturbed the general form is

$$ds^2 = \exp^{2\nu} dt^2 - \exp^{2\psi}(d\phi - \omega dt - q_2 dx^2 - q_3 dx^3)^2 - \exp^{2\mu_2}(dx^2)^2 - \exp^{2\mu_3} dx^{3^2}.$$

For the black hole $\omega = q_2 = q_3 = 0$. When perturbed we have the parameters $\omega, q_2, q_3, \delta\nu, \delta\mu_2$, $\delta\mu_3$, and $\delta\psi$. These are metric perturbations.

We can also use Newman- Penrose formalism and have perturbations of the scalars of Weyl tensors $\Psi_0, \Psi_1, \Psi_2, \Psi_3, \Psi_4$.

Chandra exploited both methods. He introduced transformation theory. This involves putting the equation for perturbation in the form

$$\frac{d^2 Z}{dr_*{}^2} + \sigma^2 Z = VZ$$

This equation, like Schrodinger equation in a barrier potential, can be solved easily numerically. Chandra studied scattering matrix and its unitarity to discuss why different potentials give the same results. He also discussed how complex potentials give conservative scattering.

Chandrasekhar considered in detail the stability of the black hole, a problem first studied by Vishveshwara in 1970 (twentyfive years ago). Chandra considered various types of black holes:
Schwarzchild charactrised by the parameter M (mass)
Reissner-Nordstrom by parameters M(mass) and Q(charge)
Kerr by M (mass) and J (angular momentum)
Kerr - Newman by M (mass), Q (charge) and J (angular momentum)

Separability of the variables like r, θ and ϕ is crucial in all these discussions. Chandra's facility in these methods is well known. Over one weekend at Princeton he separated the variables of the spin $\frac{1}{2}$ particle equation in Kerr metric.

The extensive and powerful mathematical analysis applied to this problem, in the usual Chandrasekhar way brings us again to Kip Thorne's remarks [1]: 'Insight into physical origin comes after the mathematical analysis was complete.' He also goes on to remark 'It may be tempting to deprecate Chandra's more mathematical ways, were he not so spectacularly successful.' Thorne continues 'The symmetries of the equation speak to him in a manner that they speak to nobody else I know, leading him inexorably forward toward interesting results.'

We would like to present an example of his sensitiveness to such symmetries. While discussing Schwarzschild black holes Chandra had to work in a gauge, which he called phantom gauge.(Chap. 4, sec 29 of [5]). We have the following equations in which the operators $L_n, L_n{}^+, D_0, D_2{}^+$ operate on Φ_0, Φ_1, k, s (equation numbers are those in the book).

$$L_2\Phi_0 - (D_0 + \frac{3}{r})\Phi_1 = -6Mk - - - (237)$$

$$\Delta(D_2{}^+ - \frac{3}{r})\Phi_0 + L_{-1}{}^+\Phi_1 == 6Ms - - - (238)$$

$$(D_0 + \frac{3}{r})s - L_{-1}{}^+k = \frac{\Phi_0}{r}. - - - (239)$$

A choice of gauge can be made which brings to equations above a symmetry which they lack. In these equations, the symmetry of these equations in Φ_0, k and s is only partially present in Φ_1, k and s. Equation (239) is, for example, the right equation which allows us to obtain a decoupled equation for Φ_0 after the elimination of Φ_1 between equations (237) and (238). But a similar elimination of Φ_0 does not lead to a decoupled equation for Φ_1 since we do not have a 'right' fourth equation. However exercising the freedom we have to subject the tetrad frame to an infinitesimal rotation, we can rectify the situation by supplying (ad hoc?) the needed fourth equation. Thus with the additional equation

$$\Delta(D_2{}^{+} - \frac{3}{r})k + L_2 s = \frac{2}{r}\Phi_1$$

we can eliminate Φ_0.

Chandra adds, 'we shall find (chapter 5, sec 46) that the function Φ_1 defined in this way describe Maxwell's field in Schwarzschild geometry.' Thus we have derived Maxwell's equations (appropriate for photons with spin ± 1) by finding a gauge which rectifies the truncated symmetry of equations (237) - (239) in the quantities which occur in them.

To give an example where the absence of such symmetry leads to difficulties we consider the equations in Kerr- Newman metric, which defied even Chandra in his attempts at separating the variables.The equations, there are

$$(L_2 - \frac{3ia\sin\theta}{\overline{\rho^*}})\Phi_0 - (D_0 + \frac{3}{\overline{\rho^*}})\Phi_1 = -2k[3(M - \frac{Q_*{}^2}{\rho}) + \frac{Q_*{}^2\overline{\rho^*}}{\rho^2}]$$

$$\Delta(D_2{}^{+} - \frac{3}{\overline{\rho^*}})\Phi_0 + (L_{-1}{}^{+} + \frac{3ia\sin\theta}{\overline{\rho^*}})\Phi_1 = +2s[3(M - \frac{Q_*{}^2}{\overline{\rho}}) - \frac{Q_*{}^2\overline{\rho^*}}{\rho^2}].$$

Chandrasekhar remarks 'In contrast to the simplicity of terms in the earlier case on the right hand side we now have the ugly combination of $(M - \frac{Q_*{}^2}{\rho})$ and $\frac{Q_*{}^2\overline{\rho^*}}{\rho^2}$. A separation of variables will be possible, if at all, only by contemplating equations of order 4 or higher.' That is symmetry speaking to Chandra. No wonder Kip Thorne said, while reviewing Chandra's book on Mathematical Theory of Black holes, that no student at Caltech wanted to take up the problem of separating the variables of Kerr- Newman metric after Chandra's failure to separate them.

Chandrasekhar's presentation of beauty in these complicated expressions and equations, was the reason behind the title of this talk 'Seeing beauty in the simple and the complex' The following quotation may amplify this point:

> 'The treatment of perturbations of Kerr space-time has been prolixious in its complexity. Perhaps, at a later time, the complexity will be unravelled by deeper insights. But meantime the analysis has led into a realm of the rococo: splendorous, joyful and immensely ornate.'

I looked up the meaning of rococo in the dictionary. it says ornamental; a bit, perhaps, like the doors and the dieties at the entrances and walls in South Indian temples (like say the one at Kancheepuram). The simple may, probably, be compared to the lotus shaped Bahai temple at New Delhi. If one is attuned, there is beauty to be seen in both cases.

In his treatment of colliding gravitational waves, whose metric is closely related to the stationary Kerr metric, Chandra pushed these ideas even further (went really overboard !). He compares the beauty seen in these formulations to the beauty in the series of paintings of Claude Monet. Monet has painted the same subject or scene as seen at different times of the day or in different seasons. Chandra compares this to the different viewpoints of the same metric which manifests itself as Kerr metric or colliding gravitational waves.

4 Non Radial Oscillations

The work on this subject done by Chandrasekhar in 1991 (he was more than 80 years old then) with Valeria Ferrari applies the various techniques perfected by Chandra to neutron stars. The surprising results they obtained, according to Chandra should have counterparts in the Newtonian theory too. Quotations from his paper may be of interest.

First about the work: ' We develop ab initio a complete unified version of the theory of non-radial oscillations of a spherical distribution of matter (a star!) that provides not only a different physical base for the origin and nature of these oscillations but also simpler algorithms.... for numerical evaluations of quasi normal modes ($l \geq 2$) and the real frequencies of dipole oscillations ($l = 1$).'

The mode is found by calculating the scattering cross section of a gravitational wave as a function of energy and locating the peak (resonance). A process which is familiar to particle physicists. However Chandra finds a source of puzzlement and possibly a new insight at a deeper level. We again quote:

> 'On the relativistic theory, the frequencies of oscillations of the non- radial modes (as we have shown) depend only on the distribution of the energy-density and the pressure in the static configuration and the equation of state only to the extent of its adiabatic exponent. If this is a true representation of the physical situation, then it must be valid in the Newtonian theory as well: the true nature of an object can not change with the mode and manner of one's perception. In the relativistic picture, the independence of the frequencies of the non-radial oscillations of a star, on anything except its characterization in terms of its equilibrium structure, is to be understood in terms of the scattering of incident gravitational waves by the curvature of the static space time and its matter content acting as a potential. But what are the counterparts of these same concepts in the Newtonian framework ? Perhaps they lie concealed in the meanings that are to be attached, in the Newtonian theory, to the four metric functions (and their perturbations) that describe a spherically symmetric space-time (and their polar perturbations). It is known that the Newtonian gravitational potential, in some sense, replaces the metric function g_{tt}. Are there similar meanings to be attached to $g_{rr}, g_{\theta\theta}$ and $g_{\phi\phi}$? That is the predominant question to which the present investigation seems to lead.'

To sum up, to use Chandra's own words: *'One is left speculating'*.

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