Dynamical Yukawa Couplings

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Objetivo y motivación

El campo de física de partículas se encuentra actualmente en un punto crucial. La exploración del mecanismo de rotura espontánea de simetría electrodébil (RESE) en el gran colisionador de hadrones (LHC) ha desvelado la presencia de un bosón que se asemeja al escalar de Higgs (1, 2) dada la precisión de los datos experimentales disponibles (3, 4). La descripción del Modelo Estándar (ME) de la generación de masas (5, 6, 7) ha demostrado ser acertada y la auto-interacción del bosón de Higgs que desencadena la RESE es ahora la quinta fuerza de la naturaleza, junto con la gravedad, el eletromagnetismo la interacción débil y la fuerte. Esta nueva fuerza, como el resto de las fuerzas cuantizadas, varía en intensidad dependiendo de la escala a la que se la examine, pero al contrario que la fuerza débil o fuerte, esto plantea un problema (8) ya que una escala de alta energía o corta distancia del orden de $10^{-12} fm$ el mecanismo de RESE se desestabilizaría, pues el acoplo cuártico se cancelaría (9, 10). Dicho problema podría ser resuelto por la introducción de nueva física, lo cual conduce a otra cuestión teórica, el Problema de la Jerarquía (PJ). Cualquiera sea la nueva física que se acopla a la partícula de Higgs produce una contribución radiativa al término de masa de dicho bosón del orden de la escala de nueva física, lo que significaría que la escala electrodébil es naturalmente cercana a la escala de física más alta que interacciona con los campos del ME. Las propuestas para solucionar este problema pueden ser clasificadas en soluciones de física perturbativa, siendo el paradigma la supersimetría, y ansazts de dinámica fuerte. Supersimetría es una nueva y elegante simetría entre bosones y fermiones que implica cancelaciones sistemáticas entre las contribuciones radiativas que producen cada tipo de partículas al término de masa del Higgs. Por otro lado la hipótesis de que el bosón de Higgs sea un estado ligado producido por nueva dinámica fuerte implica que el mecanismo de RESE es simplemente una descripción efectiva que debe ser completada por una teoría más fundamental. Todas estas hipótesis suponen nueva física a la escala del TeV y están siendo testeadas de manera decisiva en el LHC.

En el frente cosmológico la interacción gravitatoria ha sido la fuente de evidencia de nuevos desafíos en física de partículas. El universo está expandiéndose aceleradamente, algo que en cosmología estándar requiere la presencia de energia oscura, una energía de vacío cuya presión negativa provoca que el universo se ensanche con velocidad creciente. Cosmología y astrofísica proporcionaron la sólida evidencia de materia extra no bariónica en el universo, llamada materia oscura, como otra muestra experimental no explicable en el ME. Hay un activo programa experimental para la búsqueda de materia oscura en este agitado sector de física de partículas. La tercera evidencia de nueva física en cosmología proviene de un hecho muy familar del mundo visible, está constituido de mucha mas materia que antimateria, y aunque el ME proporciona una fuente de exceso de partículas sobre antipartículas el resultado no es suficiente para explicar la proporción observada.

La parte de nueva física que concierne más de cerca al ME es el hecho de que los neutrinos han demostrado ser masivos. La evidencia de masa de neutrinos proveniente de los datos de oscilación es una de las selectas evidencias de nueva física mas allá del ME. En este sector la búsqueda de violación leptónica de conjugación de carga y paridad (CP), transiciones de sabor de leptones cargados y la relación fundamental entre neutrinos y antineutrinos; su carácter Majorana o Dirac, tienen ambiciosos programas experimentales que producirán resultados en los próximos años.

Para completar la lista de desafíos en física de partículas, debe ser mencionado que existe la tarea pendiente de la cuantización de gravedad y el presente pobre entendimiento del vacío de QCD representado en el problema-θ. Estos temas no obstante pueden ser considerados como problemas teóricos frente a las evidencias experimentales consideradas previamente.

El tema de esta tesis es un problema horizontal: el puzle de sabor. La estructura de sabor del espectro de partículas está conectada en la teoría estándar a la RESE, y las masas de los neutrinos son parte esencial de este puzle. Estos son temas que han sido tratados en el trabajo del estudiante de doctorado en otro contexto: la fenomenología de sabor en el caso de dinámica fuerte de RESE (11, 12), la determinación del Lagrangiano bosónico general en el mismo contexto (13) y la fenomenología de un modelo para masas de neutrinos (14) han formado parte del programa de doctorado del candidato. El tema central de esta tesis está sin embargo es la exploración de una posible explicación a la estructura de sabor (15, 16, 17).

El principio gauge puede ser señalado como la fuente creadora de progreso en física de partículas, bien entendido y elegantemente implementado en el ME. Por el contrario el sector de sabor permanece durante décadas como una de las partes peor entendidas del ME. El ME muestra la estructura de sabor de una manera paramétrica, dejando sin respuesta preguntas como el origen de la fuerte jerarquía en masas de fermiones o la presencia de grandes angulos de mezcla de sabor para leptones en constraste con la pequeña mezcla del sector de quarks. El puzle de sabor permanece por lo tanto como una cuestión fundamental sin respuesta en física de partículas.

La principal guía en este trabajo es el uso de simetría para explicar el puzle de sabor. La simetría, que juega un papel central en nuestro entendimiento en física de partículas, es empleada en esta tesis para entender la estructura de sabor. Un número variado de simetrías han sido postuladas con respecto a este problema (18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29). En este estudio la simetría será seleccionada como la mayor simetría continua global posible en la teoría libre [1]. La elección está motivada por las exitosas consequencias fenomenológicas de selectionar la susodicha simetría en el caso de la hipótesis de Violación Mínima de Sabor (22, 25, 26, 27, 28), un campo en el que el autor también a trabajado (28). Debe ser destacado que los diferentes orígenes posibles para la masa de los neutrinos resultan en distintas simetrías de sabor en el sector leptónico; de especial relevancia es la eleción del carácter Dirac o Majorana. En cualquiera de los casos la simetría de sabor no es evidente en el espectro, luego debe estar escondida. En este trabajo el estudio de rotura espontánea de la simetría de sabor para leptones

[1] Alternativamente se puede definir en términos mas técnicos como la mayor simetría posible en el límite de acoplos de Yukawa ausentes (22, 25, 26).

y quarks será desarrollado con énfasis en el resultado natural contrastado con la estructura observada en la naturaleza. Se mostrará como la diferencia entre quark y leptones en la estructura de sabor resultante, en particular los ángulos de mezcla, se origina en la naturaleza Majorana o Dirac de los fermiones.

En el presente análisis, el criterio de naturalidad será la regla para decidir si la solución propuesta es aceptable o introduce puzles mas complicados que los que resuelve. Es relevante por lo tanto la acepción de naturalidad, siguiendo el criterio de t'Hooft, todos los parámetros adimensionales no restringidos por una simetría deben ser de orden uno, mientras que todos los parámetros con dimesiones se espera que sean del orden de la escala de la teoría. Exploraremos por lo tanto en qué casos este criterio permite la explicación de la estructura de masas y angulos de mezcla.

Respecto a las diferentes partes de nueva física involucradas conviene distinguir tres escalas distintas i) la escala de RESE establecida por la masa del bosón W, ii) un escala posiblemente distinta de sabor, denotada Λ_f y característica de la nueva física responsable de la estructura de sabor, iii) la escala efectiva de violación de numero leptónico M responsable de las masas de los neutrinos, en el caso de que éstas sean de Majorana.

2

Aim and Motivation

The field of particle physics is presently at a turning point. The exploration of the mechanism of electroweak symmetry breaking (EWSB) at the LHC has unveiled the presence of a boson that resembles the Higgs scalar (1, 2) with the precision of presently available data (3, 4). The Standard Model (SM) description of mass generation (5, 6, 7) has proven successful, and the Higgs self-interaction triggering EWSB stands now as the fifth force in nature, after gravity, electromagnetism, weak and strong interactions. This new force, as every other quantized force in nature, varies in strength depending on the scale at which it is probed but, unlike for strong or weak forces, this poses a problem (8) as at a high energy or short distance scale of order $10^{-12} fm$ the mechanism of electroweak symmetry breaking would be destabilized since the coupling of this force vanishes (9, 10). This problem could be solved by the introduction of new physics which brings the discussion to another theoretical issue, the Hierarchy Problem. Any new physics that couples to the Higgs particle produces generically a radiative contribution to the Higgs mass term of order of the new mass scale, which would mean that the electroweak scale is naturally close to the highest new physics scale that couples to the SM fields. Proposals to address this problem can be classified in perturbative physics solutions, the paradigm being supersymmetry, and strong dynamics ansatzs. Supersymmetry is an elegant new symmetry between bosons and fermions that implies systematic cancellations among the contributions to the Higgs mass term of these two types of particles. On the other hand the hypothesis of the Higgs boson being a bounded state produced by new strong dynamics implies that the mechanism of electroweak symmetry breaking is just an effective description to be completed by a more fundamental theory. All these hypothesis involve new physics at the TeV scale and are being crucially tested at the LHC.

In the cosmology front the gravitational interaction has been the source of the evidence of new challenges in particle physics. The universe is accelerating, something that in standard cosmology requires of the presence of Dark Energy, a vacuum energy whose negative pressure makes the universe expand with increasing rate. Cosmology together with astrophysics brought the solid piece of evidence of extra matter in the universe not in the form of baryons, the so called Dark Matter as another experimental evidence not explainable within the Standard Model. There is an active experimental program for the search of Dark Matter in this lively sector of particle physics. The third piece of evidence in cosmology stems on one very familiar fact of the visible universe: it is made out of much more matter than antimatter, and even if the SM provides a source for particle over antiparticle abundance in cosmology, this is not enough to explain the ratio observed today.

The piece of new physics that concerns more closely the Standard Model is the fact that neutrinos have shown to be massive. The neutrino mass evidence from oscillation data stands as one of the selected few sound pieces of evidence of physics beyond the SM. In this sector, the search for leptonic CP violation, charged lepton generation transitions and the fundamental relation among neutrino particles and antiparticles; their Majorana or Dirac nature, have ambitious experimental programs bound to produce results in the coming years.

To complete the list of challenges in particle physics, it shall be mentioned that there is the pending task of the quatization of gravity and the present poor understanding of the vacuum of QCD embodied in the θ problem. These issues can be regarded as theoretical problems in contrast with the experimental evidences mentioned above.

The focus of this project is a somehow horizontal problem: the flavour puzzle. The flavour structure of the particle spectrum is connected in the standard theory to EWSB, and the masses of neutrinos are an essential part the flavour puzzle. These last matters have been subject of study in a different context for the PhD candidate: the flavour phenomenology in a strong EWSB realization (11, 12), the determination of the general bosonic Lagrangian in the same scheme (13) and the flavour phenomenology of a neutrino mass model (14) are part of the author's work. The focus of this discussion is nonetheless on the exploration of a possible explanation of the flavour pattern (15, 16, 17).

The gauge principle can be singled out as the driving engine of progress in particle physics, well understood and elegantly realized in the SM. In contrast the flavour sector stands since decades as the less understood part of the SM. The Standard Model displays the flavour pattern merely parametrically, leaving unanswered questions like the origin of the strong hierarchy in fermion masses or the presence of large flavour mixing in the lepton sector versus the little overlap in the quark sector. The flavour puzzle stays therefore a fundamental open question in particle physics.

The main guideline behind this work is the use of symmetry to address the flavour puzzle. Symmetry, that plays a central role in our understanding of particle physics, is called here to explain the structure of the flavour sector. A number of different symmetries have been postulated with respect to this problem (18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29). Here the symmetry will be selected as the largest possible continuous global symmetry arising in the free theory [1]. This choice is motivated by the successful phenomenological consequences of selecting this symmetry, as in the case of the Minimal Flavour Violation (MFV) ansatz (22, 25, 26, 27, 28) , a field in which the author has also worked (28). It must be underlined that the different possible origins of neutrino masses result in different flavour symmetries in the lepton sector; of special relevance is the choice of Majorana or Dirac masses. The flavour symmetry in any case is not evident in the spectrum, ergo must be somehow hidden. In this dissertation the study of the mechanism of flavour symmetry breaking for both quark and leptons will be carried out with emphasis on its natural outcome in comparison with the observed flavour pattern in nature. It will be shown how the difference between quark and leptons in the resulting flavour structure, in particular mixing, stems on the Majorana or Dirac nature of fermions.

[1] Alternatively defined as the largest possible simmetry in the limit of vanishing Yukawa couplings (22, 25, 26), to be introduce later.

In the analysis here presented, naturalness criteria shall be the guide to tell whether the implementation is acceptable or introduces worse puzzles than those it solves. A relevant issue is what will be meant by natural; following 't Hooft's naturalness criteria, all dimensionless free parameters not constrained by a symmetry should be of order one, and all dimensionful ones are expected to be of the order of the scale of the theory. We will thus explore in which cases those criteria allow for an explanation of the pattern of mixings and large mass hierarchies.

As for the different physics involved in this dissertation, there will be three relevant scales; i) the EWSB scale set by the W mass and which in the SM corresponds to the vacuum expectation value (vev) v of the Higgs field; ii) a possible distinct flavour scale Λ_f characteristic of the new physics underlying the flavour puzzle; iii) the effective lepton number scale M responsible for light neutrinos masses, if neutrinos happen to be Majorana particles.

3

Introduction

As all pieces of the Standard Model fall into place when confronted with experiment, the last one being the discovery of a Higgs-like boson at the LHC (1, 2), one cannot help but stop and wonder at the theory the scientific community has carved to describe the majority of phenomena we have tested in the laboratory. This theory comprises both the forces we have been able to understand at the quantum level and the matter sector. The former shall be briefly reviewed first.

3.1 Forces of the Standard Model

Symmetries have shed light in numerous occasions in particle physics, in particular the understanding of local space-time or gauge symmetries stands as the deepest insight in particle physics. The gauge principle, at the heart of the SM, is as beautifully formulated as powerful and predictive for describing how particles interact through forces. The SM gauge group,

$$\mathcal{G} = SU(3)_c \times SU(2)_L \times U(1)_Y \,, \tag{3.1}$$

encodes the strong, weak and electromagnetic interactions and describes the spin 1 (referred to as vector-boson) elementary particle content that mediate these forces. The strong interactions concern those particles that transform under $SU(3)_c$ with c standing for color, and are the subject of study of quantum chromodynamics (QCD). The electroweak sector $SU(2)_L \times U(1)_Y$ comprises the weak isospin group and the abelian hypercharge group which reduce to the familiar electromagnetic gauge group and Fermi interaction below the symmetry breaking scale. This part of the theory is specified, in the unbroken phase, given the group and the coupling constants of each subgroup, here g_s for $SU(3)_c$, g for $SU(2)_L$ and g' for $U(1)_Y$ at an energy scale μ. This information is enough to know that 8 vector-boson mediate the strong interaction, the so-called gluons, and that 4 vector bosons enter the electroweak sector: the $Z, W^{\pm}$ and the photon.

The implementation of the gauge principle in a theory that allows the prediction of observable magnitudes as cross sections, decay rates etc. makes use of Quantum Field Theory (QFT). In the canonical fashion we write down the Lagrangian density denoted $\mathscr{L}$, that for the pure gauge sector of the Standard Model reads;

$$\mathscr{L}_{gauge} = -\frac{1}{4}\mathrm{Tr}\left\{F_i^{\mu\nu}F_{i,\mu\nu}\right\} , \tag{3.2}$$

which describes forces mediators and these mediators self-interaction. The field strengths are defined through the covariant derivatives:

$$D_\mu = \partial_\mu + ig_s G_\mu^i \lambda_i + ig\frac{\sigma_i}{2}W_\mu^i + ig' Q_Y B_\mu , \tag{3.3}$$

with Gell-Mann matrices λ_i acting in color space , Pauli matrices σ_i within weak isospin space, and Q_Y is the hypercharge of the field that the covariant derivative acts on. G_μ^i denote the 8 gluons, W_μ^i the three weak isospin bosons and B_μ the hypercharge mediator. The photon (A_μ) and Z are the usual combination of neutral electroweak bosons: $Z_\mu = \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu$, $A_\mu = \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu$ and the weak angle $\tan\theta_W = g'/g$. In terms of the covariant derivatives the field strengths are defined as:

$$F_{i,\mu\nu} = -\frac{i}{g_i}\left[D_\mu, D_\nu\right] . \tag{3.4}$$

However the fact that the W and Z spin-1 bosons are massive requires of the introduction of further bosonic fields in the theory. This brings our discussion to the electroweak breaking sector. Masses are not directly implementable in the theory as bare or “hard” mass terms are not allowed by the gauge symmetry. The way the SM describes acquisition of masses is the celebrated Brout-Englert-Higgs mechanism, a particularly economic description requiring the addition of a $SU(2)_L$ doublet spin-0 boson (scalar), denoted H. This bosonic field takes a

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
H	1	2	1/2

Table 3.1: The Higgs field charges under $\mathcal{G}$

vev and its interactions with the rest of fields when expanding around the true vacuum produce mass terms for the gauge bosons. The interaction of this field with the gauge fields is given by its transformation properties or charges, reported in table 3.1, the masses produced for the W and Z boson being in turn specified by the vev of the field $\langle H \rangle \equiv (0, v/\sqrt{2})^T$ together with the coupling constants g and g'. This vev is acquired via the presence of the quartic coupling of the Higgs, the fifth force, and the negative mass term. These two pieces conform the potential that triggers EWSB and imply the addition of two new parameters to the theory, explicitly;

$$\mathscr{L}_H = (D_\mu H)^\dagger D^\mu H - \lambda \left(H^\dagger H - \frac{v^2}{2} \right)^2 . \tag{3.5}$$

where the v is the electroweak scale $v/\sqrt{2} \simeq 174$GeV and λ the quartic coupling of the Higgs, which can be extracted from the measured Higgs mass $\lambda = m_h^2/(2v^2) \simeq 0.13$. Note that the potential, the second term above, has the minimum at $\left\langle H^\dagger H \right\rangle = v^2/2$.

As outlined in the previous section, the Higgs could be elementary or composite; the paradigm of composite bosons are pions, understood through the Goldstone theorem. In the pions chiral Lagrangian the relevant scale is the pion decay constant f_π associated to the strong dynamics, in the analogy with a composite Higgs the scale is denoted f which, unlike in technicolor (30, 31, 32), in Composite Higgs Models (33, 34, 35, 36, 37) is taken different from the electroweak vev v. In the limit in which these two scales are close, a more suitable parametrization of the Higgs is, alike to the exponential parametrization of the σ-model,

$$\left(\tilde{H} \, , \, H \right) = U \, \frac{\langle h \rangle + h}{\sqrt{2}} \, , \qquad\qquad U^\dagger U = U U^\dagger = 1 \, , \tag{3.6}$$

where $\tilde{H} = i\sigma_2 H^*$ with σ_2 the second Pauli matrix in weak isospin space. U is a 2×2 unitary matrix which can be thought of as a space-time dependent element of the electroweak group and consequently absorbable in a gauge transformation while $\langle h \rangle + h$ is the constant "radial" component plus the physical bosonic degree of freedom unchanged by a gauge transformation. The value of $\langle h \rangle$ is fixed by v and f.

In this way gauge invariance of the corrections to Eq. 3.5 concerns the dimensionless U matrix and its covariant derivatives whereas the series in H/f can be encapsulated in general dimensionless functions $\mathcal{F}[(\langle h \rangle + h)/f]$ different for each particular model.

Since both U and $\mathcal{F}$ are dimensionless, the expansion is in powers of momentum (derivatives) over the analogous of the chiral symmetry breaking scale (38, 39). The Lagrangian up to chiral dimension 4 in this scheme for the bosonic sector was given in (13) and the flavour phenomenology in this scenario was studied in (11, 12) as part of the authors work that however does not concern the discussion that follows.

3.2 Matter Content

The course of the discussion leads now to the matter content of the Standard Model. Completing the sequence of intrinsic angular momentum, between the spin 1 vector bosons and the spin 0 scalars the spin 1/2 ultimate constituents of matter, the elementary fermions are placed. These fermions constitute what we are made of and surrounded by. Their interactions follow from their transformation properties under the gauge group. Quarks are those fermions that sense the strong interactions and are classified in three types according of their electroweak interactions; a weak-isospin doublet Q_L and two singlets U_R, D_R . Leptons do not feel the strong but only the electroweak interaction and come in two shapes; a doublet ℓ_L, and a singlet E_R of $SU(2)_L$. The explicit transformation properties of the fermions are reported in table 3.2.

The subscripts L and R refer to the two irreducible components of any fermion; left and right-handed. Right handed fermions, in the limit of vanishing mass, have a spin projection on the direction of motion of $1/2\hbar$ whereas left-handed fermions have the opposite projection $-1/2\hbar$. These two components are irreducible in the sense that they are the smallest pieces that transform in a closed form under

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
Q_L	3	2	1/6
U_R	3	1	2/3
D_R	3	1	-1/3
ℓ_L	1	2	-1/2
E_R	1	1	-1

Table 3.2: Fermion content of the SM - Transformation properties under the gauge group $\mathcal{G}$.

the Lorentz group with a spin 1/2. The explicit description of the interaction of fermions with gauge fields is read from the Lagrangian;

$$\mathscr{L}_{matter} = i \sum_{\psi=Q_L}^{E_R} \overline{\psi} \not{D} \psi \,, \tag{3.7}$$

where $\not{D} = \gamma_\mu D^\mu$ and γ_μ are the Dirac matrices.

There is a discreet set of representations for the non-abelian groups ($SU(3)_c$ and $SU(2)_L$): the fundamental representation, the adjoint representation etc. All fermions transform in the simplest non-trivial of them [1]: the fundamental representation, hereby denoted N for $SU(N)$. For the abelian part, the representation (charge) assignation can be a priori any real number normalized to one of the fermion's charges, e.g. E_R. There is however yet another predictive feature in the SM connected to the gauge principle: the extra requirement for the consistency of the theory of the cancellation of anomalies or the conservation of the symmetry at the quantum level imposes a number of constraints. These constraints, for one generation, are just enough to fix *all* relative $U(1)_Y$ charges, leaving no arbitrariness in this sector of the SM.

Let us summarize the simpleness of the Standard Model up to this point, we have specified a consistent theory based on local symmetry described by 4 coupling constants for the 4 quantized forces of nature, a doublet scalar field acquiring a vev v and a matter content of 5 types of particles whose transformation properties or "charges" are chosen from a discreet set.

[1]The trivial representation is just not to transform, a case denoted by "1" in the first to columns of table 3.2

There is nonetheless an extra direction perpendicular to the previous which displays the full spectrum of fermions explicitly, that is, the flavour structure. Each of the fermion fields in table 3.2 appears replicated three times in the spectrum with wildly varying masses and a connection with the rest of the replicas given by a unitary mixing matrix. Explicitly:

$$Q_L^\alpha = \left\{ \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \right\}, \qquad U_R^\alpha = \{u_R, c_R, t_R\}, \tag{3.8}$$

$$\ell_L^\alpha = \left\{ \begin{pmatrix} \nu_L^e \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_L^\mu \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_L^\tau \\ \tau_L \end{pmatrix} \right\}, \qquad D_R^\alpha = \{d_R, s_R, b_R\}, \tag{3.9}$$

$$E_R^\alpha = \{e_R, \mu_R, \tau_R\} \ , \tag{3.10}$$

where e stands for the electron, μ for the muon, τ for the τ-lepton, u for the up quark, d for the down quark, c for charm, s for strange, b for bottom and t for the top quark. The flavour structure is encoded in the Lagrangian,

$$\mathscr{L}_{fermion-mass} = -\overline{Q}_L Y_U \tilde{H} U_R - \overline{Q}_L Y_D H D_R - \overline{\ell}_L Y_E E_R H + \mathcal{L}_{\nu-mass} \ , \tag{3.11}$$

where the 3×3 matrices Y_U, Y_D, Y_E have indices in flavour space.

3.2.1 Neutrino Masses

The character of neutrino masses is not yet known, however if we restrict to the matter content we have observed so far, the effective field theory approach displays a suggestive first correction to the SM. Effective field theory, implicit when discussing the Higgs sector, is a model independent description of new physics implementing the symmetries and particle content present in the known low energy theory. Corrections appear in an expansion of inverse powers of the new physics scale M. This generic scheme yields a remarkably strong result, at the first order in the expansion, the *only* possible term, produces neutrino Majorana masses after EWSB:

$$\mathscr{L}^{d=5} = \frac{1}{M}\mathcal{O}^W + h.c. \equiv \frac{1}{M}\overline{\ell}_L^\alpha \tilde{H}\, c_{\alpha\beta}\, \tilde{H}^T \ell_L^{c,\beta} + h.c. \, , \tag{3.12}$$

where c is a matrix of constants in flavour space. This operator, known as Weinberg's Operator (40), violates lepton number but this is however an accidental symmetry of the SM, the fundamental symmetries are the gauge symmetries which are compatible with lepton number violation. As to what is the theory that produces this operator, there are three possibilities corresponding to three different fields as mediators of this interaction: the type I (41, 42, 43), II (44, 45, 46, 47, 48) and III (49, 50) seesaw models. The mediator could transform as a fermionic singlet of the Standard Model (type I), a scalar triplet of $SU(2)_L$ (type II) and a fermionic triplet of $SU(2)_L$ (type III) diagrammatically depicted in Fig. 3.1. Here we will select the type I seesaw model which introduces

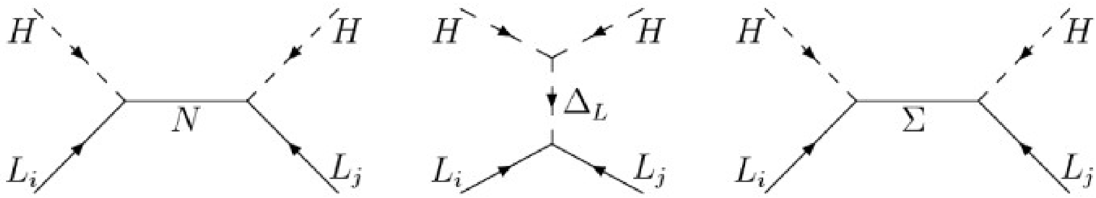

Figure 3.1: The three types of seesaw models -

right-handed neutrinos in analogy with the rest of fermions. These particles are perfect singlets under the Standard Model, see table 3.3, something that allows for their Majorana character, which is transmitted to the left-handed neutrinos detected in experiment through the Yukawa couplings. The complete Lagrangian

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
N_R	1	1	0

Table 3.3: Right-handed neutrino charges under the SM group

for the fermion masses is therefore:

$$\mathscr{L}_{fermion-mass} = \mathscr{L}_{Yukawa} + \mathscr{L}_{Majorana}\,, \tag{3.13}$$

$$\mathscr{L}_{Yukawa} = -\,\overline{Q}_L Y_U \tilde{H} U_R - \overline{Q}_L Y_D H D_R - \overline{\ell}_L Y_E E_R H - \overline{\ell}_L Y_\nu \tilde{H} N_R\,, \tag{3.14}$$

$$\mathscr{L}_{Majorana} = -\,\overline{N}^c_R M N_R\,, \tag{3.15}$$

where M is a symmetric 3×3 matrix and N_R stands for the right-handed neutrinos which now also enter the sum of kinetic terms of Eq. 3.7. The limit in which the right-handed neutrino scale M is much larger than the Dirac scale $Y_\nu v$ yields as first correction after integration of the heavy degrees of freedom the Weinberg Operator with the constants $c_{\alpha\beta}$ in Eq. 3.12 being $c_{\alpha\beta} = (Y_\nu Y_\nu^T)_{\alpha\beta}$ such that for $\mathcal{O}(1)$ Yukawas the upper bound on neutrino masses points to M around the GUT scale $\sim 10^{15}$GeV. The opposite limit is the Dirac mass limit $Y_\nu v \gg M$ in which Lepton number would be conserved and the Yukawa coupling should be tuned to 10^{-12}.

In the following we assume validity for the seesaw formula such that $Y_\nu v \ll M$.

3.2.2 The Flavour Symmetry

If the gauge part was described around the gauge group one can do the same, if only formally a priori, for the flavour side. A way to characterize it is then choosing the largest symmetry that the free theory could present given the particle content and orthogonal to the gauge group, this symmetry is that of the group (22, 25, 26):

$$\mathcal{G}_\mathcal{F} = \mathcal{G}_\mathcal{F}^q \times \mathcal{G}_\mathcal{F}^l\,,$$

$$\mathcal{G}_\mathcal{F}^q = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R} \times U(1)_B \times U(1)_{A^U} \times U(1)_{A^D}\,, \tag{3.16}$$

$$\mathcal{G}_\mathcal{F}^l = SU(3)_{\ell_L} \times SU(3)_{E_R} \times O(3)_N \times U(1)_L \times U(1)_{A^l}\,, \tag{3.17}$$

It is clear that each $SU(3)$ factor corresponds to the different gauge representation fields which do not acquire mass in the absence of interactions. Right-handed neutrinos have however a mass not arising from interactions, but present already in the free Hamiltonian. Given this fact the largest symmetry possible in this section is $O(3)$ for the degenerate case:

$$M = |M| I_{3\times 3}\,, \tag{3.18}$$

which is imposed here. The symmetry selected here can alternatively be defined as that arising, for the right-handed neutrino mass matrix of the above form, in the limit $\mathscr{L}_{Yukawa} \to 0$.

There is an ambiguity in the definition of the lepton sector symmetry and indeed other definitions are present in the literature (27, 28), in particular for the N_R fields a $U(3)_{N_R}$ symmetry is selected if the symmetry is identified with the kinetic term of the matter fields. This option leads to a complete parallelism from the symmetry point of view for leptons and quarks and would consequently lead to similar outcomes in an unsuccessful scenario.

Under the non-abelian part of $\mathcal{G}_\mathcal{F}$ the matter fields transform as detailed in table 3.4 and the abelian charges are given in table 3.5. In the non-abelian side one can identify $U(1)_B$ as the symmetry that preserves baryon number and $U(1)_L$ as lepton number which is broken in the full theory here considered. The remaining $U(1)_A$ symmetries are axial rotations in the quark and lepton sectors.

	$SU(3)_{Q_L}$	$SU(3)_{U_R}$	$SU(3)_{D_R}$	$SU(3)_{\ell_L}$	$SU(3)_{E_R}$	$O(3)_N$
Q_L	3	1	1	1	1	1
U_R	1	3	1	1	1	1
D_R	1	1	3	1	1	1
ℓ_L	1	1	1	3	1	1
E_R	1	1	1	1	3	1
N_R	1	1	1	1	1	3

Table 3.4: Representations of the fermion fields under the non-abelian part of $\mathcal{G}_\mathcal{F}$

	$U(1)_B$	$U(1)_{A^U}$	$U(1)_{A^D}$	$U(1)_L$	$U(1)_{A^l}$
Q_L	1/3	1	1	0	0
U_R	1/3	-1	0	0	0
D_R	1/3	0	-1	0	0
ℓ_L	0	0	0	1	1
E_R	0	0	0	1	-1
N_R	0	0	0	0	0

Table 3.5: Representations of the fermion fields under the abelian part of $\mathcal{G}_\mathcal{F}$

$\mathscr{L}_{Yukawa}$ is however non vanishing and encodes the flavour structure, our present knowledge about it being displayed in Eqs. 3.19-3.27. The masses for fermions range at least 12 orders of magnitude and the neutrinos are a factor 10^6 lightest than the lightest charged fermion, something perhaps connected to their possible Majorana nature. Neutrino masses are not fully determined, only the two mass squared differences and and upper bound on the overall scale are known.

The fact that one of the mass differences is only known in absolute value implies that not even the hierarchy is known, the possibilities being Normal Hierarchy (NH) $m_{\nu_1} < m_{\nu_2} < m_{\nu_3}$ and Inverted Hierarchy (IH) $m_{\nu_3} < m_{\nu_1} < m_{\nu_2}$. The mixing shape for quarks is close to an identity matrix, with deviations given by the Cabibbo angle λ_c whereas mixing angles are large in the lepton sector with all entries of the same order of magnitude. In the lepton sector the CP phase δ and the Majorana phases, if present, are yet undetermined.

$$m_d = 4.8^{+0.7}_{-0.3} MeV\,, \quad m_s = 95 \pm 5 MeV\,, \quad m_b = 4.18 \pm 0.03 GeV\,, \tag{3.19}$$
$$m_u = 2.3^{+0.7}_{-0.5} MeV\,, \quad m_c = 1.275 \pm 0.025 GeV\,, \quad m_t = 173.5 \pm 0.8 GeV\,, \tag{3.20}$$

$$m_e = 0.510998928 \pm 0.000000011 MeV\,, \tag{3.21}$$
$$m_\mu = 105.6583715 \pm 0.0000035 MeV\,, \tag{3.22}$$
$$m_\tau = 1.776.82 \pm 0.16 GeV\,, \tag{3.23}$$

$$\sum_i m_{\nu_i} \leq 0.28 eV\,, \quad \Delta m^2_{\nu_{12}} = 7.5^{+0.2}_{-0.2} 10^{-5} eV^2\,, \quad |\Delta m^2_{\nu_{23}}| = 2.42^{+0.04}_{-0.07} 10^{-3} eV^2\,, \tag{3.24}$$

$$V_{CKM} = \begin{pmatrix} 1-\lambda_c^2/2 & \lambda_c & A\lambda_c^3(\rho - i\eta) \\ -\lambda_c & 1-\lambda_c^2/2 & A\lambda_c^2 \\ A\lambda_c^3(1-\rho-i\eta) & -A\lambda_c^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda_c^4)$$
$$A\lambda_c^3(\rho + i\eta) \equiv \frac{A\lambda_c^3(\bar\rho + i\bar\eta)\sqrt{1-A^2\lambda_c^4}}{\sqrt{1-\lambda_c^2}\,(1 - A^2\lambda_c^4(\bar\rho + i\bar\eta))}\,, \qquad \lambda_c = 0.22535 \pm 0.00065\,, \tag{3.25}$$
$$A = 0.811^{+0.022}_{-0.012}\,, \qquad \bar\rho = 0.131^{+0.026}_{-0.013}\,, \qquad \bar\eta = 0.345^{+0.013}_{-0.014}\,, \tag{3.26}$$

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} e^{i\alpha_1\lambda_3 + i\alpha_2\lambda_8}$$
$$\theta_{12} = 33^{+0.88\,\circ}_{-0.78} \qquad \theta_{23} = 40 - 50^\circ \qquad \theta_{13} = 8,66^{+0.44\,\circ}_{-0.46} \tag{3.27}$$

The Majorana phases are encoded in the exponentials of the Gell-Mann matrices of Eq. 3.27. The quark data is taken from (51) and the neutrino parameters from (52, 53). The question arises of what becomes of the anomaly cancellation conditions now that the flavour structure has been made explicit. The conditions are still fixing the relative hypercharges of all generations provided all masses are different, all mixing angles nontrivial and Majorana masses for the right-handed neutrinos.

Comparison of the flavour and gauge sector will actually be useful for the introduction of the research subject of this thesis. First the ratio of certain parameters of the gauge sector, namely hypercharges, cannot take arbitrary values but are fixed due to constraints for the consistency of the theory, while the values for the flavour parameters seem all to be equally valid, at least from the point of view of consistency and stability. This brings to a second point, the inputs that are arbitrary in the gauge sector, g_s, g, g', λ are smaller but of $\mathcal{O}(1)$ at the typical scale of the theory $\sim M_Z$, whereas masses span over 6 orders of magnitude for charged leptons and including neutrinos too the orders of magnitude escalate to 12.

Because of gauge invariance particles are fitted into representations of the group, such that the dimension of the representation dictates the number of particles. There are left-handed charged leptons and left-handed neutrinos to fit a fundamental representation of $SU(2)_L$, could it be that something alike happens in the flavour sector? That is, is there a symmetry behind the flavour structure?

If this is the case, the symmetry that dictates the representation is not evident at the scale we are familiar with, so it should somehow be hidden; we can tell an electron from a muon because they have different masses. But the very same thing happens for $SU(2)_L$, we can tell the neutrino from the electron as we know that the electroweak symmetry is broken.

This comparison led neatly to the study carried out. The list of the basic ingredients here concerned has been completed; we shall assume that there is an exact symmetry behind the flavour structure, and if so necessarily broken at low energies; a breaking that we will effectively describe via a flavour Higgs mechanism. It is the purpose of this dissertation to study the mechanism responsible for the breaking of such flavour symmetry in the search for a deeper explanation of the flavour structure of elementary particles.

4

Flavour Physics

4.1 Flavour in the Standard Model + type 1 Seesaw Model

The model that serves as starting point in our discussion is the Standard Model with the addition of the type 1 Seesaw Model to account for neutrino masses, the widely accepted as simplest and most natural extension with lepton number violation. This chapter will be concerned with flavour phenomenology and the way it shapes the flavour structure of new physics at the TeV scale, aiming at the understanding from a bottom up approach of the sources of flavour violation. The way in which the flavour symmetry is violated in the theory here considered is quite specific and yields sharp experimental predictions that we shall examine next.

The energies considered in this chapter are below the electroweak scale, such that the Lagrangian of Eq. 4.1, assuming $M \gg v$, after integrating out the heavy right-handed neutrinos reads

$$\mathscr{L}_{f-mass} = -\overline{Q}_L Y_U \tilde{H} U_R - \overline{Q}_L Y_D H D_R - \overline{\ell}_L Y_E E_R H - \overline{\ell}_L \tilde{H} \frac{Y_\nu Y_\nu^T}{M} \tilde{H}^T \ell_L^c + \mathcal{O}\left(\frac{1}{M^2}\right) \tag{4.1}$$

where we recall that the flavour symmetry here considered sets $M_{ij} = M\delta_{ij}$, a case that shall not obscure the general low energy characteristics of a type 1 Seesaw Model whereas it simplifies the discussion. The flavour symmetry in this model is only broken by the above Lagrangian, including $1/M^n$ corrections. In full generality the Yukawa matrices can be written as the product of a unitary matrix, a diagonal matrix of eigenvalues and a different unitary matrix on the right end. In the case of the light neutrino mass term, it is more useful to consider the whole product $Y_\nu Y_\nu^T$ which is a transpose general matrix and therefore decomposable in a unitary matrix and a diagonal matrix in the following way:

$$Y_U = U_L^U \mathbf{y}_U U_R^U \,, \qquad Y_D = U_L^D \mathbf{y}_D U_R^D \,, \tag{4.2}$$

$$Y_E = U_L^E \mathbf{y}_E U_R^E \,, \qquad Y_\nu Y_\nu^T = U_L^\nu \mathbf{y}_\nu^2 U_L^{\nu T} \,, \tag{4.3}$$

where $U_{L,R}^{U,D,E,\nu}$ are the unitary matrices and $\mathbf{y}_{U,D,E,\nu}$ the diagonal matrices containing the eigenvalues. Even if the symmetry is broken, the rest of the SM and type 1 seesaw Lagrangian stays invariant under a transformation under the group $\mathcal{G}_\mathcal{F}$ of the fermion fields. In particular the rotation;

$$Q_L \to U_L^D Q_L \,, \qquad D_R \to U_R^{D\dagger} D_R \,, \qquad U_R = U_R^{R\dagger} U_R \,, \tag{4.4}$$

$$\ell_L \to U_L^E \,, \ell_L \qquad E_R \to U_R^{E\dagger} E_R \,, \tag{4.5}$$

simplifies the Yukawa matrices in Eqs. 4.2,4.3 after substitution in Eq. 4.1 to,

$$Y_U = U_L^{D\dagger} U_L^U \mathbf{y}_U \,, \qquad Y_D = \mathbf{y}_D \,, \tag{4.6}$$

$$Y_E = \mathbf{y}_E \,, \qquad Y_\nu Y_\nu^T = U_L^{E\dagger} U_L^\nu \mathbf{y}_\nu^2 U_L^{\nu T} U_L^{E*} \,, \tag{4.7}$$

which allows to define:

$$V_{CKM}^\dagger \equiv U_L^{D\dagger} U_L^U \,, \qquad U_{PMNS} \equiv U_L^{E\dagger} U_L^\nu \,, \tag{4.8}$$

$$\mathbf{y}_U = \text{Diag}\left(y_u, y_c, y_t\right) \,, \qquad \mathbf{y}_D = \text{Diag}\left(y_d, y_s, y_b\right) \,, \tag{4.9}$$

$$\mathbf{y}_\nu = \text{Diag}\left(y_{\nu_1}, y_{\nu_2}, y_{\nu_3}\right) \,, \qquad \mathbf{y}_E = \text{Diag}\left(y_e, y_\mu, y_\tau\right) \,, \tag{4.10}$$

with V_{CKM} being the usual quark mixing matrix and U_{PMNS} the analogous in the lepton side; the first encodes three angles and one CP-odd phase and the second two extra complex Majorana phases on top the the equivalent of the previous 4 parameters. The connection of the eigenvalues with masses will be made clear below.

There are a few things to note here. The right handed unitary matrices $U_R^{U,D,E}$ are irrelevant, the appearance of the irreducible mixing matrix in both sectors is due to the simultaneous presence of a Yukawa term for both up and down-type quarks involving the same quark doublet Q_L, and the neutrino mass term and charged lepton Yukawa where the lepton doublet ℓ_L appears. Were the mass terms to commute there would be no mixing matrix. Were the weak isospin group not present to bind together u_L with d_L and ν_L with e_L there would not either be mixing matrix. Weak interactions in conjunction with mass terms violate flavour. Although mixing matrices are there and nontrivial it is useful to have in mind this considerations to remember how they arise.

After EWSB the independent rotation of the two upper components of the weak isospin doublets

$$U_L \to V_{CKM}^\dagger U_L\,, \qquad\qquad \nu_L \to U_{PMNS}\nu_L\,, \tag{4.11}$$

takes to the mass basis yielding the Yukawa couplings diagonal, which now explicitly appear when expanding the Higgs field around the vev,

$$\mathscr{L}_{Yukawa} = -\frac{y_\alpha\,(v+h)}{\sqrt{2}}\,\overline{U}_L^\alpha U_R^\alpha - \frac{y_\beta\,(v+h)}{\sqrt{2}}\,\overline{D}_L^\beta D_R^\beta \tag{4.12}$$

$$-\frac{y_\alpha\,(v+h)}{\sqrt{2}}\,\overline{E}_L^\alpha E_R^\alpha - \frac{y_\alpha^2\,(v+h)^2}{2M}\,\overline{\nu}_L^\alpha \nu_L^\alpha + h.c.\,, \tag{4.13}$$

were h is the physical Higgs boson and the unitary gauge has been chosen.

We read from the above that the masses for the charged fermions are $m_\alpha = y_\alpha v/\sqrt{2} = y_\alpha \times 174\text{GeV}$ whereas for neutrinos $m_{\nu_\alpha} = y_{\nu_\alpha}^2 v^2/2M$. The values of masses then fix the Yukawa eigenvalues for the charged fermions to be:

$$\{y_t\,,\,y_c\,,\,y_u\} = \left\{1.0\,,\,7.3\times10^{-3}\,,\,1.3\times10^{-5}\right\}\,, \tag{4.14}$$

$$\{y_b\,,\,y_s\,,\,y_d\} = \left\{2.4\times10^{-2}\,,\,5.5\times10^{-4}\,,\,2.7\times10^{-5}\right\}\,, \tag{4.15}$$

$$\{y_\tau\,,\,y_\mu\,,\,y_e\} = \left\{1.0\times10^{-2}\,,6.0\times10^{-4}\,,\,2.9\times10^{-6}\right\}\,, \tag{4.16}$$

whereas for neutrinos only the mass squared differences are know and an upper bound $y_\nu^2 v^2/M \lesssim \text{eV}$. The values for the Yukawa eigenvalues of the charged fermions display quantitatively the hierarchies in the flavour sector, note that as dimensionless couplings of the theory they are naturally expected of $\mathcal{O}(1)$, something only satisfied by the top Yukawa. The smallness of the eigenvalues is nonetheless stable under corrections since in the limit of vanishing Yukawa eigenvalue a chiral symmetry arises, which differentiates this fine-tuning from the Hierarchy Problem.

The rest of the Lagrangian does not notice the rotation in Eq. 4.11 except for the couplings of weak isospin $+1/2$ and $-1/2$ particles;

$$\mathscr{L}_{CC} = i\frac{g}{\sqrt{2}}\overline{U}_L V_{CKM} \not{W}^+ D_L + i\frac{g}{\sqrt{2}}\overline{\nu}_L U^\dagger_{PMNS} \not{W}^+ E_L + h.c.\,. \tag{4.17}$$

The rest of couplings, which involve neutral gauge bosons, are diagonal in flavour, to order $1/M^2$. The flavour changing source has shifted therefore in the mass basis to the couplings of fermions to the gauge $W^\pm$ bosons. This is in accordance with the statement of the need of both weak isospin and mass terms for flavour violation.

This process allows to give a physical definition of the unitary matrices entering the Yukawa couplings: *mixing matrices are the change of basis from the interaction to the mass basis.* This is a more general statement than the explicit writing of Yukawa terms or the specification of the character of neutrino masses.

The absence of flavour violation in neutral currents implies the well known and elegant explanation of the smallness of flavour changing neutral currents (FCNC) of the Glashow Iliopoulos Maiani (GIM) mechanism. All neutral current flavour processes are loop level induced and suppressed by unitarity relations to be proportional to mass differences and mixing parameters, an achievement of the standard theory that helped greatly to its consolidation. At the same time this smallness of flavour changing neutral currents stands as a fire proof for theories that intend to extend the Standard Model, as we shall see next.

4.2 Flavour Beyond the Standard Model

The flavour pattern of elementary particles has been approached in a number of theoretical frameworks aiming at its explanation. Shedding light in a problem as involved as the flavour puzzle has proven not an easy task and proposed explanations are in general partial, in particular reconciling neutrino flavour data with quark and charged lepton hierarchies in a convincing common framework is a pending task in the authors view.

In the following a number of the proposed answers to explain flavour are listed,

- *Froggat Nielsen theories.* The introduction of an abelian symmetry R under which the different generation fermions with different chirality have different charges and that is broken by the vev of a field $\langle\phi_0\rangle$ can explain the hierarchies in the flavour pattern (19). In this set-up there are extra chiral fermions at a high scale which acquire a mass via the vev of a different Higgs-like R-neutral field, $\langle\phi_1\rangle$ such that the magnitude $\epsilon = \langle\phi_0\rangle / \langle\phi_1\rangle$ controls the breaking of the abelian symmetry R. Interactions among the different fermions are mediated by the field ϕ_0 at the high scale and its acquisition of a vev at the low scale implies factors of $\epsilon^{a_i+b_j}$ for the coupling of different flavour and chirality fermions Ψ_{L_i}, Ψ_{R_j} with charges $R_{L_i} = c + b_i$ and $R_{R_j} = c - a_j$. The mass matrix produced in this way contains hierarchies among masses dictated by $m_i/m_j \sim \epsilon^{a_i-a_j+b_i-b_j}$ whereas angles are given by $U_{ij} \sim (m_i/m_j)^{C_{ij}} \gtrsim (m_i/m_j)$. This symmetry based argument stands as one of the simplest and most illuminating approaches to the flavour puzzle.

- *Discreet symmetries* Discreet symmetries were studied as possible explanations for the flavour pattern in the quark sector (54) but the main focus today is on the lepton mixing pattern. The values of the atmospheric and solar angles motivated proposals of values for the angles given by simple integer ratios like the tri-bimaximal mixing pattern (55) ($\theta_{23} = \pi/4$, θ_{12} =arcsin$(1/\sqrt{3})$, $\theta_{13} = 0$) . These patterns were later shown to be obtainable with breaking patterns of relatively natural discreet symmetries like A_4(56, 57, 58), S_4 (59, 60). A discreet flavour treatment of both quark and leptons requires generally of extra assumptions like distinct breaking patterns in distinct fermion sectors which have to be kept separate, see e.g. (61, 62) These models though are now in tension with the relatively large reactor angle and new approaches are being pursued (63, 64). This approach has the advantage of avoiding goldstone bosons when breaking the discreet symmetry but the drawback of the ambiguity in choosing the group.

- *Extra Dimensions* The case of extra dimension offers a different explanation for the hierarchy in masses. In Randall-Sundrum models (65, 66) the presence of two 4 branes in a 5 dimensional space induces a metric with an overall normalization or warp factor that is exponentially decreasing with the fifth dimension and that offers an explanation of the huge hierarchy among the Planck and EW scale in terms of $\mathcal{O}(1)$ fundamental parameters. When the fermions are allowed to propagate in the fifth dimension, rather than being confined in a brane, their profile in the fifth dimension determined by the warp factor and a bulk mass term provides exponential factors for the Yukawa couplings as well, offering an explanation of the flavour pattern in terms of $\mathcal{O}(1)$ fundamental or 5th dimensional parameters (67, 68). In large extra dimensions theories, submilimiter new spacial directions can provide geometrical factors to explain the hierarchy problem (69). In this scenario, if we live on a fat brane in which the fermion profiles are localized, the mixing among generations is suppressed by the overlap of this profiles rather than symmetric arguments (70, 71, 72). In the extradimensional paradigm in general therefore the explanation of the hierarchies in flavour is found in geometry rather than symmetry.

- *Anarchy* The possibility of the flavour parameters being just random numbers without any utter reason has been also explored (73, 74), and even if the recent measurement of a "large" θ_{13} lepton mixing angle favors this hypothesis for the neutrino mass matrix (75), the strongly hierarchical pattern of masses and mixing of charged fermions is not natural in this framework.

These models introduce in general new physics coupled to the flavour sector of the Standard Model, which means modifying the phenomenological pattern too. More in general any new physics that couples to the SM flavour sector will change the predictions for experiments and shall be contrasted with data. This is examined next.

4.3 Flavour Phenomenology

Once again the effective field theory is put to use,

$$\mathscr{L} = \mathscr{L}_{SM} + \frac{1}{M}\mathcal{O}^W + \sum \frac{c_i}{\Lambda_f^2}\mathcal{O}^i + \mathcal{O}(1/\Lambda_f^3) \tag{4.18}$$

This Lagrangian can be viewed as the Standard Model theory represented by the first term above plus new physics corrections in a very general manner for the two next terms. The first correction in Eq. 4.18 has already been examined and taken into account. The next corrections have a different scale motivated by naturalness criteria. In this category we include the operators that do not break lepton number nor baryon number, listed in (76) and only recently reduced to the minimum set via equations of motion (77), and therefore need not be suppressed by the same scale. There are notheless contributions of $1/M^2$ in Eq. 4.18, but these either are too small for phenomenological purposes after applying the upper bound from neutrino masses or, in seesaw models with separate lepton number and flavour scales (78, 79, 80, 81, 82), fall in the description above (83, 84, 85). As a concrete example a possible operator at order $1/\Lambda_f^2$ is:

$$c_6\, \mathcal{O}^6 = c_{\alpha\beta\sigma\rho} \bar{Q}_L^{\alpha} \gamma_\mu Q_L^{\beta} \bar{Q}_L^{\sigma} \gamma^\mu Q_L^{\rho}\,, \tag{4.19}$$

where greek indices run over different flavours and the constants $c_{\alpha\beta\sigma\rho}$ are the coefficients different in general for each flavour combination. The modification

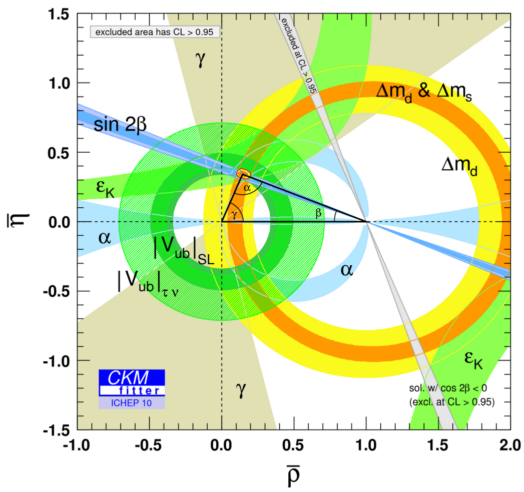

Figure 4.1: Constrains on the CKM parameters -

induced by this term in observable quantities can be computed and compared with data. A wide an ambitious set of experiments have provided the rich present amount of flavour data; from the precise branching ratios of B mesons in B factories to the search for flavour violation in the charged lepton sector, all the D and K meson observables, and if we include CP violation, the stringent electric dipole moments.

Contrast of the experimental data with expectations has led, in most occasions, to a corroboration of the Standard Model in spite of new physics, and at times certain hints of deviations from the standard theory raised hopes (86, 87, 88) that either were washed away afterwards, or stand as of today inconclusive. It is the case then that no clear proof of physics other than the SM and neutrino masses driving flavour data has been found.

Indeed the data has been not only enough to determine the flavour parameters of the SM but also to impose stress test on the theory, all faintlessly passed. Fig. 4.1 shows how all experimentally allowed regions in the mixing parameter plane of $\bar{\rho}-\bar{\eta}$, variables defined in Eq. 3.26, meet around the allowed value. The absence

Operator	Bounds on Λ_f (TeV)		Bounds on c ($\Lambda_f = 1$TeV)		Observables
	$c=1$	$c=i$	$\mathfrak{Re}(c)$	$\mathfrak{Im}(c)$	
$(s_L\gamma_\mu d_L)^2$	9.8×10^2	1.6×10^4	9.0×10^{-7}	3.4×10^{-9}	$\Delta m_K, \epsilon_K$
$(s_R d_L)(s_L d_R)$	1.8×10^4	3.2×10^5	6.9×10^{-9}	2.6×10^{-11}	$\Delta m_K, \epsilon_K$
$(c_L\gamma_\mu u_L)^2$	1.2×10^3	2.9×10^3	5.6×10^{-7}	1.0×10^{-7}	$\Delta m_D; \lvert q/p\rvert; \phi_D$
$(c_R u_L)(c_L u_R)$	6.2×10^3	1.5×10^4	5.7×10^{-8}	1.1×10^{-8}	$\Delta m_D; \lvert q/p\rvert; \phi_D$
$(b_L\gamma_\mu d_L)^2$	6.6×10^2	9.3×10^2	2.3×10^{-6}	1.1×10^{-6}	$\Delta m_{B_d}; S_{\Psi K_S}$
$(b_R d_L)(b_L d_R)$	2.5×10^3	3.6×10^3	3.9×10^{-7}	1.9×10^{-7}	$\Delta m_{B_d}; S_{\Psi K_S}$
$(b_L\gamma_\mu s_L)^2$	1.4×10^2	2.5×10^2	5.0×10^{-5}	1.7×10^{-5}	$\Delta m_{B_s}; S_{\Psi\Phi}$
$(b_R s_L)(b_L s_R)$	4.8×10^2	8.3×10^2	8.8×10^{-6}	2.9×10^{-6}	$\Delta m_{B_s}; S_{\Psi\Phi}$
$F^{\mu\nu}\bar{\mu}_R\sigma_{\mu\nu}e_L$	6.1×10^4	6.1×10^4	2.7×10^{-10}	2.7×10^{-10}	$\mu\to e\gamma$
$(\mu_L\gamma_\mu e_L)(u_L\gamma_\mu u_L)$	4.9×10^2	4.9×10^2	4.1×10^{-6}	4.1×10^{-6}	$\mu\to e(Ti)$
$(\mu_L\gamma_\mu e_L)(d_L\gamma_\mu d_L)$	5.4×10^2	5.4×10^2	3.5×10^{-6}	3.5×10^{-6}	$\mu\to e(Ti)$

Table 4.1: Bounds on the different operators, see text for details.

of new physics evidence translates in bounds on the new physics scale, reported in table 4.1. When placing the bounds, the magnitude that is constrained is the combination c/Λ_f^2 as is the one appearing in the Lagrangian of Eq. 4.18.

Naturalness criteria points at constants c of $\mathcal{O}(1)$, a case reported in table 4.1 both for CP conservation $c = 1$ (second column) and CP violation $c = i$ (third column). On the other hand if the scale is fixed at the TeV then the constants have severe upper bounds as the fourth and fifth columns in table 4.1 show. The quark bounds are taken from (89) whereas the lepton data is taken from (90, 91) and computed with the formulae of (14)

4.4 Minimal Flavour Violation

The bounds on new physics place a dilemma: either giving up new physics till the thousands of TeVs scale and with it the possibility of any direct test in laboratories, or assume that the flavour structure of new physics is highly non-generic or fined-tuned.

A solution to this dichotomy is the celebrated Minimal Flavour Violation scheme (25, 27, 28, 85) which is predictive, realistic, model independent and symmetry driven. The previous section showed that flavour phenomenology at present is explained by the SM plus neutrino masses solely, this is to say that the mass terms contain all the known flavour structure and ergo determine the flavour violation. The conclusion is that the mass terms are the only source for *all* flavour and CP violation data at our disposal. The minimality assumption of MFV is to upgrade this source to be the only one in physics Beyond the Standard Model too at low energies.

In the absence of the mass terms the theory presents a symmetry which is formally conserved if the sources of flavour violation are assigned transformation properties, in the present realization given in table 4.2. The formal restoration of

	$SU(3)_{Q_L}$	$SU(3)_{U_R}$	$SU(3)_{D_R}$	$SU(3)_{\ell_L}$	$SU(3)_{E_R}$	$O(3)_{N_R}$
Y_U	3	$\bar{3}$	1	1	1	1
Y_D	3	1	$\bar{3}$	1	1	1
Y_E	1	1	1	3	$\bar{3}$	1
Y_ν	1	1	1	3	1	3

Table 4.2: Spurious transformations of the Yukawa couplings

the flavour symmetry applied in the effective field theory set-up determines the flavour constants which shall be such as to form flavour invariant combinations with the matter fields and build up out of the sole sources of flavour violation at low energies, the Yukawas. The previous operator will serve as example now:

$$c_6\,\mathcal{O}^6 = \bar{Q}_L^\alpha \left(Y_U Y_U^\dagger\right)_{\alpha\beta} \gamma_\mu Q_L^\beta \bar{Q}_L^\sigma \left(Y_U Y_U^\dagger\right)_{\sigma\rho} \gamma^\mu Q_L^\rho\,. \tag{4.20}$$

The Yukawa couplings, can be written as in Eqs. 4.6 , 4.8 , 4.9 and therefore all parameters entering the above equation are known, they are just masses and mixings.

It should be underlined that MFV is not a model of flavour and the value of the new dynamical flavour scale Λ_f is not fixed, however the suppression introduced via the flavour parameters makes this scale compatible with the TeV, see (92) for a recent analysis. What it does predict is precise and constrained relations between different flavour transitions.

5

Spontaneous Flavour Symmetry Breaking

The previous chapter illustrated how the entire body of flavour data can be explained through a single entity, the mass terms. This has been shown to be the only culprit of flavour violation. If we pause and look at the previous sentence, it is interesting to see how the jargon itself already assumes that there is something to be violated, and implicitly a breaking idea. It has been shown that the symmetry of the matter content of the free theory here considered is the product of the gauge and flavour symmetries; $\mathcal{G} \times \mathcal{G}_{\mathcal{F}}$, and that Yukawa terms do not respect $\mathcal{G}_{\mathcal{F}}$. Subgroups of this group could also be considered, here the full $\mathcal{G}_{\mathcal{F}}$ is adopted in the general case, although in certain cases the axial abelian factors $U(1)_A$ will be dropped[1]. The case of conservation of the full $\mathcal{G}_{\mathcal{F}}$ group is also denoted *axial conserving case*, whereas assuming that the $U(1)_A$ symmetries are not exact will constitute the *explicitly axial breaking* case $\mathcal{G}_{\mathcal{F}}^{\not A} \sim SU(3)^5 \times SO(3)$. In all cases the full non-abelian group is considered.

The MFV ansatz showed the usefulness of assigning spurious transformation properties to the Yukawa couplings and having a formal flavour conservation at the phenomenological level. It is only natural to take the next step and assume the flavour symmetry is exact at some high energy scale Λ_f and the Yukawa couplings are the remains of fields that had real transformations properties under

[1]Or alternatively broken by a different mechanism, like a Froggat-Nielsen model.

this symmetry. The underlying idea of dynamical Yukawa couplings is depicted in Fig. 5.1 which resembles similar diagrams in Froggat Nielsen theories. The basic assumption is indeed already present in the literature; for example in the first formulation of MFV by Chivukula and Georgi (22), the Yukawa couplings corresponded to a fermion condensate. It should also be mentioned that a flavour breaking mechanism with different continuos non-abelian groups than the here considered has been explored (18, 24, 93, 94, 95, 96) and after the appearance of this work the quantum corrections where studied in (97, 98).

The analysis of a two generation case will serve as illustration and guide in the next chapter, for this reason it is useful and compact to introduce n_g for the number of generations. The straight-forward generalization of the flavour group is then:

$$\mathcal{G}_{\mathcal{F}} = \mathcal{G}^q_{\mathcal{F}} \times \mathcal{G}^l_{\mathcal{F}} \,,$$

$$\mathcal{G}^q_{\mathcal{F}} = SU(n_g)_{Q_L} \times SU(n_g)_{U_R} \times SU(n_g)_{D_R} \times U(1)_B \times U(1)_{A^U} \times U(1)_{A^D} \,, \quad (5.1)$$

$$\mathcal{G}^l_{\mathcal{F}} = SU(n_g)_{\ell_L} \times SU(n_g)_{E_R} \times O(n_g)_N \times U(1)_L \times U(1)_{A^l} \,. \quad (5.2)$$

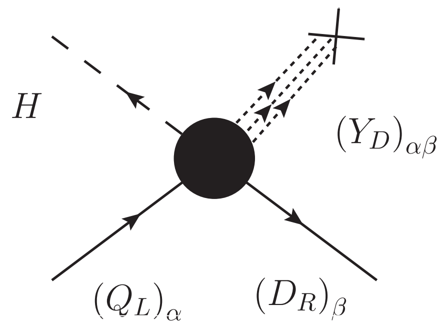

Figure 5.1: Yukawa Couplings as vevs of flavour fields -

5.1 Flavour Fields Representation

The starting point is rendering the Yukawa interaction explicitly invariant under the flavour symmetry. At the scale Λ_f of the new fields responsible for flavour breaking, the Yukawa couplings will be dynamical themselves, implying the mass dimension of the *Yukawa Operator* is now > 4.

Scalar Flavour Fields in the Bi-Fundamental

In the effective field theory expansion, the leading term is dimension 5[1]:

$$\mathscr{L}_{Yukawa} = \overline{Q}_L \frac{\mathcal{Y}_D}{\Lambda_f} D_R H + \overline{Q}_L \frac{\mathcal{Y}_U}{\Lambda_f} U_R \tilde{H} + \bar{\ell}_L \frac{\mathcal{Y}_E}{\Lambda_f} E_R H + \bar{\ell}_L \frac{\mathcal{Y}_\nu}{\Lambda_f} N_R \tilde{H} + h.c.\,, \quad (5.3)$$

where there is the need to introduce the cut-off scale ${\Lambda_f}^2$, the scalar fields $\mathcal{Y}_D$, $\mathcal{Y}_U$, $\mathcal{Y}_E$ and $\mathcal{Y}_\nu$ are dynamical fields in the bi-fundamental representation as detailed in tables 5.1,5.2, and the relation to ordinary Yukawas is:

	$SU(n_g)_{Q_L}$	$SU(n_g)_{U_R}$	$SU(n_g)_{D_R}$	$U(1)_B$	$U(1)_{A^U}$	$U(1)_{A^D}$
$\mathcal{Y}_U$	n_g	$\bar{n}_g$	1	0	2	1
$\mathcal{Y}_D$	n_g	1	n_g	0	1	2

Table 5.1: $\mathcal{G}_{\mathcal{F}}q$ representation of the quark sector bi-fundamental scalar fields for n_g fermion generations

	$SU(n_g)_{\ell_L}$	$SU(n_g)_{E_R}$	$O(n_g)_{N_R}$	$U(1)_L$	$U(1)_{A^l}$
$\mathcal{Y}_E$	n_g	$\bar{n}_g$	1	0	2
$\mathcal{Y}_\nu$	n_g	1	n_g	1	1

Table 5.2: ${\mathcal{G}_{\mathcal{F}}}^l$ representation of the lepton sector bi-fundamental scalar fields for n_g fermion generations

$$Y_D \equiv \frac{\langle \mathcal{Y}_D \rangle}{\Lambda_f}\,, \qquad Y_U \equiv \frac{\langle \mathcal{Y}_U \rangle}{\Lambda_f}\,, \qquad Y_E \equiv \frac{\langle \mathcal{Y}_E \rangle}{\Lambda_f}\,. \qquad Y_\nu \equiv \frac{\langle \mathcal{Y}_\nu \rangle}{\Lambda_f}\,. \quad (5.4)$$

[1]The expansion now differs from the EFT in the SM context since we have introduced new scalar fields

[2]The equation above could have in more generality coupling constants different for the up and down sector or equivalently a different scale for up and down, here the scale is chosen the same for simplicity

This case is hereby labeled *bi-fundamental* scenario, an the fields can be thought of as matrices whose explicit transformation is:

$$\mathcal{Y}_U(x) \xrightarrow{\mathcal{G_F}} \Omega_{Q_L}\mathcal{Y}_U(x)\,\Omega^\dagger_{U_R}\,, \qquad \mathcal{Y}_D(x) \xrightarrow{\mathcal{G_F}} \Omega_{Q_L}\mathcal{Y}_D(x)\,\Omega^\dagger_{D_R}\,, \tag{5.5}$$

$$\mathcal{Y}_E(x) \xrightarrow{\mathcal{G_F}} \Omega_{\ell_L}\mathcal{Y}_E(x)\,\Omega^\dagger_{E_R}\,, \qquad \mathcal{Y}_\nu(x) \xrightarrow{\mathcal{G_F}} \Omega_{\ell_L}\mathcal{Y}_\nu(x)\,O^T_{N_R}\,, \tag{5.6}$$

Ω_ψ (O_{N_R}) being a unitary (real orthogonal) matrix of the corresponding $\mathcal{G_F}$ subgroup: $\Omega_\psi\Omega^\dagger_\psi = \Omega^\dagger_\psi\Omega_\psi = 1$, $\psi = Q_L \ldots E_R$ ($O_{N_R}O^T_{N_R} = O^T_{N_R}O_{N_R} = 1$) .

Scalar Flavour Fields in the Fundamental

The next order in the effective field theory is a $d = 6$ Yukawa operator, involving generically two scalar fields in the place of the Yukawa couplings,

$$\mathscr{L}_{Yukawa} = \overline{Q}_L\frac{\chi^L_D\chi^{R\dagger}_D}{\Lambda^2_f}D_R H + \overline{Q}_L\frac{\chi^L_U\chi^{R\dagger}_U}{\Lambda^2_f}U_R\tilde{H} + \overline{\ell}_L\frac{\chi^L_E\chi^{R\dagger}_E}{\Lambda^2_f}E_R H + \overline{\ell}_L\frac{\chi^L_\nu\chi^{R\dagger}_\nu}{\Lambda^2_f}E_R H, \tag{5.7}$$

which provide the following relations between Yukawa couplings and vevs:

$$Y_D \equiv \frac{\left\langle\chi^L_D\chi^{R\dagger}_D\right\rangle}{\Lambda^2_f}\,, \qquad Y_U \equiv \frac{\left\langle\chi^L_U\chi^{R\dagger}_U\right\rangle}{\Lambda^2_f}\,, \qquad Y_E \equiv \frac{\left\langle\chi^L_E\chi^{R\dagger}_E\right\rangle}{\Lambda^2_f}\,, \qquad Y_\nu \equiv \frac{\left\langle\chi^L_\nu\chi^{R\dagger}_\nu\right\rangle}{\Lambda^2_f}\,, \tag{5.8}$$

The simplest assignation of charges or transformation properties of these fields is to consider each of them in the fundamental representation of a given $SU(3)_\psi$ subgroup as specified in tables 5.3,5.4.

	$SU(n_g)_{Q_L}$	$SU(n_g)_{U_R}$	$SU(n_g)_{D_R}$	$U(1)_B$	$U(1)_{A^U}$	$U(1)_{A^D}$
χ^L_U	n_g	1	1	0	1	1
χ^L_D	n_g	1	1	0	1	1
χ^R_U	1	n_g	1	0	-1	0
χ^R_D	1	1	n_g	0	0	-1

Table 5.3: Representation of the lepton sector fundamental scalar fields for n_g fermion generations

These fields are then complex n_g-vectors whose transformation under the flavour group is just a unitary or real rotation; $\chi_\psi \xrightarrow{\mathcal{G_F}} \Omega_\psi\chi_\psi$, $\chi^R_N \xrightarrow{\mathcal{G_F}} O_{N_R}\chi^R_N$.

	$SU(n_g)_{\ell_L}$	$SU(n_g)_{E_R}$	$O(n_g)_{N_R}$	$U(1)_L$	$U(1)_{A^l}$
χ_E^L	n_g	1	1	0	1
χ_ν^L	n_g	1	1	0	1
χ_E^R	1	n_g	1	0	-1
χ_N^R	1	1	n_g	0	0

Table 5.4: Representation of the lepton sector fundamental scalar fields for n_g fermion generations

From the group theory point of view this is the decomposition in the irreducible pieces needed to build up invariant Yukawa operators, and as we shall see their properties translate in an easy and clear extraction of the flavour structure.

The third case of a Yukawa operator of mass dimension 7 could arise from a condensate of fermionic fields $Y \sim \left\langle \overline{\Psi}\Psi \right\rangle / \Lambda_f^3$ (22), or as the product of three scalar fields. In both cases the simplest decomposition falls trivially into one of the previous or the assignation of representations is an otherwise unnecessarily complicated higher dimensional one.

Notice that realizations in which the Yukawa couplings correspond to the vev of an aggregate of fields, rather than to a single field, are not the simplest realization of MFV as defined in Ref. (25), while still corresponding to the essential idea that the Yukawa spurions may have a dynamical origin.

Finally, other option of dependence of the Yukawa couplings on the dynamical fields is an inverse one:

$$Y_D \equiv \frac{\Lambda_f}{\langle \mathcal{Y}_D \rangle}\,, \qquad Y_U \equiv \frac{\Lambda_f}{\langle \mathcal{Y}_U \rangle}\,, \qquad Y_E \equiv \frac{\Lambda_f}{\langle \mathcal{Y}_E \rangle}\,, \qquad Y_\nu \equiv \frac{\Lambda_f}{\langle \mathcal{Y}_\nu \rangle}\,. \tag{5.9}$$

a case in which de vev of the field rather than the scale Λ_f entering the relation is the larger one. This interesting case arises in models of gauged flavour symmetry (99, 100), in which the anomaly cancellation requirements call for the introduction of fermion fields, whose interaction in a renormalizable Lagrangian with the scalar fields and ordinary fermions suffice to constitute a self consistent theory that after the integration of the heavy states yields the relation above. The transformation properties of the fields are the same as in the bi-fundamental case.

For simplicity in the group decomposition and since they appear as the two leading terms in the effective field theory approach, we will focus the analysis here in the fundamental and bi-fundamental cases or the dimension 5 and 6 Yukawa operators, the former nonetheless also applies to relation 5.9.

5.2 The Scalar Potential

The way in which the scalar fields $\mathcal{Y}$, χ acquire a vev is through a scalar potential. This potential, must be invariant under the gauge group of the SM $\mathcal{G}$ and the flavour group $\mathcal{G}_{\mathcal{F}}$. The study is focused on the potential constituted by the flavour fields only, even if there might be some mixing with the singlet combination $H^\dagger H$ of the Higgs field, an exploration of this last case can be found in (101) in which the flavour scalar fields are postulated as Dark Matter. This case would add to the hierarchy problem but make no difference in the determination of the flavour fields minimum since the mass scale of the latter is taken larger than the Higgs vev: $\Lambda_f^2 \gg v^2$.

The goal of this work is therefore to address the problem of the determination and analysis of the general $\mathcal{G}_{\mathcal{F}}$-invariant scalar potential and its minima for the flavour scalar fields denoted above by $\mathcal{Y}$ and χ. The central question is whether it is possible to obtain the SM Yukawa pattern - i.e. the observed values of quark masses and mixings- with a “natural” potential.

It is worth noticing that the structure of the scalar potentials constructed here is more general than the particular effective realization in Eqs. (5.4) and (5.8) and it would apply also for Eq. 5.9 as it relies exclusively on invariance under the symmetry $\mathcal{G}_{\mathcal{F}}$ and on the flavon representation, bi-fundamental or fundamental.

This observation is relevant, because the case of gauged flavour symmetry leading to Eq. 5.9 addresses two problems that this approach has. Namely the presence of Goldstone bosons as a result of the spontaneous breaking of a continuous symmetry and the constraints placed on the presence of new particles carrying flavour and inducing potentially dangerous FCNC effects.

The Goldstone bosons in a spontaneously broken flavour gauge symmetry are eaten by the flavour group vector bosons which become massive. These particles even if massive would induce dangerous flavour changing processes which we expect to be suppressed by their scale. The case of gauged flavour symmetries is however such that the inverse relation of Yukawas of Eq. 5.9 translates also to the particle masses, so that the new particles inducing flavour changing in the lightest generations are the heaviest in the new physics spectrum (102). This two facts conform a possible acceptable and realistic scenario where to embed the present study, even if the analysis applies in a general set-up since it is based only on symmetries.

5.2.1 Generalities on Minimization

The variables in which we are minimizing are the parameters of the scalar fields modulo a $\mathcal{G}_\mathcal{F}$ transformation. That is, we minimize in the variables of the scalar fields that are not absorbable with a group transformation. The discussion of which are those variables in the bi-fundamental case is familiar to the particle physicist; they are the equivalent of masses and mixing angles. Indeed we can substitute in Eq. 5.4 the explicit formula for the Yukawas, Eqs. 4.6 -4.10, and express the variables of the scalar field at the minimum in terms of flavour parameters.

The equation obtained in this way is the condition of the vev of the scalar fields fixing the masses and mixings *that are measured*. It is not clear at all though that a spontaneous breaking mechanism can yield the very values that Yukawas actually have. To find this out the minimization of the potential has to be completed, such that for the next two chapters masses and mixing will be treated as variables roaming all their possible range. The question is whether at the minimum of the potential these variables can take the values corresponding to the known spectrum and if so to what cost.

The $\mathcal{G}_\mathcal{F}$ invariants out of which the potential is built will be denoted generically by I_j, while y_i stand for the physical variables of the scalar fields connected explicitly to masses and mixing. Let us call n the number of physical parameters that suffice to describe the general vev of the flavour fields, that is to say there are n variables $y_i\,, i = 1, 2, ..., n$. The following considerations can be found in (18, 93, 94)

A simple result is that there are n independent invariants I_j, since the inversion of the relation of the latter in terms of the variables[1] allows to express any new

[1]Inverse relation which is unique up to discreet choices (103)

invariant I' in terms of the independent set $\{I_j\}$; $I' = I'(y_i) = I'(y_i(I_j))$.

In terms of the set of invariants $\{I_j\}$ the stationary points of the potential, among them the true vacuum, are the solutions to the equation,

$$\sum_j \frac{\partial I_j}{\partial y_i}\frac{\partial V}{\partial I_j} = 0\,. \tag{5.10}$$

These n equations will fix the n parameters. One can regard this array of equations as a matrix $J_{ij} = \partial I_j/\partial y_i$, which is just the Jacobian of the change of "coordinates" $I_j = I_j(y_i)$, times a vector $\partial V/\partial I_j$.

This system, if the Jacobian has rank n, has only the solution of a null vector $\partial V/\partial I_j = 0$, which is the case for example for the Higgs potential of the SM.

When the Jacobian has rank smaller than n, the system of Eqs simplifies to a number of equations equal to the rank of the Jacobian. The extreme case would be a rank 0 Jacobian, which is the trivial, but always present, symmetry preserving case. This link of the smallest rank with the largest symmetry can be extended; indeed in general terms the reduction of the rank implies the appearance of symmetries left unbroken. In this sense the case of largest unbroken symmetries not being the trivial one are called maximal isotropy groups (93, 94), that is the greatest groups within the group but smaller than him. Please note that imposing a reduced rank of J is a potential-independent condition; it is a constrain depending solely on the change of basis from variables to invariants.

For a geometric comprehension of the reduction of the Jacobian's rank the manifold of possible values for the invariants can be considered (18, 93, 94), denoted *I-manifold*. The *I-manifold* can be embedded in a *n-th* dimensional real space $\mathcal{R}^n$. Whenever the Jacobian has reduced rank there exist one or more directions in which a variation in the parameters y has 0 variation in $\mathcal{R}^n$, let us denote this displacement δy_i , then this statement reads,

$$\delta I_j = \sum_i \frac{\partial I_j}{\partial y_i}\,\delta y_i = 0\,. \tag{5.11}$$

This direction is the normal to a boundary of the *I-manifold*, as displacements in this direction are not allowed. The further the rank is reduced the more reduced is the dimension of this boundary. Those points for which the rank was reduced the most while still triggering symmetry breaking, will be denoted singular were whereas in the original analysis of $SU(3) \times SU(3)$ singular stood the complete symmetry group conserving points (18).

In the general case one can expect to have a combination of both, reduced rank of the Jacobian and potential-dependent solutions. It is in any case worth examining first the Jacobian, as it is done in the next chapters.

Another relevant issue is the number of invariants that enter the potential. If one is to stop the analysis at a given operator's dimensionality as it is customary in EFT some of the invariants are left out. Does this mean there are parameters left undetermined by the potential, i. e. flat directions? We shall see that these flat directions are related to the presence of unbroken symmetries and therefore are unphysical, so rather than the potential in such cases being unpredictive is quite the opposite, it imposes symmetries in the low energy spectrum.

6

Quark Sector

This chapter will concern the analysis of flavour symmetry breaking in the quark sector through the study of the general potential in both the bi-fundamental and fundamental representation cases.

6.1 Bi-fundamental Flavour Scalar Fields

At a scale above the electroweak scale and around Λ_f we assume that the Yukawa interactions are originated by a Yukawa operator with dimension = 5 as made explicit in Eq. 5.3, the connection to masses and mixing of the new scalar fields given in Eq. 5.4. The analysis of the potential for the bi-fundamental scalar fields is split in the two and three generation case.

6.1.1 Two Family Case

The discussion of the general scalar potential starts by illustrating the two-family case, postponing the discussion of three families to the next section. Even if restricted to a simplified case, with a smaller number of Yukawa couplings and mixing angles, it is a very reasonable starting-up scenario, that corresponds to the limit in which the third family is decoupled, as suggested by the hierarchy between quark masses and the smallness of the CKM mixing angles[1] θ_{23} and θ_{13}.

[1] We follow here the PDG (51) conventions for the CKM matrix parametrization.

In this section, moreover, most of the conventions and ideas to be used later on for the three-family analysis will be introduced.

The number of variables that suffice for the description of the physical degrees of freedom of the scalar fields $\mathcal{Y}$ is the starting point of the analysis. Extending the bi-unitary parametrization for the Yukawas given in the first terms of Eqs. 4.2-4.3 to the scalar fields and performing a $\mathcal{G}_\mathcal{F}$ rotation as in Eq. 5.5 the objects left are a unitary matrix, and two diagonal matrices of eigenvalues. Out of the 4 parameters of a general unitary 2×2 matrix, three are complex phases which can be rotated away via diagonal phase rotations of $\mathcal{G}_\mathcal{F}$. The remaining variables are therefore an angle in the mixing matrix and 4 eigenvalues arranged in two diagonal matrices: a total of $n = 5$ following the notation introduced. This is no other than the usual discussion of physical parameters in the Yukawa couplings, applicable to the flavour fields since the underlying symmetry is the same.

The explicit connection of scalar fields variables and flavour parameters is,

$$\langle \mathcal{Y}_D \rangle = \Lambda_f \mathbf{y}_D = \Lambda_f \begin{pmatrix} y_d & 0 \\ 0 & y_s \end{pmatrix}, \qquad \langle \mathcal{Y}_U \rangle = \Lambda_f V_C^\dagger \mathbf{y}_U = \Lambda_f V_C^\dagger \begin{pmatrix} y_u & 0 \\ 0 & y_c \end{pmatrix}, \tag{6.1}$$

where

$$V_C = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \tag{6.2}$$

is the usual Cabibbo rotation among the first two families.

From the transformation properties in Eq. 5.5, it is straightforward to write the list of independent invariants that enter in the scalar potential. For the case of two generations that occupies us now, five independent invariants can be constructed respecting the whole $\mathcal{G}_\mathcal{F}^q$ group (103, 104):

$$I_U = \mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right), \qquad I_D = \mathrm{Tr}\left(\mathcal{Y}_D {\mathcal{Y}_D}^\dagger\right), \tag{6.3}$$

$$I_{U^2} = \mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_U \mathcal{Y}_U^\dagger\right), \qquad I_{D^2} = \mathrm{Tr}\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right), \tag{6.4}$$

$$I_{UD} = \mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D {\mathcal{Y}_D}^\dagger\right). \tag{6.5}$$

The value of these invariants at the minimum correspond to[1]:

$$I_U = \Lambda_f^2\left(y_u^2+y_c^2\right), \qquad I_D = \Lambda_f^2\left(y_d^2+y_s^2\right), \tag{6.6}$$

$$I_{U^2} = \Lambda_f^4\left(y_u^4+y_c^4\right), \qquad I_{D^2} = \Lambda_f^4\left(y_d^4+y_s^4\right), \tag{6.7}$$

$$I_{UD} = \Lambda_f^4\left[\left(y_c^2-y_u^2\right)\left(y_s^2-y_d^2\right)\cos 2\theta + \left(y_c^2+y_u^2\right)\left(y_s^2+y_d^2\right)\right]/2\,. \tag{6.8}$$

The counting of parameters required of the full $\mathcal{G}_\mathcal{F}$ group; the absence of $U(1)_A$ factors does not allow for overall phase redefinitions and therefore in the explicitly axial breaking case ($\mathcal{G}_\mathcal{F}^{\not A,q} \sim SU(n_g)^3$) two more parameters appear: the overall phases of the scalar fields. In the axial breaking case therefore the number of variables is $n=7$.

This case allows for two new invariants of dimension 2,

$$I_{\tilde{U}} = \det\left(\mathcal{Y}_U\right)\,, \qquad I_{\tilde{D}} = \det\left(\mathcal{Y}_D\right)\,, \tag{6.9}$$

the two extra parameters appearing in this case are the complex phase of the determinant for each $\mathcal{Y}$ field.

The two complex determinants together with the previous 5 operators of Eq. 6.3-6.5 add up to 9 real quantities which points to two invariants being dependent on the rest. Indeed the Cayley-Hamilton relation in 2 dimensions reads:

$$\mathrm{Tr}\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\mathcal{Y}_U\mathcal{Y}_U^\dagger\right) = \mathrm{Tr}\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\right)^2 - 2\det\left(\mathcal{Y}_U\right)\det\left(\mathcal{Y}_U^\dagger\right). \tag{6.10}$$

$$\mathrm{Tr}\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\mathcal{Y}_D\mathcal{Y}_D^\dagger\right) = \mathrm{Tr}\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^2 - 2\det\left(\mathcal{Y}_D\right)\det\left(\mathcal{Y}_D^\dagger\right). \tag{6.11}$$

The two determinants in terms of the variables read:

$$I_{\tilde{U}} = \Lambda_f^2\, y_u\, y_c\, e^{i\phi_U}\,, \qquad I_{\tilde{D}} = \Lambda_f^2\, y_d\, y_s e^{i\phi_D} - \tag{6.12}$$

The symmetry matters for the outcome of the analysis, so we shall make clear the differences in the choices of preserving the axial U(1)'s or not.

Notice that the mixing angle appears in all cases exclusively in I_{UD}, which is the only operator that mixes the up and down flavour field sectors. This is as intuitively expected: *the mixing angle describes the relative misalignment between the up and down sectors basis.* Eq. 6.8 shows that the degeneracy in any of the two

[1] Let us drop the vev symbols in $\langle I\rangle$ for simplicity in notation.

sectors makes the angle unphysical, or, in terms of the scalar fields and flavour symmetry, reabsorvable via a $\mathcal{G}_\mathcal{F}$ rotation.

Since there is one mixing parameter only in this case this invariant is related to all possible invariants describing mixing, in particular the Jarlskog invariant for two families,

$$4J = 4\det\left(\left[Y_U Y_U^\dagger, Y_D Y_D^\dagger\right]\right) = (\sin 2\theta)^2 \left(y_c^2 - y_u^2\right)^2 \left(y_s^2 - y_d^2\right)^2 ,$$

is related to I_{UD} via

$$\frac{1}{\Lambda_f^4}\frac{\partial}{\partial\theta}\mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D{}^\dagger\right) = -2\sqrt{J} . \tag{6.13}$$

The lowest dimension invariants that characterize symmetry breaking unmistakably are I_U and I_D. Indeed for $\langle I_U\rangle \neq 0$ or $\langle I_D\rangle \neq 0$, $\mathcal{G}_\mathcal{F}$ is broken, whereas if $\langle I_U\rangle = \langle I_D\rangle = 0$, $\mathcal{G}_\mathcal{F}$ remains unbroken. These invariants though only contain information on the overall scale of the breaking and make no distinction on hierarchies among eigenvalues. $I_{U,D}$ can be thought of as radii whose value gives no information on the "angular" variables. These variables can be chosen as the differences in eigenvalues, and their value at the minimum will fix the hierarchies among the different generations . The invariants that will determine these hierarchies will therefore be the those of Eqs. 6.4 , 6.5.

6.1.1.1 The Jacobian

All the work presented in this section is about to be published (17). The Jacobian of the change of coordinates from the variables to the invariants of Eqs. 6.3 6.5 is a $n \times n$ matrix. We are interested in the determinant for the location of the regions of reduced rank, or boundaries of the I-manifold. For these purpose we observe that the Jacobian has the shape:

$$J = \begin{pmatrix} \partial_{\mathbf{y}_U} I_{U^n} & 0 & \partial_{\mathbf{y}_U} I_{UD} \\ 0 & \partial_{\mathbf{y}_D} I_{D^n} & \partial_{\mathbf{y}_D} I_{UD} \\ 0 & 0 & \partial_\theta I_{UD} \end{pmatrix} \equiv \begin{pmatrix} J_U & 0 & \partial_{\mathbf{y}_U} I_{UD} \\ 0 & J_D & \partial_{\mathbf{y}_D} I_{UD} \\ 0 & 0 & J_{UD} \end{pmatrix} . \tag{6.14}$$

This structure of the Jacobian implies that the determinant simplifies to:

$$\det J = \det J_U \det J_D \det J_{UD} , \tag{6.15}$$

which is a result extensible to the 3 generation case. The third factor of this product reads:

$$\det J_{UD} = \sin 2\theta \left(y_c^2 - y_u^2\right)\left(y_s^2 - y_d^2\right)\,, \tag{6.16}$$

which signals $\theta = 0, \pi/2$ as boundaries, both of them corresponding to no mixing, we will examine this further in the next section. For the following analysis we select the $\theta = 0$ solution for illustration.

- **Axial Conserving Case:** $\mathcal{G}_{\mathcal{F}^q} \sim U(n_g)^3$ **-** The set of invariants in Eq. 6.6 , 6.6 yields:

$$J_U = \partial_{\mathbf{y}_U}\left(\mathrm{Tr}\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\right)\,,\,\mathrm{Tr}\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\mathcal{Y}_U\mathcal{Y}_U^\dagger\right)\right) = \begin{pmatrix} 2y_u & 4y_u^3 \\ 2y_c & 4y_c^3 \end{pmatrix}\,, \tag{6.17}$$

and

$$J_D = \partial_y\left(\mathrm{Tr}\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)\,,\,\mathrm{Tr}\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)\right) = \begin{pmatrix} 2y_d & 4y_d^3 \\ 2y_s & 4y_s^3 \end{pmatrix}\,, \tag{6.18}$$

so that:

$$\det J_U = y_c y_u (y_u^2 - y_c^2)\,, \qquad \det J_D = y_s y_d (y_d^2 - y_s^2)\,. \tag{6.19}$$

The solutions encoded in this can be classified according to the symmetry left unbroken,

1. $\mathcal{G}_{\mathcal{F}^q} \to U(1)_V^2 \times U(1)_A^2$ *Hierarchical spectrum for both up and down sectors*

$$\mathcal{Y}_U = \Lambda_f \begin{pmatrix} 0 & 0 \\ 0 & y \end{pmatrix}\,, \qquad \mathcal{Y}_D = \Lambda_f \begin{pmatrix} 0 & 0 \\ 0 & y' \end{pmatrix}\,. \tag{6.20}$$

2. $\mathcal{G}_{\mathcal{F}^q} \to U(1)_V^2 \times U(1)_A$

   a) *Down quarks degenerate Up quarks hierarchical*

$$\mathcal{Y}_U = \Lambda_f \begin{pmatrix} 0 & 0 \\ 0 & y \end{pmatrix}\,, \qquad \mathcal{Y}_D = \Lambda_f \begin{pmatrix} y' & 0 \\ 0 & y' \end{pmatrix}\,. \tag{6.21}$$

   b) *Up quarks degenerate Down quarks hierarchical*

$$\mathcal{Y}_U = \Lambda_f \begin{pmatrix} y & 0 \\ 0 & y \end{pmatrix}\,, \qquad \mathcal{Y}_D = \Lambda_f \begin{pmatrix} 0 & 0 \\ 0 & y' \end{pmatrix}\,. \tag{6.22}$$

3. $\mathcal{G_F}^q \to SU(2)_V \times U(1)_B$ *Down and Up quarks degenerate*

$$\mathcal{Y}_U = \Lambda_f \begin{pmatrix} y & 0 \\ 0 & y \end{pmatrix}, \qquad \mathcal{Y}_D = \Lambda_f \begin{pmatrix} y' & 0 \\ 0 & y' \end{pmatrix}. \tag{6.23}$$

The notation is such that $U(1)_V$ denote generation number and $U(1)_A$ chiral rotations, explicitly:

$$U(1)_V \;:\; \begin{cases} U(1)_{c+s}: & \begin{pmatrix} c_L \\ s_L \end{pmatrix} \to e^{ia} \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \quad c_R \to e^{ia} c_R\,, \quad s_R \to e^{ia} s_R\,, \\ U(1)_{u+d}: & \begin{pmatrix} u_L \\ d_L \end{pmatrix} \to e^{ia} \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad u_R \to e^{ia} u_R\,, \quad d_R \to e^{ia} d_R\,, \end{cases} \tag{6.24}$$

$$U(1)_A : \begin{cases} U(1)_{u_A}: & \begin{pmatrix} u_L \\ d_L \end{pmatrix} \to e^{ia} \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad c_R \to e^{-ia} c_R\,, \\ U(1)_{d_A}: & \begin{pmatrix} u_L \\ d_L \end{pmatrix} \to e^{ia} \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad d_R \to e^{-ia} d_R\,. \end{cases} \tag{6.25}$$

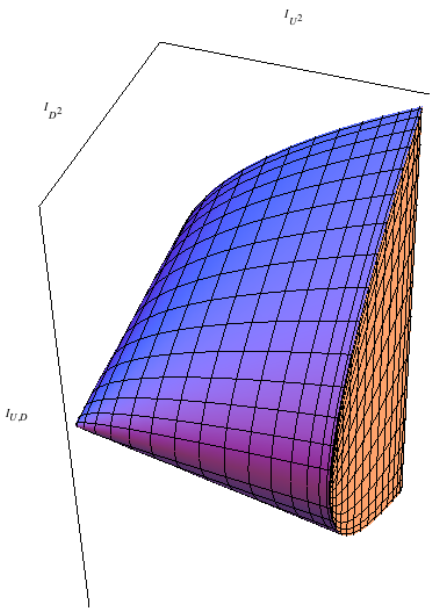

Figure 6.1: Boundaries for the *I-manifold* for fixed I_U , I_D. -

Summarizing, the total Jacobian determinant is:

$$\det J = \; y_u y_d y_s y_c \sin 2\theta \left(y_c^2 - y_u^2\right)^2 \left(y_s^2 - y_d^2\right)^2 \tag{6.26}$$

and the two largest subgroups of $\mathcal{G}_\mathcal{F}$ are $U(2)$ and $U(1)^4$ associated to the vertex point of the Fig. 6.1 and the upper corner of the same figure respectively.

- **Explicitly axial breaking case:** $\mathcal{G}_\mathcal{F}^{\not{A},q} \sim SU(n_g)^3$ **-** The invariants differ in this case and so do the Jacobians:

$$J_U = \partial_y \left(\text{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right) \,,\, |\det \mathcal{Y}_U| \right) = \begin{pmatrix} 2y_u & y_c \\ 2y_c & y_u \end{pmatrix} \,, \qquad (6.27)$$

and

$$J_D = \partial_y \left(\text{Tr}\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right) \,,\, |\det \mathcal{Y}_D| \right) = \begin{pmatrix} 2y_d & y_s \\ 2y_s & y_d \end{pmatrix} \,, \qquad (6.28)$$

so that

$$\det J_U = (y_u^2 - y_c^2) \,, \qquad \det J_D = (y_d^2 - y_s^2) \,, \qquad (6.29)$$

and the single solution associated to the pattern $\mathcal{G}_\mathcal{F}{}^q \to SU(2)_V \times U(1)_B$ survives since now no axial symmetry is present from the beginning. The third invariant related to the phase $\phi_{U,D}$ can be taken to be $\text{Arg}\,(\det \mathcal{Y}_{U,D})$, which is no other than the variable itself. Then this part of the Jacobian is block diagonal and constant, such that Jacobian determinant stays the same.

Altogether the Jacobian determinant is:

$$\det J = \sin 2\theta \, \left(y_c^2 - y_u^2\right)^2 \left(y_s^2 - y_d^2\right)^2 \,, \qquad (6.30)$$

and the only maximal subgroup is $U(2)$.

6.1.1.2 The Scalar Potential at the Renormalizable Level

The study of the Jacobian helped identify simple solutions in which some subgroup of $\mathcal{G}_\mathcal{F}$ was left unbroken corresponding to boundaries of the I-manifold. This analysis will serve as guide in the evaluation of the general scalar potential at the renormalizable level and the set of minima it allows for. The following study will reveal features obscured in the Jacobian method and will give further insight in the possible configurations and the role of unbroken symmetries. In particular the following study will reveal which of the above extrema (boundaries) correspond to minima and whether the potential allows for solutions outside of the boundaries and of what kind.

Axial preserving case: : $\mathcal{G_F}^q \sim U(n_g)^3$

The most general renormalizable potential invariant under the whole flavour symmetry group $\mathcal{G}_F^q$ can be writen in two lines by means of the introduction of the array:

$$X \equiv (I_U, I_D)^T = \left(\mathrm{Tr}\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\right), \mathrm{Tr}\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\right)\right)^T , \tag{6.31}$$

in terms of which:

$$\begin{aligned} V^{(4)} =& -\mu^2 \cdot X + X^T \cdot \lambda \cdot X + g\,\mathrm{Tr}\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\mathcal{Y}_D\mathcal{Y}_D^\dagger\right) \\ &+ h_U \mathrm{Tr}\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\mathcal{Y}_U\mathcal{Y}_U^\dagger\right) + h_D \mathrm{Tr}\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\mathcal{Y}_D\mathcal{Y}_D^\dagger\right) , \end{aligned} \tag{6.32}$$

where λ is a 2×2 real symmetric matrix, μ^2 a real 2-vector and $h_{U,D}, g$ three real parameters; a total of 8 parameters enter this potential. Strict naturalness criteria would require all dimensionless couplings λ, f, g, h to be of order 1, and the dimensionful μ-terms to be smaller or equal than Λ_f although of the same order of magnitude. The evaluation of the possible minima will reveal next nonetheless that even relaxing this condition the set of possible vacua is severly restricted.

Although is not the full solution to the minimization procedure let us consider in a first step and for illustration the first two terms in 6.32 taking the limit $g, h_{U,D} \to 0$. We can rewrite this part, if the matrix λ is invertible as:

$$-\mu^2 \cdot X + X^T \cdot \lambda \cdot X = \left(X - \frac{1}{2}\lambda^{-1} \cdot \mu^2\right)^T \lambda \left(X - \frac{1}{2}\lambda^{-1} \cdot \mu^2\right) - \mu^2 \cdot \frac{\lambda^{-1}}{4} \cdot \mu^2 \tag{6.33}$$

which is the generalization of a mexican-hat potential for two invariants. It is clear that if the vector $\frac{1}{2}\lambda^{-1} \cdot \mu^2$ takes positive values the minimum would set:

$$\begin{pmatrix} I_U \\ I_D \end{pmatrix} = \Lambda_f^2 \begin{pmatrix} y_c^2 + y_u^2 \\ y_s^2 + y_d^2 \end{pmatrix} = \frac{1}{2}\lambda^{-1} \cdot \mu^2 \tag{6.34}$$

This equation sets the order of magnitude of the Yukawa couplings as $y \sim \mu/(\Lambda_f \sqrt{\lambda})$, which signals the ratio of the mass scale of the scalar fields and the high scale Λ_f. For generic values of μ^2 and λ nonetheless the Yukawa magnitude of up and down quarks would be the same, so the two entries of $\frac{1}{2}\lambda^{-1} \cdot \frac{\mu^2}{\Lambda_f^2}$ should accommodate certain tuning, in the case that occupy us presently it would imply a $\mathcal{O}(10\%)$ ratio $y_s/y_c \simeq 10^{-1} = \sqrt{(\lambda^{-1}\mu^2)_U}/\sqrt{(\lambda^{-1}\mu^2)_D}$[1]. However let us recall here that for simplicity the coupling of the up and down scalar fields in the Yukawa operators were assumed the same, but if we were to extend this case to a two Higgs double scenario, the value of $\tan\beta$ could make this tuning disappear; as shown next it is the hierarchies within each up and down sector that the potential is unavoidably responsible for in this scheme.

For the complete minimization the extension of the above is simple, the effect of the invariants left out $I_{U,D,UD}$ adds up effectively to a modified λ and μ^2.

The stepwise strategy for minimization starts off with the minimization in those variables that appear less often in the potential, so that after solving in their minima equations the left-over potential no longer depends on them. Then we pick up the next variable which appears left often and iterate in this matrioska like fashion.

The starting point is then the angle variable, appearing in one invariant only, then follows the minimization of a variable independent from $\text{Tr}(\mathcal{Y}\mathcal{Y})$, which most often in the potential. The variables used in particular can be taken to be the difference of eigenvalues $\text{Tr}(\mathcal{Y}_{U,D}(-\sigma_3)\mathcal{Y}^\dagger_{U,D}) = \Lambda_f^2\left(y^2_{c,s} - y^2_{u,d}\right)$. The value of these variables will determine the hierarchy among the different generations, whereas $\text{Tr}(\mathcal{Y}\mathcal{Y}^\dagger)$ will have a saying on the overall magnitude of the Yukawas as shown above.

This method dictaminates therefore that we start with the mixing angle that appears in the single invariant I_{UD}. The equation for the angle is,

$$\frac{\partial V^{(4)}}{\partial\theta} = g\frac{\partial I_{UD}}{\partial\theta} = -g\,\Lambda_f^4 \sin 2\theta\left(y_c^2 - y_u^2\right)\left(y_s^2 - y_d^2\right) = 0\,. \tag{6.35}$$

The minimum of the scalar potential thus occurs for $\sin\theta = 0$ or $\cos\theta = 0$, for non-degenerate quark masses, which is the only case in which the angle makes sense. For determining which of these options is selected and to provide a very useful and general understanding of the minimization in unitary matrices parameters, the **Von Neumann trace inequality** for positive definite hermitian matrices is here reproduced:

[1]The values U, D label the to entries of μ^2: (μ^2_U, μ^2_D)

Let two hermitian positive definite $j \times j$ matrices A and B have eigenvalues of moduli $\alpha_1 \leq \alpha_2 \leq ... \leq \alpha_j$ and $\beta_1 \leq \beta_2 \leq ... \leq \beta_j$ respectively, then the following inequality holds:

$$\sum_{i=1}^{j} \alpha_{j+1-i}\, \beta_i \leq \mathrm{Tr}\,(A\, B) \leq \sum_{i=1}^{j} \alpha_i \beta_i \,. \tag{6.36}$$

The usefulness of this inequality is that it tells us that, considering the eigenvalues at a fixed value and varying the rest of parameters in the matrix, that is, the unitary matrices, the extrema are found for trivial unitary matrices. The inequality tells us that in the case of the Invariant I_{UD}:

$$y_u^2 y_s^2 + y_d^2 y_c^2 \leq \mathrm{Tr}\left(V_{CKM}^{\dagger} \mathbf{y}_U^2\, V_{CKM}\, \mathbf{y}_D^2\right) \leq y_u^2 y_d^2 + y_s^2 y_c^2 \,. \tag{6.37}$$

The two extrema are indeed given by the two solutions for the angle in Eq. 6.35 Which of these two is selected depends nonetheless on the sign of the coefficient in front of the invariant in the potential:

- $g > 0$ The potential is minimized when I_{UD} is minimized, so Eq. 6.37 dictaminates:

  $$V_C = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \tag{6.38}$$

  and the situation is such that the charm quark would couple only to the down type quark and the up to the strange, in a rather upside-down scenario.

- $g < 0$ The potential is minimized when I_{UD} is maximized, so Eq. 6.37 determines:

  $$V_C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} , \tag{6.39}$$

  This case is closer to reality, now the Cabibbo angle is set to 0 and the charm only couples to the strange quark, and the up to the down.

One can check that both these configurations leave an invariant $U(1)_V^2$ as defined in Eq. 6.24.

All in all, the straightforward lesson that follows from Eq. 6.35 is that, given the mass splittings observed in nature, *the scalar potential for bi-fundamental flavour fields does not allow mixing at the renormalizable level.*

The next step is the minimization in eigenvalues differences. The first relevant point is that only the invariants $I_{U^2}, I_{D^2}, I_{U,D}$ of Eqs. 6.7-6.8 depend on the eigenvalue squared differences $(y_{u,d}^4 + y_{c,s}^4 = (y_{u,d}^2 + y_{c,s}^2)^2/2 + (y_{u,d}^2 - y_{c,s}^2)^2/2)$ and appear linearly in the potential, Eq. 6.32.

When the operators in Eq. 6.7 have negative coefficients $h_{U,D} < 0$ the potential pushes towards the hierarchical configuration, which maximizes I_{U^2,D^2} and minimizes $-|h_{U,D}|\, I_{U^2,D^2}$. In the case of I_{UD} substitution in Eq. 6.8 and subsequently in Eq. 6.32 of the two possible solutions for the mixing at the minimum for each sign of g reveals that this term in the potential always pushes towards the hierarchical configuration. For the resemblance of nature *this configuration* (associated to case 1 of Eq. 6.20 in the Jacobian analysis) *is a good first approximation: only the heaviest family is massive so that* $y_u = y_d = 0$ *and the mixing, selecting* $g < 0$*, is vanishing.*

For completeness and illustration all the possible minima and their connection to the potential parameters are listed below:

I In this configuration a strong hierarchy arises;

$$\mathcal{Y}_U = \Lambda_f \begin{pmatrix} 0 & 0 \\ 0 & y_c \end{pmatrix}, \qquad \mathcal{Y}_D = \Lambda_f \begin{pmatrix} 0 & 0 \\ 0 & y_s \end{pmatrix}, \tag{6.40}$$

which presents an unbroken symmetry $\mathcal{G}_{\mathcal{F}}{}^q \to U(1)_V^2 \times U(1)_A^2$ and is just case 1 in the Jacobian analysis, see 6.20

II This case forbids mass for the up quark

$$\mathcal{Y}_U = \Lambda_f \begin{pmatrix} 0 & 0 \\ 0 & y_c \end{pmatrix}, \qquad \mathcal{Y}_D = \Lambda_f \begin{pmatrix} y_d & 0 \\ 0 & y_s \end{pmatrix}, \tag{6.41}$$

whereas the mass difference in the down sector is set by the relation

$$\frac{y_s^2 - y_d^2}{y_c^2} = \frac{|g|}{2h_D}, \tag{6.42}$$

and the breaking pattern is $\mathcal{G}_{\mathcal{F}}{}^q \to U(1)_V^2 \times U(1)_A$.

III The analogous of case **II** for massless down quark reads:

$$\mathcal{Y}_U = \Lambda_f \begin{pmatrix} y_u & 0 \\ 0 & y_c \end{pmatrix}, \qquad \mathcal{Y}_D = \Lambda_f \begin{pmatrix} 0 & 0 \\ 0 & y_s \end{pmatrix}, \tag{6.43}$$

$$\frac{y_c^2 - y_u^2}{y_s^2} = \frac{|g|}{2h_D}\,, \tag{6.44}$$

and again $\mathcal{G_F}^q \to U(1)_V^2 \times U(1)_A$.

IV Finally a completely degenerate scenario is possible in region **IV**

$$\mathcal{Y}_U = \Lambda_f \begin{pmatrix} y & 0 \\ 0 & y \end{pmatrix}\,, \qquad \mathcal{Y}_D = \Lambda_f \begin{pmatrix} y' & 0 \\ 0 & y' \end{pmatrix}\,, \tag{6.45}$$

having now that the potential triggers $\mathcal{G_F}^q \to SU(2)_V \times U(1)_B$, and an scenario very far from reality, but listed for completeness, and the analogous of case 3 and Eq. 6.23 in the Jacobian analysis.

These regions are shown in the $h_U - h_D$ plane in fig. 6.2.

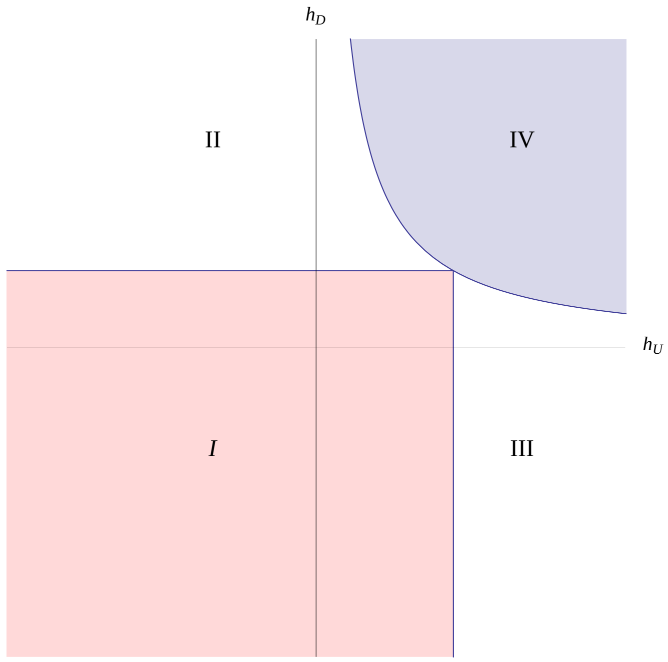

Figure 6.2: Different Regions for the Mass configuration - **I** is the region that yields a hierarchical spectrum for both up and down sectors **II** (**III**) presents a hierarchical down (up) spectrum and region **IV** results in degenerate up and down sectors

Note that the cases found here are not quite the same as the ones found in the Jacobian analysis. Case 2.a and 2.b are only present in the limiting case $g \to 0$

of **II** and **III**, so those are fine tuned cases. The reason for this is found in the symmetries, indeed cases 2.a and **II** and 2.b and **III** have the same symmetry, so from this point of view there is nothing special on having two eigenvalues degenerate when in the other sector one entry is 0, as the symmetry is the same if the two in the former sector do not coincide. The reason for the interplay of the up and down sector is the common group transformation properties under $SU(3)_L$ of $\mathcal{Y}_{U,D}$ and indeed this correlation disappears if the mixing invariant is neglected $g \to 0$, as can be checked on Eqs. 6.42-6.44.

Explicitly axial breaking case: $\mathcal{G}_{\mathcal{F}}^{\not{A},q} \sim SU(n_g)^3$

The set-up will change now with the introduction of the determinants in Eq. 6.9 when choosing to *violate* $U(1)_{A^U} \times U(1)_{A^D}$ *explicitly.* By making use of the analogous of X in this case,

$$\tilde{X} = (I_U, I_D, I_{\tilde{U}}, I_{\tilde{D}})^T \tag{6.46}$$

$$= \left(\mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right), \mathrm{Tr}\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right), |\det(\mathcal{Y}_U)|, |\det(\mathcal{Y}_D)|\right)^T, \tag{6.47}$$

the potential reads:

$$V^{(4)} = -\mu^2 \cdot \tilde{X} + \tilde{X}^T \cdot \lambda \cdot \tilde{X} + h.c. + g\, I_{UD} \tag{6.48}$$

where λ is matrix and μ^2 4-vector, the entries of these two structures are complex when they involve the determinants. The number of parameters has increased now to 14, since the symmetry is chosen less restrictive. Nonetheless the phases of the determinants are variables not observable at low energies and its minimization is of no interest here, suffice then to assume that they are set to their minimum values. Then we can effectively set it to 0 and consider all parameters in Eq. 6.48 real.

Parallel to the axial conserving case we have that, in the limit $g \to 0$, the minimum sets

$$\left\langle \tilde{X} \right\rangle = \frac{1}{2}\lambda^{-1} \cdot \mu^2, \tag{6.49}$$

if the entries of such vector are in the inside of the *I-manifold.* This now requires two conditions in the entries of $\lambda^{-1}\mu^2/2$. First all entries have to be positive, since the entries of X are always positive, and second the condition $I_U \geq 2|I_{\tilde{U}}|$

($I_D \geq 2|I_{\tilde{D}}|$) must be satisfied by the associated entries of $\lambda^{-1} \cdot \mu^2$. If this second condition is not realized the minimum is at the boundary, that is, $I_U = 2I_{\tilde{U}}$ ($I_D = 2I_{\tilde{D}}$) or equivalently $y_u = y_c$ ($y_d = y_s$).

Note also that in this case the solutions **I, II** and **III** are not present just like cases 2.a and 2.b were not either in the Jacobian analysis.

These considerations together with the distinct symmetries from which they arise lead to propose an ansazt to explain the hierarchy among the two generations of quarks.

First we start with the whole $\mathcal{G_F}$ group, so that determinants are forbidden and we chose to sit in the region **I** where the up and down are massless at this order. Then introduction of a small source of breaking of the $U(1)_A$'s would allow for the introduction of determinant terms in the potential with a naturally small coefficient since it is constrained by a symmetry.

This set-up is qualitatively explainable from symmetry considerations. In the axial preserving case the solution of hierarchical masses was present but the explicit breaking of the axial symmetry does not allow for such solutions. This means that a small perturbation on the axial symmetry breaking direction produces a small shift in the light quark masses.

6.1.1.3 The Scalar Potential at the Non-Renormalizable Level

The scalar potential at the renormalizable level in the axial preserving case allows for solutions with a strong hierarchy for both sectors of quark masses, that can be perturbed via a small breaking of the axial $U(1)'$s to displace the minimum and lift the zero masses of the lightest quarks. The Cabibbo angle was unavoidably set to 0, in this section we explore whether non-renormalizable terms in the potential may complete the picture.

Consider the addition of non-renormalizable operators to the scalar potential, $V^{(i>4)}$. It is very interesting to notice that this does *not* require the introduction of new invariants beyond those in Eqs. 6.3-6.5: all higher order traces and determinants can in fact be expressed in terms of that basis of five "renormalizable" invariants.

The lowest higher dimensional contributions to the scalar potential have dimension six. At this order, the only terms involving the mixing angle are

$$V^{(6)} \supset \frac{1}{\Lambda_f^2} \sum_{i=u,d} \left(\alpha_U I_{UD} I_U + \alpha_D I_{UD} I_D + \cdots \right) . \tag{6.50}$$

These terms, however, show the same dependence on the Cabibbo angle previously found in Eq. (6.35) and, consequently, they can simply be absorbed in the redefinition of the lowest order parameter, g. To find a non-trivial angular structure it turns out that terms in the potential of dimension eight (or higher) have to be considered, that is

$$V^{(8)} \supset \frac{\alpha}{\Lambda_f^4} I_{UD}^2 , \tag{6.51}$$

with whom the possibility of a mexican hat-like potential for I_{UD} becomes possible

$$V^{(8)} \supset \frac{\alpha}{\Lambda_f^4} \left(I_{UD} - \frac{g}{2\alpha} \Lambda_f^4 \right)^2 , \tag{6.52}$$

which would set

$$\sin^2\theta \simeq \frac{g}{2\, y_c^2 y_s^2 \alpha} . \tag{6.53}$$

Using the experimental values of the Yukawa couplings y_s and y_c, a realistic value for $\sin\theta$ can be obtained although at the price of assuming a highly fine-tuned hierarchy between the dimensionless coefficients of $d = 4$ and $d = 8$ terms, $g/\alpha \sim 10^{-10}$, that cannot be naturally justified in an effective Lagrangian approach.

The conclusion is therefore that mixing is absent in a natural 2 generation quark case.

6.1.2 Three Family Case

In this section we extend the approach discussed in the previous section to the three-family case. The two bi-triplets scalars transform explicitly under the flavour symmetry $\mathcal{G}_F^q$, as in Eq. 5.5 and the Yukawa Lagrangian is the same as that in Eq. (5.3). Once the flavons develop a vev the flavour symmetry is broken and one should recover the observed fermion masses and CKM matrix given in through Eq. (5.4).

While most of the procedure follows the steps of the 2 generation case, a few differences shall be underlined. First, the number of variables and therefore independent invariants differs. As in the two family case we can absorb three unitary matrices with $\mathcal{G}_\mathcal{F}$ rotations to leave two diagonal matrices with 3 eigenvalues each and a unitary matrix. The latter contains three angles and 6 phases; diagonal complex phase transformations allow to eliminate 5 of these so that the unitary matrix contains 4 physical parameters. In total 10 parameters describe the axial preserving case. Again this resembles closely the usual discussion of physical flavour parameters.

The higher number of variables implies that the list of invariants extends beyond mass dimension 4 and therefore not all of them will be present at the renormalizable level.

The list of invariants now grows reads (103, 104):

$$I_U = \mathrm{Tr}\left[\mathcal{Y}_U\mathcal{Y}_U^\dagger\right], \qquad I_D = \mathrm{Tr}\left[\mathcal{Y}_D\mathcal{Y}_D^\dagger\right], \tag{6.54}$$

$$I_{U^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\right)^2\right], \qquad I_{D^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^2\right], \tag{6.55}$$

$$I_{U^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\right)^3\right], \qquad I_{D^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^3\right], \tag{6.56}$$

these first 6 invariants depend only on eigenvalues while the following 4 contain mixing too,

$$I_{U,D} = \mathrm{Tr}\left[\mathcal{Y}_U\mathcal{Y}_U^\dagger\mathcal{Y}_D\mathcal{Y}_D^\dagger\right], \qquad I_{U,D^2} = \mathrm{Tr}\left[\mathcal{Y}_U\mathcal{Y}_U^\dagger\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^2\right], \tag{6.57}$$

$$I_{U^2,D} = \mathrm{Tr}\left[\mathcal{Y}_U\mathcal{Y}_U^\dagger\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^2\right], \qquad I_{(U,D)^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^2\right]. \tag{6.58}$$

Explicitly these invariants read[1]:

$$I_U = \Lambda_f^2\sum y_\alpha^2\,, \qquad I_D = \Lambda_f^2\sum y_i^2\,, \tag{6.59}$$

$$I_{U^2} = \Lambda_f^4\sum y_\alpha^4\,, \qquad I_{D^2} = \Lambda_f^4\sum y_i^4\,, \tag{6.60}$$

$$I_{U^3} = \Lambda_f^6\sum y_\alpha^6\,, \qquad I_{D^6} = \Lambda_f^6\sum y_i^6\,, \tag{6.61}$$

[1]In our convention greek letters are up-type indices and latin letters down-type indices.

$$I_{U,D} = \Lambda_f^4 \sum y_\alpha^2 V_{\alpha i} y_i^2 V_{\alpha i}^* \,, \qquad I_{U,D^2} = \Lambda_f^6 \sum y_\alpha^2 V_{\alpha i} y_i^4 V_{\alpha i}^* \,, \tag{6.62}$$

$$I_{U^2,D} = \Lambda_f^6 \sum y_\alpha^4 V_{\alpha i} y_i^2 V_{\alpha i}^* \,, \quad I_{(U,D)^2} = \Lambda_f^8 \sum y_\alpha^2 \, V_{\alpha i} \, y_i^2 \, V_{\beta i}^* \, y_\beta^2 \, V_{\beta j} \, y_j^2 \, V_{\alpha j}^* \,, \tag{6.63}$$

In the explicitly axial breaking case two complex phases add to the previous number of parameters so that 12 altogether conform the total. In this case the determinants

$$I_{\tilde{U}} = \text{Det}\left[\mathcal{Y}_U\right] \,, \qquad\qquad I_{\tilde{D}} = \text{Det}\left[\mathcal{Y}_D\right] \,, \tag{6.64}$$

substitute the invariants in Eq. 6.56 since they are connected through the relations:

$$\text{Tr}\left(\left(\mathcal{Y}_U^\dagger \mathcal{Y}_U\right)^3\right) = \frac{3}{2}\text{Tr}\left(\left(\mathcal{Y}_U^\dagger \mathcal{Y}_U\right)^2\right)\text{Tr}\left(\mathcal{Y}_U^\dagger \mathcal{Y}_U\right) - \frac{1}{2}\left(\text{Tr}\left(\mathcal{Y}_U^\dagger \mathcal{Y}_U\right)\right)^3 + 3\det \mathcal{Y}_U \det \mathcal{Y}_U^\dagger \tag{6.65}$$

$$\text{Tr}\left(\left(\mathcal{Y}_D^\dagger \mathcal{Y}_D\right)^3\right) = \frac{3}{2}\text{Tr}\left(\left(\mathcal{Y}_D^\dagger \mathcal{Y}_D\right)^2\right)\text{Tr}\left(\mathcal{Y}_D^\dagger \mathcal{Y}_D\right) - \frac{1}{2}\left(\text{Tr}\left(\mathcal{Y}_D^\dagger \mathcal{Y}_D\right)\right)^3 + 3\det \mathcal{Y}_D \det \mathcal{Y}_D^\dagger \tag{6.66}$$

and they read in terms of the variables;

$$I_{\tilde{U}} = \Lambda_f^3 e^{i\phi_U} \prod y_\alpha \,, \qquad\qquad I_{\tilde{D}} = \Lambda_f^3 e^{i\phi_U} \prod y_i \,, \tag{6.67}$$

which makes clear that the determinants of the fields $\det \mathcal{Y}$ change from mass dimension 2 to 3 in the present 3 family case.

6.1.2.1 The Jacobian

The study of the Jacobian is developed next. The Jacobian has an structure as in Eq. 6.14. For the mass terms the analysis was first carried out in (18, 105). The mixing term however is not in the literature yet (17). Let's turn first to the mixing Jacobian J_{UD}. We know that 4 parameters suffice to describe the mixing. Rather than choosing a parametrization for V_{CKM}, let us use the properties of a unitary matrix, substituting Eq. 6.1 in $I_{U,D}$:

$$\begin{aligned} I_{U,D} &= \sum_{\alpha,i}^{3} y_\alpha^2 \, V_{\alpha i} \, y_i^2 \, V_{\alpha i}^* \, , && (6.68)\\ &= \sum_{\alpha,i}^{3,2} y_\alpha^2 V_{\alpha i} \left(y_i^2 - y_b^2\right) V_{\alpha i}^* + y_b^2 \sum_\alpha y_\alpha^2 \, , && (6.69)\\ &= \sum_{\alpha,i}^{2} \left(y_\alpha^2 - y_t^2\right) V_{\alpha i} \left(y_i^2 - y_b^2\right) V_{\alpha i}^* \, , + y_b^2 \sum_\alpha y_\alpha^2 + y_t^2 \sum_i y_i^2 \, , && (6.70) \end{aligned}$$

where the terms independent of mixing elements are irrelevant for the analysis and will not be kept in the following. Note that what is achieved in using the unitarity relations is to rewrite the invariant in terms of 4 mixing elements, namely[1] $|V_{ud}|, |V_{us}|, |V_{cd}|$ and $|V_{cs}|$. The choice of these 4 is of course to one's discretion; we can choose other 4 by removing the $\alpha' th$ row and the $i' th$ column of V_{CKM}.

The same procedure for I_{U,D^2} and $I_{U^2,D}$ yields:

$$I_{U,D^2} = \sum_{\alpha,i}^{2} \left(y_\alpha^2 - y_t^2\right) V_{\alpha i} \left(y_i^2 + y_b^2\right) \left(y_i^2 - y_b^2\right) V_{\alpha i}^* + \cdots \, , \tag{6.71}$$

$$I_{U^2,D} = \sum_{\alpha,i}^{2} \left(y_\alpha^2 + y_t^2\right) \left(y_\alpha^2 - y_t^2\right) V_{\alpha i} \left(y_i^2 - y_b^2\right) V_{\alpha i}^* + \cdots \, , \tag{6.72}$$

whereas $I_{(U,D)^2}$ is more involved:

$$I_{(U,D)^2} = \sum_{\alpha,\beta,i,j}^{3} \left(y_\alpha^2 - y_t^2\right) V_{\alpha i} \left(y_i^2 - y_b^2\right) V_{\beta i}^* \left(y_\beta^2 - y_t^2\right) V_{\beta j} \left(y_j^2 - y_b^2\right) V_{\alpha j}^* + \cdots \, , \tag{6.73}$$

this equation differs from the square of $I_{U,D}$, in terms in which $\beta \neq \alpha$ and $i \neq j$, which implies they are *all* proportional to the 4 different mass differences:

$$\begin{aligned} I_{(U,D)^2} &= \left(\sum_{\alpha,i}^{3} y_\alpha^2 \, V_{\alpha i} \, y_i^2 \, V_{\alpha i}^* \right)^2 - 2 \left(y_u^2 - y_t^2\right) \left(y_c^2 - y_t^2\right) \left(y_d^2 - y_b^2\right) \left(y_s^2 - y_b^2\right) \\ &\quad \times \left(V_{ud} V_{cs} - V_{us} V_{cd}\right) \left(V_{ud}^* V_{cs}^* - V_{us}^* V_{cd}^*\right) \, . && (6.74) \end{aligned}$$

[1]These can be traded in λ, A, ρ, η in the Wolfenstein parametrization if preferred.

The first part we are not interested in as it is a function of a previously categorized invariant. The second has though a peculiar dependence on the mixing parameters. To rewrite it in terms of the four independent parameters the following relation is used:

$$\mathrm{Det}\,(V)\,\mathrm{Det}\,(V^*) = \sum_{\alpha,i}^{2} V_{\alpha i}V^*_{\alpha i} - (V_{ud}V_{cs} - V_{us}V_{cd})\,(V^*_{ud}V^*_{cs} - V^*_{us}V^*_{cd}) = 1\,. \quad (6.75)$$

Resuming, the 4 independent pieces of the invariants:

$$I'_{U,D} = \sum_{\alpha,i} \left(y_\alpha^2 - y_t^2\right)\left(y_i^2 - y_b^2\right) V_{\alpha i}V^*_{\alpha i}\,, \quad (6.76)$$

$$I'_{U,D^2} = \sum_{\alpha,i} \left(y_\alpha^2 - y_t^2\right)\left(y_i^2 + y_b^2\right)\left(y_i^2 - y_b^2\right) V_{\alpha i}V^*_{\alpha i}\,, \quad (6.77)$$

$$I'_{U^2,D} = \sum_{\alpha,i} \left(y_\alpha^2 + y_t^2\right)\left(y_\alpha^2 - y_t^2\right)\left(y_i^2 - y_b^2\right) V_{\alpha i}V^*_{\alpha i}\,, \quad (6.78)$$

$$I'_{(U,D)^2} = \prod_{\beta}\left(y_\beta^2 - y_t^2\right)\prod_{j}\left(y_j^2 - y_b^2\right)\sum_{\alpha,i}^{2} V_{\alpha i}V^*_{\alpha i}\,, \quad (6.79)$$

build up the Jacobian

$$J_{UD} = \frac{\partial \tilde{I}}{\partial |V_{\alpha,i}|} \propto \begin{pmatrix} |V_{ud}| & (y_d^2 + y_b^2)\,|V_{ud}| & (y_u^2 + y_t^2)\,|V_{ud}| & (y_c^2 - y_t^2)\,(y_s^2 - y_b^2)\,|V_{ud}| \\ |V_{us}| & (y_s^2 + y_b^2)\,|V_{us}| & (y_u^2 + y_t^2)\,|V_{us}| & (y_c^2 - y_t^2)\,(y_d^2 - y_b^2)\,|V_{us}| \\ |V_{cd}| & (y_d^2 + y_b^2)\,|V_{cd}| & (y_c^2 + y_t^2)\,|V_{cd}| & (y_u^2 - y_t^2)\,(y_s^2 - y_b^2)\,|V_{cd}| \\ |V_{cs}| & (y_s^2 + y_b^2)\,|V_{cs}| & (y_c^2 + y_t^2)\,|V_{cs}| & (y_u^2 - y_t^2)\,(y_d^2 - y_b^2)\,|V_{cs}| \end{pmatrix} \quad (6.80)$$

where the proportionality constant is different for each row; namely the product $(y_\alpha^2 - y_t^2)\,(y_i^2 - y_b^2)$. The determinant of J is

$$\mathrm{Det}\,(J_{UD}) = \left(y_u^2 - y_t^2\right)\left(y_c^2 - y_t^2\right)\left(y_u^2 - y_c^2\right)\left(y_d^2 - y_b^2\right)\left(y_s^2 - y_b^2\right)\left(y_d^2 - y_s^2\right) \quad (6.81)$$

$$\times\, |V_{ud}||V_{us}||V_{cd}||V_{cs}| \quad (6.82)$$

The analysis has turned out to be as simple as it could be. The determinant vanishes if *any* of the mass differences does, or if any of the entries of V vanishes. The rank is reduced the most for three mixing elements vanishing, which corresponds to (a permutation of) the identity.

Next the analysis of the invariants containing eigenvalues solely is presented, the axial breaking case was analyzed in (18) but is reproduced here for completeness.

- **Axial conserving case:** $\mathcal{G}^q_{\mathcal{F}} \sim U(n_g)^3$ The Jacobians are in this case,

$$J_U = \partial_y \left(\mathrm{Tr}\, \mathcal{Y}_U \mathcal{Y}_U^\dagger \,,\, \mathrm{Tr}\,(\mathcal{Y}_U \mathcal{Y}_U^\dagger)^2 \,,\, \mathrm{Tr}\,(\mathcal{Y}_U \mathcal{Y}_U^\dagger)^3 \right) = \begin{pmatrix} 2y_u & 4y_u^3 & 6y_u^5 \\ 2y_c & 4y_c^3 & 6y_c^5 \\ 2y_t & 4y_t^3 & 6y_t^5 \end{pmatrix} , \tag{6.83}$$

and

$$J_D = \partial_y \left(\mathrm{Tr}\, \mathcal{Y}_D \mathcal{Y}_D^\dagger \,,\, \mathrm{Tr}\,(\mathcal{Y}_D \mathcal{Y}_D^\dagger)^2 \,,\, \mathrm{Tr}\,(\mathcal{Y}_D \mathcal{Y}_D^\dagger)^3 \right) = \begin{pmatrix} 2y_d & 4y_d^3 & 6y_d^5 \\ 2y_s & 4y_s^3 & 6y_s^5 \\ 2y_b & 4y_b^3 & 6y_b^5 \end{pmatrix} , \tag{6.84}$$

so that:

$$\det J_U = y_c y_u y_t (y_u^2 - y_c^2)(y_c^2 - y_t^2)(y_u^2 - y_t^2)\,, \tag{6.85}$$

$$\det J_D = y_d y_s y_b (y_d^2 - y_s^2)(y_s^2 - y_b^2)(y_d^2 - y_b^2)\,. \tag{6.86}$$

There are now 4 possibilities to cancel each determinant above with ordered eigenvalues, these can be shorted in those who reduce the rank of the Jacobian to 2,

$$\mathcal{Y} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & y' \end{pmatrix} , \quad \begin{pmatrix} y & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & y' \end{pmatrix} , \quad \begin{pmatrix} y & 0 & 0 \\ 0 & y' & 0 \\ 0 & 0 & y' \end{pmatrix} , \tag{6.87}$$

and those that yield a rank 1 Jacobian

$$\mathcal{Y} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y \end{pmatrix} , \quad \begin{pmatrix} y & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & y \end{pmatrix} . \tag{6.88}$$

We will not list all the possible combinations of the up and down sector but display the two that result in maximal unbroken subgroups:

1. $\mathcal{G_F}^q \to SU(3)_V \times U(1)_B$ *Down and Up quark sectors degenerate*

$$\mathcal{Y}_U = \Lambda_f \begin{pmatrix} y & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & y \end{pmatrix} , \qquad \mathcal{Y}_D = \Lambda_f \begin{pmatrix} y' & 0 & 0 \\ 0 & y' & 0 \\ 0 & 0 & y' \end{pmatrix} . \tag{6.89}$$

2. $\mathcal{G_F}^q \to U(2)^3 \times U(1)_{t+b}$ *Down and Up quark sectors hierarchical*

$$\mathcal{Y}_U = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y \end{pmatrix}, \qquad \mathcal{Y}_D = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y' \end{pmatrix} \tag{6.90}$$

- **Explicitly axial breaking case:** $\mathcal{G}_{\mathcal{F}}^{\not{A},q} \sim SU(n_g)^3$ The Jacobians read

$$J_U = \partial_y \Big(|\det \mathcal{Y}_U|\,,\ \mathrm{Tr}\,\mathcal{Y}_U\mathcal{Y}_U^\dagger\,,\ \mathrm{Tr}\,(\mathcal{Y}_U\mathcal{Y}_U^\dagger)^2 \Big) = \begin{pmatrix} y_c y_t & 2y_u & 4y_u^3 \\ y_t y_u & 2y_c & 4y_c^3 \\ y_u y_c & 2y_t & 4y_t^3 \end{pmatrix}, \tag{6.91}$$

$$J_D = \partial_y \Big(\det \mathcal{Y}_D\,,\ \mathrm{Tr}\,\mathcal{Y}_D\mathcal{Y}_D^\dagger\,,\ \mathrm{Tr}\,(\mathcal{Y}_D\mathcal{Y}_D^\dagger)^2 \Big) = \begin{pmatrix} y_b y_s & 2y_d & 4y_d^3 \\ y_d y_b & 2y_s & 4y_s^3 \\ y_s y_d & 2y_b & 4y_b^3 \end{pmatrix}, \tag{6.92}$$

and the determinant of each Jacobian is

$$\det J_U = (y_u^2 - y_c^2)(y_c^2 - y_t^2)(y_u^2 - y_t^2)\,, \tag{6.93}$$

$$\det J_D = (y_d^2 - y_s^2)(y_s^2 - y_b^2)(y_d^2 - y_b^2)\,, \tag{6.94}$$

from where we see that the first case in 6.87 is no longer a solution.

6.1.2.2 The Potential at the Renormalizable Level

The following study will determine which of the different above unbroken symmetries (boundaries) are respected (possible) at the different minima of the potential. The renormalizable scalar potential will contain formally the same independent invariants as in the two generation case, only these invariants now depend on a higher number of variables.

Axial preserving case: $\mathcal{G}_{\mathcal{F}}^q \sim U(n_g)^3$

The most general scalar potential at the renormalizable level in this case is just the same formally as for the 2 family case: Eq. 6.32, using the vector X as defined in 6.31. Next is detailed the possible vacua permitted in this potential.

First the Von Neumann trace inequality permits the automatic minimization of the mixing term, so that we have two options;

$$g < 0 \quad V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} ; \qquad g > 0 \quad V_{CKM} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \tag{6.95}$$

the first is a good approximation to reality, whereas the second one would result in the top quark coupled only to the down type quark. These solutions leave an invariant generation number $U(1)^3_V$ defined as in Eq. 6.24 regardless for generic values of masses.

The two possibilities above are a reduced number of the various permutation matrices that the Jacobian analysis singled out. This means that the potential selects some of these boundaries, concretely those that order in an inverse or direct manner the mass eigenstates of up and down sectors.

With the same procedure as for the two family case we next minimize in the variables that will determine the hierarchy. These are now the two possible eigenvalue differences in the up sector and another two in the down sector.

The potential is formally the same as in the 2 family case and let us draw the readers attention to the fact that the "map" of Fig. 6.2 is drawn in terms of invariant magnitudes which know nothing of the dimension of the matrices involved. In this sense we expect the same map, as it will turn out. It is only left to determine what are the hierarchies in these regions.

We can anticipate, focusing on the contrast with the observed flavour pattern, that a hierarchical solution corresponding to region **I** of Fig. 6.2 where only the heaviest family is massive and the mixing matrix is the identity is a natural possible solution. Like in the two family case the resemblance with nature is good in a first sketch; *top and bottom are much heavier than the rest of quarks and the mix little (*$\sim \lambda_c^2$*) with them.*

For completeness the set of vacua is listed next:

I In this region the equivalent of the hierarchical configuration is now the case of vanishing of the lightest 4 eigenvalues,

$$\mathcal{Y}_D = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix} , \qquad \mathcal{Y}_U = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix} , \tag{6.96}$$

and an unbroken $U(2)^3 \times U(1)_{t+b}$.

II Now we have a hierarchical Yukawa for the up sector and the two lightest down-type eigenvalues are equal

$$\mathcal{Y}_D = \Lambda_f \begin{pmatrix} y & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & y_b \end{pmatrix}, \qquad \mathcal{Y}_U = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix}, \tag{6.97}$$

$$y_s = y_d = y\,, \qquad \frac{y_b^2 - y^2}{y_t^2} = \frac{|g|}{2h_D}\,, \tag{6.98}$$

leaving an unbroken an $U(2)_V \times U(2)_{U_R} \times U(1)_{t+b}$.

- **III** The analogous of the previous for the up sector is

$$\mathcal{Y}_D = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}, \qquad \mathcal{Y}_U = \Lambda_f \begin{pmatrix} y & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & y_t \end{pmatrix}, \tag{6.99}$$

$$y_c = y_u = y\,, \qquad \frac{y_t^2 - y^2}{y_b^2} = \frac{|g|}{2h_U}\,, \tag{6.100}$$

with an unbroken $U(2)_V \times U(2)_{D_R} \times U(1)_{t+b}$

- **IV** Finally the degenerate case is simply

$$\mathcal{Y}_D = \Lambda_f \begin{pmatrix} y & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & y \end{pmatrix}, \qquad \mathcal{Y}_U = \Lambda_f \begin{pmatrix} y' & 0 & 0 \\ 0 & y' & 0 \\ 0 & 0 & y' \end{pmatrix}, \tag{6.101}$$

respecting a $U(3)_V$ symmetry.

Note that none of the solutions have a single vanishing eigenvalue, so that only the case **I** could be a good approximation to reality. It is the case that the potential being the same as for two families, the picture of possible vacua in Fig. 6.2 is the same, only now the unbroken symmetry is different, but the maximal that we could choose (93, 94).

Explicitly axial breaking case: $\mathcal{G}_{\mathcal{F}}^{\not{A},q} \sim SU(n_g)^3$

The potential is now:

$$V^{(4)} = -\,\mu^2 \cdot X + X^T \cdot \lambda \cdot X + g\,\mathrm{Tr}\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\mathcal{Y}_D\mathcal{Y}_D^\dagger\right) + h_U\mathrm{Tr}\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\mathcal{Y}_U\mathcal{Y}_U^\dagger\right) \\ + h_D\mathrm{Tr}\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\mathcal{Y}_D\mathcal{Y}_D^\dagger\right) + \tilde{\mu}_U\det\mathcal{Y}_U + \tilde{\mu}_D\det\mathcal{Y}_D\,. \quad (6.102)$$

The inclusion of determinants will not change the possibilities listed as **I , II , III , IV**, since all of these configurations are also boundaries in this case. Another way of putting it is that part of the symmetries in the solutions above are still left after removing the $U(1)_A$ factors, namely $SU(2)_{D_R,E_R}$. This did not happen in the two family case as the unbroken symmetry was "$U(1)$" rather than "$U(2)$".

6.1.2.3 The Potential at the Non-Renomalizable Level

The first issue to deal with in this case is the fact that the order of magnitude of the Yukawa eigenvalues is set by the ratio $y \sim \mu/(\Lambda_f\sqrt{\lambda})$ which implies for the top Yukawa that the vev of the field $\mu/\sqrt{\lambda}$ is around the scale Λ_f signaling a bad convergence of the EFT. To cope with this first it is noted that the top Yukawa runs down with energy whereas the relation $y \sim \mu/(\Lambda_f\sqrt{\lambda})$ does not determine the overall scale. For energies of the order of 10^8 GeV (9) the top Yukawa is already smaller than the weak coupling constant allowing the usual expansion in EFT.

The case in which the two scales are or the same order can nonetheless formally be treated in the same sense as the non-linear σ-model. First the isolation in a single invariant of the problematic terms is accomplished by the set of invariants; $\{\, I_U, I_{U^2} - (I_U)^2\,, I_{U^3} - (I_U)^3 \,\}$ instead of Eqs. 6.54-6.56, such that the latter two are suppressed by one power of the second highest eigenvalue: y_c^2. Terms in I_U can be summed in a generic function in the potential $F\left(I_U/\Lambda_f^2\right) \equiv F(y_t'^2)$ and for this analysis it suffices that it has a minimum nonvanishing and around 1. The connection with Yukawas has also to be revisited

$$Y_U = \frac{\mathcal{Y}_U}{\Lambda_f} + \sum_i c_i \frac{\mathcal{Y}_U\left(\mathcal{Y}_U^\dagger\mathcal{Y}_U\right)^i}{\Lambda_f^{2i+1}} \simeq V_{CKM}^\dagger \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & f(y_t') \end{pmatrix} \quad (6.103)$$

Such that the connection with the top Yukawa coupling of the eigenvalue in $\mathcal{Y}_U$, denoted y_t', is $y_t = f(y_t')$. Then substitution in the function F yields the potential as a function of the top Yukawa coupling $F(f^{-1}(y_t))$. This means certainly a loss in predictivity since the introduced functions F , f are general, however for the present discussion it suffices that $F(f^{-1}(x))$ has a minimum at $x \simeq 1$.

In either case and to conclude this discussion, the symmetry arguments used to identify the possible vacua hold the same in this "strong interacting" scenario.

One interesting point is the possibility of non-renormalizable operators correcting the pattern of the renormalizable potential. It is a priori either a fine-tuned option like in the two family case or unsuccessful since the configurations are protected by a large unbroken symmetry. The intuitive reason for this is that for perturbations to displace the minimum they must create a small tilt in the potential via lineal dependence on the deviations from the 0-order solution; however non-renormalizable terms contain high powers of eigenvalues and therefore the corrections they introduce are not linear in the perturbations.

6.2 Flavour Scalar Fields in the Fundamental

In the simplest case from the group theory point of view, each Yukawa corresponds to two scalar fields χ transforming in the fundamental representation and the Yukawa Operator has dimension 6. This approach would *a priori* allow to introduce one new field for each component of the flavour symmetry: three fields. However, such a minimal setup leads to an unsatisfactory realization of the flavour sector as no physical mixing angle is allowed. The situation improves qualitatively, though, if two $SU(n_g)_{Q_L}$ representations are introduced, one for the up and one for the down quark sectors, the field content is detailed in table 5.3.

Before discussing the potential inspection of Eq. 5.8 will illuminate the road ahead. The hypothesis now is that Yukawas are build out of two fundamental representation. In linear algebra terms, the Yukawa matrix is made out of two vectors. This is of course a very strong assumption on the structure of the matrix. First and foremost such a matrix has rank 1, so that *by construction, there is one single eigenvalue per up and down sector different from 0.* Please note that this statement is independent of the number of generations. The situation is then a good starting approximation of a hierarchical spectrum.

Second the number of variables in the flavour fields will now not be the same as low energy flavour observables. The scalar fields are fundamental and can be thought of as complex vectors that are "rotated" under a flavour symmetry transformation. The only physical invariants that can be associated to vectors are the moduli and, if they live in the same space, their relative angles. Altogether the list of independent invariants and therefore physical variables describing the fields is,

$$Z = \left\{ \chi_U^{L\dagger} \chi_U^L\,, \qquad \chi_U^{R\dagger} \chi_U^R\,, \qquad \chi_D^{L\dagger} \chi_D^L\,, \qquad \chi_D^{R\dagger} \chi_D^R\,, \qquad \chi_U^{L\dagger} \chi_D^L \right\} \tag{6.104}$$

where the array Z will be useful for notation purposes[1].

A word on the phenomenology of this scenario is due first. Let us compare the phenomenology expected from bi-fundamental flavons (i.e. $d = 5$ Yukawa operator) with that from fundamental flavons (i.e. $d = 6$ Yukawa operators). For bi-fundamentals, the list of effective FCNC operators is exactly the same that in the original MFV proposal (25). The case of fundamentals presents some differences: higher-dimension invariants can be constructed in this case, exhibiting lower dimension than in the bi-fundamental case. For instance, one can compare these two operators:

$$\overline{D}_R\, \mathcal{Y}_D{}^\dagger\, \mathcal{Y}_U\, \mathcal{Y}_U^\dagger\, Q_L \sim [\text{mass}]^6 \qquad \longleftrightarrow \qquad \overline{D}_R\, \chi_d^R\, \chi_u^{L\dagger}\, Q_L \sim [\text{mass}]^5\,, \tag{6.105}$$

where the mass dimension of the invariant is shown in brackets; with these two types of basic bilinear FCNC structures it is possible to build effective operators describing FCNC processes, but differing on the degree of suppression that they exhibit. This underlines the fact that the identification of Yukawa couplings with aggregates of two or more flavons is a setup which goes technically beyond the realization of MFV, resulting possibly in a distinct phenomenology which could provide a way to distinguish between fundamental and bi-fundamental origin.

There is now also a clear geometrical interpretation of the Cabibbo angle: *the mixing angle between two generations of quarks is the misalignment of the χ^L flavons in the flavour space.*

[1]The index of Z will run over the five values $(U, L)\,, (U, R)\,, (D, L)\,, (D, R,)\,, (U, D)$

Let us turn now to the construction of the potential.

6.2.1 The Potential at the Renormalizable Level

Previous considerations regarding the scale separation between EW and flavour breaking scale hold also in this case, and in consequence the Higgs sector contributions will not be explicitly described. The Potential for the χ fields can be written in the compact manner;

$$V^{(4)} = -\mu_f^2 \cdot Z + Z^T \cdot \lambda_f \cdot Z + h.c.\,, \tag{6.106}$$

The total number of operators that can be introduced at the renormalizable level is 20. However, only 5 different combinations of these will enter the minimization equations. The solution

$$\langle Z \rangle = \frac{1}{2}\lambda_f^{-1}\mu_f^2\,, \tag{6.107}$$

exists if the vector $\lambda_f^{-1}\mu_f^2/2$ takes values inside the possible range of Z. The case in which this does not happen leads to a boundary of the invariant space. This occurs both when the entries turn negative in $\lambda_f^{-1}\mu_f^2$ *and* when $\chi_U^{L\dagger}\chi_U^L\chi_D^{L\dagger}\chi_D^L = \chi_D^{L\dagger}\chi_U^L\chi_U^{L\dagger}\chi_D^L$. This last case corresponds to the two vectors $\chi_{U,D}^L$ aligned, that precludes any mixing. This means that the no mixing case is a boundary to which nonetheless the minima of the potential is not restricted in general.

All these considerations make straight forward the extraction of the Yukawa configuration.

- **Two family case** From the expressions for the Yukawa matrices in Eqs. 5.8, and the previous discussion we write that the configuration for the Yukawas is

$$Y_D = \frac{\left|\chi_d^L\right|\left|\chi_d^R\right|}{\Lambda_f^2}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\,, \qquad Y_U = \frac{\left|\chi_u^L\right|\left|\chi_u^R\right|}{\Lambda_f^2} V_C \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\,. \tag{6.108}$$

$$V_C = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}\,, \tag{6.109}$$

so that quark masses are fixed via Eq. 6.107 to:

$$y_c = \sqrt{\frac{\left(\lambda_f^{-1}\mu_f^2\right)_{U,R}}{2\Lambda_f^2}\frac{\left(\lambda_f^{-1}\mu_f^2\right)_{U,L}}{2\Lambda_f^2}}\,, \qquad y_s = \sqrt{\frac{\left(\lambda_f^{-1}\mu_f^2\right)_{D,R}}{2\Lambda_f^2}\frac{\left(\lambda_f^{-1}\mu_f^2\right)_{D,L}}{2\Lambda_f^2}}\,, \tag{6.110}$$

$$\cos\theta_c = \frac{\left(\lambda_f^{-1}\mu_f^2\right)_{U,L}}{\sqrt{\left(\lambda_f^{-1}\mu_f^2\right)_{U,L}\left(\lambda_f^{-1}\mu_f^2\right)_{D,L}}}\,. \tag{6.111}$$

The vev of the moduli of the χ fields is of the same order μ for natural parameters, so that the cosine of the Cabibbo angle above is typically of $\mathcal{O}(1)$. This means that in the fundamental a natural scenario can give rise to both the strong hierarchies in quark masses and a non-vanishing mixing angle, whereas in the bi-fundamental case the mixing was unavoidably set to 0.

- **Three family case** The extension is simple, the Yukawa matrices are still of rank one and a single mixing angle arises

$$Y_D = \frac{\left|\chi_d^L\right|\left|\chi_d^R\right|}{\Lambda_f^2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad Y_U = \frac{\left|\chi_u^L\right|\left|\chi_u^R\right|}{\Lambda_f^2}V_{CKM}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{6.112}$$

$$V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix}. \tag{6.113}$$

with:

$$y_t = \sqrt{\frac{\left(\lambda_f^{-1}\mu_f^2\right)_{U,R}}{2\Lambda_f^2}\frac{\left(\lambda_f^{-1}\mu_f^2\right)_{U,L}}{2\Lambda_f^2}}\,, \qquad y_b = \sqrt{\frac{\left(\lambda_f^{-1}\mu_f^2\right)_{D,R}}{2\Lambda_f^2}\frac{\left(\lambda_f^{-1}\mu_f^2\right)_{D,L}}{2\Lambda_f^2}}\,, \tag{6.114}$$

$$\cos\theta_{23} = \frac{\left(\lambda_f^{-1}\mu_f^2\right)_{U,L}}{\sqrt{\left(\lambda_f^{-1}\mu_f^2\right)_{U,L}\left(\lambda_f^{-1}\mu_f^2\right)_{D,L}}}\,. \tag{6.115}$$

For obvious reasons, in eq. (6.112) the massive state is chosen to be that of the third generation and we have again a naturally $\mathcal{O}(1)$ angle. The flavon vevs have not broken completely the flavour symmetry, leaving a residual $U(1)_{Q_L}\times SU(2)_{D_R}\times SU(2)_{U_R}$ symmetry group. This can be seen as follows, in the three dimensional space where $SU(3)_{Q_L}$ acts, the two vectors $\chi^L_{U,D}$ define a plane, perpendicular to this plane there is the direction of the family that is completely decoupled form the rest, and in the plane we have the massive eigenstate and the eigenstate that, even if massless, can be told from the other massless states as it mixes with the massive.

If the hierarchies in mass in each up and down sectors are explained here through the very construction of the Yukawas via fundamental fields, there is still the hierarchy of masses between the top and bottom for the potential to accomodate, that is;

$$y_b^2/y_t^2 = \frac{\left(\lambda_f^{-1}\mu_f^2\right)_{D,R}\left(\lambda_f^{-1}\mu_f^2\right)_{D,L}}{\left(\lambda_f^{-1}\mu_f^2\right)_{U,R}\left(\lambda_f^{-1}\mu_f^2\right)_{U,L}} \simeq 5.7\times 10^{-4} \tag{6.116}$$

Note that the top-bottom hierarchy is explained in this context by the 4th power ratio of mass scales so that a typical ratio of $\mu_D/\mu_U \simeq 0.15$ suffices to explain the hierarchy.

One of the consequences of the strong hierarchy imposed in this scenario is that it cannot be corrected with nonrenormalizable terms to obtain small masses for the other lightest families for remember that the vanishing of all but one eigenvalues is obtained just by regarding the scalar field fundamental content. Nevertheless, the partial breaking of flavour symmetry provided by eq. (6.112) can open quite interesting possibilities from a model-building point of view. Consider as an example the following multi-step approach. In a first step, only the minimal number of fundamental fields are introduced: i.e. χ^L, χ^R_U and χ^R_D. Their vevs break $SU(3)^3$ down to $SU(2)^3$, originating non-vanishing Yukawa couplings only for the top and the bottom quarks, without any mixing angle (as we have only one left-handed flavon). As a second step, four new triplet fields $\chi'^{L,R}_{u,d}$ are added, whose contributions to the Yukawa terms are suppressed relatively to the previous flavons. If their vevs point in the direction of the unbroken flavour subgroup $SU(2)^3$, then the residual symmetry is further reduced. As a result, non-vanishing charm and strange Yukawa couplings are generated together with a mixing among the first two generations:

$$
\begin{aligned}
Y_u &\equiv \frac{\chi^L\,\chi_U^{R\dagger}}{\Lambda_f^2} + \frac{\chi_U'^L\,\chi_U'^{R\dagger}}{\Lambda_f^2} = \begin{pmatrix} 0 & \sin\theta\, y_c & 0 \\ 0 & \cos\theta\, y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}\,, \\
Y_d &\equiv \frac{\chi^L\,\chi_D^{R\dagger}}{\Lambda_f^2} + \frac{\chi_D'^L\,\chi_D'^{R\dagger}}{\Lambda_f^2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}\,.
\end{aligned}
\tag{6.117}
$$

The relative suppression of the two sets of flavon vevs correspond to the hierarchy between y_c and y_t (y_s and y_b)[1]. Hopefully, a refinement of this argument would allow to explain the rest of the Yukawas and the remaining angles. The construction of the scalar potential for such a setup would be quite model dependent though, and beyond the scope of this discussion.

6.3 Combining fundamentals and bi-fundamentals

Until now we have considered separately Yukawa operators of dimension $d = 5$ and $d = 6$. It is, however, interesting to explore if some added value from the simultaneous presence of both kinds of operators can be obtained. This is a sensible choice from the point of view of effective Lagrangians in which, working at $\mathcal{O}(1/\Lambda_f^2)$, contributions of three types may be included: i) the leading $d = 5$ $\mathcal{O}(1/\Lambda_f)$ operators; ii) renormalizable terms stemming from fundamentals (i.e. from $d = 6$ $\mathcal{O}(1/\Lambda_f^2)$ operators; iii) other corrections numerically competitive at the orders considered here. We focus here as illustration on the impact of i) and ii):

$$
\mathcal{L}_Y = \overline{Q}_L \left[\frac{\mathcal{Y}_D}{\Lambda_f} + \frac{\chi_D^L\chi_D^{R\dagger}}{\Lambda_f^2}\right] D_R H + \overline{Q}_L \left[\frac{\mathcal{Y}_U}{\Lambda_f} + \frac{\chi_U^L\chi_U^{R\dagger}}{\Lambda_f^2}\right] U_R \tilde{H} + h.c.\,, \tag{6.118}
$$

As the bi-fundamental flavons arise at first order in the $1/\Lambda_f$ expansion, it is suggestive to think of the fundamental contributions as a "higher order" correction. Let us then consider the case in which the flavons develop vevs as follows:

$$
\frac{\mathcal{Y}_{U,D}}{\Lambda_f} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_{t,b} \end{pmatrix}\,, \qquad \frac{\chi_{U,D}^L}{\Lambda_f^2} \sim \begin{pmatrix} 0 \\ y_{c,s} \\ 0 \end{pmatrix}\,, \tag{6.119}
$$

[1] Alternatively, all flavon vevs of similar magnitude with different flavour scale would lead to the same pattern.

and $\chi^R_{u,d}$ acquire arbitrary vev values of order Λ_f, for all components. Finally,

$$Y_U = \begin{pmatrix} 0 & \sin\theta_c\, y_c & 0 \\ 0 & \cos\theta_c\, y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}, \qquad Y_D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}. \tag{6.120}$$

This seems an appealing pattern, with masses for the two heavier generations and one sizable mixing angle, that we chose to identify here with the Cabibbo angle[1]. As for the lighter family, non-vanishing masses for the up and down quarks could now result from non-renormalizable operators.

The drawback of these combined analysis is that the direct connection between the minima of the potential and the spectrum is lost and the analysis of the potential would be very involved.

[1]Similar constructions have been suggested also in other contexts as in (95).

7

Lepton Sector

The lepton sector is at the moment in a dynamical and exciting state. The determination of the fundamental nature of neutrino masses through neutrinoless double beta decay (106) will explore one very fundamental question: are there fermions in nature which are their own antiparticle? With the recent measure of a sizable θ_{13} mixing angle in the lepton sector (107, 108), all angles of the mixing matrix are determined and the race for discovery of CP violation in the lepton sector has started (109). At the same time there is an ambitious experimental search for flavour violation in the charged lepton sector (110, 111, 112, 113) which could pour light in possible new physics beyond the SM, and provide a new probe of the magnitude of the seesaw scale (14), whereas on the cosmology side recent data seems to favor 3 only light species of neutrinos (114).

For the present theoretical analysis the nature of neutrino masses is crucial. If neutrinos happen to be Dirac particles, the analysis of the flavour symmetry breaking mechanism is completely analogous to that for the quark case: all conclusions drawn are directly translated to the lepton case and negligible mixing would be favored for the simplest set-up in which each Yukawa coupling is associated to a field in the bifundamental of the flavour group. As for quarks, sizable mixing would be allowed, though, for setups in which the Yukawas are identified with (combinations of) fields in the fundamental representation of the flavour group, implying a strong hierarchy for neutrinos.

We turn here instead to the case in which neutrinos are Majorana particles and more concretely generated by a type I seesaw model. It has been previously found (78, 79, 80, 81, 82) that for type I seesaw scenarios which exhibit approximate Lepton Number conservation, interesting seesaw models arise in which the effective scale of Lepton Number is distinct from the flavour scale yielding an interesting phenomenology (82, 115, 116, 117, 118), and it was first in this setup that we identified the patterns (16) to be established with more generality in the next sections. Let us consider in this chapter the general seesaw I scenario with degenerate heavy right-handed neutrinos as outlined in the introduction.

With our hypothesis of dynamical Yukawa couplings we introduce to scalar fields in parallel to the two Yukawa matrices that are bifundamentals of $\mathcal{G}_F$ as detailed in table 5.2.

7.1 Two Family Case

The counting of physical parameters is simple. It is known (83) that for two families with heavy degenerate neutrinos, the number of physical parameters describing the lepton sector is eight: six moduli and two phases.

Indeed, after using the freedom to choose the lepton charged matrix diagonal, as in Eq. 4.7, Y_ν is still a priori a general complex matrix with 8 parameters. Two phases can be reabsorbed through left-handed field $U(1)$ rotations, though, and an $O(2)$ rotation on the right of the neutrino Yukawa coupling (see Eq. 4.1), reduces to five the number of physical parameters in Y_ν, so that altogether $n = 7$ parameters suffice to describe the physical degrees of freedom in the lepton Yukawas, with the eight physical parameter being the heavy neutrino mass M. Below, for the explicit computation we will use either the so-called Casas-Ibarra parametrization (119) of the neutrino Yukawa couplings to maintain explicit the connection with masses and mixing,

$$Y_E = \begin{pmatrix} y_e & 0 \\ 0 & y_\mu \end{pmatrix}, \quad Y_\nu = \frac{\sqrt{M}}{v} U \begin{pmatrix} \sqrt{m_{\nu_1}} & 0 \\ 0 & \sqrt{m_{\nu_2}} \end{pmatrix} R, \tag{7.1}$$

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{-i\alpha} \end{pmatrix}, \qquad R = \begin{pmatrix} \cosh\omega & i\sinh\omega \\ -i\sinh\omega & \cosh\omega \end{pmatrix}. \tag{7.2}$$

In order to extend the parametrization above to the fields $\mathcal{Y}_E$, $\mathcal{Y}_\nu$, it is convenient to use the definitions

$$y_{\nu_i} \equiv \frac{M}{v^2} m_{\nu_i}\,, \tag{7.3}$$

leading to

$$\mathcal{Y}_\nu = \Lambda_f \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{i\alpha}\sqrt{y_{\nu_1}} & 0 \\ 0 & e^{-i\alpha}\sqrt{y_{\nu_2}} \end{pmatrix} \begin{pmatrix} \cosh\omega & -i\sinh\omega \\ i\sinh\omega & \cosh\omega \end{pmatrix}, \tag{7.4}$$

$$\mathcal{Y}_E = \Lambda_f \mathbf{y}_E = \Lambda_f \begin{pmatrix} y_e & 0 \\ 0 & y_\mu \end{pmatrix}. \tag{7.5}$$

It is the case nonetheless that the minimization procedure is optimized when selecting a different parametrization, the bi-unitary in analogy with quarks Eq. 4.2:

$$\mathcal{Y}_\nu = \Lambda_f U_L \mathbf{y}_\nu U_R\,, \qquad \mathcal{Y}_E = \Lambda_f \mathbf{y}_e\,; \qquad\qquad U_L U_L^\dagger = 1\,, \quad U_R U_R^\dagger = 1\,, \tag{7.6}$$

with $\mathbf{y}_E$ as defined above, $U_{L,R}$ being unitary matrices and $\mathbf{y}$ containing the eigenvalues of the neutrino Yukawa matrix $\mathbf{y} \equiv \mathrm{Diag}(y_1, y_2)$, distinct from neutrino masses. The connection with the latter is:

$$m_\nu = Y_\nu \frac{v^2}{M} Y_\nu^T = \frac{v^2}{M} U_L \mathbf{y}_\nu U_R U_R^T \mathbf{y}_\nu U_L^T\,. \tag{7.7}$$

None of the unitary matrices above corresponds to U_{PMNS}, but U_{PMNS} is the the matrix such that diagonalizes the matrix above, that is

$$m_\nu = U_{PMNS} \mathbf{m}_\nu U_{PMNS}^T\,. \tag{7.8}$$

The expression of mixing and masses in terms of the bi-unitary parameters is involved but the usefulness of this method is that we will not need it. The potential will select particularly simple points of this parametrization with an easy connection to low energy parameters.

In the following we will use the Casas-Ibarra parametrization for the Jacobian and mixing analysis and move to the bi-unitary to simplify matters in the mass hierarchy analysis of the potential.

The scalar potential for the $\mathcal{Y}_E$ and $\mathcal{Y}_\nu$ fields must be invariant under the SM gauge symmetry and the flavour symmetry $\mathcal{G}_\mathcal{F}$. The possible independent invariant terms reduce to precisely seven terms, e.g.:

$$I_E = \mathrm{Tr}\left[\mathcal{Y}_E\mathcal{Y}_E^\dagger\right]\,, \qquad I_\nu = \mathrm{Tr}\left[\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\right]\,, \tag{7.9}$$

$$I_{E^2} = \mathrm{Tr}\left[(\mathcal{Y}_E\mathcal{Y}_E^\dagger)^2\right]\,, \qquad I_{\nu^2} = \mathrm{Tr}\left[(\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger)^2\right]\,, \tag{7.10}$$

$$I_{\nu'} = \mathrm{Tr}\left[\mathcal{Y}_\nu^\dagger\mathcal{Y}_\nu\mathcal{Y}_\nu^T\mathcal{Y}_\nu^*\right]\,, \qquad I_{\nu,E} = \mathrm{Tr}\left[\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\mathcal{Y}_E\mathcal{Y}_E^\dagger\right]\,, \tag{7.11}$$

$$I_{\nu',E} = \mathrm{Tr}\left[\mathcal{Y}_\nu\mathcal{Y}_\nu^T\mathcal{Y}_\nu^*\mathcal{Y}_\nu^\dagger\mathcal{Y}_E\mathcal{Y}_E^\dagger\right]\,. \tag{7.12}$$

In terms of the variables defined above, the invariants read:

$$I_E = \Lambda_f^2\left(y_e^2 + y_\mu^2\right)\,, \qquad I_\nu = \Lambda_f^2\left(y_{\nu_1} + y_{\nu_2}\right)\cosh 2\omega\,, \tag{7.13}$$

$$I_{E^2} = \Lambda_f(y_e^4 + y_\mu^4)\,, \qquad I_{\nu^2} = \Lambda_f^4((y_{\nu_1} - y_{\nu_2})^2 + (y_{\nu_1} + y_{\nu_2})^2\cosh 4\omega)/2\,, \tag{7.14}$$

$$I_{\nu'} = \Lambda_f^4\left(y_{\nu_1}^2 + y_{\nu_2}^2\right)\,, \tag{7.15}$$

$$\begin{aligned} I_{\nu,E} = &\Lambda_f^4[\left(y_\mu^2 - y_e^2\right)\left(y_{\nu_1} - y_{\nu_2}\right)\cos 2\theta\cosh 2\omega + \left(y_e^2 + y_\mu^2\right)\left(y_{\nu_1} + y_{\nu_2}\right) \\ &+ 2\left(y_\mu^2 - y_e^2\right)\sqrt{y_{\nu_1}y_{\nu_2}}\sin 2\alpha\sin 2\theta\sinh 2\omega]/2\,, \end{aligned} \tag{7.16}$$

$$I_{\nu',E} = \Lambda_f^6\left[\left(y_\mu^2 - y_e^2\right)\left(y_{\nu_1}^2 - y_{\nu_2}^2\right)\cos 2\theta + \left(y_e^2 + y_\mu^2\right)\left(y_{\nu_1}^2 + y_{\nu_2}^2\right)\right]/2\,. \tag{7.17}$$

These results apply to any general seesaw I construction with heavy degenerate neutrinos. Note the different dependence in the mixing angle in the last two equations. Crucial to this difference are non trivial values of $\omega \neq 0$ and $\sin 2\alpha \neq 0$, which will be shown below to be natural minima of the system.

Again, for the explicitly axial breaking case ($\mathcal{G}_F \sim SU(n_g)^2 \times SO(n_g)$) two new invariants would appear

$$I_{\tilde{E}} = \mathrm{Det}\left[\mathcal{Y}_E\right]\,, \qquad I_{\tilde{\nu}} = \mathrm{Det}\left[\mathcal{Y}_\nu\right]\,, \tag{7.18}$$

which would substitute the invariants in Eq. 7.10 as for the quark case, see Eqs. 6.10-6.11.

Finally, the determinants in Eqs. 7.18 can be expressed as

$$I_{\tilde{E}} = \Lambda_f^2 y_e y_\mu e^{i\phi_E}, \qquad I_{\tilde{\nu}} = \Lambda_f^2\sqrt{y_{\nu_1}y_{\nu_2}}e^{i\phi_\nu}. \tag{7.19}$$

7.1.1 The Jacobian

The Jacobian can be factorized as follows:

$$J = \begin{pmatrix} J_E & 0 & \partial_{y_E} I_{(\nu,E),(\nu',E)} \\ 0 & J_\nu & \partial_{y_\nu,\omega} I_{(\nu,E),(\nu',E)} \\ 0 & 0 & \partial_{\theta,\alpha} I_{(\nu,E),(\nu',E)} \end{pmatrix} . \tag{7.20}$$

With respect to the mixing variables, the sub-Jacobian is given by

$$\partial_{\theta,\alpha}\left(I_{\nu,E}\,,\,I_{\nu',E}\right) = \partial_{\theta,\alpha}\left(\text{Tr}\left[\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\mathcal{Y}_E\mathcal{Y}_E^\dagger\right]\,,\,\text{Tr}\left[\mathcal{Y}_\nu\mathcal{Y}_\nu^T\mathcal{Y}_\nu^*\mathcal{Y}_\nu^\dagger\mathcal{Y}_E\mathcal{Y}_E^\dagger\right]\right)\,, \tag{7.21}$$

$$\propto \begin{pmatrix} 2\sqrt{y_{\nu_1}y_{\nu_2}}\sinh 2\omega\sin 2\alpha\cos 2\theta - (y_{\nu_1}-y_{\nu_2})\cosh 2\omega\sin 2\theta & \left(y_{\nu_1}^2 - y_{\nu_2}^2\right)\sin 2\theta \\ 2\sqrt{y_{\nu_1}y_{\nu_2}}\sinh 2\omega\sin 2\theta\cos 2\alpha & 0 \end{pmatrix} \tag{7.22}$$

with subdeterminant given by

$$\det J_{\theta,\alpha} = \left(y_\mu^2 - y_e^2\right)\left(y_{\nu_1}^2 - y_{\nu_2}^2\right)\sinh 2\omega\,\sin^2 2\theta\,\cos 2\alpha \tag{7.23}$$

This last equation shows the fundamental difference with respect to the quark (or more in general Dirac) case: reducing the rank can be accomplished by choosing $\alpha = \pi/4$. It will be shown later on, through an explicit example, how this solution comes along with mass degeneracy for light neutrinos.

Let us next consider the analysis the Jacobian for the mass sector

- **Axial preserving case:** $\mathcal{G}_\mathcal{F}^l \sim U(n_g)^2 \times O(n_g)$

$$J_\nu = \partial\left(\text{Tr}\left[\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\right]\,,\,\text{Tr}\left[(\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger)^2\right]\,,\,\text{Tr}\left[\mathcal{Y}_\nu\mathcal{Y}_\nu^T\mathcal{Y}_\nu^*\mathcal{Y}_\nu^\dagger\right]\right) \tag{7.24}$$

$$= \begin{pmatrix} \cosh 2\omega & y_{\nu_1}\cosh^2 2\omega + y_{\nu_2}\sinh^2 2\omega & 2y_{\nu_1} \\ \cosh 2\omega & y_{\nu_1}\sinh^2 2\omega + y_{\nu_2}\cosh^2 2\omega & 2y_{\nu_2} \\ 2(y_{\nu_1}+y_{\nu_2})\sinh 2\omega & (y_{\nu_1}+y_{\nu_2})^2\sinh 4\omega & 0 \end{pmatrix}, \tag{7.25}$$

The determinant of this matrix is:

$$\det J_\nu = 8(y_{\nu_1}+y_{\nu_2})^2(y_{\nu_1}-y_{\nu_2})\sinh 2\omega\,, \tag{7.26}$$

whereas for charged leptons it results, in analogy with the quark case:

$$\det J_E = y_e y_\mu\left(y_e^2 - y_\mu^2\right)\,. \tag{7.27}$$

- **Explicitly Axial breaking case:** $\mathcal{G}_{\mathcal{F}}^{\not{A},l} \sim SU(n_g)^2 \times SO(n_g)$- The Jacobian reads now,

$$J_\nu = \partial \left(\det \mathcal{Y}_\nu \,,\ \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right] \,,\ \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger\right]\right) \,, \tag{7.28}$$

$$= \begin{pmatrix} \sqrt{y_{\nu_2}/y_{\nu_1}} & \cosh 2\omega & 2y_{\nu_1} \\ \sqrt{y_{\nu_1}/y_{\nu_2}} & \cosh 2\omega & 2y_{\nu_2} \\ 0 & 2(y_{\nu_1}+y_{\nu_2})\sinh 2\omega & 0 \end{pmatrix} , \tag{7.29}$$

with determinant

$$\det J_\nu = \frac{(y_{\nu_1}+y_{\nu_2})^2 (y_{\nu_1}-y_{\nu_2})}{\sqrt{y_{\nu_1} y_{\nu_2}}} \sinh 2\omega \,, \tag{7.30}$$

and for charged leptons

$$\det J_E = \left(y_e^2 - y_\mu^2\right) \,. \tag{7.31}$$

7.1.2 The Potential at the Renormalizable Level

In this section the study of the renormalizable potential will reveal that all possible vacua retain some unbroken symmetry and in turn correspond to some of the boundary regions identified in the previous section. Nonetheless the allowed boundaries are not arbitrary, the potential selects only certain of these and in particular the potential does not restrict neccesarily to the smallest dimension non-trivial boundaries, such that one can have certain parameters adjustable by the potential. This section will treat by default of the axial preserving case, unless stated otherwise.

At the renormalizable level the most general potential respecting $\mathcal{G}_F$ is

$$V = -\mu^2 \cdot \mathbf{X}^2 + \left(\mathbf{X}^2\right)^\dagger \lambda\, \mathbf{X}^2 + h_E\, \mathrm{Tr}\left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2 + g\, \mathrm{Tr}\left(\mathcal{Y}_E \mathcal{Y}_E^\dagger \mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right) \tag{7.32}$$
$$+ h_\nu\, \mathrm{Tr}\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)^2 + h'_\nu\, \mathrm{Tr}\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger\right) .$$

In this equation $\mathbf{X}^2$ is a two-component vector defined by

$$\mathbf{X}^2 \equiv \left(\mathrm{Tr}\left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right) , \mathrm{Tr}\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu\right)\right)^T ,$$

μ^2 is a real two-component vector, λ is a 2×2 Hermitian matrix and all other coefficients are real parameters, a total of 9 parameters, one more than in the quark case since the new invariant I'_ν is allowed by the symmetry. The full scalar potential includes in addition Higgs-$\mathcal{Y}_E$ and Higgs-$\mathcal{Y}_\nu$ cross-terms, but they do not affect the mixing pattern and will thus be obviated in what follows.

Consider now the fermion masses fixed at their physical values and focus on the mixing pattern allowed at the minimum of the potential. Since mixing arises from the misalignment in flavour space of the charged lepton and the neutrino flavons, the only relevant invariant at the renormalisable level is $I_{\nu,E}$ whose explicit dependence is shown in 7.16 and we reproduce here

$$\begin{aligned}\mathrm{Tr}\left(\mathcal{Y}_E\mathcal{Y}_E^\dagger\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\right) =&\Lambda_f^4[\left(y_\mu^2-y_e^2\right)\left(y_{\nu_1}-y_{\nu_2}\right)\cos 2\theta\cosh 2\omega+\left(y_e^2+y_\mu^2\right)\left(y_{\nu_1}+y_{\nu_2}\right)\\ &+2\left(y_\mu^2-y_e^2\right)\sqrt{y_{\nu_1}y_{\nu_2}}\sin 2\alpha\sin 2\theta\sinh 2\omega]/2\,,\end{aligned}\tag{7.33}$$

for comparison with the quark case analogous

$$\mathrm{Tr}\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\mathcal{Y}_U\mathcal{Y}_U^\dagger\right)=\Lambda_f^4\left[\left(y_c^2-y_u^2\right)\left(y_s^2-y_d^2\right)\cos 2\theta+\left(y_c^2+y_u^2\right)\left(y_s^2+y_d^2\right)\right]/2\,.\tag{7.34}$$

The first term in Eq. (7.33) for leptons corresponds to that for quarks in Eq. (7.34): the only difference is the linear -instead of quadratic- dependence on neutrino masses, as befits the seesaw realisation. The second line in Eq. (7.33) has a strong impact on the localisation of the minimum of the potential and is responsible for the different results in the quark and lepton sectors: it contains the Majorana phase α and therefore connects the Majorana nature of neutrinos to their mixing.

This formula also shows explicitly the relations expected on physical grounds, between the mass spectrum and non-trivial mixing: i) the dependence on the mixing angle disappears in the limit of degenerate charged lepton masses; ii) it also vanishes for degenerate neutrino masses if and only if $\sin 2\alpha = 0$; iii) on the contrary, for $\sin 2\alpha \neq 0$ the dependence on the mixing angle remains, as it is physical even for degenerate neutrino masses; iv) the α dependence vanishes when one of the two neutrino masses vanishes or in the absence of mixing, as α becomes then unphysical.

The minimisation with respect to the Majorana phase and the mixing angle leads to the constraints:

$$\sinh 2\omega \sqrt{m_{\nu_2} m_{\nu_1}} \sin 2\theta \cos 2\alpha = 0\,, \tag{7.35}$$

$$\mathrm{tg} 2\theta = \sin 2\alpha \tanh 2\omega \frac{2\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}}\,. \tag{7.36}$$

Where we have restored neutrino masses explicitly since the formula stays the same. The first condition predicts then that the *Majorana phase is maximal,* $\alpha = \{\pi/4, 3\pi/4\}$*, for non-trivial mixing angle.* The relative Majorana phase between the two neutrinos is therefore $2\alpha = \pm\pi/2$ which implies no CP violation due to Majorana phases. On the other hand, Eq. 7.36 establishes a link between the mixing strength and the type of spectrum, which indicates *a maximal angle for degenerate neutrino masses, and a small angle for strong mass hierarchy.*

Using the Von Neumann trace inequality we have that the previous result corresponds to the configurations in which the eigenvalues of $\mathcal{Y}_E \mathcal{Y}_E^\dagger$ and $\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger$, are coupled in direct or inverse order:

$$\begin{cases} I_{E\nu}\Big|_{min} \propto m_e^2\, m_+ + m_\mu^2\, m_-\,, & g > 0\,, \\ I_{E\nu}\Big|_{min} \propto m_e^2\, m_- + m_\mu^2\, m_+\,, & g < 0\,, \end{cases} \tag{7.37}$$

where the eigenvalues of $\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger$ are,

$$m_\pm \equiv a_\nu \pm \sqrt{a_\nu^2 - c_\nu^2}\,, \tag{7.38}$$
$$a_\nu = (m_{\nu_2} + m_{\nu_1}) \cosh 2\omega\,,\;\; c_\nu = 4\sqrt{m_{\nu_2} m_{\nu_1}} (\cosh 2\omega + \sinh 2\omega)\,.$$

This two family scenario resulted in a remarkable connection of mass degeneracy and large angles, for an attempt at a realistic case we must wait to the three family case.

The minimization for the rest of the potential will fix masses and ω but it will not allow for arbitrary values of these. The procedure leads to 4 types of vacua . The details for the procedure of finding this minimum are not detailed here, suffice to say that there are two types of solutions one of them not leading to mixing, and equivalent to the quark case. This corresponds to $\omega = 0$ which is listed as one of the solutions for a vanishing Jacobian. One can see how this solution leads to no mixing just substituting in Eq. 7.33. The other solution which does lead to mixing corresponds to degenerate neutrinos $m_{\nu_1} = m_{\nu_2}$, which corresponds to a boundary, and through Eq. 7.36 correspond to maximal mixing $\theta = \pi/4$, and $\alpha = \pi/4$. In this case the Yukawa, turning now to the bi-unitary parametrization, have a structure:

$$\mathcal{Y}_\nu = \Lambda_f \begin{pmatrix} y_1 & 0 \\ 0 & y_2 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ -1 & i \end{pmatrix} \tag{7.39}$$

where the values y_1 y_2 are not proportional to masses and define in Eq. 7.6; if we write the Majorana mass matrix:

$$m_\nu = \frac{v^2}{M} \begin{pmatrix} 0 & y_1 y_2 \\ y_2 y_1 & 0 \end{pmatrix}, \tag{7.40}$$

we realize that the neutrinos are degenerate by construction. Even the values of y_1 and y_2 are not arbitrary but the possible configurations come along with certain hierarchies of charged lepton Yukawas, like in the quark case. Before we discuss the possible vacua let us pause for examining more closely 7.39. Is there something special about such a configuration? There is, it leaves certain symmetry unbroken. For determining it we perform a transformation of $O(2)_{N_R}$:

$$\mathcal{Y}_\nu \xrightarrow{O(2)} \mathcal{Y}_\nu \, e^{i\sigma_2 \theta} = \begin{pmatrix} \frac{y_1}{\sqrt{2}} & \frac{i y_1}{\sqrt{2}} \\ -\frac{y_2}{\sqrt{2}} & \frac{i y_2}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} e^{-i\theta} & 0 \\ 0 & e^{i\theta} \end{pmatrix} \begin{pmatrix} \frac{y_1}{\sqrt{2}} & \frac{i y_1}{\sqrt{2}} \\ -\frac{y_2}{\sqrt{2}} & \frac{i y_2}{\sqrt{2}} \end{pmatrix}. \tag{7.41}$$

It is clear now that a simultaneous rotation of the left handed group $SU(2)_{\ell_L}$ generated by σ_3 compensates these phases such that we have an unbroken U(1) that we call $SO(2)_V$ since it would be the equivalent of $SU(2)_V$ in the quark case.

The allowed ratios of eigenvalues are constrained like in the quark case. The minimization in these variables shows that one possible solution resembling nature sets:

$$\mathcal{Y}_E = \Lambda_f \begin{pmatrix} 0 & 0 \\ 0 & y_\mu \end{pmatrix}, \qquad \mathcal{Y}_\nu = \Lambda_f \begin{pmatrix} \frac{y_1}{\sqrt{2}} & \frac{i y_1}{\sqrt{2}} \\ -\frac{y_2}{\sqrt{2}} & \frac{i y_2}{\sqrt{2}} \end{pmatrix}, \tag{7.42}$$

with a breaking pattern $\mathcal{G}^l_{\mathcal{F}} \to U(1)_{e_R} \times SO(2)_V$. *In this scenario the electron is massless and the two neutrinos have the same absolute value for the mass while* *the mixing angle is maximal $\theta = \pi/4$ in a tantalizing first approximation to the lepton flavour pattern.*

The rest of possible vacua are listed in what follows:

I This hierarchical solution sets the electron massless and forbids Majorana masses for the neutrinos,

$$\mathcal{Y}_E = \Lambda_f \begin{pmatrix} 0 & 0 \\ 0 & y_\mu \end{pmatrix}, \qquad \mathcal{Y}_\nu = \Lambda_f \begin{pmatrix} 0 & 0 \\ -\frac{y_2}{\sqrt{2}} & \frac{iy_2}{\sqrt{2}} \end{pmatrix}, \tag{7.43}$$

since the breaking pattern is $\mathcal{G}_f^l \to U(1)_{LN} \times U(1)_e \times U(1)_A$. Even if there is no Majorana mass for the neutrinos, the muon neutrino mixes with the heavy right handed and produces flavour effects. The spectrum has then a massless neutrino, which is mostly active and a heavy Dirac neutrino.

II This case yields a massless electron and two degenerate majorana neutrinos;

$$\mathcal{Y}_E = \Lambda_f \begin{pmatrix} 0 & 0 \\ 0 & y_\mu \end{pmatrix}, \qquad \mathcal{Y}_\nu = \Lambda_f \begin{pmatrix} \frac{y_1}{\sqrt{2}} & \frac{iy_1}{\sqrt{2}} \\ -\frac{y_2}{\sqrt{2}} & \frac{iy_2}{\sqrt{2}} \end{pmatrix}, \tag{7.44}$$

with the relation;

$$\frac{y_2^2 - y_1^2}{y_\mu^2} = \frac{|g|}{2(h_\nu - |h'_\nu|)}, \tag{7.45}$$

and the symmetry pattern; $\mathcal{G}_\mathcal{F}^l \to U(1)_{e_R} \times SO(2)_V$.

III The two leptons have a mass and the neutrinos sector has a single massive Dirac fermion.

$$\mathcal{Y}_E = \Lambda_f \begin{pmatrix} y_e & 0 \\ 0 & y_\mu \end{pmatrix}, \qquad \mathcal{Y}_\nu = \Lambda_f \begin{pmatrix} 0 & 0 \\ -\frac{y_2}{\sqrt{2}} & \frac{iy_2}{\sqrt{2}} \end{pmatrix}, \tag{7.46}$$

satisfying

$$\frac{y_\mu^2 - y_e^2}{y_\mu^2} = \frac{|g|}{2h_E}, \tag{7.47}$$

the unbroken symmetry is $U(1)_e \times U(1)_{LN}$.

IV The degenerate case now corresponds to a configuration of the Yukawas of the type

$$\mathcal{Y}_E = \Lambda_f \begin{pmatrix} y & 0 \\ 0 & y \end{pmatrix}, \qquad \mathcal{Y}_\nu = \Lambda_f y' \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{pmatrix}, \tag{7.48}$$

which preserves $SO(2)_V$.

From these set of possible minima we learn that all the vacua found at the renormalizable level have an unbroken symmetry. Like in the quark case the introduction of determinants will disrupt those configurations that have a chiral $U(1)_A$. This fact can be used to lift the zero eigenvalues through a small determinant coefficient like in the quark case.

Finally we remark that all cases with nontrivial mixing, result in *sharp predictions: a maximal mixing angle and degenerate neutrinos with a $\pi/2$ relative majorana phase.*

7.2 Three Family case

The scalar fields are taken to be bi-triplets as detailed in table 5.2 and are connected proportionally to Yukawas as seen in Eq. 5.4.

For the number of parameters that suffice to parametrize such scalar fields modulo the symmetry above, starting as in the 2 family case from diagonal $\mathcal{Y}_E$, $\mathcal{Y}_\nu$ is a complex matrix with a priori 18 parameters. An $O(3)_{N_R}$ rotation can eliminate 3 of these, and there are still the residual symmetry of complex phase redefinitions to absorb 3 complex phases , leaving 12 parameters (83). These parameters can be encoded in 3 masses for the light neutrinos, two majorana phases, 4 mixing parameters like for the quark mixing matrix and 3 complex angles in the orthogonal R-matrix in the Casas-Ibarra parametrization.

This parametrization nonetheless proved not very useful in the 2 generation scenario, instead a parametrization that unfolds minima easily is the bi-fundamental parametrization of Eq. 7.6, where now $\mathbf{y} \equiv \mathrm{Diag}(y_1, y_2, y_3)$. The parameters in 7.6 are distributed as follows; 4 in the CKM-like matrix U_L, 3 in

U_R, the three moduli of the eigenvalues in $\mathbf{y}$ and two relative complex phases of these eigenvalues.

Without further delay we list the 15 invariants that constitute a complete basis. The first 6,

$$I_e = \text{Det}\left[\mathcal{Y}_E\right] , \qquad I_{y_\nu} = \text{Det}\left[\mathcal{Y}_\nu\right] , \tag{7.49}$$

$$I_{e^2} = \text{Tr}\left[\mathcal{Y}_E\mathcal{Y}_E^\dagger\right] , \qquad I_{y_\nu^2} = \text{Tr}\left[\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\right] , \tag{7.50}$$

$$I_{e^4} = \text{Tr}\left[\left(\mathcal{Y}_E\mathcal{Y}_E^\dagger\right)^2\right] , \qquad I_{y_\nu^4} = \text{Tr}\left[\left(\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\right)^2\right] , \tag{7.51}$$

depend on eigenvalues only. The following 7

$$I_L = \text{Tr}\left[\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\mathcal{Y}_E\mathcal{Y}_E^\dagger\right] , \qquad I_R = \text{Tr}\left[\mathcal{Y}_\nu^\dagger\mathcal{Y}_\nu\mathcal{Y}_\nu^T\mathcal{Y}_\nu^*\right] , \tag{7.52}$$

$$I_{L^2} = \text{Tr}\left[\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\left(\mathcal{Y}_E\mathcal{Y}_E^\dagger\right)^2\right] , \qquad I_{R^2} = \text{Tr}\left[\left(\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\right)^2\mathcal{Y}_\nu^T\mathcal{Y}_\nu^*\right] , \tag{7.53}$$

$$I_{L^3} = \text{Tr}\left[\mathcal{Y}_E\mathcal{Y}_E^\dagger\left(\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\right)^2\right] , \qquad I_{R^3} = \text{Tr}\left[\left(\mathcal{Y}_\nu^\dagger Y_L\mathcal{Y}_\nu^T\mathcal{Y}_\nu^*\right)^2\right] , \tag{7.54}$$

$$I_{L^4} = \text{Tr}\left[\left(\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\mathcal{Y}_E\mathcal{Y}_E^\dagger\right)^2\right] , \tag{7.55}$$

depend on U_L and $U_R U_R^T$ only respectively. Note that the quark analysis goes through the same for these terms (with the subtlety of considering three elements of $U_R U_R^T$, as $(U_R U_R^T)_{ij} = (U_R U_R^T)_{ji}$). Finally the two remaining invariants that will fix the relative complex phases are

$$I_{LR} = \text{Tr}\left[\mathcal{Y}_\nu\mathcal{Y}_\nu^T\mathcal{Y}_\nu^*\mathcal{Y}_\nu^\dagger\mathcal{Y}_E\mathcal{Y}_E^\dagger\right] , \quad I_{RL} = \text{Tr}\left[\mathcal{Y}_\nu\mathcal{Y}_\nu^T\mathcal{Y}_E^*\mathcal{Y}_E^T\mathcal{Y}_\nu^*\mathcal{Y}_\nu^\dagger\mathcal{Y}_E\mathcal{Y}_E^\dagger\right] . \tag{7.56}$$

7.2.1 The Jacobian

The number of variables and invariants has scaled up to 15, in this sense the Casas Ibarra parametrization becomes hard to handle specially due to the orthogonal matrix. In the context of the bi-unitary parametrization though we can make use of the previously derived Jacobians, in particular, the unitary relations we employed for finding the mixing subjacobian hold for both U_L and U_R. In this parametrization the structure of the Jacobian reads:

$$J = \begin{pmatrix} \partial_{y_E} I_{e^n} & 0 & 0 & \partial_{y_E} I_{L^n} & \partial_{y_E} I_{LR} \\ 0 & \partial_{y_\nu} I_{\nu^n} & \partial_{y_\nu} I_{R^n} & \partial_{y_\nu} I_{L^n} & \partial_{y_\nu} I_{LR} \\ 0 & 0 & \partial_{U_R} I_{R^n} & 0 & \partial_{U_R} I_{LR} \\ 0 & 0 & 0 & \partial_{U_L} I_{L^n} & \partial_{U_L} I_{LR} \\ 0 & 0 & 0 & 0 & \partial_{U_L U_R} I_{LR} \end{pmatrix} . \tag{7.57}$$

Luckily from the above shape we reduce the calculation of the 15×15 determinant to the product of 5 subdeterminants, those of the diagonal. We are already familiar with the first two, in the axial preserving scenario

$$\det J_E = y_e y_\mu y_\tau (y_e^2 - y_\mu^2)(y_\mu^2 - y_\tau^2)(y_e^2 - y_\tau^2) \,, \tag{7.58}$$

$$\det J_\nu = y_{\nu_1} y_{\nu_2} y_{\nu_3} (y_{\nu_1}^2 - y_{\nu_2}^2)(y_{\nu_2}^2 - y_{\nu_3}^2)(y_{\nu_3}^2 - y_{\nu_1}^2) \,, \tag{7.59}$$

whereas in the axial breaking case,

$$\det J_E = (y_e^2 - y_\mu^2)(y_\mu^2 - y_\tau^2)(y_e^2 - y_\tau^2) \,, \tag{7.60}$$

$$\det J_\nu = (y_{\nu_1}^2 - y_{\nu_2}^2)(y_{\nu_2}^2 - y_{\nu_3}^2)(y_{\nu_3}^2 - y_{\nu_1}^2) \,. \tag{7.61}$$

For the U_L in analogy with quarks:

$$\begin{aligned} \det (J_{U_L}) = & \left(y_{\nu_1}^2 - y_{\nu_2}^2\right)\left(y_{\nu_2}^2 - y_{\nu_3}^2\right)\left(y_{\nu_3}^2 - y_{\nu_1}^2\right)\left(y_e^2 - y_\mu^2\right)\left(y_\mu^2 - y_\tau^2\right)\left(y_\tau^2 - y_e^2\right) \\ & |U_L^{e1}||U_L^{e2}||U_L^{\mu 1}||U_L^{\mu 2}| \,. \end{aligned} \tag{7.62}$$

For U_R the dependence on in the invariants looks like

$$I_R = \mathrm{Tr}\left(\mathbf{y}_\nu^2 U_R U_R^T \mathbf{y}_\nu^2 U_R^* U_R^\dagger\right) \,, \qquad I_{R^2} = \mathrm{Tr}\left(\mathbf{y}_\nu^4 U_R U_R^T \mathbf{y}_\nu^2 U_R^* U_R^\dagger\right) \,, \tag{7.63}$$

$$I_{R^3} = \mathrm{Tr}\left(\mathbf{y}_\nu^4 U_R U_R^T \mathbf{y}_\nu^4 U_R^* U_R^\dagger\right) \,, \tag{7.64}$$

and the Jacobian:

$$J_{U_R} \propto \begin{pmatrix} 1 & y_{\nu_1}^2 + y_{\nu_3}^2 & \left(y_{\nu_1}^2 + y_{\nu_3}^2\right)^2 \\ 1 & y_{\nu_2}^2 + y_{\nu_3}^2 & \left(y_{\nu_1}^2 - y_{\nu_3}^2\right)^2 \\ 2 & y_{\nu_1}^2 + y_{\nu_1}^2 + 2y_{\nu_3} & 2\left(y_{\nu_1}^2 + y_{\nu_3}^2\right)\left(y_{\nu_2}^2 + y_{\nu_3}^2\right) \end{pmatrix} , \tag{7.65}$$

where the proprotinality is different for each row and equal to $\left(y_{\nu_1}^2 - y_{\nu_3}^2\right)^2$, $\left(y_{\nu_2}^2 - y_{\nu_3}^2\right)^2$ and $\left(y_{\nu_1}^2 - y_{\nu_3}^2\right)\left(y_{\nu_2}^2 - y_{\nu_3}^2\right)^2$ respectively. Then the determinant is;

$$\det J_{U_R} = \left(y_{\nu_1}^2 - y_{\nu_2}^2\right)^3 \left(y_{\nu_2}^2 - y_{\nu_3}^2\right)^3 \left(y_{\nu_3}^2 - y_{\nu_1}^2\right)^3 |\left(U_R U_R^T\right)_{11}||\left(U_R U_R^T\right)_{22}||\left(U_R U_R^T\right)_{12}| \tag{7.66}$$

Last in line are the two invariants I_{LR} that in terms of the bi-unitary parametrization read:

$$I_{LR} = \mathrm{Tr}\left(\mathbf{y}_\nu U_R U_R^T \mathbf{y}_\nu^2 U_R^* U_R^\dagger \mathbf{y}_\nu U_L^\dagger \mathbf{y}_e^2 U_L\right) , \tag{7.67}$$

$$I_{RL} = \mathrm{Tr}\left(\mathbf{y}_\nu U_R U_R^T \mathbf{y}_\nu U_L^T \mathbf{y}_E^2 U_L^* \mathbf{y}_\nu U_R^* U_R^\dagger \mathbf{y}_\nu U_L^\dagger \mathbf{y}_e^2 U_L\right) , \tag{7.68}$$

Let's parametrize the two remaining degrees of freedom as

$$\mathbf{y}_\nu \to \mathbf{y}_\nu e^{i\alpha_3\lambda_3} e^{i\alpha_8\lambda_8} , \tag{7.69}$$

we have then that the Jacobian built with the four terms:

$$\frac{\partial I_{LR}}{\alpha_3} = i\mathrm{Tr}\left(\left[\lambda_3 \ , \ \mathbf{y}_\nu U_R U_R^T \mathbf{y}_\nu^2 U_R^* U_R^\dagger \mathbf{y}_\nu\right] U_L^\dagger \mathbf{y}_e^2 U_L\right) , \tag{7.70}$$

$$\frac{\partial I_{LR}}{\alpha_8} = i\mathrm{Tr}\left(\left[\lambda_8 \ , \ \mathbf{y}_\nu U_R U_R^T \mathbf{y}_\nu^2 U_R^* U_R^\dagger \mathbf{y}_\nu\right] U_L^\dagger \mathbf{y}_e^2 U_L\right) , \tag{7.71}$$

$$\frac{\partial I_{RL}}{\alpha_3} = 2i\mathrm{Tr}\left(\left[\lambda_3 \ , \ U_L^T \mathbf{y}_E^2 U_L^* \mathbf{y}_\nu U_R^* U_R^\dagger \mathbf{y}_\nu\right] U_L^\dagger \mathbf{y}_e^2 U_L \mathbf{y}_\nu U_R U_R^T \mathbf{y}_\nu\right) , \tag{7.72}$$

$$\frac{\partial I_{RL}}{\alpha_8} = 2i\mathrm{Tr}\left(\left[\lambda_8 \ , \ U_L^T \mathbf{y}_E^2 U_L^* \mathbf{y}_\nu U_R^* U_R^\dagger \mathbf{y}_\nu\right] U_L^\dagger \mathbf{y}_e^2 U_L \mathbf{y}_\nu U_R U_R^T \mathbf{y}_\nu\right) , \tag{7.73}$$

and the determinant of this part:

$$J_{LR} = \frac{\partial I_{LR}}{\alpha_8}\frac{\partial I_{RL}}{\alpha_3} - \frac{\partial I_{LR}}{\alpha_3}\frac{\partial I_{RL}}{\alpha_8} \tag{7.74}$$

which vanishes if $\mathbf{y}_\nu U_R U_R^T \mathbf{y}_\nu$, $U_L^\dagger \mathbf{y}_e^2 U_L$ or their product is diagonal.

7.2.2 The Potential at the Renormalizable Level

The number of boundaries or subgroups of the flavour group has grown sensibly complicating the Jacobian analysis, the study of the potential will help clarify which of these configurations are realized and how at the renormaliable level.

The potential including all possible terms respecting the full flavour group looks just like the two family case Eq 7.32 and the counting of potential parameters goes like the same; they add up to 9. We shall examine next the way in which this potential will fix the vev of the scalar fields. For the same reason as in the previous chapter the minimization process will start on those variables that appear less often in the potential. In this case the paremeters of the unitary matrices, which will in turn determine U_{PMNS}.

The left handed matrix U_L appears in the term:

$$g\mathrm{Tr}\left(\mathcal{Y}_E\mathcal{Y}_E^\dagger\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\right) = g\Lambda_f^4\mathrm{Tr}\left(\mathbf{y}_E^2U_L\mathbf{y}_\nu^2U_L^\dagger\right) \,, \tag{7.75}$$

the Von Neumann trace inequality solves in a line the minimization:

$$g<0,\quad U_L = \begin{pmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix};\quad g\Lambda_f^4\mathrm{Tr}\left(\mathbf{y}_E^2U_L\mathbf{y}_\nu^2U_L^\dagger\right) = g\Lambda_f^4\sum_{i=1}^3 y_{E,i}y_{\nu,i}\,, \tag{7.76}$$

$$g>0,\quad U_L = \begin{pmatrix} 0 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{pmatrix};\quad g\Lambda_f^4\mathrm{Tr}\left(\mathbf{y}_E^2U_L\mathbf{y}_\nu^2U_L^\dagger\right) = g\Lambda_f^4\sum_{i=1}^3 y_{E,i}y_{\nu,4-i}\,, \tag{7.77}$$

Under the same reasoning, U_R appears only in:

$$h'_\nu\,\mathrm{Tr}\left(\mathcal{Y}_\nu\mathcal{Y}_\nu^T\mathcal{Y}_\nu^*\mathcal{Y}_\nu^\dagger\right) = h'_\nu\,\mathrm{Tr}\left(\mathbf{y}_\nu^2U_RU_R^T\mathbf{y}_\nu U_R^*U_R^\dagger\right) \tag{7.78}$$

then the U_R has two discreet possible solutions

A For a negative coefficient we have

$$h'_\nu<0\quad U_RU_R^T = \begin{pmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\quad h_\nu\mathrm{Tr}\left(\mathbf{y}_\nu^2U_RU_R^T\mathbf{y}_\nu U_R^*U_R^\dagger\right) = g\Lambda_f^4\sum_{i=1}^3 y_{\nu,i}^4 \tag{7.79}$$

B Whereas for a positive coefficient,

$$h'_\nu>0\quad U_RU_R^T = \begin{pmatrix} 0 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{pmatrix}\quad h_\nu\mathrm{Tr}\left(\mathbf{y}_\nu^2U_RU_R^T\mathbf{y}_\nu U_R^*U_R^\dagger\right) = g\Lambda_f^4\sum_{i=1}^3 y_{\nu,i}^2y_{\nu,4-i}^2 \tag{7.80}$$

If we recall the expression for the neutrino mass matrix in 7.7 contains precisely the combination $U_RU_R^T$. A quick look at the four possible combinations of products of minima for $U_{L,R}$ reduce to two, since both configurations of U_L leave the neutrino mass matrix unchanged. Nonetheless if the configuration $U_RU_R^T=1$

has trivialy no mixing since everything is already diagonal, *possibility B for* $U_R U_R^T$ *implies a maximal angle*. Indeed the diagonalization reads:

$$\frac{v^2}{M}\begin{pmatrix} 0 & 0 & y_3 y_1 \\ 0 & y_2^2 & 0 \\ y_3 y_1 & 0 & 0 \end{pmatrix} = U_{PMNS}\begin{pmatrix} m_{\nu_1} & 0 & 0 \\ 0 & m_{\nu_2} & 0 \\ 0 & 0 & m_{\nu_1} \end{pmatrix} U_{PMNS}^T \,,$$

with a mixing matrix and masses

$$U_{PMNS} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \end{pmatrix}, \qquad m_{\nu_1} = m_{\nu_3} = \frac{v^2}{M} y_1 y_3 \,, \quad m_{\nu_2} = \frac{v^2}{M} y_2^2 \,. \tag{7.81}$$

At this point we do not know which mass is greater than the other. If these cases are hierarchical, they correspond to either normal or inverted hierarchy in a first rough approximation ($\Delta m^2_{sol} = 0$) and the maximal angle lies always among the two degenerate neutrinos, meaning $\theta_{sol} \simeq \pi/4$; on the other hand if the spectrum is quasidegenerate, the mixing angle correspondence is unclear and the perturbations for splitting masses shall be studied.

Remember that all these conclusion were drawn from the minimization in two terms of the potential only and they hold quite generally.

Another question is whether the configuration of off-diagonal $U_R U_R^T$ has any special property from the symmetry point of view. Recalling the two family case the generalization is straight forward

$$\begin{pmatrix} \frac{y_1}{\sqrt{2}} & 0 & \frac{iy_1}{\sqrt{2}} \\ 0 & y_2 & 0 \\ -\frac{y_3}{\sqrt{2}} & 0 & \frac{iy_3}{\sqrt{2}} \end{pmatrix} e^{i\theta\lambda_5} = e^{i\theta/2(\lambda_3+\sqrt{3}\lambda_8)} \begin{pmatrix} \frac{y_1}{\sqrt{2}} & 0 & \frac{iy_1}{\sqrt{2}} \\ 0 & y_2 & 0 \\ -\frac{y_3}{\sqrt{2}} & 0 & \frac{iy_3}{\sqrt{2}} \end{pmatrix} . \tag{7.82}$$

So that a simultaneous rotation in the direction λ_5 of $O(3)_N$ and an opposite sign transformation in the direction $(\lambda_3 + \sqrt{3}\lambda_8)/2$ of $SU(3)_{\ell_L}$ constitute a preserved $U(1)$ symmetry. It is interesting to note that on the other hand, the configuration of diagonal $\mathcal{Y}_\nu$ has no symmetry for generic $y_{1,2,3}$, we shall see how this fits in the general picture of the possible minima. It is nonetheless evident that for 2 degenerate $y_{1,2,3}$ there is a $SO(2)_V$ symmetry unbroken and that for a configuration proportional the identity $\mathcal{Y}_\nu \propto 1$ a vectorial $SO(3)_V$ arises. So one can wonder if this happens for case B, Eq. 7.80, in the case of all eigenvalues degenerate.

The result is that there is an unbroken $SO(3)$ in this case as well. The two new relations,

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \end{pmatrix} e^{i\theta_2\lambda_2} = e^{-i\theta_2\frac{1}{\sqrt{2}}(\lambda_2+\lambda_7)} \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \\ 0 & y_2 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \end{pmatrix}, \tag{7.83}$$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \end{pmatrix} e^{i\theta_3\lambda_7} = e^{i\theta_3(\lambda_1+\lambda_6)/\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \\ 0 & y_2 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \end{pmatrix}, \tag{7.84}$$

provide two new directions of conserved symmetry. This is however not enough to prove that we have $SO(3)$ and not just $U(1)^3$. For this the basis

$$\left\{\frac{1}{2}(\lambda_3 + \sqrt{3}\lambda_8)\,,\, -\frac{1}{\sqrt{2}}(\lambda_2 + \lambda_7)\,,\, \frac{1}{\sqrt{2}}(\lambda_1 + \lambda_6)\right\} = \tag{7.85}$$

$$\left\{ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & \frac{i}{\sqrt{2}} & 0 \\ -\frac{i}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \\ 0 & -\frac{i}{\sqrt{2}} & 0 \end{pmatrix}, \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 \end{pmatrix} \right\} \tag{7.86}$$

can be shown to have the commutation relations of $SO(3)$, that is structure constants ϵ_{ijk}.

The emphasis will be on case B, Eq. 7.80, since it gives a maximal mixing angle, but first a few words on the other case. If both Yukawas are diagonal, as in case A, and for arbitrary eigenvalues, there is no symmetry left unbroken at all. Nonetheless, when $h'_\nu < 0$, after minimizing in U_R the structure of I_R is just like that of I_ν^2, so that the effective coupling of I_ν^2 can be taken to be $h'_\nu + h_\nu$. Then the analysis of quarks holds just the same and we find the type of solution listed in section 5.1.2.2, but all of these have at least one pair of eigenvalues degenerate, this implies that there is indeed always at least one $SO(2)_V$ in the minimum.

This same reasoning applied to case B will reveal new freedom in the possible eigenvalues of the Yukawas, since now the symmetry reported in Eq. 7.82, is present for arbitrary entries.

Before entering the details on the complete set of vacua, for the reader interested in the closest solution to nature we report here a new kind of solution with respect to the quark case:

$$\mathcal{Y}_E = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}, \qquad \mathcal{Y}_\nu = \Lambda_f \begin{pmatrix} y_{\nu_1}/\sqrt{2} & 0 & -iy_{\nu_1}/\sqrt{2} \\ 0 & y_{\nu_2} & 0 \\ y_{\nu_3}/\sqrt{2} & 0 & iy_{\nu_3}/\sqrt{2} \end{pmatrix}, \tag{7.87}$$

the two different entries for the charged leptons are in agreement with the larger mass of the muon an tau leptons whereas in the neutrino sector there is one maximal angle and the three massive neutrinos can be quasidegenerate leading to an appealing set up in which small corrections produce another large mixing angle (120).

Explicitly the types of vacua found are;

- **I** The hierarchical solution for the eigenvalues translates now into Yukawas of the type

$$\mathcal{Y}_E = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix}, \qquad \mathcal{Y}_\nu = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1/\sqrt{2} & 0 & i/\sqrt{2} \end{pmatrix}, \tag{7.88}$$

  and a pattern $\mathcal{G}_f^q \to U(2)^2 \times U(1)_{LN}$. There are no light neutrinos in this scenario, but flavour effects are present.

- **II** The equivalent of case **II** in the 2 family case differs from the extension of this case in the quark case from 2 to 3 generations. We have now a hierarchical set-up for charged leptons and arbitrary entries for the eigenvalues,

$$\mathcal{Y}_E = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix}, \qquad \mathcal{Y}_\nu = \Lambda_f \begin{pmatrix} y_{\nu_1}/\sqrt{2} & 0 & -iy_{\nu_1}/\sqrt{2} \\ 0 & y_{\nu_2} & 0 \\ y_{\nu_3}/\sqrt{2} & 0 & iy_{\nu_3}/\sqrt{2} \end{pmatrix}, \tag{7.89}$$

  and the breaking pattern is $\mathcal{G}_f^q \to U(2)_{E_R} \times U(1)_{\tau-e}$. The reason for $y_{\nu_1} \neq y_{\nu_2}$ now is that the degeneracy of these two parameters leads to no extra symmetry, so their equality is not protected.

- **III** The third kind of solution stands the same as in the quark case

$$\mathcal{Y}_E = \Lambda_f \begin{pmatrix} y & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & y_\tau \end{pmatrix}, \qquad \mathcal{Y}_\nu = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1/\sqrt{2} & 0 & i/\sqrt{2} \end{pmatrix}, \tag{7.90}$$

for now the identity $y_e = y_\mu$ yields the breaking structure $\mathcal{G}_f^q \to U(2)_V \times U(1)_{LN}$, were the unbroken group would be different if the two first eigenvalues of $\mathcal{Y}_E$ were to differ.

- **IV** The completely degenerate configuration is

$$\mathcal{Y}_E = \Lambda_f \begin{pmatrix} y & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & y \end{pmatrix}, \qquad \mathcal{Y}_\nu = \Lambda_f \begin{pmatrix} 1/\sqrt{2} & 0 & -i/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & i/\sqrt{2} \end{pmatrix}, \tag{7.91}$$

we have now that $\mathcal{G}_f^q \to SO(3)_V$ with the vectorial group as pointed out in Eqs. 7.82-7.86. In this case nonetheless the mixing loses meaning since the charged leptons are degenerate.

V New configurations are now possible as

$$\mathcal{Y}_E = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}, \qquad \mathcal{Y}_\nu = \Lambda_f \begin{pmatrix} y_{\nu_1}/\sqrt{2} & 0 & -iy_{\nu_1}/\sqrt{2} \\ 0 & y_{\nu_2} & 0 \\ y_{\nu_3}/\sqrt{2} & 0 & iy_{\nu_3}/\sqrt{2} \end{pmatrix}, \tag{7.92}$$

with $\mathcal{G}_f^q \to U(1)_V \times U(1)_{e_R}$

VI The presence of arbitrary charged lepton masses is present when two neutrinos are massless,

$$\mathcal{Y}_E = \Lambda_f \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}, \qquad \mathcal{Y}_\nu = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_{\nu_2} & 0 \\ y_{\nu_3}/\sqrt{2} & 0 & iy_{\nu_3}/\sqrt{2} \end{pmatrix}, \tag{7.93}$$

with $\mathcal{G}_f^q \to U(1)_\tau \times U(1)_e$ since the neutrinos that the electron and tau couple to are massless.

VII Finally the case **II** leaves and extended symmetry if two neutrinos are massless

$$\mathcal{Y}_E = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix}, \qquad \mathcal{Y}_\nu = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_{\nu_2} & 0 \\ y_{\nu_3}/\sqrt{2} & 0 & iy_{\nu_3}/\sqrt{2} \end{pmatrix}, \tag{7.94}$$

with $\mathcal{G}_f^q \to U(2)_{E_R} \times U(1)_e \times U(1)_\tau$

The possibilities for the vacua have grown sensibly. This is related to the flavour group. The presence of the new invariant at the renormalizable level I_R gave rise to the maximal angle solution. In turn this choice resulted in a term in the potential which was not present in the quark case, unlike the no mixing case. This invariant then produces new configurations for the values of Yukawa eigenvalues. Indeed in the limit $h'_\nu \to 0$ all this different cases recombine in the ones for the quark case.

In this scenario the introduction of small breaking terms of the axial symmetry, that is determinants, could produce a hierarchy by lifting the 0 eigenvalues in the new configurations **V VI VII**.

For a realistic scenario at this level, the quasidegenerate scenario for neutrino masses would be a good starting point and simultaneously the charged lepton spectrum can be chosen hierarchical (case **II**) or semi-hierarchical (case **V**). One can imagine perturbations in this scenario correcting the pattern; these corrections should give rise to one other large mixing angle and the "small" reactor angle, such that the largest of the three is related to Δm^2_{atm}. Lifting the electron mass from 0 is possible in case **V** as outlined.

The general conclusion is therefore that in first approximation a *maximal mixing angle is obtained in the lepton sector* whereas *for the quark case no mixing is allowed* in this same level of approximation. This stands as a tantalizing framework for explaining the differences in mixing matrices in the two sectors in *a common framework for quarks and leptons*. The solution of the *maximal angle* can be traced back to the *presence of an orthogonal group* in the flavour symmetry of the lepton sector, which is in turn *related to the Majorana nature of neutrino masses*.

8

Resumen y Conclusiones

En esta tesis la estructura de sabor de las partículas elementales ha sido examinada desde el punto de vista de una posible simetría de sabor implícita. La simetría de sabor considerada es la simetría global que presenta el ME en ausencia de masa para los fermiones. La extensión necesaria del ME para acomodar masas de neutrinos introduce no obstante una dependencia en el modelo elegido. Por simplicidad el escenario del Seesaw con neutrinos pesados (conocido como tipo I o tipo III) es considerado cuando se trata de leptones, asumiendo la existencia de n_g generaciones ligeras y pesadas. La simetría de sabor es entonces seleccionada como la mayor simetría posible de la teoría libre, esquemáticamente $\mathcal{G}_{\mathcal{F}} \sim U(n_g)^5 \times O(n_g)$, en dónde $O(n_g)$ está asociado a neutrinos pesados degenerados, cuya masa es la única presente en la teoría libre, mientras que cada factor $U(n_g)$ corresponde a cada campo con distinta carga en el ME.

Sin espicificar un modelo de sabor es posible explorar la posibilidad de que, a bajas energías, los Yukawas sean las fuentes de sabor en el ME y la teoría que lo completa; esta suposición está en acuerdo con los datos experimentales y se encuetra en el centro del éxito fenomenológico de la hipótesis de MFV, implementada a través de técnicas de Lagrangianos efectivos. Prosiguiendo este camino, hemos explorado las consecuencias de un carácter dinámico de los acoplos de Yukawa mediante la determinación, en una base general, de los posibles extremos del conjunto de invariantes (gauge y de sabor) que pueden ser construidos con éstos. Existen tantos invariantes independientes como parámetros físicos, y un conjunto de invariantes completo e independiente ha sido determinado y examinado. Hemos demostrado que, mientras para quarks los extremos de los invariantes apuntan hacia la ausencia de mezcla, para leptones grandes ángulos correlacionados con un carácter de Majorana no trivial resultan ser los extremos naturales. Éste puede ser un motivador y sugerente primer paso en la empresa del entendimiento del origen de sabor, dado que este esquema resulta muy similar al obesrvado en la naturaleza.

Un verdadero origen dinámico de los acoplos de Yukawa sugiere un paso más: considerar que corresponden a campos dinámicos, o agregados de éstos, que poseen sabor y han adquirido un vev. La simetría de sabor sería manifiesta en el Lagrangiano total de alta energía, a una escala Λ_f. Tras la rotura espontánea de simetría, los acoplos de Yukawa de bajas energías resultarían de operadores efectivos de dimension $d > 4$ invariantes bajo la simetría de sabor, que involucran uno o mas campos de sabor junto con los campos usuales del ME.

Solo un escalar (o cunjunto de campos en una configuracion escalar) puede tomar un vev, que deberá corresponder al mínimo de un potencial. ¿Cuál es el potencial escalar para estos campos escalares de sabor? ¿Puede alguno de sus mínimos corresponder naturalmente al espectro observado de masas y ángulos? Estas preguntas son respondidas en el presente trabajo. El análisis del potencial está relacionado con los extremos de los invariantes mencionados antes, pero va mas allá dado que la presencia simultánea de varios invariantes no tiene por qué producir mínimos que coincidan con los extremos hallados mediante la consideración independiente de invariantes.

La realización mas simple de este tipo se obtene via una correspondencia uno a uno de cada acoplo de Yukawa (up, down, elecrón y neutrino) con un único campo escalar perteneciente a la representación bi-fundamental del grupo de sabor $\mathcal{G}_\mathcal{F}$. En el lenguaje de Lagrangianos efectivos este caso corresponde al orden más bajo en la expansión de sabor: operadores de Yukawa de dimension $d = 5$ construidos por un campo escalar y los campos del ME usuales. El potencial escalar general para campos escalares bi-fundamentales ha sido construido para quarks y leptones en el caso de dos y tres familias. Formalmente, se construye con los invariantes mencionados arriba y no obstante de su combinación surgen nuevos mínimos.

Al determinar el potencial escalar, primero se demostró que imponer la simetría de sabor representa una condición muy restrictiva: al nivel renormalizable sólo ciertos términos son permitidos en el potencial, e incluso al nivel renormalizable estructuras constreñidas deben ser respetadas.

En el caso de quarks, al nivel renormalizable, en el mínimo del potencial solo ángulos nulos son permitidos. Respecto a jerarquías de masa, uno de los posibles mínimos presenta masas nulas para todos los quarks excepto los pertenecientes a la familia más pesada, esto es, un quark tipo down y otro tipo up con masa solamente tanto en dos como en tres familias. Exite por lo tanto una solución incial que se asemeja en primera aproximación a la naturaleza: un espectro jerárquico sin mezcla. Dicha solución puede ser pertubada al nivel renormalizable para obtener masas para las familias más ligeras mediante términos de rotura explícita de la parte abeliana de ${\mathcal{G}_\mathcal{F}}^q$, es decir $U(1)^3$. Esta opción no está presente en el caso de tres familias dado que la configuración jerárquica está protegida por una mayor simetría no rota: $SU(2)^3$. La introducción de términos no renormalizables en el potencial permite una rotura mayor de la simetría, al precio de enormes ajustes finos, que son inaceptables en nuestra opinión en el espíritu de la teoría efectiva de campos.

En el sector leptónico la misma realización de correspondencia Yukawa-campo, escalares bi-fundamentales, condujo a resultados soprendentemente diferentes. En el caso de dos y tres familias, fases de Majorana y ángulos de mezcla no triviales pueden ser seleccionados por el mínimo del potencial, indicando una nueva conexión en la estructura de masas de neutrinos: i) grandes ángulos de mezcla son posibles; ii) hay una fuerte correlación entre ángulos de mezcla grandes y espectro degenerado de masas; iii) la fase de Majorana relativa es predicha como máxima, $2\alpha = \pi/2$, aunque no implica violación de conjugación de carga y paridad.

Las soluciones exactas del potencial renomalizable condujentes a mezcla no trivial muestran un único ángulo máximo entre dos neutrinos degenerados pero distinguibles tanto para el caso de dos como el de tres familias. Esto conduce, para el caso de jerarquía normal e invertida, a el ángulo máximo siendo el solar en lugar del atmosférico, numéricamente compatible con un valor máximo. En el caso de los tres neutrino ligeros degenerados, permitido por el potencial renormalizable, la asignación del ángulo depende de las correcciones al espectro de masas, pero parecen indicar la posibilidad de un segundo gran ángulo de mezcla en un escenario más prometedor, actualmente bajo estudio (17).

Otra avenida explorada en este trabajo asocia dos campos a cada acoplo de Yukawa, esto es $Y \sim \chi^L \chi^{R\dagger}/\Lambda_f^2$. Esta situación es atrayente dado que mientras que los Yukawas son objetos compuestos, los nuevos campos están en la representación fundamental. Dichos campos podrían ser escalares o fermiónicos: aqui nos centramos exclusivamente en escalares. Desde el punto de vista de Lagrangianos efectivos, este caso podría corresponder al siguiente al primer orden en la expansión: operadores de Yukawa efectivos de dimension 6, como fuentes totales o parciales de los Yukawas de baja energía. Hemos construído el potencial escalar general para campos escalares en la representación fundamental para los casos de dos y tres familias de quarks, aunque las conclusiones se transladan de manera directa a leptones. Por construcción este escenario resulta inevitablemente en una fuerte jerarquía de masas: solamente un quark en cada sector up y down obtiene masa: los quarks top y bottom. Una mezcla no trivial requiere dos campos escalares de sector up y down (neutrino y electrón) transformando bajo el grupo $SU(3)_{Q_L}$. En consequencia el contenido mínimo es de cuatro campos $\chi^L_{U\,(\nu)}$, $\chi^L_{D\,(E)}$, $\chi^R_{U\,(\nu)}$ and $\chi^R_{D\,(E)}$ y la mezcla surge de la interacción entre los dos primeros. En resumen, para escalares en la fundamental en un modo natural se obtiene: i) una fuerte jerarquía entre quarks de la misma carga, señalando un quark distinguible por su mayor masa en cada sector; ii) un ángulo de mezcla no trivial, que puede ser identificado tanto para quarks como para leptones con el del sector 23 en el caso de tres familias.

Finalmente, como una posible corrección a los patrones discutidos previamente, se ha discutido brevemente la posibilidad de introducir simultáneamente escalares bi-fundamentales y fundamentales. Es una posibilidad muy sensata, desde el punto de vista de Lagrangianos efectivos, considerar operadores de Yukawa de orden $d = 5$ y $d = 6$ trabajando a orden $\mathcal{O}(1/\Lambda_f^2)$. Sugiere que el término de $d = 5$, que acarrea bi-fundamentales, podría proporcionar la contribución dominante, mientras que el operador de $d = 6$, que trae consigo los campos en la fundamental, proporciona correcciones para inducir masas no nulas para las dos familias ligeras junto con ángulos no triviales.

En general, es destacable que el requisito de invarianza bajo la simetría de sabor constriña fuertemente el potencial escalar y consequentemente los mínimos y patrones de ruptura de simetría. De entre los resultados obtenidos uno sobresale de entre los demás. En el mínimo del potencial, al nivel renormalizable, los ángulos de mezcla para quarks son nulos a primer orden, mientras que la mezcla en los leptones resulta ser máxima. La presencia de mezcla máxima es debida al factor $O(n_g)$ del grupo de sabor, que está a su vez relacionado con la naturaleza Majorana de los neutrinos. La explicación de la diferente estructura de mixing entre quarks y leptones en este escenario es, en última instancia, la distinta naturaleza de los dos tipos de fermiones: Dirac y Majorana.

9

Summary and Conclusions

In this dissertation the flavour pattern of the elementary particles was examined from the point of view of its possible underlying flavour symmetry. The flavour symmetry considered is the global flavour symmetry which the SM possesses in the limit of massless fermions. The necessary extension of the SM to accommodate Majorana neutrino masses introduces nevertheless a model dependence in the neutrino sector; for simplicity the seesaw scenario with heavy neutrinos (known as type I or type III) is considered here when dealing with leptons, assuming n_g generations in both the light and heavy sectors. *The largest possible flavour symmetry of the free theory* for both quark and lepton sectors is then, schematically, $\mathcal{G}_{\mathcal{F}} \sim U(n_g)^5 \times O(n_g)$, with $O(n_g)$ associated to heavy degenerate neutrinos, whose mass is the only one present in the free theory, and each $U(n_g)$ factor for each SM fermion field[1].

Without particularizing to any concrete flavour model, it is possible to explore the possibility that, at low energies, the Yukawas may be the sources of flavour in the SM and beyond; this assumption is well in agreement with data and lies at the heart of the phenomenological success of the MFV ansatz, implemented through effective Lagrangian techniques. Walking further on this path, we have explored the consequences of an hypothetical dynamical character for the Yukawa couplings themselves by determining, on general grounds, the possible extrema of the (gauge and flavour) invariants that can be constructed out of them. There

[1]The flavour group can alternatively be defined as the largest flavour group in the absence of Yukawa interactions.

are as many independent invariants as physical parameters, and a complete set of independent invariants has been determined. We have shown that, while for quarks the extrema of the invariants point to no mixing, for leptons large mixings correlated with a non-trivial Majorana character turn out to be natural extrema. This may be a very encouraging and suggestive first step in the quest for the understanding of the origin of flavour, as that pattern resembles closely the mixings observed in nature.

A true dynamical origin for the Yukawa couplings suggests a further step: to consider them as corresponding to dynamical fields, or aggregate of fields, that carry flavour and have taken a vev. Flavour would be a manifest symmetry of the total, high energy Lagrangian, at a flavour scale Λ_f. After spontaneous symmetry breaking, the low-energy Yukawa interactions would result from effective operators of dimension $d > 4$ invariant under the flavour symmetry, which involve one or more flavour fields together with the usual SM fermionic and Higgs fields.

Only a scalar field (or an aggregate of fields in a scalar configuration) can get a vev, which should correspond to the minimum of a potential. What is the scalar potential for those scalar flavour fields? May some of its minima naturally correspond to the observed spectra of masses and mixing angles? These questions have been addressed in this work. The analysis of the potential is related to the extrema of the invariants mentioned above, but it goes beyond since the simultaneous presence of various invariant terms need not result in minima associated to the extrema that their independent consideration yields.

The simplest realization of this kind is obtained by a one-to-one correspondence of each Yukawa coupling with a single scalar field transforming in the bi-fundamental of the flavour group $\mathcal{G}_\mathcal{F}$. In the language of effective Lagrangians, this may correspond to the lowest order terms in the flavour expansion: $d = 5$ effective Yukawa operators made out of one flavour field plus the usual SM fields. The general scalar potential for bi-fundamental flavor scalar fields was constructed for quark and leptons in the two and three family case. Formally, it can be simply built out of the same Yukawa invariants mentioned above: from their combination new minima may a priori follow.

When determining the scalar potential, it was first shown that the underlying flavour symmetry is a very restrictive constraint: at the renormalizable level only a few terms are allowed in the potential, and even at the non-renormalizable level quite constrained patterns have to be respected.

For the quark case at the renormalizable level, at the minimum of the potential only vanishing mixing angles are allowed. Regarding mass hierarchies, one of the possible minima allows vanishing Yukawa couplings for all quarks but those in the heaviest family, both for the two and three generation cases. There is therefore an staring solution in the quark case which resembles in first approximation nature: a hierarchical spectrum with no mixing. This solution in the two family case can be perturbed at the renormalizable level to provide masses for the light families, by means of small explicit breaking terms of the abelian part of $\mathcal{G}_{\mathcal{F}}^{q}$, that is $U(1)^3$. This option is not present in the three family case since the hierarchical configuration is protected by a larger unbroken symmetry: $SU(2)^3$. The introduction of non-renormalizable terms in the potential allowed for further breaking of the symmetry, at the price of large fine-tunings, which are in our opinion unacceptable in the spirit of and effective field theory approach.

For the lepton sector, the same realization one Yukawa-one field, that is, of scalar bi-fundamental fields led to strikingly different results. In the two and three family cases non-trivial Majorana phases and mixing angles may be selected by the potential minima and indicates a novel connection with the pattern of neutrino masses: i) large mixing angles are possible; ii) there is a strong correlation between mixing strength and mass spectrum; iii) the relative Majorana phase among the two massive neutrinos is predicted to be maximal, $2\alpha = \pi/2$, for non-trivial mixing angle; moreover, although the Majorana phase is maximal, it does not lead to CP violation, as it exists a basis in which all terms in the Lagrangian are real.

The exact solutions of the renormalizable potential leading to non-trivial mixing showed one maximal mixing angle only among two degenerate but distinct neutrinos for both two and three generations. This scenario leads in the case of normal or inverted hierarchies to the maximal angle being the solar instead of the atmospheric angle. In the case of all three neutrinos degenerate, allowed by the renormalizable potential, the assignation of the angle depends on the corrections on the spectrum of masses, in a more promising scheme currently under exploration (17).

Another avenue explored in this work associates two vector flavour fields to each Yukawa spurion, i.e. a Yukawa $Y \sim \chi^L \chi^{R\dagger}/\Lambda_f^2$. This is an attractive scenario in that while Yukawas are composite objects, the new fields are in the fundamental representation of the flavour group, in analogy with the case of quarks. Those flavour fields could be scalars or fermions: we focused exclusively on scalars. From the point of view of effective Lagrangians, this case could correspond to the next-to leading order term in the expansion: $d = 6$ effective Yukawa operators as total or partial sources of the low-energy Yukawa couplings. We have constructed the general scalar potential for scalar flavour fields in the fundamental representation, both for the case of two and three families of quarks, although conclusions translate straightforwardly to leptons. By construction, this scenario results unavoidably in a strong hierarchy of masses: at the renormalizable level only one quark gets mass in each sector: they could be associated with the top and bottom quark. Non-trivial mixing requires as expected a misalignment between the flavour fields associated to the up and down (neutrino and electron) left-handed quarks (leptons). In consequence, the minimal field content corresponds to four fields $\chi^L_{U\,(\nu)}$, $\chi^L_{D\,(E)}$, $\chi^R_{U\,(\nu)}$ and $\chi^R_{D\,(E)}$, and the physics of mixing lies in the interplay of the first two. In resume, for fundamental flavour fields it follows in a completely natural way: i) a strong mass hierarchy between quarks of the same charge, pointing to a distinctly heavier quark in each sector; ii) one non-vanishing mixing angle, which can be identified with the with the rotation in the 23 sector for both quark and leptons in the three generation case.

Finally, as a possible correction to the patterns above, we briefly explored the possibility of introducing simultaneously bi-fundamentals and fundamentals flavour fields. It is a very sensible possibility from the point of view of effective Lagrangians to consider both $d = 5$ and $d = 6$ Yukawa operators when working to $\mathcal{O}(1/\Lambda_f^2)$. It suggests that $d = 5$ operators, which bring in the bi-fundamentals, could give the dominant contributions, while the $d = 6$ operator - which brings in the fundamentals - should provide a correction inducing the masses of the two lighter families and non-zero angles.

Overall, it is remarkable that the requirement of invariance under the flavour symmetry strongly constraints the scalar potential. Furthermore, one result of the analysis stands out among the rest. In the minimum of the potential, at the renormalizable level the quark mixing angles vanish at leading order, whereas lepton mixing is found to be maximal. The presence of the maximal angle in the lepton case is due to the $O(n_g)$ factor of the flavour group, which is in turn related of the Majorana nature of neutrinos. The explanation of the different mixing patterns in quarks and leptons in this scheme is, utterly, the different fundamental nature of the two types of fermions: Dirac and Majorana.

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