

BEAM ENLARGEMENT BY MISMATCHING THE ENERGY-DISPERSION FUNCTION†

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Summary

The natural beam size at the interaction point of a storage ring can be increased by operating the machine in a configuration in which the energy-dispersion function (η -function) is nonrepetitive in the periodic cells of the machine lattice. This mode of operation has been studied in SPEAR and is being considered in the PEP design. The usual method of beam enlargement in SPEAR has been to operate the machine in a configuration with a large value of the η -function at the interaction point, but with periodic η -function in the cells. Some of the results of this study will be described.

Introduction

For a given operating configuration and energy, the maximum luminosity \mathcal{L}_m is proportional to the product of the beam currents at the beam-beam limit as characterized by the value of the maximum incoherent tune shift $\Delta\nu_m$. Since the value of $\Delta\nu_m$ is proportional to the beam current density, higher values of \mathcal{L}_m can be obtained for configurations with larger natural beam size at the interaction point. The natural beam size is determined by two terms: one depends on the square of the value of energy dispersion η^* at the interaction point and the other depends on the value of beam emittance due to betatron motion. It can be shown that configurations with nonperiodic η -functions in the cells (η mismatched) give higher values of beam emittance than those with periodic η -functions (η matched).

In order to get higher luminosity at the lower operating energies, SPEAR has been operated in high- η^* configurations with η matched. Recently we have tried operating SPEAR in a mismatched- η configuration with $\eta^*=0$. The maximum value of luminosity obtained for this mismatched-configuration has been found to be comparable to those of the normal high- η^* configuration. In practice, operation in the mismatched- η mode is limited by the available machine aperture in the lattice since the η -function varies over a large range of values and the betatron oscillation amplitudes are increased by the greater emittance.

In the following sections, we will describe the effect of mismatching η upon the luminosity, present some configurations for SPEAR with η mismatched and discuss some experimental results.

Luminosity

The dependence of luminosity upon the various machine parameters will be studied in this section. It will be shown that luminosity can be increased by mismatching the η -function in the lattice cells.

Consider a storage ring consisting of several superperiods. Each superperiod is composed of periodic cells and insertions. An example of a superperiod can be seen in Fig. 1 which shows the lattice for half of a superperiod of SPEAR.

For an electron storage ring with N_b equal bunches in each beam, the luminosity is¹

$$\mathcal{L} = \frac{N_b N_f^2 f}{4\pi \sigma_{xt}^* \sigma_{yt}^*} \quad (1)$$

where f is the revolution frequency, N is the number of electrons in a bunch, σ_{xt}^* and σ_{yt}^* are the effective beam widths and height in the interaction region. At the beam-beam limit, the number of electrons as limited by horizontal tune shift is given by

$$N_m = \frac{2\pi \Delta\nu_m}{r_e \beta_x^*} \left(\frac{E}{mc^2} \right) (\sigma_{xt}^* + \sigma_{yt}^*) \sigma_{xt}^* \quad (2a)$$

or the vertical tune shift

$$N_m = \frac{2\pi \Delta\nu_m}{r_e \beta_y^*} \left(\frac{E}{mc^2} \right) (\sigma_{yt}^* + \sigma_{xt}^*) \sigma_{yt}^* \quad (2b)$$

where r_e is the classical electron radius, β^* is the value of the β -function at the interaction point and (E/mc^2) is the beam energy.

For simplicity, we will assume that there are no vertically bending fields and that all the horizontally bending fields are uniform. Under this condition it can be shown (see Appendix) that the maximum value of luminosity for a given $\Delta\nu_m$ is given by

$$\mathcal{L}_m = \frac{\pi N_b f \Delta\nu_m^2}{r_e^2} \left(\frac{E}{mc^2} \right)^2 \left(\frac{1}{\beta_x^*} + \frac{1}{\beta_y^*} \right) \left(\epsilon_0 + \frac{\eta^{*2} \delta^2}{\beta_x^*} \right) \quad (3)$$

where ϵ_0 = the horizontal beam emittance in the absence of x-y betatron coupling = $\sigma_{x\beta}^{*2}(0)/\beta_x^*$
 δ = energy spread in the beam = $(\Delta E/E)$.

In particular, for a storage ring with M identical zero-gradient bending magnets of length ℓ ,

$$\mathcal{L}_m = \frac{\pi N_b f \Delta\nu_m^2}{r_e^2 \rho} \left(\frac{E}{mc^2} \right)^4 \left(\frac{\hbar}{mc} \right) \frac{55}{32\sqrt{3}} \left(\frac{1}{\beta_x^*} + \frac{1}{\beta_y^*} \right) \left(\langle \bar{H} \rangle + \frac{\eta^{*2}}{2\beta_x^*} \right), \quad (4)$$

where ρ is the radius of curvature in the bend and (\hbar/mc) is the reduced Compton wavelength;

$$\langle \bar{H} \rangle = \frac{1}{2\pi\rho} \sum_i \int_i H(\eta, \eta') ds \quad ; \quad (5)$$

$$H(\eta, \eta') = \frac{1}{\beta_x^*} \left[\eta^2 + \left(\beta_x^* \eta' - \frac{1}{2} \beta_x^* \eta \right)^2 \right]. \quad (6)$$

The \sum denotes summation over all of the bending magnets and \int_i denotes integration over the i th bending magnet.

To see the effect of mismatching η on the value \mathcal{L}_m , it will suffice to consider its effects on $\langle \bar{H} \rangle$. For this purpose we let

$$\eta(s) = \eta_0(s) + \eta_1(s) \quad (7)$$

where η_0 is the η -function for the corresponding storage ring composed of only the repeated cells. In the cells, both η and η_0 satisfy the same inhomogeneous differential

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equation

$$\eta''(s) - K(s) \eta(s) = \frac{1}{\rho} \quad (8)$$

where $K(s)$ is the focusing function in the cells.

The function $\eta_1(s)$ satisfies the homogeneous differential equation

$$\eta_1''(s) - K(s) \eta_1(s) = 0$$

so that the function $H(\eta_1, \eta_1')$ is a well-known invariant;³ i.e.,

$$H_1 = H(\eta_1, \eta_1') = \text{constant.}$$

It can be shown¹ that the function $\eta_0(s)$ varies approximately as $\sqrt{\beta_x(s)}$ so that

$$\beta_x(s) \eta_0'(s) - \frac{1}{2} \beta_x'(s) \eta_0(s) \approx 0$$

and the function $H(\eta_0, \eta_0')$ is also approximately an invariant; i.e.,

$$H_0 = H(\eta_0, \eta_0') \approx \text{constant.}$$

Making use of these properties of η_0 and η_1 , we find for the cells

$$H(\eta, \eta') \approx H_0 + H_1 + \frac{2\eta_0(s) \eta_1(s)}{\beta_x(s)} \quad (9)$$

For a machine with many cells, the last term gives a small contribution to the value of $\langle \bar{H} \rangle$ because η_1 oscillates about η_0 , and most of the contribution to the value of $\langle \bar{H} \rangle$ comes from the cells. In addition, if there is a point of symmetry in the cell lattice ($\beta_{xs}' = \eta_s' = 0$) then the value of $\langle \bar{H} \rangle$ is approximately given by

$$\langle \bar{H} \rangle \approx \frac{\eta_{0s}^2}{\beta_{xs}} + \frac{\eta_{1s}^2}{\beta_{xs}} \quad (10)$$

with the subscript s denoting the symmetric point. Since $\eta_{1s} = \eta_s - \eta_{0s}$, η_{1s} is a measure of how much η is mismatched in the cells. The increase in luminosity varies as η_{1s}^2 for a mismatched configuration.

Computed Results

The values of luminosity for a family of mismatched- η configurations have been computed for SPEAR using Eq. (4). Since SPEAR has a superperiodicity of two and each superperiod is symmetric about its midpoint, it is convenient to characterize these configurations by the value of η at that symmetry point as discussed in the previous section. Figure 1 shows a schematic layout of the magnets in SPEAR for half of a superperiod starting at the interaction point and ending at the symmetry point. There are five cells in each superperiod. Each cell contains two focusing quadrupole magnets, one defocusing magnet and two bending magnets. The values of η at different points along the machine for a typical matched- η configuration (A) and a typical mismatched- η configuration (C) are shown in Fig. 1. It can be seen that the η -function is periodic in the cells for configuration A with $\eta^* = 1.75$ m. The η -function for the configuration C has no periodicity within a superperiod of the machine, and $\eta^* = 0.0$ m.

Figure 2 shows a plot of \mathcal{L}_m as a function of η_s for a family of configurations with $\Delta\nu_m = 0.025$, $E = 1.5$ GeV,

$\beta_x^* = 1.2$ m, $\beta_y^* = 0.1$ m, $\nu_x = 5.15$, $\nu_y = 5.11$ and $\eta^* = 0, 1.0, 1.5, 1.75$ and 2 m. The values of \mathcal{L}_m for the matched- η configuration lie along the curve between points A and B. It can be seen that for the matched configurations \mathcal{L}_m

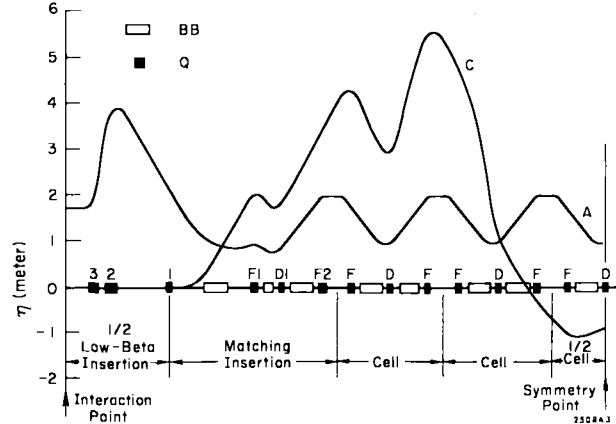


FIG. 1--A schematic layout of the magnets in SPEAR for half of a superperiod. The matching section consists of quadrupole magnets F1, D1 and F2. The low-beta insertion consists of quadrupole magnets 1, 2 and 3. A cell is composed of quadrupole magnets F, D and F.

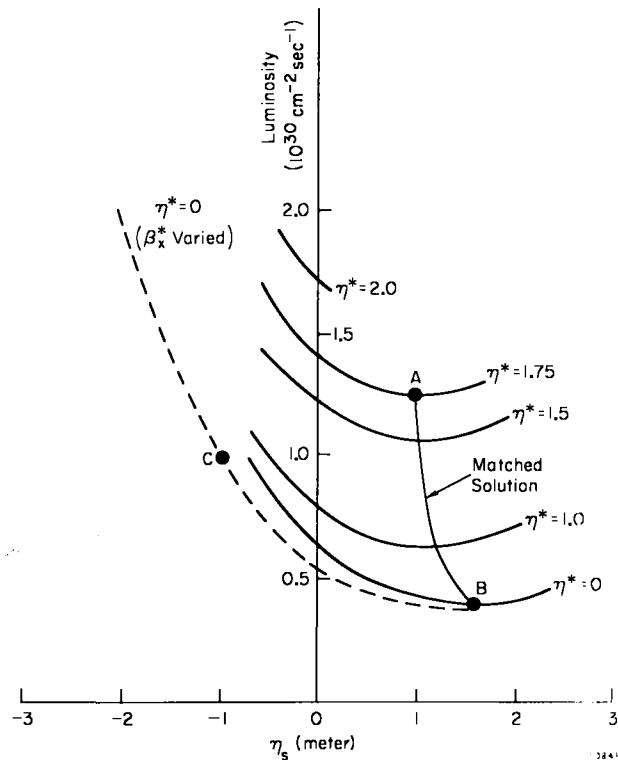


FIG. 2--Computed values for the maximum value of luminosity for some SPEAR configurations with $\Delta\nu_m = .025$. The machine parameters for these configurations are: $\nu_x = 5.15$, $\nu_y = 5.11$, $\beta_x^* = 1.2$ m, $\beta_y^* = .1$ m and different values of η^* and η_s . For the values on the dashed line, the value of β_x^* varies linearly from 1.2 m at the matched- η configuration B to 4.5 m at the mismatched- η configuration C. ($E = 1.5$ GeV.)

increases as η^* increases as expected. Furthermore, for a given value of η^* , the value of \mathcal{L}_m is very close to a minimum as a function of η_s at the matched configuration. Although SPEAR has only five cells per superperiod, the gain in \mathcal{L}_m by using the mismatched- η configurations and the approximate validity of Eq. (10) can be seen from this plot.

From Eqs. (4) and (10), it can be seen that the value of \mathcal{L}_m is not appreciably affected by the value of β_x^* for $\eta^*=0$ configurations if $\beta_x^* \gg \beta_y^*$. The dashed line in Fig. 2 shows the values of \mathcal{L}_m for configurations with $\eta^*=0$ and with β_x^* increasing linearly from 1.2 to 4.5 m between configurations B and C. It has been observed that as the value of η_s gets more negative, maximum β values in the matching section become very large. However, by using larger values of β_x^* , it is possible to keep the maximum beam size within the allowable machine aperture up to some large values of η -mismatch.

Since the rate of change of the machine damping time constant with RF frequency depends on the momentum compaction factor α , Fig. 3 shows a plot of α for some of the configurations shown in Fig. 2. It can be seen that α is relatively independent of β_x^* for $\eta^*=0.0$ configurations.

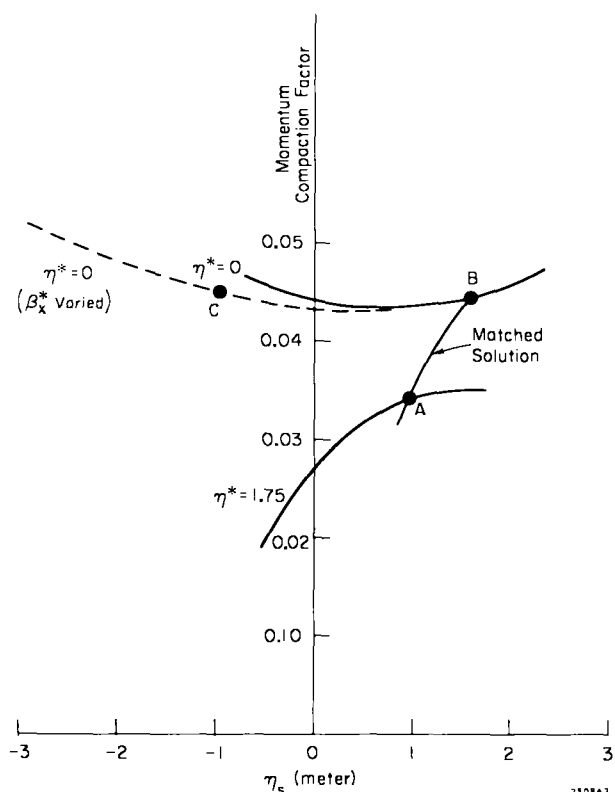


FIG. 3--Values of momentum compaction factor for some typical configurations shown in Fig. 2.

Experimental Results

Luminosity for different beam currents has been measured for configurations A, B and C at 1.5 GeV. The results are shown in Fig. 4. Configuration A is a matched- η configuration with $\eta^* = 1.75$ m, B is also a matched- η configuration with $\eta^* = 0.0$ and C is a mismatched- η configuration with $\eta^* = 0.0$ and $\beta_x^* = 4.5$ m. The values of \mathcal{L}_m at the beam-beam limit for these cases are shown in Fig. 4. \mathcal{L}_m is largest for A and smallest for B, which is consistent with the computed results (see Fig. 2). It may be interesting to note that at the limit the ratio of \mathcal{L}_m to beam

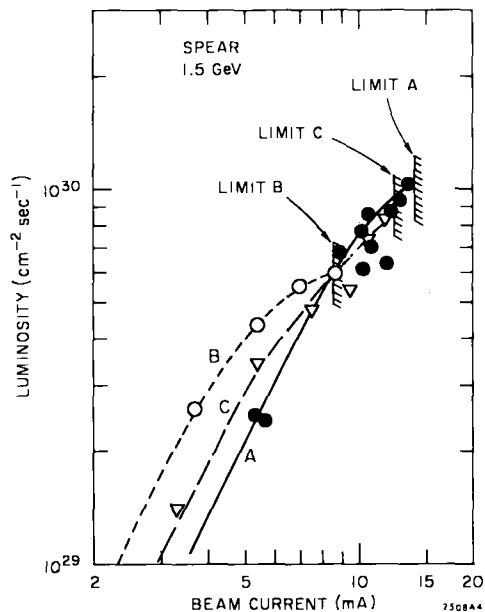


FIG. 4--Measured values of luminosity for three configurations A, B and C. The values of machine parameters for these configurations are given in Fig. 2.

current is nearly the same for all three cases; i.e., the values of $\Delta\nu_m$ are independent of configurations.

The beam width at one point in the cell has been measured for A and C. The measured values are in general agreement with the computed values. The variation of damping time constant for different values of RF frequency has been measured also for A. The result is consistent with the computed results in the order of magnitude. We expect from the computed result that the change in damping time with frequency should be nearly the same for both A and C. Further decrease in the values of η_s beyond the value for C has been tried. Beam life-time becomes poorer for these configurations as we approach the limit imposed by the machine aperture at locations of large η and β values.

Conclusions

It may be concluded from this study that mismatched- η configurations can produce higher luminosity values than matched- η configurations and are a viable alternative to matched- η , high- η^* configurations, provided that there is sufficient machine aperture to allow for the increase in beam width.

For a given machine the optimum operating configuration may be one with mismatched- η , finite η^* and a judicious choice for the value of β_x^* .

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Appendix

The expressions for the maximum value of luminosity [Eqs. (3) and (4)] will be derived in this section. For this derivation, we assume that there are no vertically bending fields in the machine and that the horizontally bending fields are uniform within each magnet. The effective beam width and height are then given by

$$\sigma_{xt}^* = \sqrt{\sigma_{x\beta}^{*2} + \eta^{*2} \delta^2} \quad (\text{A. 1})$$

and

$$\sigma_{yt}^* = \sigma_{y\beta}^* \quad (\text{A. 2})$$

where * denotes the values at the interaction point, the subscript β denotes the contribution to the beam dimension due to betatron motion, and δ is the energy spread in the beam ($\Delta E/E$). The value of $\sigma_{y\beta}^*$ comes from the coupling between the x and y betatron motion of the particles. Let the coupling constant be defined by

$$A = \frac{\sigma_{y\beta}^*}{\sigma_{x\beta}^*} \sqrt{\frac{\beta_x^*}{\beta_y^*}} \quad (\text{A. 3})$$

If we assume¹ that

$$\frac{\sigma_{x\beta}^{*2}}{\beta_x^*} + \frac{\sigma_{y\beta}^{*2}}{\beta_y^*} = \frac{\sigma_{x\beta}^{*2}(0)}{\beta_x^*} \quad (\text{A. 4})$$

where $\sigma_{x\beta}^{*2}(0)$ is the effective beam width for zero coupling, then

$$\sigma_{x\beta}^* = \frac{1}{\sqrt{1+A^2}} \sigma_{x\beta}^{*}(0) \quad (\text{A. 5})$$

and

$$\sigma_{y\beta}^* = \frac{A}{\sqrt{1+A^2}} \sigma_{x\beta}^{*}(0) \sqrt{\frac{\beta_y^*}{\beta_x^*}} \quad (\text{A. 6})$$

The expression for luminosity may be written in terms of $\sigma_{x\beta}^{*}(0)$ and A as

$$\mathcal{L} = \frac{N_b N_f^2 \sqrt{1+A^2}}{4\pi A \sigma_{x\beta}^{*}(0) \sqrt{\sigma_{x\beta}^{*}(0)^2 + \eta^{*2} \delta^2 (1+A^2)}} \quad (\text{A. 7})$$

The maximum value of luminosity \mathcal{L}_m is determined by the maximum number of electrons in a bunch as given by Eq. (2).

If we assume that the value of the incoherent tune shift for both x and y oscillations are the same, we find from Eq. (2) that at the beam-beam limit,

$$\frac{\sigma_{xt}^*}{\beta_x^*} = \frac{\sigma_{yt}^*}{\beta_y^*} \quad (\text{A. 8})$$

Substituting the expression for σ_{xt}^* and σ_{yt}^* into Eq. (A. 8) and solving for the value of A, we obtain

$$A = \sqrt{\frac{\beta_y^*}{\beta_x^*}} \sqrt{\frac{(\sigma_{x\beta}^{*2}(0) + \eta^{*2} \delta^2)}{(\sigma_{x\beta}^{*2}(0) - \eta^{*2} \delta^2 \frac{\beta_y^*}{\beta_x^*})}} \quad (\text{A. 9})$$

Combining Eqs. (2), (A. 7), and (A. 9), we find at the beam-beam limit the expression for \mathcal{L}_m in terms of ϵ_0 and δ as given by Eq. (3). The values $\sigma_{x\beta}^{*2}(0)$ and δ may be expressed in terms of the synchrotron integrals² I_2 , I_3 , I_4 and I_5

$$\epsilon_0 = \frac{\sigma_{x\beta}^{*2}(0)}{\beta_x^*} = \frac{55}{32\sqrt{3}} \left(\frac{\hbar}{mc}\right) \left(\frac{E}{mc^2}\right)^2 \frac{I_5}{I_2 - I_4}; \quad (\text{A. 10})$$

$$\delta^2 = \frac{55}{32\sqrt{3}} \left(\frac{\hbar}{mc}\right) \left(\frac{E}{mc^2}\right)^2 \frac{I_3}{2I_2 + I_4}, \quad (\text{A. 11})$$

where \hbar/mc is the reduced Compton wavelength.

For the special case of equal zero gradient bending magnets of length l :

$$I_2 = 2\pi/\rho, \quad I_3 = 2\pi/\rho^2, \quad I_4 \approx -\frac{\pi l^2}{6\rho^3} \ll I_2,$$

$$I_5 = \langle \bar{H} \rangle \frac{2\pi}{\rho},$$

we find the expression for \mathcal{L}_m as given by Eq. (4).

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