Spanish Relativity Meeting (ERE 2009) IOP Publishing
Journal of Physics: Conference Series **229** (2010) 012069 doi:10.1088/1742-6596/229/1/012069

Stellar oscillations in TeVeS

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Abstract. In this work we examine the oscillation spectra of neutron stars in TeVeS. As a result with Cowling approximation, we find that the frequencies of fundamental modes in TeVeS could become lager than those expected in general relativity, while the dependence of frequency of higher overtone on gravitational theory is stronger than that of lower modes. Additionally, as a results of calculations with the assumption that the vector field is unperturbed, we also find that the dependences of frequencies of spacetime oscillations (w modes) on the stellar compactness are almost independent from the adopted equation of state and the parameter in TeVeS, and that these dependences of frequencies of axial w modes in TeVeS is obviously different from those expected in the general relativity. These imprints of TeVeS make it possible to distinguish the gravitational theory in strong-field regime via the observations of gravitational waves, which can provide unique confirmation of the existence of scalar field.

1. Introduction

The tests of gravitational theories in the strong-field regime are quite important, because the gravitational theories in the strong-field regime are still largely unconstrained by observations in contrast to those in the weak-field regime. However, owing to the developments in observational technology, it is becoming possible to observe compact objects with high accuracy. Via the observations of X-rays and/or gamma rays emitted from compact objects, one can know the properties of compact objects and could use as a direct test of the gravitational theory in the strong-field regime [1]. As an alternative way to observe compact objects, the gravitational waves are also expected to obtain the raw information of the compact objects [2, 3].

The tensor-vector-scalar (TeVeS) theory has attracted considerable attention, which is proposed by Bekenstein [4] as a relativistic theory of Modified Newtonian Dynamics (MOND) [5]. In TeVeS, it is possible to explain many galactic and cosmological observations without the need for dark matter [4]. While, in the strong-field regime, the Schwarzschild solution in TeVeS and the Reissner-Nordström solution in TeVeS have been found [6, 7], while Lasky *et al.* derived the TOV equations and they showed the possibility of distinguishing TeVeS from GR by way of redshift observations [8]. In this work, we examine whether observations of gravitational waves associated with the neutron star oscillations can provide an alternative way of probing the gravitational theory in the strong-field regime. The more detailed study can be seen in [9, 10].

2. Stellar model in TeVeS

TeVeS is based on three dynamical gravitational fields, such as an Einstein metric $g_{\mu\nu}$, a timelike 4-vector field $\mathcal{U}^\mu$, and a scalar field φ, in addition to a nondynamical scalar field σ. By varying the total action with respect to $g_{\mu\nu}$, $\mathcal{U}_\mu$, and φ, one obtains the field equations for the metric, vector, and scalar fields, which include two positive dimensionless parameters, k and $\mathcal{K}$. To construct the stellar models in TeVeS, three parameters, k, $\mathcal{K}$ and φ_c, are introduced with respect to GR, where φ_c denotes the cosmological value of the scalar field. The value of k has a tightly constraint as $k \sim 0.03$ by both cosmological models and also planetary motions in the solar system, while the cosmological considerations imply that the value of φ_c is restricted to $0 \leq \varphi_c \ll 1$ [4]. With respect to the value of φ_c, it was shown that φ_c could have a minimum value of around 0.001, which is based on the causality issues inside the star [8]. Anyway, since it was also found in [8] that the neutron star models are almost independent from the values of k and φ_c, in this work we make examinations with $k = 0.03$ and $\varphi_c = 0.003$. While, so far there is no severe restriction on $\mathcal{K}$ except that $\mathcal{K}$ should be in the range of $0 < \mathcal{K} < 2$ [7, 8]. Thus we adopt various values of $\mathcal{K}$ in the above range and study the dependence of gravitational waves on $\mathcal{K}$. Furthermore, as equilibrium stellar models, we adopt the similar models in [9].

3. Fluid Oscillations

First, with the Cowling approximation we examine the fluid oscillations of neutron stars. Especially, we focus on the stellar models whose central density is in the range from $\tilde{\rho}_0 = 10^{14}$ g/cm^3 up to the value given the maximum ADM mass. In general, the oscillation spectrum is directly related to the stellar parameter, but the frequencies of fundamental oscillation modes can be connected to the stellar average density. Actually, in GR, it is well known the empirical formula for the frequency of f mode as a function of stellar average density, which is almost independent from the adopted EOS to construct the stellar models. While, the frequencies of f mode for the stellar models in TeVeS with above two EOS are shown in Fig.1. The deviation from GR is clearly cognized for typical neutron stars and depending on the value of parameter $\mathcal{K}$, the frequencies become around 20 % larger than those expected in a general relativistic neutron star. This can be an observable effect and one might distinguish the gravitational theory in strong gravitational field by using the observations of gravitational waves.

The possibility to probe the gravitational theory by using observations of gravitational waves can be also seen in Fig.2, where we plot the normalized frequencies of f and p_1 modes as functions of ADM mass. One can easily observe that the frequencies expected in TeVeS are quite different from those in GR. Since this distinction results from the difference of gravitational theory, which creates due to the presence of scalar field, observing more than one mode of gravitational wave could tell us the existence of the scalar field. Through this figure, we can find that with a help of observation of stellar mass, it is possible to probe the gravitational theory in the strong-field regime by using observations of gravitational waves.

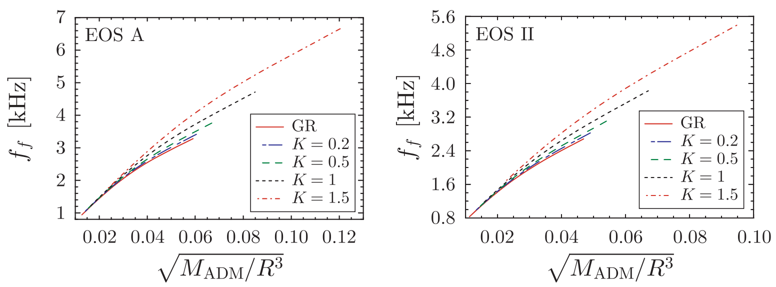

Figure 1. The frequency of f mode as a function of the stellar average density.

4. Spacetime Oscillations

Next, with the assumption that the vector field is fixed, we examine the spacetime oscillations. Similar to the fluid modes, in GR it is well known that the frequencies of w and $w_{\rm II}$ modes can

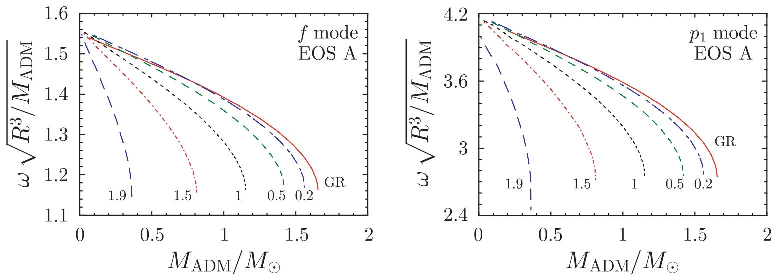

Figure 2. The frequencies of first two fluid modes are plotted as functions of the ADM mass.

be described as a function of stellar compactness $M_{\rm ADM}/R$. So, we will examine the dependence of frequencies of $w_{\rm II}$ and w modes on the stellar compactness. Figs.3 and 4 show the frequencies of axial $w_{\rm II}$ and w_1 as functions of the total stellar compactness, where the solid symbols are results with EOS A and the open ones are those with EOS II. From Fig.3, we can see that the dependence of frequencies of $w_{\rm II}$ modes on the stellar compactness are almost independent from the values of $\mathcal{K}$ and the adopted EOS. Additionally, it is found that the deviation from the frequencies expected in GR is very little. We can see little difference between the expectations in GR and in TeVeS in the damping rates (imaginary part of complex frequencies) for the stars with larger compactness and in the oscillation frequencies (real part of complex frequencies) for the stars with weaker compactness. For the oscillation frequencies in TeVeS, we can get the empirical formula, such as

$$Re\,(\omega M_{\rm ADM}) = -0.0412 + 0.7084\left(\frac{M_{\rm ADM}}{R}\right) + 3.305\left(\frac{M_{\rm ADM}}{R}\right)^2, \tag{1}$$

and the oscillation frequencies can be in very good agreement with this empirical formula.

On the other hand, in Fig.4 for the frequencies of w_1 modes, we can observe same feature as the frequencies of $w_{\rm II}$ modes, i.e., if we see those frequencies as functions of the stellar compactness, those are almost independent from the values of $\mathcal{K}$ and the adopted EOS. However, in the case of w_1 modes, there exists the crucial different feature in contrast to the $w_{\rm II}$ modes. That is, the dependence of the frequencies in TeVeS is obviously different from those expected in GR. In other words, with this different dependence of frequencies on the gravitational theory, one can distinguish the gravitational theory in strong-field regime by using the gravitational waves observations. In fact, the oscillation frequencies of axial w_1 modes in GR and TeVeS can be expected with high accuracy via the following empirical formula

$$Re\,(\omega R) = \alpha - \beta\left(\frac{M_{\rm ADM}}{R}\right), \tag{2}$$

where $(\alpha, \beta) = (2.797, 4.255)$ in GR and $(\alpha, \beta) = (2.533, 3.714)$ in TeVeS.

5. Conclusion

In this work, we examine the effect of the Tensor-Vector-Scalar (TeVeS) Theory on the oscillation spectra of neutron stars. Depending on the parameter of TeVeS, the frequencies of fundamental oscillation could be off the well-known empirical formula in GR and they become lager than those expected in GR. We can also see the deviation from GR in the frequencies of higher overtones and they have stronger dependence on the parameter $\mathcal{K}$ than the lower oscillation modes. Additionally, by calculating the complex eigenfrequencies of axial $w_{\rm II}$ and w modes, where the real and imaginary parts of complex frequencies are corresponding to the oscillation

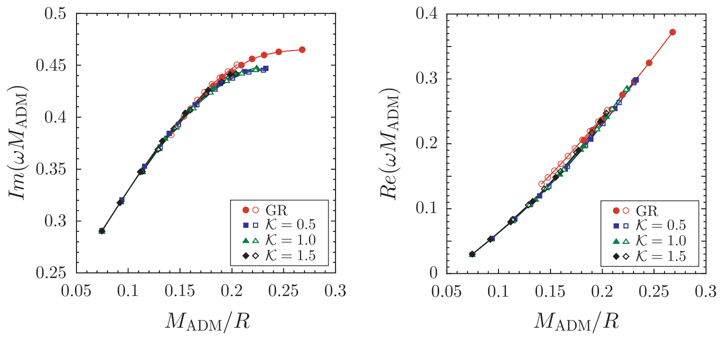

Figure 3. For $w_{\rm II}$ modes, the damping rates (left panel) and the oscillation frequencies (right panel) as functions of stellar compactness $M_{\rm ADM}/R$. The solid symbols correspond to the results for EOS A and the open ones to those for EOS II.

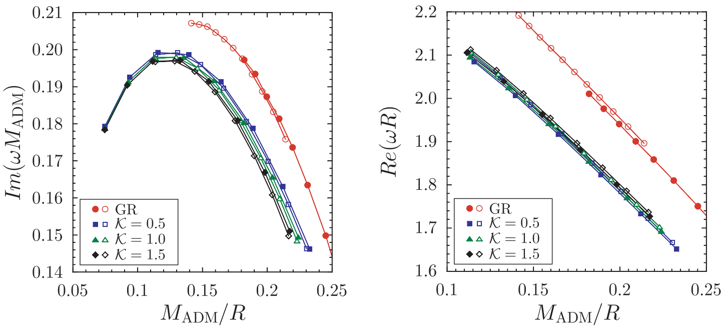

Figure 4. Similar to Fig.3, but for w_1 modes.

frequencies and the damping rates of the emitted gravitational waves, respectively, we find that the dependences of the both frequencies of $w_{\rm II}$ and w_1 modes in TeVeS on the stellar compactness are almost independent from the parameter $\mathcal{K}$ and the adopted EOS. The dependences of w_1 modes are obviously different from that expected in GR, while the dependences of $w_{\rm II}$ modes are similar to that in GR. Owing to these differences of dependence on the gravitational theory, one can distinguish the gravitational theory in the strong-field regime by using the direct observations of gravitational waves emitted from the neutron stars. Since these imprints of TeVeS come from the presence of scalar field, by using the observations of gravitational waves associated with the stellar oscillations, it will be possible not only to distinguish the gravitational theory in the strong-field regime, but also to probe the existence of the scalar field.

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