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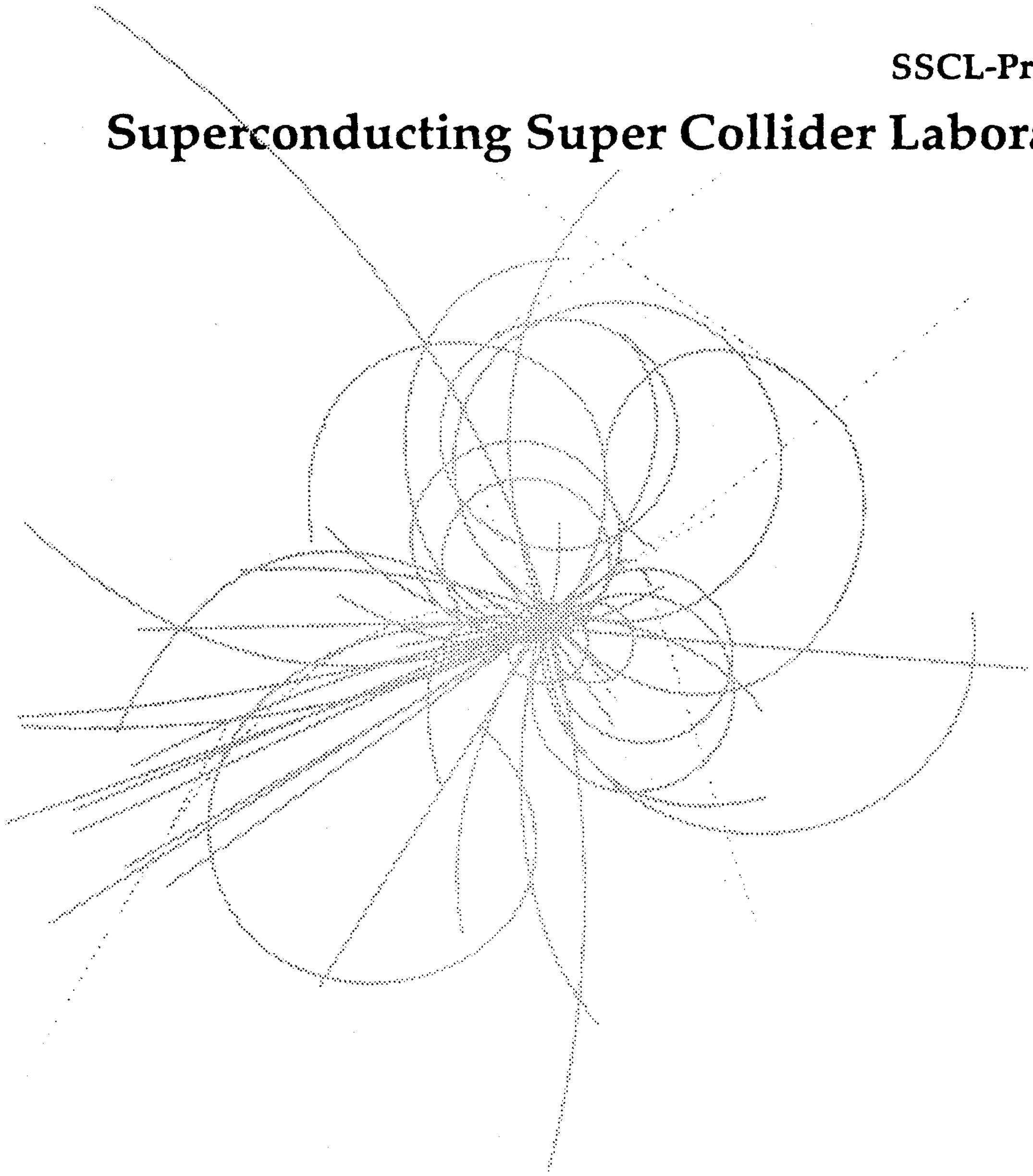

Brief Comment on One-Turn Maps for Long-Term Tracking

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October 1991

To be published by American Institute of Physics. SSCL-Preprint-16

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October 1991

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* Presented at the Advanced Beam Dynamics Workshop on Effects of Errors in Accelerators, Their Diagnosis and Corrections, Corpus Christi, Texas, October 3–8, 1991.

† Operated by the Universities Research Association, Inc., for the U.S. Department of Energy under Contract No. DE-AC35-89ER40486.

Brief Comment on One-Turn Maps for Long-Term Tracking

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There is no doubt that one-turn maps are valuable for analysis of short-term beam dynamics. However, it has always been a controversial issue as to whether one can use a one-turn map for long-term tracking of large storage rings and thereby save enormous computer time. Based on some recent works,[1–5] the author wishes to state tentatively that for certain cases—in particular, for the Superconducting Super Collider (SSC) dynamic aperture study—one-turn-map trackings can be made nearly as reliable as element-by-element trackings. The author also wishes to make the following general statement: If a one-turn map is symplectic and accurate to five or more digits in one-turn tracking for the phase-space coordinates of particles within the interest region (inside or around the dynamic aperture), then the one-turn map is nearly as reliable as its associated element-by-element tracking for predicting the "global" long-term stability of the storage ring.

Before proceeding further, to avoid possible ambiguities, the term "one-turn map" needs to be clarified. In this comment, whenever "a one-turn map" is mentioned, it means a one-turn differential algebraic Taylor map or one of its descendants (maps symplectified from the one-turn differential algebraic Taylor map).

Each element-by-element tracking program can be easily translated into a map extraction program using the existing differential-algebraic library such as the da-package,[6] the Mxyzpplk,[7] or Zlib.[8] For example, a program at the SSC Laboratory called Zmap can extract various one-turn maps for the element-by-element tracking program Teapot.[9,10] Also, at CERN, Schmidt has a map-extraction program associated with the element-by-element tracking program Sixtrack.[11] For some cases, such as the SSC, a one-turn, differential-algebraic Taylor map of modest order (e.g., 8th-to-11th order) can be obtained with reasonable use of computer time; such a map, relative to the associated element-by-element tracking, can offer accuracy to five or more digits in one-turn tracking for the phase-space coordinates of particles within the interest region. It is the author's opinion that for design and commissioning purposes, inaccuracy at the 6th digit in one-turn tracking would be negligible. It is also the author's opinion that, excluding extreme cases, a one-turn map that offers accuracy of five or more digits in one-turn tracking would have included all the important driving terms (in terms of Taylor expansions) governing the "global" long-term behavior of the nonlinear system. Therefore, the author wishes to make the first tentative conclusion: Excluding extreme cases, a one-turn map that—relative to its associated element-by-element tracking—can offer accuracy to five or more digits in one-turn tracking for the phase-space coordinates of particles within the interest region carries nearly as much reliable global (not necessarily detailed) physical information as the nonlinear detailed lattice for the associated element-by-element tracking.

A one-turn, differential-algebraic Taylor map of modest order, extracted from a symplectic tracking program, inherits symplectic properties up to the order of the Taylor map, but it is not symplectic in general because of the truncation of higher orders. Thus, it can be used for tracking only up to a certain number of turns, depending on its degree of symplecticity.

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However, it is computationally economic to make the map symplectic through Lie transformations (Dragt-Finn factorization) of the differential-algebraic Taylor map such that, excluding extreme cases, the one-turn map represented by the series of Lie transformations has nearly the same degree of accuracy within the interest region.[12] Note that a map represented by a series of Lie transformations is always symplectic. Therefore, the author wishes to make a second tentative conclusion: For some cases, such as the SSC, one can, with reasonable computer time, obtain a symplectic one-turn map represented by a series of Lie transformations that carries nearly as much reliable "global" physical information as the nonlinear detailed lattice for the element-by-element tracking.

Generally, a map represented by a Lie transformation cannot be used for tracking directly. This is rather unfortunate. Nonetheless, a map represented by a series of Lie transformations can always be converted into a kick map represented by another series of Lie transformations—called kick factorizations—for tracking so that, with care, the degree of accuracy at the borderline of the interest region may be maintained at about 10% of the original accuracy (for example, from 6-digit accuracy to 5-digit accuracy).[13–15] A series of Lie transformations can also be economically re-expanded into a higher-order Taylor map to gain a higher degree of symplecticity without losing the degree of accuracy; thus, such a Taylor map can be used for longer-turn tracking.[4] For example, we have used the 11th- and the 12th-order Taylor map for 10^6-turn trackings and have obtained the same dynamic aperture estimate as that of the element-by-element trackings.[2–4] Note that, including one rf cavity for synchrotron oscillation or acceleration, using the Zlib subroutine "zpmtrk" for 10,000-turn tracking of a particle requires 2.6 Cray-2 CPU seconds with an 11th-order Taylor map or 2 Cray-2 CPU minutes with a 16th-order Taylor map. Although there is no *a priori* way of checking whether the map is reliable (a disadvantage), one can always check afterward whether the kick-map or the Taylor-map tracking results are correct without needing to compare them with the element-by-element tracking results, because—excluding extreme cases—increasing (decreasing) each order of the extracted Taylor map would enhance (reduce) the degree of accuracy in one-turn tracking by a few times within the interest region (that is, error is dominated by the lowest truncated order). The following example may help clarify the above description.

Step 1: Extract a 10th-order, one-turn, differential-algebraic Taylor map for an SSC injection lattice with the map extraction program Zmap that is associated with the element-by-element tracking program Teapot.

Step 2: Symplectify the map by performing Lie transformations of the 10th-order Taylor map.

Step 3: Convert the map represented by the Lie transformations into a kick map.

Step 4: Perform the kick-map tracking up to 10^7 turns. Obtain a survival plot and estimate the dynamic aperture.

Step 5: Perform one-turn trackings with the kick map, the 10th-order Taylor map, the 9th-order Taylor map, and the 8th-order Taylor map for particles around the dynamic aperture estimated in Step 4. Make sure that errors in the truncated Taylor-map trackings are indeed dominated by the lowest truncated order and that one-turn tracking results from the 10th-order and the 9th-order Taylor maps agree up to five digits. The degree of accuracy for the kick-map tracking is thus confirmed to be sufficient if the one-turn tracking results from the kick map and from the 10th-order Taylor map agree up to five digits. The degree of symplecticity is automatically sufficient because the kick map is symplectic.

In conclusion, it is the author's opinion that long-term trackings using one-turn maps (obtained from symplectifying one-turn differential algebraic Taylor maps) are worth pursuing for large storage rings such as the SSC collider rings.

In writing this brief comment, the author owes a great deal to E. Forest and T. Sen for their collaboration and to A. Chao for his support during the last years. The author thanks T. Garavaglia, J. Peterson, and F. Schmidt for helpful discussions, and M. Syphers for continuous encouragement and support.

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