Can dissipative effects help the MSSM inflation?

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Abstract

We study the dissipative effects on the MSSM inflation, which suffers from severe fine-tuning problems on potential parameters and initial conditions. Dissipative effects appear as a consequence of interactions between an inflaton and other fields and they act as a friction term in its equation of motion. Of course it also can emerge in the case of MSSM inflation. However, we find that in the inflationary stage it can neither overwhelm the friction term that is due to the cosmic expansion nor affect the primordial fluctuations. Therefore it cannot relax the fine-tuning problem of the MSSM inflation.

1 Introduction

MSSM inflation [1] is a model of inflation that is based on an extension of the Standard Model (SM) that is motivated by the supersymmetry (SUSY), the Minimal Supersymmetric Standard Model (MSSM). In this model, the inflaton is the flat direction in the MSSM, a gauge invariant combination of either squark or slepton fields. Flat directions can be lifted by the SUSY-breaking effect and the non-renormalizable potential. If a fine-tuning of the potential parameters is realized, the potential for the flat direction has a saddle point and inflation can occur near the point. If we take the inflaton as the $\bar{u}\bar{d}\bar{d}$ or the $LL\bar{e}$ flat direction, the saddle point locates at $\varphi \sim 10^{14}$ GeV and the Hubble parameter during inflation is about $1-10$ GeV. Here φ is the inflaton. Its most interesting feature is that the inflaton couplings to particles in the SM are known and, at least in principle, measurable in laboratory experiments such as the Large Hadron Collider or a future Linear Collider. We do not need new physics beyond the MSSM.

Despite these attractive features described above, the MSSM inflation has some drawbacks, such as the fine-tuning of the potential parameters and worse, the fine-tuning of the initial condition. In particular, the latter is a very difficult problem. Because the slow-roll region for the MSSM inflation is extremely narrow, if the former is solved by some mechanism, the inflaton must reach the slow-roll region with an extremely small velocity in order to expand the Universe exponentially.

However, previous analyses have been neglected the interactions between the inflaton and other MSSM particles, which we know well, when they consider the dynamics of the inflaton. In general, interactions of the scalar fields with other fields can cause the dissipative effect. This phenomenon has been discussed in the high temperature regime [2, 3], in particular, in the context of the warm inflation [4]. It acts as a friction term in the equation of motion of the inflaton and can affects its dynamics. Therefore it has a possibility of relaxation of the fine-tuning of the MSSM inflation.

We investigate the interaction between the MSSM inflaton and other fields and the resultant dissipative effect carefully. As a consequence, dissipative effects arise but cannot change the dynamics of the MSSM inflaton in the inflationary stage. Moreover, it does not affect the primordial fluctuations. Therefore dissipative effects cannot relax the fine-tuning problem of the MSSM inflation.

2 MSSM inflation

Let us summarize the main features of the MSSM inflation [1] briefly. We adopt a non-renormalizable superpotential of the form

$$W_{\rm non} = \frac{\lambda}{6M_G^3}\Phi^6. \tag{1}$$

Here M_G is the reduced Planck mass and Φ is a supermultuplet whose scalar component ϕ parameterizes the flat direction. in the case where the $\bar{u}\bar{d}\bar{d}$ flat direction acts as an inflaton, ϕ represents the following field configuration.

$$\tilde{\bar{u}}_i^{\alpha} = \tilde{\bar{d}}_j^{\beta} = \tilde{\bar{d}}_k^{\gamma} = \frac{1}{\sqrt{3}}\phi, \quad (j \neq k, \alpha \neq \beta \neq \gamma), \tag{2}$$

where i, j and k are the family indices and α, β and γ are the color indices. We assume that λ is parameter of order unity. The flat direction is lifted by the superpotential (1). Hereafter we consider the case when the scalar component of ϕ acquires a large expectation value.

Including the SUSY breaking effect from the hidden sector, the scalar potential is found to be

$$V(\varphi) = \frac{1}{2}m_\phi^2\varphi^2 - \frac{A\lambda}{24M_G^3}\varphi^6 + \frac{\lambda^2}{32M_G^6}\varphi^{10}, \tag{3}$$

where $m_\phi \sim$100 GeV - 1 TeV is the soft-breaking mass of ϕ and A is a parameter whose amplitude is also $|a| \simeq \mathcal{O}(100)$ GeV - 1 TeV. This is justified when SUSY-breaking is gravity-mediated. Here we minimized the potential along the angular direction of ϕ, and rewriting the flat direction as $\phi = \frac{1}{\sqrt{2}}\varphi e^{i\theta}$, with φ being a real quantity,

If A is fine-tuned as $A^2 = 20m_\phi^2(1+\alpha^2/4)$, with $\alpha^2 \ll 1$, $V(\varphi)$ has a inflection point φ_0 at

$$\varphi_0 = \left(\frac{2AM_G^3}{5\lambda}\right)^{1/4}(1+\mathcal{O}(\alpha^2)), \quad V''(\varphi_0) = 0, \tag{4}$$

where

$$V(\varphi_0) = \frac{4}{15}m_\phi^2\varphi_0^2(1+\mathcal{O}(\alpha^2)). \quad V'(\varphi_0) = m_\phi^2\varphi_0 \times \mathcal{O}(\alpha^2), \tag{5}$$

Therefore inflation can occur near the inflection point and its Hubble parameter is

$$H_{\rm MSSM} \simeq \frac{2m_\phi}{3\sqrt{5}M_G}\varphi_0 \simeq 10^{-16}\left(\frac{m_\phi}{10^3{\rm GeV}}\right)\varphi_0. \tag{6}$$

The potential for MSSM inflation $V(\varphi)$ can be expanded around the inflection point as

$$V(\varphi) = \frac{4}{15}m_\phi^2\varphi_0^2 + \frac{8}{3}\frac{m_\phi^2}{\varphi_0}(\varphi-\varphi_0)^3. \tag{7}$$

The slow-roll parameters, $\epsilon \equiv (M_G^2/2)(V'/V)^2$ and $\eta \equiv M_G^2(V''/V)$, can be calculated from Eq. (7). We find that the required degree of fine-tuning is

$$|\alpha| \ll 10^{-9} \tag{8}$$

in order to generate the correct primordial perturbations [1].

Moreover we find that slow roll region is very narrow, $|\varphi - \varphi_0|/\varphi_0 \ll 10^{-10}$ even if the above fine-tuning is accomplished by some unknown mechanism. Therefore the inflaton must reach the slow-roll region with an extremely small velocity, $H^{-1}d(\log\varphi)/dt|_{\varphi=\varphi_{c+}} < \varphi_0^2/20M_G^2$. Otherwise the inflaton passes through the slow-roll region within a time interval of H^{-1} because the time interval of passing is shorter than the time scale of deceleration of the inflaton. If the above condition is accomplished, the Universe goes through the self-reproducing regime when exactly $\varphi = \varphi_0$ and we have enough number of e-folds and correct primordial perturbations.

3 Dissipative effect

Next, we consider the dissipative effects. So far, we have not considered the interactions between the inflaton and other fields, which might cause dissipative effects and modify the dynamics of the inflaton. As a consequence, the narrowness of the slow-roll region could be relaxed if dissipative effects turned out to be strong enough. Here, we see whether it can relax the problems of the MSSM inflation or not.

3.1 Modification to inflation

When an inflaton ϕ has interaction with other fields, especially when they are in thermal bath, a dissipative phenomenon takes place and the inflaton feels a damping force. Although often a thermal correction to the potential also causes and might affect the slow-roll conditions, in the context of the MSSM inflation, it is difficult to consider such a case. For the fields that couple to the inflaton acquire large mass from the vacuum expectation value of the inflaton. Such fields are hard to be in thermal bath. Moreover, if they are in thermal bath, their energy density will overwhelm that of the inflaton. Consequently, inflaton does not occur. Instead, we consider the case where the inflaton couples to the radiation catalyzed by heavy fields [5]. In such a case, we do not have to worry about the thermal correction to the potential.

According to the discussion above, we take into account dissipative effects and write down the equation of motion for the inflaton ϕ approximately as

$$\ddot{\phi} + (3H + F_r)\dot{\phi} + \frac{\partial V}{\partial \phi} = 0. \tag{9}$$

Here F_r is the dissipative coefficient representing the dissipative effect, which may enhance inflation [6]and depends on ϕ and the temperature of radiation T. We estimate the value of F_r in the next subsection. The relative strength of the dissipative effect compared to the friction term from the expansion can be described by a parameter r,

$$r \equiv \frac{F_r}{H}. \tag{10}$$

If r is much larger than unity, the dynamics of inflaton would be modified. In order to estimate it quantitatively, we evaluate the dynamics of the inflaton and other components of the Universe further.

Other equations that governs the universe are,

$$3M_G^2 H^2 = V(\phi) + \rho_\gamma, \tag{11}$$

$$\dot{\rho}_\gamma + 4H\rho_\gamma = F_r \dot{\phi}^2, \tag{12}$$

where ρ_γ is the energy density of the radiation, that is, $\rho_\gamma = (\pi^2/30) g_* T^4$. Here g_* is the effective number of relativistic degree of freedom. (11) is the Friedman equation and (12) is the Boltzmann equation for the radiation.

The slow-roll conditions in this case are, then, the conditions that the time variations of H, $\dot{\phi}$ and T is negligible in comparison to the time scale of the cosmic expansion. Using equations (9), (11) and (12), if $r \gg 1$, the slow-roll condition changes as [7]

$$\epsilon \ll r, \quad \eta \ll r, \quad \beta \ll r. \tag{13}$$

Here β is the new slow-roll parameter introduced in order to take into account the time variation of F_r,

$$\beta \equiv M_G^2 \left(\frac{V_{,\phi} F_{r,\phi}}{V F_r} \right). \tag{14}$$

We can see from (13) that slow-roll condition is relaxed by the factor of r. [1]

If we apply this effect to the MSSM inflation, the slow-roll region is determined by the condition $\eta \ll r$. As a consequence, the slow-roll region will be enhanced,

$$\frac{|\varphi - \varphi_0|}{\varphi_0} \ll 10^{-10} r. \tag{15}$$

3.2 Dissipative coefficients for the MSSM inflaton

Then, we estimate the dissipative coefficients of the MSSM inflaton. The relevant part of the potential is, then,

$$\begin{aligned} V_{\rm int} =& h_1^2(|\phi|^2|\chi_1|^2 + |\phi|^2|\chi_2|^2 + |\chi_1|^2|\chi_2|^2) + h_1 h_2(\phi\chi_2 y_1^* y_2^* + \text{H.c.}) \\ &+ h_2(|\chi_1|^2|y_1|^2 + |\chi_1|^2|y_2|^2 + |y_1|^2|y_2|^2) \\ &+ h_1\left(\phi\bar{\psi}_\chi P_L \psi_\chi + \phi^* \bar{\psi}_\chi P_R \psi_\chi\right) + h_2\left(\chi_1 \bar{\psi}_y P_L \psi_y + \chi_1^* \bar{\psi}_y P_R \psi_y\right), \end{aligned} \tag{16}$$

[1] Strictly speaking, the degree of relaxation is dependent on the form of the potential V and the dissipative coefficient F_r.

Here ϕ, χ and y are the scalar fields. $P_L(P_R) = 1 - \gamma_5(1+\gamma_5)$ is left-(right-)handed projection operators. ψ_χ and ψ_y are the four-component Dirac spinor . As derived in [2], the dissipative coefficients are calculated using the in-in or the closed time-path formalism,

$$F_r = h_1^4 \varphi_c(t)^2 \int_{-\infty}^{t} dt'(t'-t) \int \frac{d^3q}{(2\pi)^3} \mathrm{Im}\left[G_{\chi_i}^{++}(\boldsymbol{q}, t-t') G_{\chi_i}^{++}(\boldsymbol{q}, t'-t)\right], \tag{17}$$

where $G_\chi^{++}(\boldsymbol{k}, t)$ is the Fourier transform of the Feynman propagator of χ [2]

In order to include the dissipative process, we dress the χ propagator with ψ loop. The imaginary part of its self energy generate non-zero value of F_r. At zero temperature, although there exists an imaginary part of its self energy, its contribution cancels out. At non-zero temperature non-zero value of the dissipative coefficient appears[5],

$$F_r = C g_* \frac{T^3}{\varphi^2}. \tag{18}$$

Here C is a numerical constant of $\mathcal{O}(10)$ and $g_* \simeq \mathcal{O}(10^2)$ is the relativistic degree of freedom. However, the radiation must not overwhelm the vacuum energy, $(\pi^2/30) g_* T^4 < (4/15) m_\phi^2 \varphi_0^2$. As a result, the dissipative coefficient F_r is smaller than $3H$, the friction coefficient from cosmic expansion unless g_* is extremely large. [2] Therefore, dissipative effects cannot relax the fine-tuning problem or the problem of narrowness of the slow-roll region.

4 Conclusion

We have studied the effect of the interaction of the MSSM inflaton. The interaction between the MSSM inflaton and the other fields can cause the dissipative effect. However, this effect is very weak and cannot change the dynamics of the MSSM inflaton. It also does not change the primordial perturbations that the MSSM inflaton generates. Therefore, there still remains the problem of the narrowness of the slow-roll region of the MSSM inflaton. The fine-tuning problem of the MSSM inflation must be solved by other mechanisms.

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