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Searching for Planck scale effects from the free expansion of a Bose–Einstein Condensate

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Abstract. The free expansion of a Bose–Einstein condensate is analyzed, assuming that the single particle energy spectrum is modified due to some quantum structure of space time. The condensate’s free velocity expansion leads in a natural way to a deformed uncertainty principle. We show that large expansion times are required in order to amplify some possible traces of the Planck scale regime. The analysis presented in this work opens up the opportunity to use Bose–Einstein condensates as tools in *quantum gravity phenomenology*, with a solid experimental basis.

1. Introduction

It is generally accepted that the single particle energy spectrum could be modified due to the possible quantum structure of space time. In ordinary units the non-relativistic form of the aforementioned modified dispersion relation can be expressed as follows [1, 2]

$$\epsilon \simeq mc^2 + \frac{p^2}{2m} + \frac{1}{2M_p} \left(\xi_1 mcp + \xi_2 p^2 + \xi_3 \frac{p^3}{mc} \right). \quad (1)$$

The parameters ξ_1 , ξ_2 , and ξ_3 , are model dependent [1, 2, 3], and should take positive or negative values close to 1 (see Ref. [4] for more details). In fact, the form of the energy dispersion relation (1), was recently constrained by using high precision atom–recoil frequency measurements [1, 2]. In this scenario, bounds for the deformation parameters of order $\xi_1 \sim -1.8 \pm 2.1$ and $-3.8 \times 10^9 < \xi_2 < 1.5 \times 10^9$ were obtained.

It is important to note here that the use of N-body systems as test tools in the search for possible manifestations of Planck scale physics has become a very interesting line of research [4, 5, 6, 7, 8, 9, 10, 11].

These works suggest that some relevant properties associated with Bose–Einstein condensates could be used to obtain representative bounds on the deformation parameters associated with quantum gravity models [9, 12] or for certain areas in the parameter space it is possible to explore the sensitivity of these systems to Planck scale effects [4, 5, 10, 11, 13, 14].

With this in mind, it is well known that when the trapping potential that confines the condensate is turned off, the free velocity expansion of the cloud corresponds to the velocity predicted by Heisenberg’s uncertainty principle [15, 16, 17]. This fact is one of several reasons



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why Bose–Einstein condensates are relevant systems in the analysis and estimation of possible Planck scale effects, since quantum gravity models suggest modifications to this principle [18, 19, 20].

In Refs. [4, 5] we have analyzed the free velocity expansion of a Bose–Einstein condensate, assuming that the energy per particle is modified due to some possible quantum structure of space–time. As a consequence, we were able to deduce a modified uncertainty principle, of the form:

$$\Delta x \Delta p \geq \frac{\hbar}{2}(1 + \beta^*) - \alpha^* x + \dots, \quad (2)$$

where $\alpha^* = 2\xi_1 m^2 c / 3\sqrt{\pi} M_p$, M_p is the Planck mass, c is the speed of light, and m is the mass of the particle and $\beta^* \equiv \xi_2 m / M_p$. We must mention that, as far as we know, this linear modification had not previously been reported in the literature [18, 19, 20].

With this at hand, we are able to extend the analysis of the free expansion to more general and realistic scenarios, in which the interaction between particles are present in the system. Moreover, numerical analyses at different time scales in the free expansion were performed, allowing us to determine the time scales in which possible effects related to the quantum structure of space time could be amplified. The analysis of the free expansion of the condensate at different time scales shows that the more promising scenario corresponds to large expansion times, of order of a few seconds, that can be in principle achieved in free fall experiments [17].

2. Modified uncertainty principle and free expansion

Let us calculate the *modified* energy associated with the system. The total energy of the system with contributions of the deformation parameters α and β is given by

$$\begin{aligned} E(\psi) = & \int d\mathbf{r} \left[\frac{\hbar^2}{2m} |\nabla \psi(\mathbf{r})|^2 + V(\mathbf{r}) |\psi(\mathbf{r})|^2 + \frac{U_0}{2} |\psi(\mathbf{r})|^4 \right. \\ & \left. + \hbar \alpha |\psi(\mathbf{r})| \sqrt{|\nabla|^2} |\psi(\mathbf{r})| + \beta \hbar^2 |\nabla \psi(\mathbf{r})|^2 \right], \end{aligned} \quad (3)$$

where ψ is the wave function of the condensate or the so-called order parameter, $V(r) = m\omega_0^2 r^2 / 2$ is the external potential. The term $U_0 = 4\pi\hbar^2 a / m$ models the interatomic potential, a being the s-wave scattering length, i.e. only two–body interactions are taken into account. Additionally, we have assumed that $\xi_3 = 0$.

In order to calculate the energy of the system at any time let us employ, as usual, the following *ansatz* [15]

$$\psi(\mathbf{r}) = \frac{N^{1/2}}{\pi^{3/4} R^{3/2}} \exp(-r^2/2R^2) \exp[i\phi(r)], \quad (4)$$

where N is the corresponding number of particles and $\phi(r)$ is a phase related to particle flows in the system. If the external potential $V(r)$ is turned off at $t = 0$, there is a force that changes R and produces an expansion of the cloud [15]. It is straightforward to obtain the kinetic energy E_F by using the *ansatz* Eq. (4), with the result $E_F = 3R^2 N m / 4$. Moreover, assuming that the energy is conserved at any time, we obtain the following energy conservation condition associated with our system

$$\frac{3m\dot{R}^2}{4} + \frac{3\hbar^2}{4mR^2} + \frac{U_0 N}{2(2\pi)^{3/2} R^3} - \alpha \frac{2\hbar}{\sqrt{\pi} R} + \beta \frac{3\hbar^2}{2R^2} = \frac{3\hbar^2}{4mR_0^2} + \frac{U_0 N}{2(2\pi)^{3/2} R_0^3} - \alpha \frac{2\hbar}{\sqrt{\pi} R_0} + \beta \frac{3\hbar^2}{2R_0^2}, \quad (5)$$

where the dot stands for derivative with respect to time and R_0 is the radius of the condensate at time $t = 0$, which is approximately equal to the oscillator length $a_{ho} = (\hbar/m\omega_0)^{1/2}$ in the

non-interacting case. Otherwise, when interactions are present, we will assume that the initial radius corresponds to the result for an isotropic trap [15]

$$R_0 = \left(\frac{2}{\pi}\right)^{1/10} \left(\frac{Na}{a_{ho}}\right)^{1/5} a_{ho}. \quad (6)$$

We remark that R is function of time, i.e., R corresponds to the radius at time t , see (5). If we set $\alpha = \beta = 0$ then we recover the usual solution in the non interacting case [15] which is given by

$$R^2(t) = R_0^2 + \left(\frac{\hbar}{mR_0}\right)^2 t^2. \quad (7)$$

Notice that in the usual case, $\alpha = \beta = 0$, $v_0 = \hbar/mR_0$ is defined as the velocity expansion of the condensate, corresponding to the velocity predicted by Heisenberg's uncertainty principle for a particle confined within a distance R_0 [15]. Thus, in the usual case $\alpha = \beta = 0$, the width of the cloud at time t can be written in its usual form $R^2(t) = R_0^2 + (v_0 t)^2$.

It is interesting to note that when interactions are neglected we are able to obtain an analytical solution for Eq. (5) when $R \gg R_0$ together with $\alpha \ll 1$ and $\beta \ll 1$. In such a scenario we obtain

$$R_{\alpha,\beta}^2(t) = R_0^2 + \left[\frac{\hbar^2}{m^2 R_0^2} (1 + 2m\beta)^2 - \alpha \frac{8}{3\sqrt{\pi}} \frac{\hbar}{mR_0} \right] t^2. \quad (8)$$

If we set $\beta = 0$, the result obtained in Ref. [4] is recovered. Thus, we may recognize the free velocity expansion as a function of the deformation parameters α and β , which is given by

$$(v_0^{\alpha,\beta})^2 = \frac{\hbar^2}{m^2 R_0^2} (1 + 2m\beta)^2 - \alpha \frac{8}{3\sqrt{\pi}} \frac{\hbar}{mR_0}. \quad (9)$$

Since the corrections caused by α and β are small the following expansion is justified:

$$(v_0^{\alpha,\beta}) \approx \frac{\hbar}{mR_0} (1 + 2m\beta) - \alpha \frac{4}{3\sqrt{\pi}}. \quad (10)$$

Then, the velocity expansion corresponds to the following deformed Heisenberg uncertainty principle

$$\Delta x \Delta p \geq \frac{\hbar}{2} (1 + \beta^*) - \alpha^* x + \dots, \quad (11)$$

where we have defined $\beta^* \equiv \xi_2 m/M_p$ and $\alpha^* \equiv \xi_1 2m^2 c/3\sqrt{\pi} M_p$ together with $R_0 = x$.

Let us now analyze the velocity expansion of our condensate by solving Eq. (5) numerically at any time and taking into account the interactions among the particles within the system. In doing so, we will use some experimental values of interest related to the condensate, i.e., $N \sim 10^4 - 10^6$ particles, $\omega_0 \sim 10 - 10^6$ Hz, $a \sim 10^{-9}$ m, and $m \sim 10^{-26}$ kg [21]. Together with $M_p \simeq 2.18 \times 10^{-8}$ kg, $\hbar \sim 6.623 \times 10^{-34}$ Js, and $c \sim 3 \times 10^8$ m/s. Finally, we assume that the values for the deformation parameters are those reported in Refs. [1, 2].

There are three cases of interest: a) When α, β and U_0 are different from zero. b) For $\alpha = \beta = 0$ and $U_0 = 4\pi\hbar^2 a/m$ and c) when $\alpha, \beta, U_0 = 0$

In Figure 1 the numerical solutions for the modified velocity for the cases a) and b) are illustrated. The velocity in case c) is the usual result for an ideal gas and so we show only the cases a) and b).

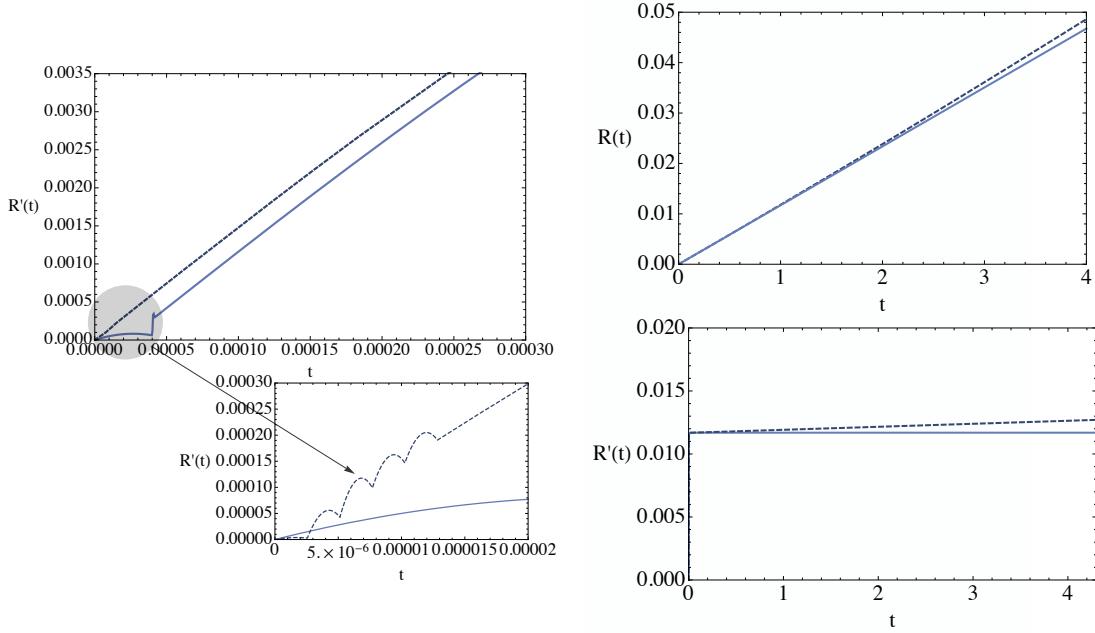


Figure 1. *Left:* Velocity expansion of the condensate at early times for the cases a) and b) are shown in the upper plot. Scenario a) is represented by the solid line and scenario b) is represented by the dashed line. The plot below represents the evolution at early times. *Right:* The velocity expansion at large time scales of order $t \sim 4$ sec is plotted above, the plot below represents the free expansion with the same time scale (Figures taken from Ref. [5]).

3. Discussion

Summarizing, we note that at very early expansion times, there is a period in which the velocity seems to be dominated by the deformation parameters. However, we estimate that this short period of expansion of order $t \sim 7.7 \times 10^{-6}$ sec may not be accessible from the experimental point of view, since the results of [16] offer accessibility at the order of milliseconds, i.e., three orders of magnitude bigger than the expansion time obtained here. Conversely, for large expansion times up to $t \sim 4$ sec, there is a region in which the presence of the deformation parameters modified the velocity expansion in a way that may be significant, even when interactions are present.

Concerning the experiment performed in [17], it was proven that for sufficiently large expansion times, the system operates deeply in the linear regime, i.e., almost as in the non-interacting case. In this experiment it was shown by measuring the free velocity expansion at large times that the evolution of the condensate can be independent of interactions during extended free fall experiments. Each of the above scenarios shows that the *modified* free velocity expansion leads to deformations of Heisenberg's uncertainty principle which are around two orders of magnitude smaller than the typical case [5]. This fact could be tested, in principle, in the laboratory, by searching for deviations in the free velocity expansion of the condensate. However, we remark that according to our results, large expansion times are required. This analysis opens a very important branch of research concerning the search for traces of quantum gravity in low energy earth based experiments.

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