

# Theoretical analysis of physical properties of a mirror for interferometric gravitational wave detection

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## Abstract.

We present a theoretical and computational study of a compression-bias thermally-actuated mirror (CBM) for candidate application in interferometric gravitational wave detectors. We investigate the mechanical properties of a fused silica mirror assembled within an aluminum outer ring inspired by the work of Cao et al. [1]. In our research, however, the assembly is studied under the influence of radial forces that would mimic the thermal compression studied in Cao et al. [1] Using CAD-based modelling (Autocad and SolidWorks), we analyzed the simulated deformation. The results indicate that strain is more evenly distributed at an axial offset of  $\delta z = 3.0\text{mm}$ , while displacement achieves a more homogeneous distribution at  $\delta z = 4.5\text{mm}$ , particularly reducing stress in the mirror. These results provide a first validation of this CAD-based approach to the assembly's behavior and establish a framework for further simulations using multiphysics software, like COMSOL and Elmer Fem.

## 1 Introduction

Gravitational waves (GW) were theoretically discovered as a part of Albert Einstein's work in the Theory of Relativity [2], and after a century since the original publication, it became possible to have interferometric detectors such as LIGO, KAGRA, and Virgo to detect GW directly. This type of detector relies on mirrors with extreme optical and mechanical stability to achieve the sensitivity required for astrophysical observations. The design of mirror suspensions and mounting techniques is crucial, as mechanical stress and deformation can introduce noise sources that limit detector performance.

In the work by Cao et al. [1] a compression-bias mirror (CBM) was studied for candidate application in interferometric GW detectors. In that work the CBM model was analyzed with a multiphysics software (COMSOL) using finite element analysis (FEA), in regard to a thermal, shrink-fitting capture process of the mirror. Motivated by this system and the involved questions, our idea takes a different approach. We analyzed the assembly's response to a compressive mechanical load that mimics the thermal process of the mirror. We used SOLIDWORKS and AutoCAD to investigate the compression pattern when applying radial forces on the assembly.



### 1.1 Simulation setup

The two elements in the assembly were: a disc (mirror) composed by solid, fused silica with 25.4 mm in radius and 6 mm thick; and an outer ring, solid, made of Al6061, with an outer radius of 55 mm and 6 mm thick. The elements are allowed to have an axial offset given by the parameter  $\delta z$ .

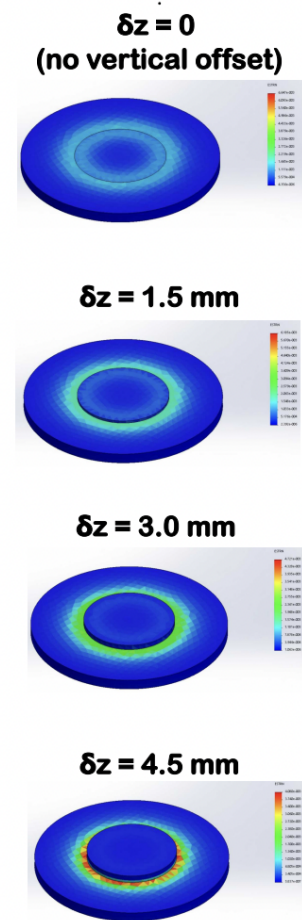
The mirror is captured by the outer ring through the application of a uniform, radial force along the outer radius of the ring. The shrink-fitting thermal effect investigated by Cao et al.[1] was mimicked by these compressive radial forces.

SolidWorks simulation was used to build the geometries and analyze the mechanical response of the components. A fine mesh was applied with approximately 50,000 elements to ensure convergence. We investigated both strain (deformation) and displacement fields under the applied loads, aiming to identify conditions that minimize stress concentration in the mirror while maintaining structural stability.

## 2 Results and discussion

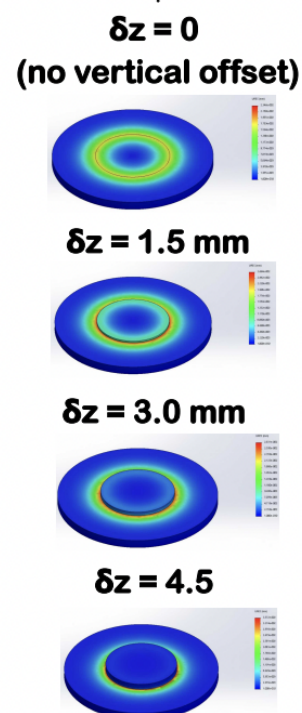
The displacements were evaluated at 4 values of the offset:  $\delta z = 0$  mm, 1.5 mm, 3.0 mm, and 4.5 mm. The simulated contraction forces generated both strain and displacement in the two components of the assembly. Figures 1a and 1b show the strain and displacement distributions for different axial offsets.

### Strain (deformation)



(a) Strain.

### Displacement (mm)



(b) Displacement.

Figure 1: Comparison among 4 different values of the axial offset for the two components of the assembly, the disc (mirror) and the outer ring (support). Two physical properties were targeted: (a) strain and (b) displacement distributions in the mirror-ring assembly under the different axial offsets  $\delta z$ .

The analysis of Figures 1a and 1b indicates that the displacements and strains induced by the radial forces are not uniform throughout the mirror-ring assembly. It is noted that regions near the interface between the two materials exhibit stress concentrations, suggesting a significant mechanical coupling between the mirror and the ring. Moreover, maximum displacements occur at locations different from maximum deformations, highlighting that the structural response of the assembly depends not only on the material properties, but also on geometry and axial misalignment  $\delta z$ . These results emphasize the importance of carefully considering the axial adjustment and the design of the support ring to minimize localized stresses and ensure the stability of the mirror under operational conditions.

### 3 Conclusion and outlook

We analyzed the assembly's response through CAD-based simulations to a compressive mechanical load that mimicked a thermal shrink-fitting process of the mirror. The simulated contraction forces produced both strain and displacements in the two components. Strain was most evenly distributed for an axial offset of  $\delta z = 3.0$  mm, while displacement was more uniformly distributed for  $\delta z = 4.5$  mm, although it remained particularly low in the mirror. The latter offset implies that the contact region between the two bodies would reduce to 1.5 mm. Since the mirror is expected to be positioned vertically, further studies should be pursued to analyze whether this contact area would be enough to ensure structural stability to the system.

Using a mechanical approach, this study provides preliminary validation of the compression-biased thermal approach proposed by Cao et al.[1] and opens new directions for more detailed modeling. Future work is been planned to include multiphysics analyses using the software Elmer FEM [3] and/or COMSOL.

### References

- [1] H. T. Cao, S. W. S. Ng, M. Noh, A. Brooks, F. Matichard, and P. J. Veitch. Enhancing the dynamic range of deformable mirrors with compression bias. *Optics Express*, 28:38112–38125, 2020.
- [2] Charles W. Misner, Kip S. Thorne, and John Archibald Wheeler. *Gravitation*. W. H. Freeman and Company, San Francisco, 1973.
- [3] Elmer fem software. <http://www.elmerfem.org>. Accessed: 2025.