

Systematics of mean resonance spacing and average radiative width from random forest regression

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Abstract. The interaction of a nucleus with photons plays a key role in understanding a competition in nuclear reactions. The photon strength function (PSF) gives the average response of a nucleus to an electromagnetic probe. In this study, an estimation is performed of the mean resonance spacing and average radiative width of 22 nuclei in the range $A = 46 - 90$. The calculations for this analysis are done in TALYS. In addition, a machine-learning driven approach for determining experimentally obtained model parameters from the neutron/proton and mass number is given. A reasonable agreement within the uncertainties is achieved between the model dependent calculations, using TALYS, and results from random forest regression.

1. Introduction

The mean resonance spacing (D_0) and average radiative width (Γ_γ) of 22 nuclei are estimated from the proton number, Z , and mass number, A , by random forest regression [1]. The motivation of this study is to develop a consistent framework of systematics for experimental nuclear physics. The outcomes of this study are mainly focused on applications in nuclear astrophysics, nuclear structure and data-driven approaches in nuclear physics.

2. Theory

The average s-wave or p-wave spacing is a quantity used in the nuclear structure to describe the mean spacing of nuclear levels in the quasi-continuum region. The $D_{l=0/1}$ is calculated from the nuclear level density (NLD). The Γ_γ is used in nuclear structure to describe the continuous transition probabilities of the nuclear states of an excited nucleus. The average radiative width is calculated, in part, from the photon strength function (PSF), which like the NLD is also given by phenomenological and microscopic models. An overview of the NLD models is given in the review papers by Heenens et al. [2] and Koning et al. [3]. A comprehensive summary of PSFs is given in the review paper by Goriely et al. [4]. The theoretical calculations of these quantities is done in TALYS [5]. The models used in TALYS to calculate the theoretical D_0 are the Fermi-gas model (CT+FGM) and the Hartree-Fock-Bogoliubov model with Gogny (HFB+Gogny) or Skyrme (HFB+Skyrme) force interaction [2]. The model used to calculate the theoretical Γ_γ is the Brink-Axel PSF [4].

Random forest regression (RFR) is a technique of supervised ensemble learning whereby a random partition of a data set is uniformly distributed onto a decision tree. By a sequence of binary decisions based on an input, a unit output is cast by the tree. For an ensemble of trees,



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the unit outputs are aggregated to determine a RFR estimation [1]. The RFR model used is a forest with 500 trees and a depth of 100, this was chosen based on a grid-search. The RFR model was not further tuned. RFR models are built on the RIPL-3 database that contains experimental D_0 and Γ_γ values from various experimental techniques [6]. An RFR model is built based on a random training partition of the RIPL-3 data set. The model's predictions are then validated on the test partition, kept separate from the training partition. The experimental errors of the RIPL-3 data was not used nor propagated in the training of the model; instead an estimation error based on the RFR procedure is used from the paper by Wager et al. [7]. The RFR model is then used to carry out predictions on studied nuclei for which the PSF has been measured following a (p, γ) reaction [4]. The RFR predictions are then compared to model calculations done in TALYS.

3. Results

The figures 1¹ and 2, show the ratio of the RFR estimations ($D_0^{\text{cal.}}$) to experimental values of the D_0 and Γ_γ , respectively. A table of the 22 nuclei can be found in the appendix of [8].

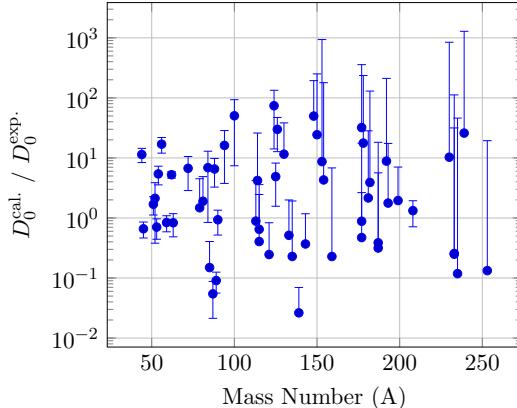


Figure 1. Ratio plot of the mean resonance spacing RFR estimation ($D_0^{\text{cal.}}$) to the experimental value ($D_0^{\text{exp.}}$).

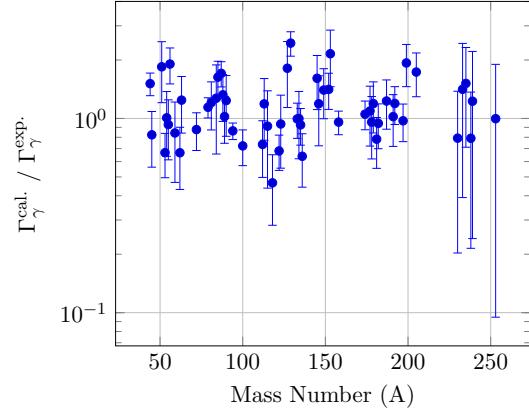


Figure 2. Ratio plot of the average radiative width RFR estimation ($\Gamma_\gamma^{\text{cal.}}$) to the experimental value ($\Gamma_\gamma^{\text{exp.}}$).

The results of Γ_γ show better agreement than D_0 . Overall, the RFR models are in strong agreement and can be used to make further predictions on the 22 nuclei. These predictions are carried out on the D_0 and Γ_γ . For all but one (^{51}V) of the 22 nuclei have no experimental values in RIPL-3, the results of the RFR estimation are thus compared to theoretical calculations from TALYS. The results are given in figures 3 and 4. The results for the 22 studied nuclei show fair agreement. The estimations of the mean resonance spacing are off by a factor of $\sim 10 - 15$ while the average radiative width is off by a factor of ~ 0.5 . The phenomenological models show better agreement with the RFR estimation over the microscopic models. The results show that systematics can be applied to determine resonance parameters for nuclei for which no experimental data exists.

¹ The data points without negative error bars have errors on the log-scale that extend beyond zero.

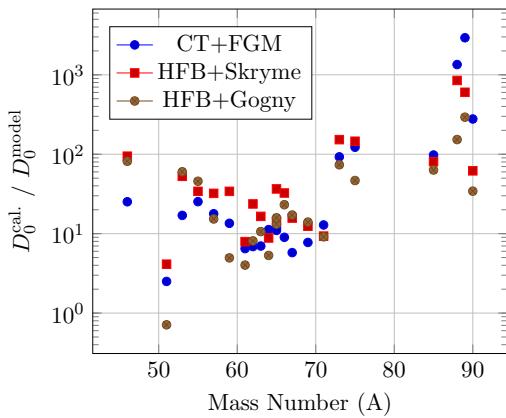


Figure 3. Ratio plot of the mean resonance spacing RFR estimation ($D_0^{\text{cal.}}$) to the model calculation (D_0^{model}).

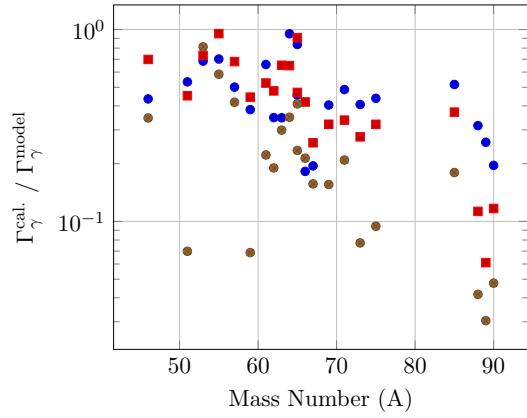


Figure 4. Ratio plot of the average radiative width RFR estimation ($\Gamma_\gamma^{\text{cal.}}$) to the model calculation ($\Gamma_\gamma^{\text{model}}$).

4. Conclusions

The results of the mean resonance spacing estimation to the experimental data show strong agreement while the estimation of the 22 nuclei is off by a factor of $\sim 10 - 15$. The results of the average radiative width estimation to the experimental data also show strong agreement with the estimation of the 22 nuclei off by a factor of ~ 0.5 and in fair agreement. The average radiative width estimations are more in agreement than the mean resonance spacing. This approach is feasible and with more robust machine-learning techniques the estimations will possibly improve and provide consistent, data-driven, systematics.

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