Non-locality in quadrupolar gravitational radiation

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ABSTRACT: General Relativity suffers for two main problems which have not yet been overcome: it predicts spacetime singularities and cannot be formulated as a perturbative renormalizable theory. In particular, many attempts have been made for avoiding singularities, such as considering higher order or infinite derivative theories. The price to pay in both cases is to give up locality and therefore they are known altogether as non-local theories of gravity. In this paper, we investigate how to recognize the presence of non-local effects by exploiting the power emitted by gravitational waves in a binary system in presence of non-local corrections as $R\Box^{-1}R$ to the Hilbert-Einstein action. After solving the field equations in terms of the source stress-energy tensor $T_{\mu\nu}$ and obtaining the gravitational wave stress-energy pseudo-tensor, $\tau_{\mu\nu}$, we find that the General Relativity quadrupole formula is modified in a non-trivial way, making it feasible to find a possible signature of non-locality. Our final results on the gravitational wave stress-energy pseudo-tensor could also be applied to several astrophysical scenarios involving energy or momentum loss, potentially providing multiple tests for non-local deviations from General Relativity. We finally discuss the detectability of the massless transverse scalar mode, discovering that, although this radiation is extremely weak, in a small range around the model divergence, its amplitude could fall within the low-frequency Einstein Telescope sensitivity.

KEYWORDS: Gravitational waves in GR and beyond: theory , gravitational waves / theory, gravity, modified gravity

Contents

1 Introduction

Modern physics is based on two main pillars, i.e. General Relativity and Quantum Field Theory. General Relativity (GR) describes the gravitating systems and non-inertial frames on large scales, while Quantum Field Theory (QFT) holds at high energy or, equivalently, small scale regimes where a classical description breaks down. However, QFT assumes that space-time is flat and even its extensions, such as QFT in curved space time, consider space-time as a classical background, never allowing a quantized version of the latter. GR, on the other hand, does not take into account the quantum nature of matter because the source of the gravitational field simply is the (classical) Stress-Energy tensor $T_{\mu\nu}$ and hence, in its formulation, gravity is a *local* interaction. Although a theory of Quantum Gravity remains unknown, it is legitimate to ask for what happens when a strong gravitational field is present at quantum scales. Because of its weakness compared to other interactions, the characteristic scale under which one would expect to experience full non-classical effects, relevant to gravity, the Planck scale is quite small, i.e. 10^{-33} cm, which is not accessible by any current experiment.

Furthermore, the Einstein Equivalence Principle is based on the assumption that an accelerated observer in Minkowski spacetime, at each event along its world line, is physically equivalent to a momentarily identical comoving inertial observer. Non-locality comes out taking also the past history of the accelerated observer into account. Such history-dependent theories have recently been developed with the purpose of see a quantum feature arising from GR, while addressing, at the same time, some of the most important issues of modern Cosmology, namely dark energy or dark matter. It is not a new fact that Quantum Mechanics shows non-local aspects: non-locality is a manifestation of entanglement and the latter has been repeatedly demonstrated in laboratory experiments; the Bell theorem, furthermore, demonstrated that locality is violated in some quantum systems. After that, non-locality has been investigated in QFT (see e.g. [1–7]), as well as in string theory [8–11], where it mainly

emerges as a 'side' effect of the introduction of quantum corrections to GR the purpose of which was to cure the singularity problem. Indeed, applications to Cosmology showed that non-local ghost-free higher-derivative modifications of the Einstein gravity, in the ultraviolet regime, can admit non-singular bouncing solutions for the Universe (in place of the Big Bang singular solution) and non-singular Schwarzschild metrics for black holes [12]. Another motivation for considering non-local gravity is the possibility of achieving renormalizability without the appearance of ghost modes [13]. A road towards Quantum Gravity, by considering non-local corrections to the Hilbert-Einstein action, has been drawn in [14–17]. The introduction of non-local terms have also been considered in alternative theories of gravity, such as teleparallel gravity [18] and revealed extremely useful in cosmology [19–21]. However, it must be said that the non-local theories of gravity themselves can be considered as Extended Theories of Gravity [22], meaning that when the non-local terms are cancelled out, the GR limit is recovered.

More precisely, non-local theories of gravity are described by Lagrangians composed by a finite sum of products between fields and their derivatives evaluated at different points x and y of the spacetime while metric $g_{\mu\nu}$ and/or other fields are described by integro-differential equations, implying that the value of the field at one point depends on its value at another point of the spacetime, weighted by a function called nucleus or kernel. There are essentially three different ways to implement non-locality from a mathematical point of view. The first approach (the most studied in the literature) is by means of a convergent series expansion with real coefficients c_n of an analytic non-polynomial function F of the d'Alembert operator $\Box$, known as Infinite Derivative Gravity (IDG) [3, 23–25]:

$$F(\Box) = \sum_{n=1}^{\infty} c_n \frac{\Box^n}{M^{2n}} \,,$$

where M is the mass scale associated to non-locality effects. Notice that, in this case, non-locality appears only at the level of interaction, and not at the level of a free field propagation without a source. Recently, it was shown that, in gravity theories containing such class of non-local terms, the linearized Ricci tensor $R_{\mu\nu}$ and Ricci scalar R are not vanishing in the region of non-locality, i.e. at short distance from a source, due to the smearing of the source induced by the presence of non-local gravitational interactions. It follows that, unlike in the Einstein gravity, the Riemann tensor is not traceless and it does not coincide with the Weyl tensor, which, however, vanishes at short distances, implying that the (static) metric is conformally flat in that region [23], implying a possible deviation from $1/r$ potential drop at very short distance.[1]

In the second way, that we shall follow here, the non-locality manifests itself in non-analytic operators such as [28, 29]

$$\Box^{-n} R(x) = \phi_0(x) + \int_{\Omega} d^4 y \sqrt{-g(y)} G(x,y) R(y) \,, \tag{1.1}$$

where $G(x,y)$ is the retarded Green function of the operator $\Box^n$, $\Omega \subseteq \mathbb{R}^4$ and ϕ_0 is the solution of $\Box^{-n}\phi_0(x) = 0$. It was shown that the application of the non-local operator

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[1]A decay as $1/r$ of the gravitational potential has only been verified up to $\sim 10^{-5}$ m, which is thirty orders of magnitude away from the Planck length [26, 27].

$\square^{-1}$ to the scalar curvature R gives rise to the late-time cosmic expansion of the Universe without invoking any Dark Energy contribution. For an overview on non-local cosmology see also [19, 30, 31]. Finally, non-locality can enter through a constitutive relation on the (linearized) gravitational field involving a causal kernel determined via observational data, in the spirit of non-local electrodynamics of media [32, 33].

In all these approaches, it is important to study the linearized versions of the theories and to derive gravitational waves (GWs). For IDG, they have been studied in [34] and [35], while, for higher order theories with Lagrangian $\mathcal{L} = R + \sum_{h=1}^{n} a_h R\square^{-h}R$, with n fixed, they are discussed in [28]. Indeed, gravitational radiation allows to detect possible effects of non-local gravity [36] as well as to classify the degrees of freedom of a given theory.

In this paper, we compute the gravitational wave stress-energy tensor (GW-SET) for the non-local gravity theory $\mathcal{L} = R + aR\square^{-1}R$ with a an adimensional constant and derive, from it, the power emitted by a gravitational system in terms of quadrupole momenta, in view of possible astrophysical applications. This gravitational action is a peculiar case of models considered in [37], whose cosmological implications have been studied in [38]. A modified quadrupole formula with non-local terms has been studied for binary pulsars, see [39]. A derivation of gravitational pseudo-tensor in higher order curvature-based and torsion-based gravity has been taken into account in [40, 41]. In particular, in section 2 we obtain the field equations and the Noether current using a localized version of the Lagrangian, in order to avoid integro-differential equations. Then, in section 3, we solve the non-homogeneous system of equations in terms of the (source) stress-energy tensor $T_{\mu\nu}$, whose solutions in the far region condition only depends on the quadrupole momenta. Finally, we close the manuscript with a summary and an outlook in section 4. More detailed accounts of some computations are given in the appendices.

In this work, we adopt natural units $\hbar = c = 1$, and we define the Planck mass as $M_{pl}^2 = 8\pi G$. The negative metric signature $(+,-,-,-)$ is also adopted. The Greek indices are the coordinate ones contracted with the metric tensor $g_{\mu\nu}$, while the Latin indices (spatial coordinates) are contracted with the Kronecker delta. Finally, space vectors are indicated in **bold**.

2 Non-local gravity as an extension of General Relativity

Let us derive the field equations for an extended theory of gravity given by the action

$$S[g] = \frac{1}{2\chi} \int d^4x \sqrt{-g} R\Big(1 + f(\square^{-1}R)\Big) + \int d^4x \sqrt{-g} \mathcal{L}_m[g] \,, \qquad (2.1)$$

where $\chi = 8\pi G$ is the gravitational coupling, $\mathcal{L}_m$ is the matter Lagrangian, and $f(\square^{-1})$ is an analytic function of $\square^{-1}$ which is the inverse of d'Alembert operator $\square = \nabla^\mu\nabla_\mu$, being $\square^{-1}\square = \mathbb{1}$. A similar action has been considered in [37]. According to the form of $f(\square^{-1})$, it shows Noether symmetries [42]. An interesting case presents a non-local correction like $\sim R\exp\{a\square^{-1}R\}$ where a is an adimensional constant. However, we can simplify the previous action by expanding f in terms of $\square^{-1}R$ and considering only the first term, which, in the quadrupole approximation we are going to discuss, is the dominant one. It is

$$f(\square^{-1}) \approx a\square^{-1}R \,. \qquad (2.2)$$

Therefore the action (2.1) becomes [43]

$$S[g] = \frac{1}{2\chi} \int d^4x \sqrt{-g} \left(R + aR\Box^{-1}R \right) + \int d^4x \sqrt{-g} \mathcal{L}_m[g] \,, \tag{2.3}$$

In view to derive the field equations, we can recast action (2.1) in a *localized* version. It is

$$S_g[g_{\mu\nu}, \phi, \lambda] = \frac{1}{2\chi} \int d^4x \sqrt{-g} \left[R(1 + a\phi) + \lambda(\Box\phi - R) \right] \,, \tag{2.4}$$

where the auxiliary field $\phi = \Box^{-1}R$ has been introduced. Here λ is a Lagrange multiplier. See ref. [30] and references therein for details. After integration by parts, eq. (2.4) becomes

$$S[g_{\mu\nu}, \phi, \lambda] = \frac{1}{2\chi} \int d^4x \sqrt{-g} \left[R(1 + a\phi - \lambda) - \nabla^\alpha \lambda \nabla_\alpha \phi \right]. \tag{2.5}$$

The variation with respect to the metric $g^{\mu\nu}$ gives the equations of motion, i.e.

$$G_{\mu\nu} + (G_{\mu\nu} + g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)(a\phi - \lambda) - \nabla_{(\mu}\phi\nabla_{\nu)}\lambda + \frac{1}{2}g_{\mu\nu}\nabla^\sigma\phi\nabla_\sigma\lambda = \chi T_{\mu\nu} \,, \tag{2.6}$$

while variations with respect to both the scalar fields give the two constraints

$$\Box\lambda = -aR \,, \quad \Box\phi = R \,. \tag{2.7}$$

The stress-energy tensor is $T_{\mu\nu} = -\dfrac{2}{\sqrt{-g}} \dfrac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}}$, and the Einstein tensor is $G_{\mu\nu} = R_{\mu\nu} - \dfrac{1}{2}g_{\mu\nu}R$. Tracing eq. (2.6), one obtains the scalar equation

$$(1 + a\phi - 6a - \lambda)R - \nabla^\sigma\phi\nabla_\sigma\lambda = -\chi T \,, \tag{2.8}$$

where $T = g^{\mu\nu}T_{\mu\nu}$ and we used the relation

$$\Box(a\phi - \lambda) = 2aR \,. \tag{2.9}$$

An important issue is whether it is possible to eliminate or not the non-local terms from the Lagrangian (2.3) by redefining the metric tensor as done, for example, in ref. [44]. This is not possible for the following mathematical and physical reasons. In fact, from a mathematical point of view, non-locality is linked to the integro-differential character of the field equations and therefore, from eq. (1.1), the non-local operators are integral operators where the value of a physical quantity at a point depends on its value assumed at other points modulated by the kernel. Thus it is not possible to eliminate the integral character of physical quantities through a redefinition of the metric which is a local tensor object. On the other hand, from a physical point of view, non-locality is in this theory an intrinsic property of gravity related to finite characteristic lengths and masses. Also if a "localization" process is performed (see e.g. [17, 30]) auxiliary scalar fields emerge pointing out that non-locality introduces further degrees of freedom.

In order to analyze the gravitational radiation, we consider the first-order perturbations $h_{\mu\nu}$, φ and w. The metric perturbation is around the flat metric $\eta_{\mu\nu}$ and the perturbations

of the two scalar fields ϕ and λ are around their (constant) Minkowskian values ϕ_0 and λ_0, respectively, i.e.

$$
\begin{aligned}
g_{\mu\nu} &= \eta_{\mu\nu} + h_{\mu\nu} \,, \\
\phi &= \phi_0 + \varphi \,, \\
\lambda &= \lambda_0 + w \,.
\end{aligned}
\tag{2.10}
$$

At first order in $h_{\mu\nu}$ and without imposing any gauge, the Ricci tensor $R_{\mu\nu}$ and Ricci scalar R are [45]

$$
R^{(1)}_{\mu\nu} = \frac{1}{2}\left(\partial_\rho\partial_\mu h^\rho_\nu + \partial_\rho\partial_\nu h^\rho_\mu - \partial_\mu\partial_\nu h - \Box h_{\mu\nu}\right) \,,
\tag{2.11}
$$

$$
R^{(1)} = \partial_\mu\partial_\nu h^{\mu\nu} - \Box h \,.
\tag{2.12}
$$

In GR, the gravitational pseudo-tensor can be obtained in different ways, however, the related physical information has to be the same. This statement holds also in modified gravity as showed in [46]. Here, we follow the Noether current method, based on the symmetries of the action, which lead to conserved currents. The gravitational stress-energy tensor comes out by varying, on shell, the second-order Lagrangian density $\mathcal{L}^{(2)}$ with respect to the coordinates. In our case, it is

$$
\mathcal{L}^{(2)} = \frac{1}{2\chi}\left[(\sqrt{-g}R)^{(2)}\left(1 + a\phi_0 - \lambda_0\right) + R^{(1)}\left(a\varphi - w\right) - \partial^\mu\varphi\partial_\mu w\right] \,.
\tag{2.13}
$$

Defining $\psi_0 \doteq 1 + a\phi_0 - \lambda_0 \neq 0$ and $\psi^{(1)} \doteq a\varphi - w$ its perturbation, and introducing the new field

$$
\theta_{\mu\nu} \doteq h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h - \frac{\eta_{\mu\nu}}{\psi_0}\psi^{(1)},
\tag{2.14}
$$

such that, in our reference frame, it is $\partial_\mu\theta^{\mu\nu} = 0$ (the Lorenz gauge), eq. (2.13) becomes

$$
\begin{aligned}
\mathcal{L}^{(2)} = \frac{\psi_0}{32\pi G}\Bigg[&\frac{1}{4}\partial_\alpha\theta\partial^\alpha\theta + \partial_\alpha\theta_{\beta\gamma}\partial^\beta\theta^{\alpha\gamma} - \frac{1}{2}\partial_\gamma\theta_{\alpha\beta}\partial^\gamma\theta^{\alpha\beta} + \frac{(6a - 2\psi_0)}{\psi_0^2}\partial^\alpha\varphi\partial_\alpha w \\
&- \frac{3}{\psi_0^2}\left(a^2\partial_\alpha\varphi\partial^\alpha\varphi + \partial_\alpha w\partial^\alpha w\right)\Bigg] \,.
\end{aligned}
\tag{2.15}
$$

See appendix A for details. From this Lagrangian, the Euler-Lagrange equations of motion, in the Lorenz gauge and in vacuum, are

$$
\Box\theta_{\mu\nu} = 0 \,,
\tag{2.16}
$$

$$
K\Box w - \frac{6a}{\psi_0^2}\Box\varphi = 0 \,,
\tag{2.17}
$$

$$
K\Box\varphi - \frac{6a}{\psi_0^2}\Box w = 0 \,,
\tag{2.18}
$$

as already found in [43]. In eqs. (2.17) and (2.18), we have defined the constant $K \doteq \dfrac{(6a - 2\psi_0)}{\psi_0^2}$. Notice that the last two are coupled differential equations for $\Box\varphi$ and $\Box w$. When $a = 0$ they implies $\Box\varphi = 0 = \Box w$ and therefore $\varphi = 0 = w$, that is GR is recovered.

The trivial solution of the system formed by the equations (2.17) and (2.18) is obtained by imposing that the determinant is different from zero, that is

$$6a \neq \psi_0, \tag{2.19}$$

which implies

$$\Box\varphi = \Box w = 0, \tag{2.20}$$

or

$$\Box\psi^{(1)} = 0. \tag{2.21}$$

This means that $\phi^{(1)}$ is a massless scalar field added to the two tensor fields of the field eq. (2.16). In other words, gravity linearly perturbed presents three degrees of freedom.

The Noether current for our non-local gravity model is defined as

$$j_\beta^\alpha \doteq -\frac{\partial \mathcal{L}^{(2)}}{\partial(\partial_\alpha \theta_{\mu\nu})}\partial_\beta \theta_{\mu\nu} - \frac{\partial \mathcal{L}^{(2)}}{\partial(\partial_\alpha \varphi)}\partial_\beta \varphi - \frac{\partial \mathcal{L}^{(2)}}{\partial(\partial_\alpha w)}\partial_\beta w + \delta_\beta^\alpha \mathcal{L}^{(2)}, \tag{2.22}$$

which, after some computations (see appendix A), becomes

$$
\begin{aligned}
j_\beta^\alpha = \frac{\psi_0}{32\pi G}\Big[&-2\partial^\nu \theta^{\mu\alpha}\partial_\beta \theta_{\mu\nu} + \partial^\alpha \theta^{\mu\nu}\partial_\beta \theta_{\mu\nu} - K\partial^\alpha w \partial_\beta \varphi \\
&+ \frac{6}{\psi_0^2}a^2 \partial^\alpha \varphi \partial_\beta \varphi - K\partial^\alpha \varphi \partial_\beta w + \frac{6}{\psi_0^2}\partial^\alpha w \partial_\beta w \\
&+ \delta_\beta^\alpha \partial_\nu \theta_{\mu\gamma}\partial^\gamma \theta^{\mu\nu} - \frac{1}{2}\delta_\beta^\alpha \partial_\gamma \theta_{\mu\nu}\partial^\gamma \theta^{\mu\nu} + \delta_\beta^\alpha K\partial^\gamma \varphi \partial_\gamma w - \delta_\beta^\alpha \frac{3a^2}{\psi_0^2}\partial^\gamma \varphi \partial_\gamma \varphi - \delta_\beta^\alpha \frac{3}{\psi_0^2}\partial^\gamma w \partial_\gamma w \Big],
\end{aligned}
$$

where we have already dropped terms with θ since they would be zero because of the TT gauge. For similar reasons, the first and the fifth-from-last terms are zero,[2] while the fourth is zero thanks to the equation of motion (2.16). Furthermore, the last three terms in the above equation are also zero. Integrating by parts, they can be recast as

$$
\begin{aligned}
\delta_\beta^\alpha\Big[&K\partial^\gamma \varphi \partial_\gamma w - \frac{3a^2}{\psi_0^2}\partial^\gamma \varphi \partial_\gamma \varphi - \frac{3}{\psi_0^2}\partial^\gamma w \partial_\gamma w \Big] \\
&= \delta_\beta^\alpha\Big[-K\varphi\Box w + \frac{3a^2}{\psi_0^2}\varphi\Box\varphi + \frac{3}{\psi_0^2}w\Box w \Big] \\
&= \delta_\beta^\alpha\Big[-\frac{1}{2}K\varphi\Box w - \frac{1}{2}Kw\Box\varphi + \frac{3a^2}{\psi_0^2}\varphi\Box\varphi + \frac{3}{\psi_0^2}w\Box w \Big],
\end{aligned}
\tag{2.23}
$$

where, in the last line, we have simply written $K\varphi\Box w$ as the sum of two equal terms and integrated the second one. Then, the final result in eq. (2.23) can be seen as a combination[3]

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[2] For our purpose, the Noether current should be considered under average over several wavelenghts, $\langle j_\beta^\alpha \rangle$, in order to obtain a gauge-invariant measure of physical quantities which we are interested in (such as energy and momentum). This imply that single derivatives are zero, $\langle \partial_\mu f \rangle = 0$, and hence we are empowered to integrate by parts under the averaging brackets.

[3] Multiplying eq. (2.17) by a factor $-\varphi/2$ and eq. (2.18) by a factor $-w/2$, the sum is equal to eq. (2.23).

of eqs. (2.17) and (2.18) and therefore it is always zero. By deleting all the null terms, what remains is

$$j_\beta^\alpha = \frac{\psi_0}{32\pi G}\left[\partial^\alpha\theta^{\mu\nu}\partial_\beta\theta_{\mu\nu} + \frac{6}{\psi_0^2}\partial^\alpha\psi^{(1)}\partial_\beta\psi^{(1)} + \frac{4}{\psi_0}\partial^\alpha\varphi\partial_\beta w\right], \tag{2.24}$$

which holds only in the TT gauge and under averaging brackets.

The gravitational wave stress-energy tensor (GW-SET) is then defined as $\tau_\beta^\alpha \doteq \langle j_\beta^\alpha\rangle$, therefore

$$\tau_\beta^\alpha = \frac{\psi_0}{32\pi G}\left\langle \partial^\alpha\theta_{(TT)}^{\mu\nu}\partial_\beta\theta_{\mu\nu}^{(TT)} + \frac{6}{\psi_0^2}\partial^\alpha\psi^{(1)}\partial_\beta\psi^{(1)} + \frac{4}{\psi_0}\partial^\alpha\varphi\partial_\beta w\right\rangle. \tag{2.25}$$

where we have stressed the gauge condition on $\theta_{\mu\nu}$ and brackets means average over all wavelengths. Notice that the above expression is symmetric in the indices α, β and it is not invariant under diffeomorphisms, but only under affine transformations. In this sense, it is a pseudo-tensor. Therefore, to make it a real tensor, the average procedure is essential. Furthermore, as we can see, in addition to GR term (the first) and a scalar field contribution from $\psi^{(1)} = a\varphi - w$, a mixed term also appears. It is not possible to recast it as a term involving only ψ. We would expect a pseudo-tensor including only terms in $\theta_{\mu\nu}$ and $\psi^{(1)}$ as in the case of scalar-tensor theories [46] or $f(R)$ gravity [47]. The non-zero mixed term is a peculiarity of non-local gravity which could constitute a signature for such theories.

As a final remark, we point out that combining relations in eqs. (2.7), it turns out that

$$\Box(\lambda + a\psi) = 0, \tag{2.26}$$

and then the first-order perturbation gives

$$\Box(w + a\psi) = 0, \tag{2.27}$$

whose trivial solution is $w = -a\psi$. This means that the two fields w and ψ can be chosen as redefinitions of each other and that, therefore, it is necessary to add only one additional degree of freedom to the two standard tensor degrees of freedom of linearized GR. Based on this, the $f(R)$-like field $\psi^{(1)}$ can be written as $\psi^{(1)} = 2a\varphi$. It is clear, from the above considerations and in the following sections, that the three scalar fields are related each other and non-local gravity implies only a further mode in GWs [36].

3   Gravitational waves with source

In order to get physical quantities from the pseudo-tensor (2.25), we have to link each of the fields $\theta_{\mu\nu}, \varphi, w$ to the source of gravitational waves, that is to $T_{\mu\nu}$. This implies to solve the equations of motions not in vacuum, but with $T_{\mu\nu} \neq 0$. The field equations with source, in the Lorentz gauge, are[4]

$$\Box\theta_{\mu\nu} = -2\chi T_{\mu\nu}, \tag{3.1}$$

$$\Box\varphi = \frac{1}{2}\Box\theta + 3\Box\psi^{(1)}, \tag{3.2}$$

$$\Box w = -a\left[\frac{1}{2}\Box\theta + 3\Box\psi^{(1)}\right]. \tag{3.3}$$

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[4]Since our background space is Minkowski, we have $\phi_0 = \Box^{-1}R^{(0)} = 0$ and, similarly, $\lambda_0 = 0$. Furthermore, notice that in solving non-vacuum equations, the traceless condition is not allowed.

Here the matter energy-momentum tensor $T_{\mu\nu}$ stands for the unperturbed one because the continuity equation $\partial_\mu T^{\mu\nu} = 0$ must be fulfilled. The proportionality between $\Box\varphi$ and $\Box w$ is obvious and it comes directly from eq. (2.7). The trace equation $\Box\theta = -2\chi T$ can be recast in the form

$$\Box\psi^{(1)} = \frac{1}{3}\Big[\psi_0 R^{(1)} + \chi T\Big] , \tag{3.4}$$

where we have used $G^{(1)} = -R^{(1)}$ with (see eq. (2.12))

$$R^{(1)} = -\frac{1}{2}\Box h + \frac{1}{\psi_0}\Box\psi^{(1)} . \tag{3.5}$$

with $\psi_0 = 1$. Eq. (3.4) is analogous to the trace equation of $f(R)$ gravity where it is usually recasted as a Klein-Gordon equation for the scalar field [22]. We will see later that this is not the case for non-local gravity, i.e. it is not possible to obtain a Klein-Gordon equation for the scalar field $\psi^{(1)}$. Notice also that eqs. (3.2) and (3.3) are not identical to eqs. (2.17) and (2.18) since now $\Box\theta \neq 0$.

The above non-homogeneous linear system of equations has two possible solutions according to the value of a. For $a \neq 1/6$, the linearized field equation admits the standard Einsten gravitational waves plus massless transverse scalar radiation while the non-local $\Box^{-1}$-theory, for the value of the coupling constant $a = 1/6$, shows a different behavior. It admits massive scalar waves as solutions and reproduces the Polyakov two-dimensional conformal effective action that occurs in string theory. See [28, 29]. If $a = 1/6$, it is:

$$a = 1/6 \quad \Rightarrow \quad \left\{T = 0 , \ \Box w = -\frac{1}{6}\Box\varphi\right\} . \tag{3.6}$$

The traceless condition at zero order $T_{\mu\nu}$ is not suitable for our purposes: it is not physically acceptable for a binary system where the linear approximation holds and, more importantly, it would not allow to obtain a solution for the fields φ and w depending on the source. In other words, a contribution from the non-local term $aR\Box^{-1}R$ to the quadrupole formula of the gravitational radiation directly generated by the matter is not admissible if $a = 1/6$ for the slowly moving compact binaries, where the background space-time curvature is negligible. However, this fact does not exclude the presence of lower multipoles (monopole and dipole) in gravitational radiation in addition to the quadrupole and higher multipole contributions, in presence of strong-field sources. Hence, we will not consider this case any more.

On the other hand, when $a \neq 1/6$, the solution of the system is the following:

$$a \neq 1/6 \quad \Rightarrow \quad \left\{\Box\varphi = \frac{1}{6a-1}\chi T , \ \Box w = \frac{-a}{6a-1}\chi T\right\}, \tag{3.7}$$

which, once solved in turn, clearly give explicit solutions for the scalar fields φ, w in terms of the source. Differently from the case $f(R)$, (3.7) are not standard Klein-Gordon equations, because no mass term appears, according to the fact that, for $a \neq 1/6$, the non-local $\Box^{-1}$-gravity shows only massless scalar modes but not massive ones. One might see the effect of non-locality in the manifestation of an effective energy-momentum tensor, namely $\tilde{T} \doteq -T/[2(6a-1)]$. Moreover, since $\Box\psi^{(1)} = 2a\Box\varphi$, getting solutions from (3.7) is equivalent to find a solution for $\psi^{(1)}$ as well. Based on these facts, in the following we will focus only on the case (3.7).

3.1 Explicit solutions

We want now to seek for explicit solutions for $\theta_{\mu\nu}$, φ and w with the assumption of far wave zone, i.e. for a source far enough from the observer, and we also assume a source in non-relativistic motion in order to simplify the orbital description of the binary system.

In analogy with GR, eq. (3.1) immediately returns

$$\theta_{\mu\nu}(t, \mathbf{x}) = 4G \int \frac{T_{\mu\nu}(t - |\mathbf{x} - \mathbf{y}|, \mathbf{y})}{|\mathbf{x} - \mathbf{y}|} d^3\mathbf{y} := J_{\mu\nu} \,, \tag{3.8}$$

which, in the hypothesis of far-field, i.e. $|\mathbf{x} - \mathbf{y}|^{-1} \simeq 1/r$, becomes

$$\theta_{ij}(t, \mathbf{x}) \simeq \frac{4G}{r} \int T_{ij}(t - r, \mathbf{y}) d^3\mathbf{y} \,. \tag{3.9}$$

In the above equation, we considered only spatial components since the remaining ones can be computed exploiting the gauge condition as in GR. We call $t' \doteq t - r$ the retarded time and, using the conservation of energy-momentum $\partial_\nu T^{i\nu} = 0$ and the relation

$$\partial_\mu \partial_\nu \left(x^i x^j T^{\mu\nu} \right) = 2T^{ij}, \tag{3.10}$$

eq. (3.8) turns into

$$\theta_{ij}(t, \mathbf{x}) = \frac{2G}{r} \frac{d^2 Q_{ij}(t')}{d^2 t} \,, \tag{3.11}$$

where we have defined the quadrupole tensor

$$Q_{ij}(t) \doteq \int d^3 y T^{00}(t, \mathbf{y}) y^i y^j \,, \tag{3.12}$$

with integration over the entire space of the source.[5] The above quantity is closely linked to the geometry and masses involved in the astrophysical system and it is therefore a known quantity. Solution (3.11), on the other hand, gives the gravitational tensor field as a function of the quadrupole time-derivative, to be replaced into eq. (2.25).

From eq. (3.7), it turns out that the relation $w = -a\varphi$ holds, and therefore it is sufficient to obtain a solution for only one scalar field. More precisely they are

$$\varphi(t) \simeq \frac{2G}{(1 - 6a)r} \int T(t - r, \mathbf{y}) d^3\mathbf{y} = \frac{\eta^{\mu\nu}}{2(1 - 6a)} J_{\mu\nu} \,, \tag{3.13}$$

$$w(t) \simeq -\frac{2aG}{(1 - 6a)r} \int T(t - r, \mathbf{y}) d^3\mathbf{y} = -\frac{a\eta^{\mu\nu}}{2(1 - 6a)} J_{\mu\nu} \,. \tag{3.14}$$

In obtaining the above solutions, we assumed the solution of the homogeneous equations to be zero, as in GR. In other words, we are assuming that when the source is off, all the fields are zero, just as for $h_{\mu\nu}$, and this is legitimate if we want such fields to be generated by matter. Apart from this, when $a \neq 1/6$, the GW-SET (2.25) reduces to

$$\tau_\beta^\alpha = \frac{\psi_0}{32\pi G} \left\langle \partial^\alpha \theta_{(TT)}^{\mu\nu} \partial_\beta \theta_{\mu\nu}^{(TT)} + 4a(6a - 1) \partial^\alpha \varphi \partial_\beta \varphi \right\rangle, \tag{3.15}$$

---

[5]In linearized gravity, $T_{\mu\nu}$ is a zero-order quantity. This implies that the contribution to the energy-momentum tensor comes only from ordinary matter and not from gravitational waves. It turns out that $T_{\mu\nu}$ is zero outside and on the boundary of the source.

where only one scalar field appears. Notice that the above expression is always symmetric and gauge-invariant.[6] The trace J in eq. (3.13) can be developed as

$$J(t) = \frac{4G}{r}\left[M + n^i \dot{D}_i(t') + \frac{1}{2}n_i n_j \ddot{Q}^{ij}(t')\right] + \frac{2G}{r}\ddot{Q}(t')\,, \tag{3.16}$$

with $Q \doteq \delta_{ij}Q^{ij}$, overdot means derivative with respect to time t', and $n_i \doteq \hat{x}_i = x_i/r$. See also [48]. Therefore, using conservation laws $\dot{M}$ and $\ddot{D}^i = 0$, we arrive at (see appendix B)

$$\partial_0\varphi\partial_r\varphi = \frac{-1}{4(1-6a)^2}\left[\frac{2G}{r}\left(n_i n_j \ddot{Q}^{ij}(t') + \ddot{Q}\right)\right]^2\,. \tag{3.17}$$

3.2 The emitted power

The quadrupole radiation is the emitted power (or luminosity) in the far region condition, and it is given by [49]

$$P_{\text{tot}} = -r^2 \int d\Omega \langle\tau^{0i}\rangle n_i := P_{\text{tot}}^{(GR)} + P_{\text{tot}}^{(NL)}\,, \tag{3.18}$$

with integration on a spatial surface at spatial infinity. The subscript 'NL' labels the non-local contribution in addition to the GR result, which corresponds to the first term of eq. (3.15). It is well-known that the latter is given by [45, 49]

$$P_{\text{tot}}^{(GR)} = \frac{G}{5c^5}\left\langle \dddot{Q}_{ij}\dddot{Q}^{ij} - \frac{1}{3}\dddot{Q}^2 \right\rangle\,, \tag{3.19}$$

where we have restored the factor c^5 at the denominator. Using eq. (3.17), after long but straightforward calculations, the non-local contribution to the energy rate reads as

$$P_{\text{tot}}^{(NL)} = \frac{aG}{15(6a-1)}\langle \dddot{Q}^{jk}\dddot{Q}_{jk} + 13\dddot{Q}^2 \rangle\,, \tag{3.20}$$

which clearly cancels out when $a = 0$. Substituting eqs. (3.19) and (3.20) into eq. (3.18), the following formula for quadrupole radiation in non-local gravity is found:

$$P_{\text{tot}} = \frac{G}{5c^5}\left[\left(1 + \frac{a}{3(6a-1)}\right)\langle \dddot{Q}^{ij}\dddot{Q}_{ij}\rangle + \frac{1+7a}{3(6a-1)}\langle \dddot{Q}^2 \rangle\right]\,. \tag{3.21}$$

Just to give a preliminary example, we can consider a Keplerian binary system consisting in two stars of mass M in a circular orbit at distance R from their common center of mass, then eq. (3.21) predicts an emitted power of

$$P_{\text{tot}} = \frac{128GM^2R^4w^6}{5c^5}\left[1 - \frac{a}{3(1-6a)}\right]\,, \tag{3.22}$$

---

[6]The first term, corresponding to the tensor contribution, is invariant under gauge transformation only considering the average procedure, as in GR. The second term, i.e. the non-local correction, is manifestly covariant. Explicitly, under a transformation $x'^\mu = x^\mu + \epsilon^\mu(x)$, it turns out that $\partial_\mu\varphi \to \partial_\mu\varphi + \partial_\sigma\varphi\partial_\mu\epsilon^\sigma$, and therefore $\partial_\alpha\varphi\partial_\beta\varphi \;\to\; \partial_\alpha\varphi\partial_\beta\varphi + \partial_\alpha\varphi\partial_\rho\varphi\partial_\beta\epsilon^\rho + \partial_\beta\varphi\partial_\rho\varphi\partial_\alpha\epsilon^\rho$, i.e. $\partial_\alpha\varphi\partial_\beta\varphi$ transforms like a 2-rank tensor.

where $w = \sqrt{\frac{GM}{4R^3}}$ is the keplerian angular frequency of the binary system. In general, for binary systems with different masses m_1 and m_2, on elliptic orbits of semi-major axis R and eccentricity e, from the Peters-Mathews formula [49], it turns out that

$$P_{\text{tot}} = \frac{32G^4 m^2 \mu^2}{5c^5 R^5} f(e, a) \,, \tag{3.23}$$

where $m = m_1 + m_2$ is the total mass and $\mu = m_1 m_2 / (m_1 + m_2)$ is the reduced mass, and we have defined the function

$$f(e, a) \doteq \frac{e^4(241a - 37) + 4e^2(469a - 73) + 608a - 96}{96(6a - 1)(1 - e^2)^{7/2}} \,. \tag{3.24}$$

From eq. (3.22) it is evident that non-local gravity induces a modification in quadrupole approximation of emitted power from binary systems, therefore resulting eventually observable. Since we expect a small value for a (any correction to GR in any extended theory of gravity should be rather slight), the above result shows that non-local corrections are, at least in principle, compatible with orbit decays by quadrupole radiation in binary systems, paving the way for new observational constraints on non-local corrections compared to those already present in the literature [31, 38, 42]. Of course, to find constraints on the non-local parameter a we need to apply eq. (3.21) to realistic astrophysical systems. This way to proceed, i.e. constraining parameters after the selection of the functional form for the correction, is an alternative to a more general approach, where the functional form itself can be selected by fitting the observations.

3.3 Amplitude of the scalar mode

Finally, let us evaluate the amplitude of the gravitational wave. As mentioned before, the tensor mode of the gravitational wave is given by the field θ_{ij} defined in eq. (2.14). From an observational point of view, it would be desirable to separate the purely metric contribution h_{ij} from the field contribution given by $\psi^{(1)} = a\varphi - w$, since we just have eq. (3.11) which holds for the full field θ_{ij}. For this purpose, one could assume φ and w as distinct propagating (scalar) modes, according to eqs. (3.13) and (3.14), where we have

$$\eta^{\mu\nu} J_{\mu\nu} \equiv J \simeq \frac{4G}{r}\left[M + \frac{1}{2}\ddot{Q}\right] \,, \tag{3.25}$$

approximating eq. (3.16) considering only $(1/r)$-terms. Therefore, φ and w could be seen as breathing modes in this theory of gravity. In order to estimate the amplitude of such modes, we first notice that the trace of the quadrupole tensor for a binary system is given by $Q = L^2 \mu$, where we called $L = 2R$ the separation between the two bodies. Here μ is the reduced mass. Based on this assumption, a scalar contribution will be present only if L is a function of time, as assumed in [47]. In particular, considering a trend like $L = L_0 \left(1 - \frac{t}{t_{\text{coal}}}\right)^{1/4}$, it turns out that

$$\ddot{Q}(t) = -\frac{1}{4}\frac{\mu L_0^2}{t_{\text{coal}}^2}\left(1 - \frac{t'}{t_{\text{coal}}}\right)^{-3/2} \,, \tag{3.26}$$

where $t_{\rm coal}$ is the coalescence time and we have evaluated the quantity at the retarded time $t' = t - r/c$. Halfway through the life of the binary system, i.e. at $t' = t_{\rm coal}/2$, the modulus will be[7]

$$|\varphi| \sim \frac{G\mu L_0^2}{r(1-6a)c^4 t_{\rm coal}^2} \, , \qquad (3.27)$$

which clearly depends on the non-local parameter a. For estimating the strain, we first need an estimate of the coalescence time in non-local gravity. Assuming a circular orbit[8] for simplicity, the period is $T = 2\pi\sqrt{\frac{R^3}{GM_{\rm tot}}}$ then

$$\frac{\dot{T}}{T} = \frac{3}{2}\frac{\dot{R}}{R} \quad \rightarrow \quad \dot{R}(t) = -\tilde{k}R^{-3}(t) \, , \qquad (3.28)$$

where we define

$$\tilde{k} \doteq \frac{192}{15}\frac{G^3 \mu M_{\rm tot}^2}{c^5}f(e,a) \, . \qquad (3.29)$$

In obtaining the above equation for $R(t)$, we used the relation between the variation of the period and the variation of the gravitational energy, i.e. $\frac{\dot{T}}{T} = -\frac{3}{2}\frac{\dot{E}}{E}$, where $E = -(GM\mu)/(2R)$ and $\dot{E} = -P_{\rm tot}$ given in the previous section. For circular orbits, $f(e,a) \approx 1$ in analogy with the GR case, hence the solution of the above differential relation is

$$R(t) = \sqrt{2}(c_1 - \tilde{k}t)^{1/4} \, , \qquad (3.30)$$

where c_1 is found by implying that $R(0) = R_{\rm now}$, with $R_{\rm now}$ the current value for R. It turns out that $c_1 = R_{\rm now}^4/4$. With this value, the coalescence time, $t_{\rm coal}$, is the time at which $a(t_{\rm coal}) = 0$, giving

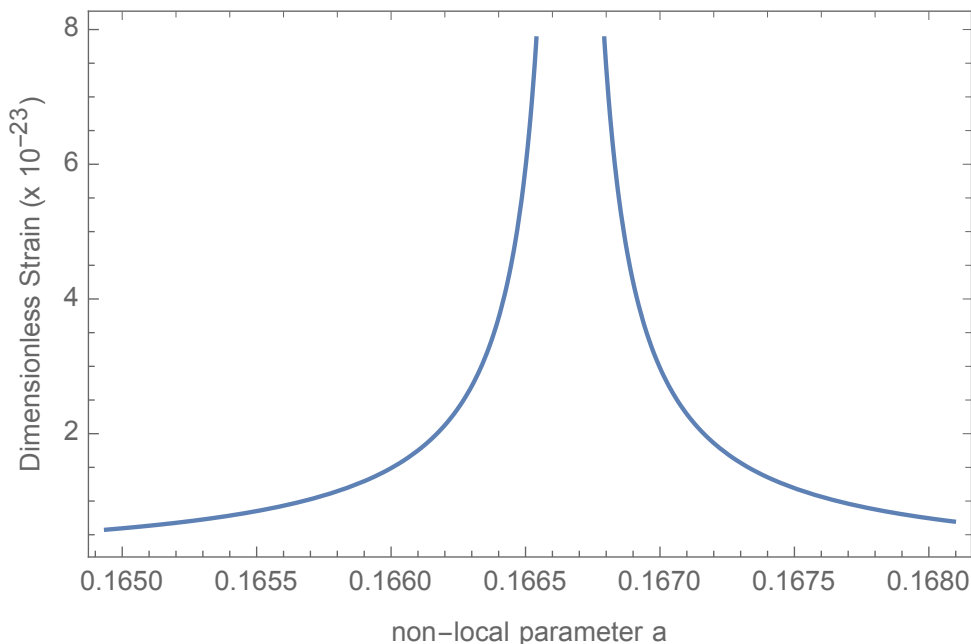$$t_{\rm coal} = \frac{5}{256}\frac{c^5 R_{\rm now}^4}{G^3 \mu M_{\rm tot}^2} \, . \qquad (3.31)$$

which is the well-known result in GR. This is enough to obtain an order-of-magnitude estimate for $t_{\rm coal}$ valid also for the present non-local theory. In particular, for two black holes with masses of the order $10 M_\odot$, at a semi-distance $R_{\rm now} = L_0/2 = 5\,{\rm AU}$, the time for merging will be $t_{\rm coal} \sim 10^9\,{\rm yr}$. This implies a (dimensionless) strain[9] of the order 10^{-50} for a source at a distance $r = 100\,{\rm Mpc}$. A more promising scenario is offered by the merging of supermassive black holes, as is the case of SDSS J153636.22 + 044127.0 at a redshift $z \simeq 0.4$ (i.e. $\simeq 1700\,{\rm Mpc}$) [51]. With masses of $10^9 M_\odot$ and $10^7 M_\odot$ and a semi-distance of 0.05 pc, the frequency[10] and coalescence time would be $\sim 10^{-9}\,{\rm Hz}$ and $\sim 4 \times 10^8\,{\rm yr}$ respectively, thus implying an amplitude of 10^{-34}, which is higher than the stellar-mass black-hole binaries,

---

[7]The monopole term, proportional to the (total) mass M, in eq. (3.25) is static and therefore it is negligible w.r.t. the quadrupole radiation.

[8]The following argument could easily be generalized to elliptical orbits. We also denote with $M_{\rm tot}$ the total mass of the binary system in order to generalize to the case of different masses.

[9]See [50] for details on different quantities for expressing the sensitivity curve of GWs detectors.

[10]The frequency f can be estimated remembering that the velocity is $v \simeq \sqrt{GM_{\rm tot}/L}$ and $f \simeq v/(\pi L)$. More precisely, the frequency of the gravitational radiation would be twice that value.

Figure 1. Dimensionless strain of the scalar mode for a black-hole binary system with masses $100 M_\odot$ and orbital distance of 0.01 AU as a function of the dimensionless non-local parameter a. In this range of values, the non-local massless transverse scalar mode could fall within the low frequency sensitivity of the Einstein Telescope.

but still very faint for all current and future gravitational wave observatories, including advanced LIGO, SKA, LISA and the Einstein Telescope, regardless of the value of the non-local parameter. The most prosperous case would be that of two black holes of 100 $M_\odot$ at an initial distance of $L_0 = 0.01\,\mathrm{AU}$.[11] The frequency, in this case, would be $\sim 2\,\mathrm{Hz}$. From figure 1, it is clear that the dimensionless strain explodes as soon as $a \to 1/6$, where the theory presents a divergence.

In a very narrow range, for the non-local coupling constant, it could fall within the low frequency sensitivity of the Einstein Telescope, thus representing a case of scientific interest for the detection of a massless transverse scalar mode in the very near future. See figure 1.

4 Discussion and conclusions

Although a theory of quantum gravity remains unknown, it is legitimate to ask what happens when a strong gravitational field is present at quantum scales. Because of its weakness compared to other interactions, the characteristic scale under which one would expect to experience non-classical effects relevant to gravity, the Planck scale, is quite

---

[11]The ratio between the Schwarzschild radius R_s, corresponding to the total mass of the system, and the orbital separation L_0 is an indicator of the gravitational strength. With our assumptions, it gives $R_s/L_0 \simeq 0.04$. For a self-gravitating system, the virial theorem implies $v/c \sim \sqrt{R_s/d} \simeq 0.2$, meaning that relativistic corrections to the Newtonian motion of the binary system, although not negligible, do not affect very much our estimates. At closer distances, i.e. very near to the merging, significant corrections or numerical relativity techniques should be adopted.

small, i.e. 10^{-33} cm, which is not accessible by any current experiment. It is not a new fact that Quantum Mechanics shows non-local aspects: non-locality is a manifestation of entanglement and the latter has been repeatedly demonstrated in laboratory experiments; Bell's theorem, furthermore, demonstrated that locality is violated in quantum systems. In gravitational physics, non-locality mainly emerges as a 'side' effect of the introduction of quantum corrections to GR in view to cure the singularity problem. Indeed, applications to Cosmology showed that non-local ghost-free higher-derivative modifications of the Einstein gravity in the ultraviolet regime can admit non-singular bouncing solutions for the Universe (in place of the Big Bang singular solution) and non-singular Schwarzschild metrics for black holes [12]. Another motivation for considering non-local gravity is the possibility to achieve renormalizability without the appearance of ghost modes [13]. More precisely, non-local theories of gravity can be considered as Extended Theories of Gravity [22], meaning that when the non-local terms are canceled out, GR is restored.

In this paper, we studied a possible implementation of non-local gravity consisting in the addition of a non-local term of the form $R\Box^{-1}R$ to the Hilbert-Einstein Lagrangian. We derived, from it, the power emitted by a gravitational system in terms of the quadrupole radiation, in view of possible future astrophysical detections. In particular, we obtained the field equations (using a localized version of the Lagrangian) of the theory and solved them in terms of the (source) stress-energy tensor $T_{\mu\nu}$, which, in the far region condition, only depends on the quadrupole momenta. Finally, from the expression of the Noether current, we computed the gravitational wave stress-energy pseudo-tensor, (GW-SET), $\tau_{\mu\nu}$, finding that the GR quadrupole formula is modified in a non-trivial way. This fact make it feasible to find possible signatures of non-locality. Incidentally, we have found no mass connected to non-local terms and this implies no length scale associated to non-locality, at least in the class of theories considered here. This is different from the other class of non-local gravity theories, namely the so called Infinite Derivative Gravity theories, where it results a lower bound on the non-local energy scale of $M \sim 10^{14}$ GeV [52]. Even if we expect a very small value for the non-local parameter a (any correction to GR in any extended theory of gravity should be rather slight), a preliminary application to a simple astrophysical scenario (a Keplerian binary system in circular orbit) showed that non-local corrections are compatible with GR predictions, but the deviation cloud be, at least in principle, appreciable, paving the way for new observational constraints on non-local corrections compared with those already present in the literature [31, 38, 42, 53].

In a forthcoming paper, we will apply our final result (3.21) to realistic astrophysical scenarios (such as the Hulse-Taylor binary system) in order to find constraints on the non-local parameter a. Another point to focus on will be considering the power emitted by a binary system in an *elliptical* orbit, with the aim of computing how the emitted power depends on the eccentricity of the orbit in presence of non-local corrections. We expect a strong dependence on eccentricity, as in GR, but a different enhancement factor. Other possible continuations of this work could concern the extension to higher order corrections like $R\Box^{-n}R$ with $n > 1$ [28]. In particular, models with $R\Box^{-2}R$-corrections seems to be very promising as discussed in [54, 55].

A The Noether current for non-local gravity

The Lagrangian density for our model of non-local gravity in eq. (2.4) is similar to a non-minimally coupled scalar-tensor theory. The presence of scalar fields allows us to avoid integro-differential equations, relatively simplifying the calculations. However, the latter are very involved and long; so in this appendix, we report only the main steps that led first to eq. (2.15) and then to eq. (2.24).

The starting point is to write the second-order Lagrangian density, as given in eq. (2.13). It is

$$
\mathcal{L}^{(2)} = \frac{1}{2\chi} \Big[(\sqrt{-g}R)^{(2)} \Big(1 + a\phi_0 - \lambda_0 \Big) + R^{(1)} \Big(a\varphi - w \Big) - \partial^\mu \varphi \partial_\mu w \Big] , \qquad (A.1)
$$

where $\psi_0 = 1 + a\varphi - \lambda_0$ and $\psi^{(1)} = a\varphi - w$. The first term in (A.1) is the second-order correction of the Ricci scalar. We have

$$
\frac{(\sqrt{-g}R)^{(2)}}{2\chi} = \frac{1}{64\pi G} \Big[\partial_\alpha h \partial^\alpha h + 2\partial_\alpha h_{\beta\gamma} \partial^\beta h^{\alpha\gamma} - 2\partial^\alpha h \partial_\beta h_\alpha^\beta - \partial_\gamma h_{\alpha\beta} \partial^\gamma h^{\alpha\beta} \Big] . \qquad (A.2)
$$

It has to be rewritten in terms of the gauge field $\theta_{\alpha\beta}$ given in eq. (2.14). The second term of eq. (A.1), written in $\theta_{\alpha\beta}$, is

$$
\begin{aligned}
\frac{R^{(1)} \psi^{(1)}}{2\chi} &= -\partial_\nu h^{\mu\nu} \partial_\mu \psi^{(1)} + \partial_\alpha h \partial^\alpha \psi^{(1)} \\
&= -\frac{1}{32\pi G} \Big[2\partial_\alpha \psi^{(1)} \partial_\beta \theta^{\alpha\beta} + \partial_\mu \psi^{(1)} \partial^\mu \theta + \frac{6}{\psi_0} \partial_\mu \psi^{(1)} \partial^\mu \psi^{(1)} \Big] ,
\end{aligned}
$$

where in the first line we integrated by parts, while, in the second line, we used the definition of $\theta_{\mu\nu}$. Putting all together in eq. (A.1), we have

$$
\frac{32\pi G}{\psi_0} \mathcal{L}^{(2)} = \frac{1}{4} \partial_\alpha \theta \partial^\alpha \theta + \partial_\alpha \theta_{\beta\gamma} \partial^\beta \theta^{\alpha\gamma} - \frac{3}{\psi_0^2} \partial_\alpha \psi^{(1)} \partial^\alpha \psi^{(1)} - \frac{1}{2} \partial_\gamma \theta_{\alpha\beta} \partial^\gamma \theta^{\alpha\beta} - \frac{2}{\psi_0} \partial_\beta \varphi \partial^\beta w . \text{ (A.3)}
$$

From the definition of $\psi^{(1)}$, it is

$$
\partial_\alpha \psi^{(1)} \partial^\alpha \psi^{(1)} = a^2 \partial_\alpha \varphi \partial^\alpha \varphi + \partial_\alpha w \partial^\alpha w - 2a \partial_\alpha w \partial^\alpha \varphi , \qquad (A.4)
$$

and hence the final expression for the second order Lagrangian $\mathcal{L}^{(2)} = \mathcal{L}^{(2)}(\theta_{\alpha\beta}, \varphi, w)$ is given by

$$
\begin{aligned}
\frac{32\pi G}{\psi_0} \mathcal{L}^{(2)} = & \frac{1}{4} \partial_\alpha \theta \partial^\alpha \theta + \partial_\alpha \theta_{\beta\gamma} \partial^\beta \theta^{\alpha\gamma} - \frac{1}{2} \partial_\gamma \theta_{\alpha\beta} \partial^\gamma \theta^{\alpha\beta} \\
& + \frac{(6a - 2\psi_0)}{\psi_0^2} \partial_\alpha w \partial^\alpha w - \frac{3}{\psi_0^2} \Big(a^2 \partial_\alpha \varphi \partial^\alpha \varphi + \partial_\alpha w \partial^\alpha w \Big) ,
\end{aligned}
$$

which is equal to eq. (2.15). Now, we notice that $\frac{\partial \mathcal{L}^{(2)}}{\partial \theta_{\alpha\beta}} = 0$, while

$$
\frac{\partial \mathcal{L}^{(2)}}{\partial (\partial_\gamma \theta_{\alpha\beta})} = 2\partial^{(\beta} \theta^{\alpha)\gamma} - \partial^\gamma \theta^{\alpha\beta} + \frac{1}{2} \eta^{\alpha\beta} \partial^\gamma \theta , \qquad (A.5)
$$

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and hence the Euler-Lagrange equation for the field $\theta_{\alpha\beta}$ is

$$\frac{\partial \mathcal{L}^{(2)}}{\partial \theta_{\alpha\beta}} - \partial_\gamma \frac{\partial \mathcal{L}^{(2)}}{\partial(\partial_\gamma \theta_{\alpha\beta})} = 0 \,,$$

$$-\Box\theta^{\alpha\beta} + \frac{1}{2}\eta^{\alpha\beta}\Box\theta = 0 \quad \Rightarrow \quad \Box\theta^{\alpha\beta} = 0 \,.$$

where in the second line we took advantage of the Lorentz gauge and exploited the trace equation. In a similar way, we arrive at the other two equations of motion (2.17) and (2.18) for the remaining fields φ and w. In particular, for this purpose, we needed the following partial derivatives:

$$\frac{\partial \mathcal{L}^{(2)}}{\partial(\partial_\gamma \varphi)} = \frac{\partial}{\partial(\partial_\gamma \varphi)}\left[\frac{(6a - 2\psi_0)}{\psi_0^2}\partial_\alpha\varphi\partial^\alpha w - \frac{3}{\psi_0^2}a^2\partial_\alpha\varphi\partial^\alpha\varphi\right]$$

$$= \frac{(6a - 2\psi_0)}{\psi_0^2}\delta_\alpha^\gamma\partial^\alpha w - \frac{6}{\psi_0^2}a^2\partial^\gamma\varphi \,,$$

and

$$\frac{\partial \mathcal{L}^{(2)}}{\partial(\partial_\gamma w)} = \frac{\partial}{\partial(\partial_\gamma w)}\left[\frac{(6a - 2\psi_0)}{\psi_0^2}\partial_\alpha\varphi\partial^\alpha w - \frac{3}{\psi_0^2}a^2\partial_\alpha w\partial^\alpha w\right]$$

$$= \frac{(6a - 2\psi_0)}{\psi_0^2}\delta_\alpha^\gamma\partial^\alpha\varphi - \frac{6}{\psi_0^2}\partial^\gamma w \,.$$

The above expressions have also been used in eq. (2.22), which becomes

$$j_\beta^\alpha = \frac{\psi_0}{32\pi\varphi}\left(-2\partial^\nu\theta^{\mu\alpha} + \partial^\alpha\theta^{\mu\nu}\right)\partial_\beta\theta_{\mu\nu} - \frac{\psi_0}{32\pi G}\left[\frac{(6a_1 - 2\psi_0)}{\psi_0^2}\partial^\alpha w - \frac{6}{\psi_0^2}a^2\partial^\alpha\varphi\right]\partial_\beta\varphi$$

$$- \frac{\psi_0}{32\pi G}\left[\frac{(6a - 2\psi_0)}{\psi_0^2}\partial^\alpha\varphi - \frac{6}{\psi_0^2}\partial^\alpha w\right]\partial_\beta w + \frac{\psi_0\delta_\beta^\alpha}{32\pi G}\left[\frac{1}{4}\partial_\alpha\theta\partial^\alpha\theta + \partial_\nu\theta_{\mu\gamma}\partial^\gamma\theta^{\mu\nu}\right.$$

$$\left. - \frac{1}{2}\partial_\gamma\theta_{\mu\nu}\partial^\gamma\theta^{\mu\nu} + \frac{(6a - 2\psi_0)}{\psi_0^2}\partial^\gamma\varphi\partial_\gamma w - \frac{3}{\psi_0^2}\left(a^2\partial_\mu\varphi\partial^\mu\varphi + \partial_\mu w\partial^\mu w\right)\right] \,.$$

Using the Lorenz gauge condition, the field equations and integrating by parts when necessary, the above equations simplify to

$$j_\beta^\alpha = \frac{\psi_0}{32\pi G}\left[\partial^\alpha\theta^{\mu\nu}\partial_\beta\theta_{\mu\nu} - K\partial^\alpha w\partial_\beta\varphi + \frac{6}{\psi_0^2}a^2\partial^\alpha\varphi\partial_\beta\varphi - K\partial^\alpha\varphi\partial_\beta w + \frac{6}{\psi_0^2}\partial^\alpha w\partial_\beta w\right.$$

$$\left. + \delta_\beta^\alpha K\partial^\gamma\varphi\partial_\gamma w - \delta_\beta^\alpha\frac{3a^2}{\psi_0^2}\partial^\gamma\varphi\partial_\gamma\varphi - \delta_\beta^\alpha\frac{3}{\psi_0^2}\partial^\gamma w\partial_\gamma w\right] \,,$$

where we defined $K = (6a - 2\psi_0)/\psi_0^2$. After noticing that the last three terms are zero as well (see section 2), we easily obtain eq. (2.24).

B The non-local energy rate

In this appendix, we briefly report the calculations leading to eq. (3.20). We need to write eq. (3.15), i.e.

$$\tau_\beta^\alpha = \frac{\psi_0}{32\pi G}\left\langle \partial^\alpha\theta_{(TT)}^{\mu\nu}\partial_\beta\theta_{\mu\nu}^{(TT)} + 4a(6a - 1)\partial^\alpha\varphi\partial_\beta\varphi \right\rangle \,, \tag{B.1}$$

in terms of $J_{\mu\nu}$, defined in eq. (3.8), and then $J_{\mu\nu}$ in terms of the quadrupole momentum Q_{ij}, defined in eq. (3.12). First, we expand $J_{\mu\nu}$ in Taylor series around $t' = t - r$, i.e. [48]

$$J^{\mu\nu}(\mathbf{x}, t) = \frac{4}{r}\left[\int d^3\mathbf{x}' T^{\mu\nu}(\mathbf{x}', t') + \hat{x}\int d^3\mathbf{x}\mathbf{x}'\frac{\partial T^{\mu\nu}(\mathbf{x}, t)}{\partial t'}\right.$$
$$\left. + \frac{1}{2}\int d^3\mathbf{x}'(\hat{x}\cdot\mathbf{x}')^2\frac{\partial^2 T^{\mu\nu}(\mathbf{x}, t)^2}{\partial t'}\right], \tag{B.2}$$

where we assumed that, in the far region, it is $|\mathbf{x} - \mathbf{x}'|^{-1} \simeq \frac{1}{r}$ and $|\mathbf{x} - \mathbf{x}'| \simeq r - \hat{x}\cdot\mathbf{x}'$, with $r = |\mathbf{x}|$. Using eq. (3.11) rewritten in the TT gauge, one finds that the GR contribution to the GW-SET is

$$\tau_{0r}^{(GR)} = \frac{1}{32\pi G}\left\langle\partial_0\theta_{(TT)}^{ij}\partial_r\theta_{ij}^{(TT)}\right\rangle = -\frac{G}{8\pi r^2}\left\langle\dddot{Q}_{ij}^{(TT)}\dddot{Q}_{(TT)}^{ij}\right\rangle. \tag{B.3}$$

Following the standard computations in GR [45, 49], one immediately[12] obtains eq. (3.19).

For the second term of eq. (B.1), we use eq. (3.13) and notice that, in our metric signature, it is $J = J_\mu^\mu = J^{00} - J^{11} - J^{22} - J^{33}$. Using eq. (B.2), after long but straightforward computations, we get eq. (3.16), whose time derivative reads as

$$\dot{J}(t') = \frac{2G}{r}\left[n_i n_j\dddot{Q}^{ij} + \dddot{Q}\right], \tag{B.4}$$

where we used mass and angular momentum conservation. It must be emphasized that although usually the component J_{00} is approximated as $J_{00} \simeq \frac{4GM}{r} + \mathcal{O}(\frac{1}{r^3})$, here it is necessary to retain higher order terms as well in order not to lose its quadrupole contribution,

$$J_{00} \simeq \frac{4G}{r}\left[M + n_i\dot{D}^i + \frac{1}{2}n_i n_j\ddot{Q}^{ij}\right]. \tag{B.5}$$

Since for the computation of the gravitational power P_{tot} only the component τ_{0r} is involved, we just need to compute the term $\partial_0\varphi\partial_r\varphi$, with φ given by eq. (3.13). In particular, it will be

$$\partial_0 J = \frac{4G}{r}\left[\frac{1}{2}n_i n_j\dddot{Q}^{ij}\right] + \frac{2G}{r}\dddot{Q}, \tag{B.6}$$

and

$$\partial_r J \simeq -\frac{2G}{r}n_i n_j\dddot{Q}^{ij} - \frac{2G}{r}\dddot{Q}, \tag{B.7}$$

where in $\partial_r J$ we neglected $1/r^2$-terms. Putting all together we obtain eq. (3.17). Therefore, to arrive at eq. (3.20), we need to perform the integral

$$P_{\text{tot}}^{(NL)} = \frac{aG}{8\pi(6a-1)}\int d\Omega\left[n_i n_j\dddot{Q}_{ij} + \dddot{Q}\right]^2, \tag{B.8}$$

which explicitly reads as

$$P_{\text{tot}}^{(NL)} = \frac{aG}{8\pi(6a-1)}\left[\frac{4\pi}{15}\left(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)\dddot{Q}^{ij}\dddot{Q}^{kl} + 4\pi\dddot{Q}^2 + \frac{8\pi}{3}\delta_{ij}\dddot{Q}^{ij}\dddot{Q}^2\right], \tag{B.9}$$

and simplifying

$$P_{\text{tot}}^{(NL)} = \frac{aG}{8(6a-1)}\left\langle\frac{8}{15}\dddot{Q}^{jk}\dddot{Q}_{jk} + \frac{104}{15}\dddot{Q}^2\right\rangle. \tag{B.10}$$

It is evident that, this equation, after a trivial simplification, reduces to eq. (3.20).

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[12]One needs to write $Q_{ij}^{(TT)}$ as $Q_{ij}^{(TT)} = \bar{Q}_{ij}^{(TT)}$ where $\bar{Q}_{ij} = Q_{ij} - \frac{1}{3}\delta_{ij}\delta^{kl}Q_{kl}$ and then use the projector tensors (see Chapter 7 in [45]) to expand the TT gauge components.

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