



Simultaneous explanation of the R_K and $R(D^{(*)})$ puzzles



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ABSTRACT

At present, there are several hints of lepton flavor non-universality. The LHCb Collaboration has measured $R_K \equiv \mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-) / \mathcal{B}(B^+ \rightarrow K^+ e^+ e^-)$, and the BaBar Collaboration has measured $R(D^{(*)}) \equiv \mathcal{B}(\bar{B} \rightarrow D^{(*)+} \tau^- \bar{\nu}_\tau) / \mathcal{B}(\bar{B} \rightarrow D^{(*)+} \ell^- \bar{\nu}_\ell)$ ($\ell = e, \mu$). In all cases, the experimental results differ from the standard model predictions by $2\text{--}3\sigma$. Recently, an explanation of the R_K puzzle was proposed in which new physics (NP) generates a neutral-current operator involving only third-generation particles. Now, assuming the scale of NP is much larger than the weak scale, this NP operator must be made invariant under the full $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group. In this Letter, we note that, when this is done, a new charged-current operator can appear, and this can explain the $R(D^{(*)})$ puzzle. A more precise measurement of the double ratio $R(D)/R(D^*)$ can rule out this model.

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To date, the standard model (SM) has been extremely successful in describing experimental data. There are, however, a few measurements that are in disagreement with the predictions of the SM. For example, the LHCb Collaboration recently measured the ratio of decay rates for $B^+ \rightarrow K^+ \ell^+ \ell^-$ ($\ell = e, \mu$) in the dilepton invariant mass-squared range $1 \text{ GeV}^2 \leq q^2 \leq 6 \text{ GeV}^2$ [1]. They found

$$R_K \equiv \frac{\mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-)}{\mathcal{B}(B^+ \rightarrow K^+ e^+ e^-)} = 0.745_{-0.074}^{+0.090} (\text{stat}) \pm 0.036 (\text{syst}), \quad (1)$$

which is a 2.6σ difference from the SM prediction of $R_K = 1 \pm O(10^{-4})$ [2]. As another example, the BaBar Collaboration with their full data sample has reported the following measurements [3,4]:

$$\begin{aligned} R(D) &\equiv \frac{\mathcal{B}(\bar{B} \rightarrow D^+ \tau^- \bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \rightarrow D^+ \ell^- \bar{\nu}_\ell)} = 0.440 \pm 0.058 \pm 0.042, \\ R(D^*) &\equiv \frac{\mathcal{B}(\bar{B} \rightarrow D^{*+} \tau^- \bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \rightarrow D^{*+} \ell^- \bar{\nu}_\ell)} = 0.332 \pm 0.024 \pm 0.018, \end{aligned} \quad (2)$$

where $\ell = e, \mu$. The SM predictions are $R(D) = 0.297 \pm 0.017$ and $R(D^*) = 0.252 \pm 0.003$ [3,5], which deviate from the BaBar mea-

surements by 2σ and 2.7σ , respectively. (The BaBar Collaboration itself reported a 3.4σ deviation from SM when the two measurements of Eq. (2) are taken together.) These two measurements of lepton flavor non-universality, respectively referred to as the R_K and $R(D^{(*)})$ puzzles, may be providing a hint of the new physics (NP) believed to exist beyond the SM.

In addition, we note that the three-body decay $B^0 \rightarrow K^* \mu^+ \mu^-$ by itself offers a large number of observables in the kinematic and angular distributions of the final-state particles, and it has been argued that some of these distributions are less affected by hadronic uncertainties [6]. Interestingly, the measurement of one of these observables shows a deviation from the SM prediction [7]. However, the situation is not clear whether this anomaly is truly a first sign of new physics. There are unknown hadronic uncertainties that must be taken into account before one can draw this conclusion [8–10]. We therefore do not discuss this measurement further.

To search for an explanation of R_K , in Ref. [11] Hiller and Schmaltz perform a model-independent analysis of $b \rightarrow s \ell^+ \ell^-$. They consider NP operators of the form $(\bar{s} \mathcal{O} b)(\bar{\ell} \mathcal{O}' \ell)$, where $\mathcal{O}$ and $\mathcal{O}'$ span all Lorentz structures. They find that the only NP operator that can reproduce the experimental value of R_K is $(\bar{s} \gamma_\mu P_L b)(\bar{\ell} \gamma^\mu P_L \ell)$. This is consistent with the NP explanations for the $B \rightarrow K^{(*)} \mu^+ \mu^-$ angular distributions measured by LHCb [9].

In Ref. [12], Glashow, Guadagnoli and Lane (GGL) note that lepton flavor non-universality is necessarily associated with lepton flavor violation (LFV). With this in mind, they assume that the NP

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couples preferentially to the third generation, giving rise to the operator

$$G(\bar{b}'_L \gamma_\mu b'_L)(\bar{\tau}'_L \gamma^\mu \tau'_L), \quad (3)$$

where $G = O(1)/\Lambda_{\text{NP}}^2 \ll G_F$, and the primed fields are the fermion eigenstates in the gauge basis. The gauge eigenstates are related to the physical mass eigenstates by unitary transformations involving U_L^d and U_L^ℓ :

$$d'_{L3} \equiv b'_L = \sum_{i=1}^3 U_{L3i}^d d_i, \quad \ell'_{L3} \equiv \tau'_L = \sum_{i=1}^3 U_{L3i}^\ell \ell_i. \quad (4)$$

With this, Eq. (3) generates an NP operator that contributes to $\bar{b} \rightarrow \bar{s} \mu^+ \mu^-$:

$$G \left[U_{L33}^d U_{L32}^{d*} |U_{L32}^\ell|^2 (\bar{b}_L \gamma_\mu s_L) (\bar{\mu}_L \gamma^\mu \mu_L) + h.c. \right]. \quad (5)$$

Because the coefficient of this operator involves elements of the mixing matrices, which are unknown, one cannot make a precise evaluation of the effect of this operator on $\mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-)$, and hence on R_K . Still, GGL note that the hierarchy of the elements of Cabibbo–Kobayashi–Maskawa quark mixing matrix, along with the apparent preference of the NP for muons over electrons, suggests that $|U_{L33}^{d,\ell}| \simeq 1$ and $|U_{L31}^{d,\ell}|^2 \ll |U_{L32}^{d,\ell}|^2 \ll 1$. Furthermore, there are limits on some ratios of magnitudes of matrix elements. Taken together, GGL find that the observed value of R_K can be accommodated with the addition of the NP operator in Eq. (5).

In any case, GGL's main point is not so much to offer Eq. (3) as an explanation of R_K , but rather to stress that the NP responsible for the lepton flavor non-universality will generally also lead to an enhancement of the rates for lepton-flavor-violating processes such as $B \rightarrow K \mu e$, $K \mu \tau$ and $B_s \rightarrow \mu e$, $\mu \tau$. In the case of Eq. (3), it is clear how LFV arises. This operator is written in terms of the fermion fields in the gauge basis and does not respect lepton-flavor universality. In transforming to the mass basis, the GIM mechanism [13] is broken, and processes with LFV are generated.

In fact, this behavior is quite general. In writing down effective Lagrangians, it is usually only required that the operators respect $SU(3)_C \times U(1)_{em}$ gauge invariance. However, it was argued in Refs. [11,14] that if the scale of NP is much larger than the weak scale, the operators generated when one integrates out the NP must be invariant under the full $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group. In the same vein, the operators should be written in terms of the fermion fields in the gauge basis – after all, above the weak scale, the mass eigenstates do not (yet) exist. If these operators break lepton universality, lepton-flavor-violating interactions will appear at low energy when one transforms to the mass basis. (Note, however, that in explicit models one can avoid lepton flavor non-universality and lepton flavor violation through the imposition of additional symmetries. One such example can be found in Ref. [15].)

There have been a number of analyses, both model-independent and model-dependent, examining explanations of the R_K puzzle. (Sometimes the data from the $B \rightarrow K^{(*)} \mu^+ \mu^-$ angular distributions were also included.) In all cases, the low-energy operators were written in terms of mass eigenstates, and lepton-flavor-violating operators were not included. However, as argued above, such operators will appear when lepton universality is broken. Now, the model-independent analyses [9,11,14,16] will be little changed by the inclusion of such operators. However, considerations of such lepton-flavor-violating interactions would be useful in the context of model-dependent analyses. Leptoquarks [11,17] and R-parity-violating SUSY [18] have been proposed as possible solutions to the R_K puzzle. In both cases, it would be interesting to examine the predictions for the lepton-flavor-violating processes.

Coming back to the GGL operator of Eq. (3), it too must be made invariant under $SU(3)_C \times SU(2)_L \times U(1)_Y$. There are two consequences. First, the left-handed fermion fields must be replaced by $SU(2)_L$ doublets: $b'_L \rightarrow Q'_L$ and $\tau'_L \rightarrow L'_L$, where $Q' \equiv (t', b')^T$ and $L' \equiv (\nu'_\tau, \tau')^T$. Second, there are two NP operators that are invariant under $SU(2)_L$ and contain Eq. (3):

$$\begin{aligned} \mathcal{O}_{\text{NP}}^{(1)} &= G_1 (\bar{Q}'_L \gamma_\mu Q'_L) (\bar{L}'_L \gamma^\mu L'_L), \\ \mathcal{O}_{\text{NP}}^{(2)} &= G_2 (\bar{Q}'_L \gamma_\mu \sigma^I Q'_L) (\bar{L}'_L \gamma^\mu \sigma^I L'_L), \end{aligned} \quad (6)$$

where G_1 and G_2 are both $O(1)/\Lambda_{\text{NP}}^2$ (but not equal to one another), and σ^I are the Pauli matrices (the generators of $SU(2)$). Using the identity

$$\sigma_{ij}^I \sigma_{kl}^I = 2\delta_{il}\delta_{kj} - \delta_{ij}\delta_{kl}, \quad (7)$$

where i, j are $SU(2)_L$ indices, the second operator can be written as

$$\mathcal{O}_{\text{NP}}^{(2)} = G_2 \left[2(\bar{Q}'_L{}^i \gamma_\mu Q'_L{}^j) (\bar{L}'_L{}^j \gamma^\mu L'_L{}^i) - (\bar{Q}'_L \gamma_\mu Q'_L) (\bar{L}'_L \gamma^\mu L'_L) \right]. \quad (8)$$

The two operators correspond to different types of underlying NP. Specifically, $\mathcal{O}_{\text{NP}}^{(1)}$ contains only neutral-current (NC) interactions, while $\mathcal{O}_{\text{NP}}^{(2)}$ contains both neutral-current and charged-current (CC) interactions. $\mathcal{O}_{\text{NP}}^{(2)}$ therefore offers the potential to simultaneously explain both the R_K and $R(D^{(*)})$ puzzles, and we examine the effects of including this NP operator.

Writing $\mathcal{O}_{\text{NP}}^{(2)}$ explicitly in terms of the up-type and down-type fields, there are four NC operators and one CC operator:

$$\mathcal{O}_{\text{NP}}^{(2)} = O_{tt\nu_\tau\nu_\tau} + O_{bb\tau\tau} + O_{tt\tau\tau} + O_{bb\nu_\tau\nu_\tau} + O_{tb\tau\nu_\tau}, \quad (9)$$

with

$$\begin{aligned} O_{tt\nu_\tau\nu_\tau} &= G_2 (\bar{t}'_L \gamma_\mu t'_L) (\bar{\nu}'_{\tau L} \gamma^\mu \nu'_{\tau L}), \\ O_{bb\tau\tau} &= G_2 (\bar{b}'_L \gamma_\mu b'_L) (\bar{\tau}'_L \gamma^\mu \tau'_L), \\ O_{tt\tau\tau} &= -G_2 (\bar{t}'_L \gamma_\mu t'_L) (\bar{\tau}'_L \gamma^\mu \tau'_L), \\ O_{bb\nu_\tau\nu_\tau} &= -G_2 (\bar{b}'_L \gamma_\mu b'_L) (\bar{\nu}'_{\tau L} \gamma^\mu \nu'_{\tau L}), \\ O_{tb\tau\nu_\tau} &= 2G_2 (\bar{t}'_L \gamma_\mu b'_L) (\bar{\tau}'_L \gamma^\mu \nu'_{\tau L}) + h.c. \end{aligned} \quad (10)$$

If both $\mathcal{O}_{\text{NP}}^{(1)}$ and $\mathcal{O}_{\text{NP}}^{(2)}$ are present then the NC interactions receive contributions from both NP operators.

Above, we see that the NC part of $\mathcal{O}_{\text{NP}}^{(2)}$ contains $O_{bb\tau\tau}$, which is the GGL operator of Eq. (3). In transforming to the mass basis, the GGL piece therefore contributes to $\bar{b} \rightarrow \bar{s}$ transitions through the quark-level decays $\bar{b} \rightarrow \bar{s} \ell^+ \ell^-$ and $\bar{b} \rightarrow \bar{s} \ell^+ \ell'^-$. These generate the meson-level decays $B \rightarrow K^{(*)} \mu^+ \mu^-$, $B \rightarrow K^{(*)} \mu^\pm e^\mp$, $B \rightarrow K^{(*)} \mu^\pm \tau^\mp$, $B_s \rightarrow \mu^+ \mu^-$, $B^0 \rightarrow X_s \mu^+ \mu^-$, $B_s^0 \rightarrow \mu^+ \mu^- \gamma$, etc. (Many of these decays are discussed by GGL.) The largest effects will be an enhancement of the SM contribution to $\bar{b} \rightarrow \bar{s} \tau^+ \tau^-$, and the generation of the lepton-flavor-violating decays $\bar{b} \rightarrow \bar{s} \tau^\pm \mu^\mp$ [19].

We begin by discussing the effect of $\mathcal{O}_{\text{NP}}^{(2)}$ on R_K . The amplitude for $\bar{b} \rightarrow \bar{s} \ell_1^+ \ell_2^-$ ($\ell_1 = e, \ell_2 = \mu$) can be expressed as

$$\begin{aligned} A_{\ell_i} &= A^{\text{SM}} \left(1 + V_L^{bs\ell_i} \right), \quad V_L^{bs\ell_i} = \frac{\kappa}{C_9} \frac{U_{L33}^d U_{L32}^{d*}}{V_{tb} V_{ts}^*} |U_{L3i}^\ell|^2, \\ \kappa &= \frac{4\pi}{\alpha_{\text{EM}}} \frac{g_2^2}{g^2} \frac{M_W^2}{\Lambda_{\text{NP}}^2}. \end{aligned} \quad (11)$$

Here A^{SM} is the lepton-flavor-universal (SM) contribution, the V_{ij} are Cabibbo–Kobayashi–Maskawa (CKM) matrix elements, C_9 is a Wilson coefficient, and we have written $G_2 = g_2^2/\Lambda_{\text{NP}}^2$. Neglecting the masses of the leptons we then arrive at the following result:

$$R_K = \frac{1 + 2 \text{Re}[V_L^{bs\mu}] + |V_L^{bs\mu}|^2}{1 + 2 \text{Re}[V_L^{bse}] + |V_L^{bse}|^2} \approx 1 + \frac{8\pi}{C_9 \alpha_{\text{EM}}} \frac{g_2^2}{g^2} \frac{M_W^2}{\Lambda_{\text{NP}}^2} \frac{U_{L32}^d |U_{L32}^\ell|^2}{\lambda^2}, \quad (12)$$

where λ is the sine of the Cabibbo angle. We have assumed the usual hierarchy of CKM matrix elements and ignored all CP-violating phases. The 5σ limit on R_K from LHCb then implies

$$-2 \times 10^{-4} \lesssim \frac{1}{C_9} \frac{g_2^2}{g^2} \frac{M_W^2}{\Lambda_{\text{NP}}^2} \frac{U_{L32}^d |U_{L32}^\ell|^2}{\lambda^2} \lesssim 7 \times 10^{-5}. \quad (13)$$

It is clear that the LHCb measurement constrains the magnitudes of the down-type and lepton mixing-matrix elements. However, a further set of constraints will be obtained below.

In addition to the decays produced by the GGL operator, one now also has the quark-level decay $\bar{b} \rightarrow \bar{s} \nu \bar{\nu}$ that contributes to $B \rightarrow K^{(*)} \nu \bar{\nu}$. The amplitude for $\bar{b} \rightarrow \bar{s} \nu_i \bar{\nu}_j$ can be expressed as

$$A_{ij} = C_{ij} (\bar{b}_L \gamma_\mu s_L) (\bar{\nu}_i \gamma^\mu \nu_{jL}). \quad (14)$$

The SM contributes only to terms diagonal in neutrino flavor ($i = j$), while the NP operator also gives rise to off-diagonal terms that violate lepton flavor ($i \neq j$). We have

$$C_{ij} = \kappa_{\text{SM}} \left(\delta_{ij} - \frac{\kappa}{C_L^{\text{SM}}} \frac{U_{L32}^d U_{L32}^{d*}}{V_{tb} V_{ts}^*} U_{L3i}^{*\nu} U_{L3j}^\nu \right), \quad (15)$$

where

$$\kappa_{\text{SM}} = \frac{\sqrt{2} G_F \alpha_{\text{EM}}}{\pi} V_{tb} V_{ts}^* C_L^{\text{SM}}. \quad (16)$$

In the above, C_L^{SM} is a Wilson coefficient [10]. The square of the amplitude for the process is thus proportional to

$$\sum_{i,j} |C_{ij}|^2 = 3 |\kappa_{\text{SM}}|^2 \left(1 - \frac{2\kappa}{3} \text{Re}[x] + \frac{\kappa^2}{3} |x|^2 \right), \quad (17)$$

where $x = (U_{L32}^d U_{L32}^{d*}) / (V_{tb} V_{ts}^*)$.

We ignore all CP-violating phases, so that x is real. Taking $|U_{L32}^d| \sim 1$, we have $x \sim U_{L32}^d / \lambda^2$. The decay rate for $B \rightarrow K^{(*)} \nu \bar{\nu}$ is given by

$$\Gamma = \Gamma_{\text{SM}} \left(1 - \frac{2\kappa U_{L32}^d}{3\lambda^2} + \frac{(\kappa U_{L32}^d)^2}{3\lambda^4} \right). \quad (18)$$

The SM decay rate can be expressed as follows:

$$\Gamma_{\text{SM}} = \frac{m_B |\kappa_{\text{SM}}|^2}{64\pi^3} \int_0^{q^2_{\text{max}}} \rho_{K^{(*)}}(q^2) dq^2, \quad (19)$$

where q represents the sum of four momenta of the neutrino and the antineutrino, and $\rho_{K^{(*)}}$ is the appropriate $B \rightarrow K^{(*)}$ transition form factor. (Note that we have treated the neutrinos as massless particles.) Thus we see that the NP term simply modifies the SM rate for $B \rightarrow K \nu \bar{\nu}$ by an overall numerical factor.

One can use the above result to get an estimate of how large the NP couplings and mixing matrix elements can be. A precise

calculation of the SM branching ratio for $B^+ \rightarrow K^+ \nu \bar{\nu}$ was performed in Ref. [10]. It was found that

$$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})_{\text{SM}} = (4.20 \pm 0.33 \pm 0.15) \times 10^{-6}. \quad (20)$$

The strongest experimental bounds from the BaBar Collaboration [20] at present only set an upper limit of 1.7×10^{-5} at the 90% confidence level. Thus there is still room for the measured decay rate to be a factor of five larger than the SM prediction. Taking $C_L^{\text{SM}} \approx -6.13$ [10], we have $\kappa \approx -281 (g_2/g)^2 (M_W/\Lambda_{\text{NP}})^2$. A factor of five enhancement in the decay rate due to the NP operator $\mathcal{O}_{\text{NP}}^{(2)}$ would then imply

$$-1.6 \times 10^{-2} \lesssim \frac{g_2^2}{g^2} \frac{M_W^2}{\Lambda_{\text{NP}}^2} \frac{U_{L32}^d}{\lambda^2} \lesssim 9.3 \times 10^{-3}. \quad (21)$$

If $\Lambda_{\text{NP}} \approx 10 M_W$ then $(g_2^2/g^2)(U_{L32}^d/\lambda^2)$ must be $O(1)$. In this case, a NP coupling of the same order as that of the SM will still allow a reasonably large value for U_{L32}^d . For example, if $g/2 \lesssim g_2 \lesssim g$, one can have $\lambda \gtrsim U_{L32}^d \gtrsim \lambda^2$. In addition, we can now combine Eqs. (13) and (21). Since C_9 is an $O(1)$ number, this implies that an $O(10^{-1})$ value for $|U_{L32}^\ell|$ is still allowed. A more precise measurement of both R_K and $B^+ \rightarrow K^+ \nu \bar{\nu}$ will put stricter bounds on both the down-type and lepton mixing-matrix elements.

Finally, the neutral-current part of $\mathcal{O}_{\text{NP}}^{(2)}$ also contributes to the decays $t \rightarrow c \ell^+ \ell^-$, $t \rightarrow c \ell^+ \ell'^-$ and $t \rightarrow c \nu \bar{\nu}$. The branching ratios for these decays are negligible in the SM, so any observation would be a clear sign of NP. For decays to charged leptons, the most promising is $t \rightarrow c \tau^+ \tau^-$. In the mass basis, the contributing NP operator is

$$G \left[U_{L32}^{u*} U_{L33}^u |U_{L33}^\ell|^2 (\bar{c}_L \gamma^\mu t_L) (\bar{\tau}_L \gamma_\mu \tau_L) + h.c. \right], \quad (22)$$

which gives a partial width of

$$\frac{g_2^4 |U_{L32}^u|^2 |U_{L33}^u|^2 |U_{L33}^\ell|^4}{16 \Lambda_{\text{NP}}^4} \frac{m_t^5}{48 \pi^3}. \quad (23)$$

Taking $g_2 \sim g$, $|U_{L33}^u| \simeq |U_{L33}^\ell| \simeq 1$, $|U_{L32}^u| \simeq \lambda$, and $\Lambda_{\text{NP}} = 800$ GeV, this gives

$$\Gamma(t \rightarrow c \tau^+ \tau^-) = 1 \times 10^{-7} \text{ GeV}. \quad (24)$$

The full width of the t quark is 2 GeV, so this corresponds to a branching ratio of 5×10^{-8} . This is much larger than the SM branching ratio ($O(10^{-16})$), but is still tiny. The branching ratio for $t \rightarrow c \nu \bar{\nu}$ takes the same value, while those for all other $t \rightarrow c \ell^+ \ell^-$ and $t \rightarrow c \ell^+ \ell'^-$ decays are considerably smaller. Thus, while the branching ratios for these decays can be enormously enhanced compared to the SM, they are still probably unmeasurable. (This point is also noted in Ref. [11].)

Another process involving t quarks that could potentially reveal the presence of NP with LFV is $pp \rightarrow t \bar{t}$, followed by the radiation of a $\tau^\pm \mu^\mp$ pair. At the LHC with a 13 TeV center-of-mass energy, gluon fusion dominates the production of $t \bar{t}$ pairs. We use MadGraph 5 [21] to calculate the cross section for $gg \rightarrow t \bar{t} \tau^\pm \mu^\mp$, taking $g_2 \sim g$. We find $\sigma_{t \bar{t} \tau \mu} \approx 0.4 |U_{L32}^\ell|^2$ fb. By contrast, the SM cross section for $t \bar{t}$ pair production is $\sigma_{t \bar{t}} \approx 450$ pb, so that $\sigma_{t \bar{t} \tau \mu} / \sigma_{t \bar{t}} \approx 10^{-6} |U_{L32}^\ell|^2$, which is extremely small. With a luminosity of $100 \text{ fb}^{-1} / \text{year}$ at the 13 TeV LHC [22], we therefore expect about 40 events/year for $gg \rightarrow t \bar{t} \tau^\pm \mu^\mp$ if $|U_{L32}^\ell| \sim 1$, or about two events/year if $|U_{L32}^\ell| \sim \lambda$. Thus, even though the final-state signal is striking, $pp \rightarrow t \bar{t} \tau^\pm \mu^\mp$ is probably unobservable.

Turning to the charged-current interactions, these contribute to both b and t semileptonic decays. Even with the enhancement

from NP, the decay $t \rightarrow b\tau\bar{\nu}_\tau$ will still be difficult to observe, as it is swamped by the two-body decay $t \rightarrow bW$. On the other hand, the decay $b \rightarrow c\tau\bar{\nu}_i$ ($i = \tau, \mu, e$) is particularly interesting, since it contributes to the decay $\bar{B} \rightarrow D^{(*)+}\tau^-\bar{\nu}_\tau$ and the $R(D^{(*)})$ puzzle [Eq. (2)], and provides a source of lepton flavor non-universality in such decays.

In the SM, the effective Hamiltonian for the quark-level transition $b \rightarrow c\tau\bar{\nu}_\tau$ is

$$\mathcal{H}_{\text{eff}} = \frac{4G_F V_{cb}}{\sqrt{2}} (\bar{c}_L \gamma_\mu b_L) (\bar{\tau}_L \gamma^\mu \nu_{\tau L}) + h.c. \quad (25)$$

Now, if $\mathcal{O}_{\text{NP}}^{(2)}$ is also present, in addition to $\tau\bar{\nu}_\tau$ in the final state, the NP operator also produces $\tau\bar{\nu}_\mu$ and $\tau\bar{\nu}_e$. However, as the final-state neutrino is not observed, we have to sum over the neutrino species. That is, the squared-amplitude for $b \rightarrow c\tau^-\bar{\nu}_i$ can be written as

$$|A|^2 = \sum_{i=\tau, \mu, e} |A_i|^2, \quad (26)$$

with

$$A_i = \frac{4G_F V_{cb}}{\sqrt{2}} [\delta_{i\tau} + V_L^{cb\tau\nu_i}],$$

$$V_L^{cb\tau\nu_i} = 4 \frac{g_2^2 M_W^2}{g^2 \Lambda_{\text{NP}}^2} \frac{U_{L33}^d U_{L32}^u U_{L33}^\ell U_{L3i}^\nu}{V_{cb}}. \quad (27)$$

As was done above, we have written $G_2 \equiv g_2^2/\Lambda_{\text{NP}}^2$ and used $G_F/\sqrt{2} = g^2/8M_W^2$. One then has

$$|A|^2 = |A|_{\text{SM}}^2 [1 + 2 \text{Re}(V_L^{cb\tau\nu_\tau}) + |V_L^{cb\tau}|^2], \quad (28)$$

where

$$|V_L^{cb\tau}|^2 \equiv \sum_i |V_L^{cb\tau\nu_i}|^2 = \left| 4 \frac{g_2^2 M_W^2}{g^2 \Lambda_{\text{NP}}^2} \frac{U_{L33}^d U_{L32}^u U_{L33}^\ell}{V_{cb}} \right|^2. \quad (29)$$

(Here we have used the fact that $\sum_i |U_{L3i}^\nu|^2 = 1$.) The addition of the NP operator thus has the effect of modifying the SM prediction for $\Gamma(b \rightarrow c\tau\bar{\nu}_i)$ by an overall factor that is lepton flavor non-universal. In fact, if the elements of the charged-lepton mixing matrix obey the hierarchy suggested by GGL, namely $|U_{L33}^\ell| \simeq 1$ and $|U_{L31}^\ell|^2 \ll |U_{L32}^\ell|^2 \ll 1$, then $b \rightarrow c\tau\bar{\nu}_i$ is affected by the NP, but $b \rightarrow c\mu\bar{\nu}_i$ and $b \rightarrow ce\bar{\nu}_i$ are basically unchanged from the SM.

We now have the simple prediction

$$\left[\frac{R(D)}{R(D^*)} \right]_{\text{exp}} = \left[\frac{R(D)}{R(D^*)} \right]_{\text{SM}}. \quad (30)$$

Using Eq. (2), we have

$$\left[\frac{R(D)}{R(D^*)} \right]_{\text{exp}} = 1.33 \pm 0.24, \quad \left[\frac{R(D)}{R(D^*)} \right]_{\text{SM}} = 1.2 \pm 0.07. \quad (31)$$

So this model is consistent with experiment, but a careful measurement of the double ratio can rule it out. The double ratio in the SM is also likely to have less uncertainty from hadronic form factors. Furthermore, all angular asymmetries, such as the D^* polarization, forward-backward asymmetries, and the azimuthal angle asymmetries including the triple products, will show no deviation from the SM as these asymmetries probe non-SM operator structures.

If the ratios $R(D^{(*)})$ are defined with respect to the $B \rightarrow D^{(*)}\mu\nu$ decay mode, we can also write

$$\left[\frac{R(D^{(*)})_{\text{exp}}}{R(D^{(*)})_{\text{SM}}} \right] = \left[\frac{R(D)_{\text{exp}}}{R(D)_{\text{SM}}} \right] = \frac{[1 + 2 \text{Re}(V_L^{cb\tau\nu_\tau}) + |V_L^{cb\tau}|^2]}{[1 + 2 \text{Re}(V_L^{cb\mu\nu_\mu}) + |V_L^{cb\mu}|^2]}. \quad (32)$$

Again assuming a hierarchy in the mixing matrix, to leading order we have

$$\left[\frac{R(D^{(*)})_{\text{exp}}}{R(D^{(*)})_{\text{SM}}} \right] = \left[\frac{R(D)_{\text{exp}}}{R(D)_{\text{SM}}} \right] \approx \left[1 + 8 \frac{g_2^2 M_W^2}{g^2 \Lambda^2} \frac{U_{L32}^u}{V_{cb}} \right]. \quad (33)$$

Averaging $[R(D^{(*)})_{\text{exp}}/R(D^{(*)})_{\text{SM}}]$ and $[R(D)_{\text{exp}}/R(D)_{\text{SM}}]$, we get

$$8 \frac{g_2^2 M_W^2}{g^2 \Lambda^2} \frac{U_{L32}^u}{V_{cb}} \approx 0.4. \quad (34)$$

Taking $g/2 \lesssim g_2 \lesssim g$ and $\Lambda \sim 10M_W$, this gives $0.8 \lesssim U_{L32}^u \gtrsim \lambda$.

There have been numerous analyses examining NP explanations of the $R(D^{(*)})$ measurements [5,23]. Above, in the context of R_K , we noted that, assuming the scale of NP is much larger than the weak scale, all NP operators must be invariant under the full $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group. This same argument applies also to NP proposed to explain $R(D^{(*)})$. Such considerations were applied to the semileptonic $b \rightarrow c$ transitions in Ref. [24], but they could have important implication for the various NP explanations of the $R(D^{(*)})$ puzzle.

To sum up, the recent measurement of $R_K \equiv \mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-)/\mathcal{B}(B^+ \rightarrow K^+ e^+ e^-)$ by the LHCb Collaboration differs from the SM prediction of $R_K = 1$ by 2.6σ . And the BaBar Collaboration has measured $R(D^{(*)}) \equiv \mathcal{B}(\bar{B} \rightarrow D^{(*)+} \tau^- \bar{\nu}_\tau)/\mathcal{B}(\bar{B} \rightarrow D^{(*)+} \ell^- \bar{\nu}_\ell)$ ($\ell = e, \mu$), finding discrepancies with the SM of 2σ ($R(D)$) and 2.7σ ($R(D^*)$). The R_K and $R(D^{(*)})$ puzzles exhibit lepton flavor non-universality, and therefore hint at new physics (NP).

Recently, Glashow, Guadagnoli and Lane (GGL) proposed an explanation of the R_K puzzle. They assume that the NP couples preferentially to the third generation, and generates the neutral-current operator $(\bar{b}'_L \gamma_\mu b'_L)(\bar{\tau}'_L \gamma^\mu \tau'_L)$, where the primed fields denote states in the gauge basis. When one transforms to the mass basis, one obtains operators that give rise to decays that violate lepton universality (and lepton flavor conservation).

It is known that, assuming the scale of NP is much larger than the weak scale, all NP operators must be made invariant under the full $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group. In this Letter, we find that, when this is applied to the GGL operator, there are two types of fully gauge-invariant NP operators that are possible. And one of these contains both neutral-current and charged-current interactions. While GGL has shown that the neutral-current piece of this NP operator can explain the R_K puzzle, we demonstrate that the charged-current piece can simultaneously explain the $R(D^{(*)})$ puzzle. We also show that this model makes a prediction for the double ratio $R(D)/R(D^*)$, so that it can be ruled out with a more precise measurement of this quantity.

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