

Relativistic Effects in Nuclear Physics

Relativistic effects in nuclear physics, especially those in nuclear structure, are discussed. While general arguments are given which exclude several recently proposed effects, two important relativistic effects remain: (1) A strongly density dependent repulsive term in the energy per particle $(\delta\epsilon)_{\text{rel}} \cong 2(\rho/\rho_0)^{8/3}$ MeV, where ρ_0 is nuclear matter density, arises from virtual pair terms; this term is very important in saturating nuclear matter. (2) The nucleon–nucleon spin-orbit interaction is enhanced, since the nucleon mass which enters into this interaction is effectively changed locally by scalar fields which connect to negative energy states.

INTRODUCTION

The past few years have witnessed steadily increasing activity on the role of relativity in nuclear physics. Two extensive reviews are about to appear. The first one, by Serot and Walecka,¹ summarizes the developments of relativistic nuclear mean field theories. The second one, a book by Celenza and Shakin,² reviews approaches aiming at a theoretical foundation of the highly successful Dirac phenomenology.^{3,4} A recent Comment by Negele⁵ reviews the situation. Our presentation here is somewhat different in spirit.

The purpose of this Comment is twofold: first, to point out some conceptual problems that one encounters in relativistic many-body theories; second, to present a critical discussion of some of the so-called large relativistic effects in nuclear structure, once they are seen in the broader context of consistency requirements imposed by Fermi liquid theory. Despite their phenomenological successes, when applying relativistic models to nuclear structure problems,

one must be careful not to disregard basic results of many-body theory. We shall focus here primarily on two issues related to effects of relativity in nuclei: first, a genuine repulsive contribution to the nuclear binding energy; second, the detailed structure of the spin-orbit coupling in nuclei. Other relativistic effects discussed in the literature, such as the enhancement of magnetic moments, will be shown to disappear almost completely if treated consistently.

CONCEPTS

The generalization of nonrelativistic many-body theory to a relativistic one is far from trivial, as pointed out long ago by Brown and Ravenhall⁶ and recently by Sucher.⁷ The problem is the following: consider an A -particle product wave function of the Dirac–Hartree type, as often used in relativistic mean field theories. This wave function is generally written without reference to the filled Dirac sea of negative energy states. When a two-body interaction is introduced which has nonzero matrix elements between positive and negative energy states, there exists an infinite number of A -particle states with identical energy, in which one particle goes to a positive energy continuum state and the other one to a negative energy state. Consequently, one cannot write down a localized, normalizable A -particle wave function, because it will dissolve into the continuum, spreading over all space.

While this problem does not exist in phenomenological mean field approaches, further extensions towards “Quantum Hadrodynamics,”¹ with inclusion of two-body interactions, become meaningful only when accompanied by projection operators specifying that negative energy states are filled. (In field-theoretical equations such as the Bethe–Salpeter equation, projection operators are correctly handled.)

A relativistic (but not covariant) many-body Hamiltonian can be introduced as follows:

$$H = \sum_{i=1}^A H_D(i) + \frac{1}{2} \sum_{i \neq j} \Lambda_+(i) \Lambda_+(j) V_{\text{eff}}(i,j) \Lambda_+(i) \Lambda_+(j), \quad (1)$$

where

$$H_D = -i\boldsymbol{\alpha} \cdot \boldsymbol{\nabla} + \beta M \quad (2)$$

is the free nucleon Dirac Hamiltonian. The effective interaction $V_{\text{eff}}(i,j)$ operates only in positive energy states (including the small components of their wave functions), owing to the Casimir projection operators

$$\Lambda_{\pm}(p) = \frac{p_0 \pm H_D}{2p_0} \quad \text{with } p_0 = \sqrt{\mathbf{p}^2 + M^2}.$$

The eigenfunctions of the Hamiltonian (1) are now well defined. In particular, in such an approach, it is possible to establish connections with the nonrelativistic many-body problem and to make use of the great amount of experience gained in this area.

Relativistic effects occur in the effective interaction V_{eff} by couplings to negative energy intermediate states $|n^{(-)}\rangle$

$$V_{\text{eff}} = \nu + \sum_{n^{(-)}} \nu |n^{(-)}\rangle \frac{1}{E - E_n^{(-)}} \langle n^{(-)} | \nu + \dots \quad (3)$$

where ν is the two-body interaction and $E_n^{(-)}$ is in the negative energy continuum. Our point here is that relativistic effects can be understood within perturbation theory, which converges rapidly, given that the energy denominators in Eq. (3) are of order $2M$. In this way we may study relativistic effects by a well-defined procedure.

PERTURBATION THEORY AND THE MANY-BODY PROBLEM

Let us now consider the case of nuclear matter and assume that the two-body interaction ν in Eq. (3) has an attractive scalar and a repulsive vector component⁸ which in the static limit takes the form:

$$\nu(1,2) = -\frac{g_s^2}{4\pi} \frac{e^{-m_s r}}{r} \beta_1 \beta_2 + \frac{g_v^2}{4\pi} \frac{e^{-m_v r}}{r} (1 - \boldsymbol{\alpha}_1 \cdot \boldsymbol{\alpha}_2). \quad (4)$$

In the mean field (Hartree) approximation, Fig. 1, this leads to

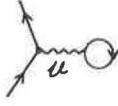


FIGURE 1 Mean field (Hartree) approximation with the two-body interaction U .

the one-body equation familiar from Dirac phenomenology:

$$[\boldsymbol{\alpha} \cdot \mathbf{p} + \beta(M + U_s) - (\epsilon_p - U_v)]\psi_p = 0 \quad (5)$$

with

$$U_s = -\frac{g_s^2}{m_s^2} \rho_s, \quad U_v = \frac{g_v^2}{m_v^2} \rho_v, \quad (6)$$

where

$$\rho_s = \sum_n \psi_n^\dagger \beta \psi_n \quad \text{and} \quad \rho_v = \sum_n \psi_n^\dagger \psi_n \quad (7)$$

are the scalar and fourth component vector densities, respectively. Typical values of the scalar and vector mean fields appearing in the literature are $U_s = -400 \text{ MeV}$ (ρ/ρ_0) and $U_v = 300 \text{ MeV}$ (ρ/ρ_0) with $\rho_0 = 0.16 \text{ fm}^{-3}$ as nuclear matter density. The resulting Dirac “effective mass” is

$$\bar{M} = M + U_s \cong 540 \text{ MeV} \quad (8)$$

and the leading term in the Dirac single particle potential, at $\rho = \rho_0$, is $U_s + U_v \cong -100 \text{ MeV}$. The mass \bar{M} is not to be confused with the effective mass M^* of nonrelativistic many-body theory, which arises from the velocity dependence of the interaction, although, as we discuss below, these two masses are related in nuclear matter.

Let us now investigate, using first-order perturbation theory, how the relativistic effects, especially the effective mass \bar{M} , tie in with virtual pair terms. We begin with noninteracting positive energy wave functions ψ_{+i} . The relation of small to large components here is given by

$$\Lambda_-(p_i)\psi_{+i} = 0 \quad (9)$$

so that

$$\psi_{+i} = \sqrt{\frac{p_0 + M}{2p_0}} \begin{bmatrix} \chi_{1/2}^m \\ \frac{\boldsymbol{\sigma} \cdot \mathbf{p}_i}{p_0 + M} \chi_{1/2}^m \end{bmatrix} \cong \begin{bmatrix} \chi_{1/2}^m \\ \frac{\boldsymbol{\sigma} \cdot \mathbf{p}_i}{2M} \chi_{1/2}^m \end{bmatrix}; \quad (10)$$

the negative-energy solutions are of the form

$$\psi_{-i} \cong \begin{bmatrix} \frac{-\boldsymbol{\sigma} \cdot \mathbf{p}_i}{2M} \phi_{1/2}^m \\ \phi_{1/2}^m \end{bmatrix}. \quad (11)$$

Here $p_0 = \sqrt{\mathbf{p}^2 + M^2}$ and $\chi_{1/2}^m$ and $\phi_{1/2}^m$ are two component Pauli spin functions with spin projection $m = \pm 1/2$; it is sufficiently accurate for our work to consider $p_0 \cong M$. Note that p_0 does not include any interaction. With U_s and U_v included, the wave functions (10) and (11) are modified only by replacing M by $\tilde{M} = M + U_s$. Note that this replacement tends to enhance the small components of (10) and (11) by a factor $\sim M/\tilde{M}$ which is ~ 1.7 at nuclear density.

The mean field, Fig. 1, will now admix negative-energy states into ψ_{+i} , as shown in Fig. 2. The correction to the positive energy wave function is

$$\delta\psi_i = \frac{\Lambda_-}{2\sqrt{\mathbf{p}_i^2 + M^2}} (\beta U_s + U_v) \psi_{i+}. \quad (12)$$

Since $\Lambda_- \psi_{i+} = 0$, we find, approximating $\sqrt{\mathbf{p}_i^2 + M^2}$ by M ,

$$\begin{aligned} \delta\psi_i &\cong \frac{1}{2M} [\Lambda_-, (\beta U_s + U_v)] \psi_{i+} \\ &\cong \frac{\beta \boldsymbol{\alpha} \cdot \mathbf{p}_i U_s}{2M^2} \psi_{i+}, \end{aligned} \quad (13)$$

since only the $\boldsymbol{\alpha} \cdot \mathbf{p}$ term in H_D , Eq. (2), does not commute with

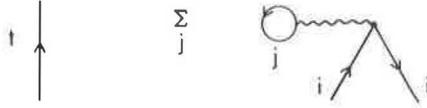


FIGURE 2 Tadpole diagram which in mean-field approximation admixes negative-energy states into ψ_{i+} . The upward line on the right represents a positive-energy wave function; the downward one, a negative-energy one.

$(\beta U_s + U_v)$. Thus, only the scalar interaction U_s connects positive and negative energy states. Since

$$\alpha = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix}, \quad \beta\alpha = \begin{pmatrix} 0 & \sigma \\ -\sigma & 0 \end{pmatrix} \quad (14)$$

in our representation, the correction to the large components of ψ_{+i} can be neglected and

$$\delta\psi_i \cong - \left[\frac{U_s \sigma \cdot \mathbf{p}_i}{2M^2} \chi_{i/2}^m \right]. \quad (15)$$

Note that this is just the first-order correction to ψ_{i+} , Eq. (10), if the M in the denominator is replaced by $\bar{M} = M + U_s$ (as in the mean-field equation (5)). The expectation value of α in ψ_{+i} is

$$\psi_{+i}^\dagger \alpha \psi_{+i} = \frac{\mathbf{p}}{M}. \quad (16)$$

When M is replaced by \bar{M} in (10) this expectation value increases by the factor M/\bar{M} . This does not, however, mean that the current is increased by this factor, as we discuss in the next section.

The energies of the positive energy solutions to (5) are

$$\epsilon_p = U_v + \sqrt{\mathbf{p}^2 + (M + U_s)^2}. \quad (17)$$

To order U_s^2 the energy becomes

$$\epsilon_p = U_v + p_0 + \frac{MU_s}{p_0} + \frac{\mathbf{p}^2}{2p_0^3} U_s^2 + \dots \quad (18)$$

Let us see how the final term in (18) can be understood from diagrammatic perturbation theory. In second order the tadpole term, Fig. 2, with its matrix element $-(U_s/M)\boldsymbol{\sigma} \cdot \mathbf{p}$ enters squared, as in Fig. 3(a), to be multiplied by the energy denominator $(2M)^{-1}$ of the intermediate pair state. The leading relativistic correction of this process to the energy, for small $|\mathbf{p}| \ll M$, is therefore

$$\delta\epsilon_p = \left[\frac{U_s}{M} \right]^2 \frac{\mathbf{p}^2}{2M}, \quad (19)$$

which agrees with (18). Given that the average kinetic energy per nucleon is

$$\bar{T} = \left\langle \frac{\mathbf{p}^2}{2M} \right\rangle \cong 23 \text{ MeV} \left(\frac{\rho}{\rho_0} \right)^{2/3}, \quad (20)$$

and that $U_s \cong -400 \text{ MeV} (\rho/\rho_0)$, we see that the average correction is

$$\bar{\delta\epsilon} = \left[\frac{U_s}{M} \right]^2 \bar{T} \cong 4.2 \text{ MeV} \left[\frac{\rho}{\rho_0} \right]^{8/3}. \quad (21)$$

Celenza and Shakin, taking into account exchange effects and distortions, find $\bar{\delta\epsilon} = 3.6 (\rho/\rho_0)^{2.4}$, reasonably close to Eq. (21).

As we see from Fig. 3(b), the relativistic effects^{9,10} can be simply interpreted as a small density dependent correction to the mass m_s of the scalar (σ)-boson, in other words, as a medium correction. In the vacuum, the σ has self-energy contributions due to its coupling to virtual nucleon-antinucleon pairs. In the presence of nuclear matter, processes with momenta $p < p_F$ are forbidden by the

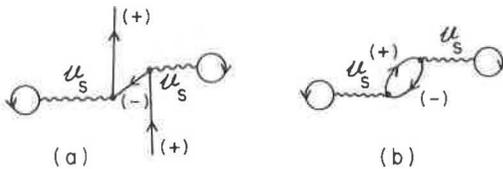


FIGURE 3 (a) Relativistic correction to the single particle propagation of a positive energy state; (b) corresponding correction to the energy.

Pauli principle. This gives rise to the repulsive contribution (21) to the energy per particle.

Corrections due to correlations and exchange terms bring $\overline{\delta\epsilon}$ of Eq. (21), in the case of the HM1 version of the Bonn potential,¹¹ down to

$$\overline{\delta\epsilon} \cong 2 \text{ MeV} \left[\frac{\rho}{\rho_0} \right]^{8/3}. \quad (22)$$

This correction tends to move the equilibrium density of nuclear matter to lower values, a desirable effect. At the same time it considerably stiffens the nuclear equation of state. A strictly $\rho^{8/3}$ -behavior of the mean energy per particle would imply an adiabatic index $\gamma = 11/3$ at high densities (the adiabatic index is $\gamma = \rho/P dP/d\rho$, where P is the pressure). In fact, the density dependence of $(\rho/\rho_0)^{8/3}$ obtained in perturbation theory is increased¹¹ to $(\rho/\rho_0)^{3.4}$ with inclusion of higher-order effects^{10,12}; this higher density dependence, implicit in the work of Ref. 9, arises because the $\delta\epsilon_p$ calculated in perturbation theory goes as M^{-3} (see Eq. (17)). Carrying higher-order terms, this M^{-3} becomes \tilde{M}^{-3} , which is substantially larger than M^{-3} , and has a large density dependence.

Simply adding the relativistic correction to the energies calculated in a nonrelativistic framework does not solve the nuclear matter problem. Detailed many-body calculations,¹³ which have reached a much higher degree of sophistication than the relativistic ones, underbind nuclear matter by several MeV at normal nuclear matter density ρ_0 , and give saturation at about $2\rho_0$ with roughly the correct binding energy. Obviously several MeV of attraction are needed. For example, Jackson, Rho and Krotschek⁹ have invoked the three-body term, in Fig. 4, which appears in chiral models, to counterbalance part of the relativistic correction $\overline{\delta\epsilon}$, Eqs. (21) and (22), and provide the small additional binding at nuclear matter density. Conventional three-body forces involving isobars in intermediate states also give additional binding of 1–2 MeV per particle at nuclear matter density with a weak density dependence.¹⁴

Considerations within nonlinear chiral models show that the process, Fig. 3, is only one of a number that must be considered.

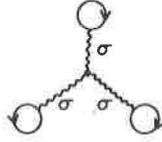


FIGURE 4 Three-body contribution to the energy of nuclear matter as it arises in chiral models.

Nevertheless, the result (22), together with a small attractive three-body term of the type in Fig. 4, appears to give a reasonable description of many-body effects, provided that the density is not too high.

CURRENTS

Let us now discuss relativistic effects on the current and on magnetic moments. In Fermi liquid theory the isoscalar current is given by

$$\mathbf{j} = \sum_{\sigma, \tau} \int \frac{d^3 p}{(2\pi)^3} n_p (\nabla_p \epsilon_p) \quad (23)$$

where n_p is the quasiparticle number in momentum state \mathbf{p} . Consider an n_p not far from equilibrium:

$$n_p = n_p^{(0)} + \delta n_p. \quad (24)$$

In this case

$$\begin{aligned} \mathbf{j} &= \sum_{\sigma, \tau} \int \frac{d^3 p}{(2\pi)^3} (n_p^{(0)} + \delta n_p) \nabla_p \left[\epsilon_p^{(0)} + \sum_{\sigma', \tau'} \int \frac{d^3 p'}{(2\pi)^3} f_{pp'} \delta n_{p'} \right] \\ &\equiv \sum_{\sigma, \tau} \int \frac{d^3 p}{(2\pi)^3} \left[\delta n_p (\nabla_p \epsilon_p^{(0)}) - \sum_{\sigma', \tau'} \int \frac{d^3 p'}{(2\pi)^3} f_{pp'} \delta n_{p'} \nabla_p n_p^{(0)} \right] \end{aligned} \quad (25)$$

where we have used the fact that the current is zero in the equi-

librium distribution and have integrated by parts to obtain the last term. Here $f_{pp'}$ is the Fermi-liquid interaction. Now

$$\nabla_p n_p^{(0)} = \frac{\partial n_p^{(0)}}{\partial \epsilon_p} \frac{\partial \epsilon_p}{\partial p} \hat{p} \quad (26)$$

and

$$\frac{\partial n_p^{(0)}}{\partial \epsilon_p} = -\delta(\epsilon_p - \epsilon_f) \quad (27)$$

where ϵ_f is the Fermi energy. Interchanging the labels p and p' in the last term of Eq. (25) and using

$$\sum_{\sigma', \tau'} \int \frac{d^3 p'}{(2\pi)^3} \delta(\epsilon_{p'} - \epsilon_f) f_{p'p} \hat{p}' = \frac{F_1}{3} \hat{p} \quad (28)$$

where

$$F_1 \equiv \frac{2p_f^2}{\pi^2} \left(\frac{\partial p}{\partial \epsilon_p} \right)_{p_f} f_1 \quad (29)$$

and f_1 is the first multipole of $f_{pp'}$, we obtain

$$\mathbf{j} = \sum_{\sigma, \tau} \int \frac{d^3 p}{(2\pi)^3} \delta n_p \left[1 + \frac{F_1}{3} \right] (\nabla_p \epsilon_p^{(0)}). \quad (30)$$

Note that F_1 is defined with the full 2 spin state, 2 isospin state density of states at the Fermi surface. We next use Eq. (13) of Ref. 15, obtained from Lorentz invariance:

$$\left[\frac{\partial p}{\partial \epsilon} \right]_{p_f} = \frac{\mu}{p_f} \left[1 + \frac{F_1}{3} \right] \quad (31)$$

where μ is the quasiparticle chemical potential. Note that the quantity $\mu(1 + F_1/3)$ plays the role of the effective mass M^* at the Fermi surface; $(\partial \epsilon / \partial p)_{p_f} = v_f$. Equation (31) shows that the density of states at the Fermi surface, which is proportional to $(\partial p / \partial \epsilon)_{p_f}$,

goes as $[\mu(1 + F_1/3)]$, replacing M^* in the nonrelativistic theory. Inserting (31) into (30) gives the isoscalar current

$$\mathbf{j} = \sum_{\sigma,\tau} \int \frac{d^3p}{(2\pi)^3} \delta n_p \frac{\mathbf{p}}{\mu}. \quad (32)$$

This is the relativistic generalization of the nonrelativistic expression, with μ replacing the bare mass M in the denominator.

The first term on the right-hand side of (30) involves $\nabla_p \epsilon_p^{(0)}$, which is the quasiparticle velocity \mathbf{v}_p . Because of the interactions, the moving quasiparticle causes a backflow given by the term involving F_1 . The sum of the two terms differs from the current $\mathbf{j} = \mathbf{p}/M$ of a quasiparticle in a noninteracting system only through binding energy effects, since $\mu - M$ is just the binding energy per particle at saturation density. In the nuclear case this is ~ 8 MeV for nuclei, ~ 16 MeV for nuclear matter, so this is a 1–2% effect. (In many articles in the literature, the backflow has been left out.) Modifications from interactions enter the current only through their effect on μ . The isovector current can be calculated similarly and we find

$$\mathbf{j}_{\text{isovector}} = \sum_{\sigma,\tau} \int \frac{d^3p}{(2\pi)^3} \frac{\mathbf{p}}{\mu} \frac{1 + F_1'/3}{1 + F_1/3} \tau_3 \quad (33)$$

where $F_1' \tau_1 \cdot \tau_2$ is the (dimensionless) Fermi-liquid isospin dependent interaction. The electromagnetic current is the average of the scalar and isovector terms (32) and (33):

$$\mathbf{j}_{\text{em}} = \frac{1}{2} \sum_{\sigma,\tau} \int \frac{d^3p}{(2\pi)^3} \frac{\mathbf{p}}{\mu} \left[(1 + \tau_3) + \frac{(F_1' - F_1)/3}{1 + F_1/3} \tau_3 \right]. \quad (34)$$

This is just the relativistic generalization of the corresponding expression in Migdal's book, Ref. 17. Note that the isospin-dependent part of the backflow in the electromagnetic current involves the velocity dependent effective proton–neutron interaction $F_1^{np} = F_1 - F_1'$.

Although it is easy to calculate the current from Eqs. (32)–(34), the way in which one arrives at these results can be quite com-

plicated in relativistic field theory. Bentz *et al.*¹⁶ have derived the results for the currents starting from the Ward identities associated with baryon and charge conservation. Let us consider how (32) can be derived diagrammatically in the relativistic field theory. As noted earlier, the small components of the Dirac single-particle wave functions are enhanced by a factor $\sim(M/\bar{M})$. Thus, the expectation value of the Dirac operator α which gives the quasiparticle velocity $v_f = (\partial\epsilon_p^{(0)}/\partial p)_{\epsilon_f}$ in (30) includes this factor. The backflow term, with coefficient $(1/3)F_1$ in Eq. (30), actually arises¹⁶ in the relativistic field theory from vacuum polarization, the lowest-order graph of which is shown in Fig. 5.

To see how the $(1/3)F_1$ correction arises from this diagrammatic calculation, we note that the bubble in Fig. 5(a) is just the current-current correlation function $\chi'_{ij}(q_0, q)$ which in the limit $|\mathbf{q}| \rightarrow 0$ involves a sum over virtual pairs with the Dirac operator α_i at one vertex, α_j at the other. In calculating the current in general, one should take the limit $|\mathbf{q}| \rightarrow 0$ first, then¹⁷ $q_0 \rightarrow 0$. (This procedure is followed because one wishes to guarantee that $\partial Q/\partial t = 0$, where Q is the total charge of the system, i.e., the $\mathbf{q} = 0$ component of the charge density ρ .) Keeping only the large components of the ψ_{+i} and ψ_{-i} , Eqs. (10) and (11), and using (19) for α , we find that the product of these matrix elements becomes $\text{Tr}[\sigma_i\sigma_j] = 2\delta_{ij}$ when one sums over spins. A further factor of 2 comes from the two

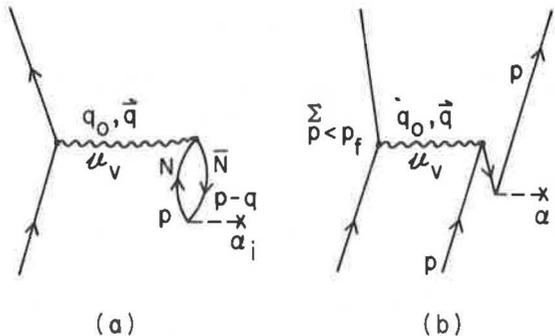


FIGURE 5 Vacuum polarization corrections to the current in lowest order. The α_i is the i th component of the Dirac α . Of the full vacuum polarization shown in (a) only the part modified by the medium—that resulting from blocking of virtual nucleon production by nucleons in nuclear matter—should be kept. That part is redrawn in (b).

possible time orderings and the denominator is $-2M$. Now, as in our treatment of virtual pairs admixed by the scalar field (see Fig. 3(b)), one should include only the modification of the vacuum polarization by the medium; the modification due to the presence of nucleons is to block virtual pairs for all particles $p > p_f$. The net result is

$$\chi'_{ij}(q_0, q) \equiv \frac{2\delta_{ij}}{M} \sum_{\tau} \int^{p_f} \frac{d^3p}{(2\pi)^3} = \frac{\rho}{M} \delta_{ij}.$$

The vector exchange interaction furnishes a factor $-g_v^2/m_v^2$ so that the contribution of the process in Fig. 5(b) to the current is

$$\delta \mathbf{j} = (\psi_{+i}^{\dagger} \boldsymbol{\alpha} \psi_{+i}) \left(-\frac{g_v^2}{m_v^2} \frac{\rho}{M} \right). \quad (35)$$

This is just the first-order correction to \mathbf{j} expected from vector-meson exchange on the basis of Eq. (32), since

$$\mu = M + \frac{g_v^2}{m_v^2} \rho \quad (36)$$

if only the vector coupling is present. From (30) we can identify

$$\frac{F_1}{3} = -\frac{g_v^2}{m_v^2} \frac{\rho}{M}. \quad (37)$$

To lowest order this is just the result of Baym and Chin,¹⁵ and Matsui.¹⁸ The full expression was obtained by Matsui¹⁸ by summing bubbles:

$$\frac{F_1}{3} = -\frac{g_v^2 \rho}{m_v^2} \frac{1}{\sqrt{p_f^2 + M^2} + g_v^2 \rho / m_v^2}. \quad (38)$$

The existence of an F_1 signifies a velocity-dependent interaction. Indeed, this F_1 arises directly from the velocity-dependent interaction (at zero frequency)

$$\delta V = -\frac{g_v^2}{4\pi r} \boldsymbol{\alpha}_1 \cdot \boldsymbol{\alpha}_2 e^{-m_v r} \quad (39)$$

in the vector-meson exchange. Clearly this term should be included in calculations.

One therefore sees how Fermi liquid parameters familiar from nonrelativistic theory emerge in a relativistic treatment. Relativistic effects in the current are small, of order B/M where B is the binding energy. These corrections are easily treated as small perturbations of the nonrelativistic theory. Note that we could have anticipated this result from the observation that the main part of the problem can be formulated in the space of positive-energy states, with corrections involving negative-energy states introduced by perturbation theory. In the positive-energy space, the individually large scalar and vector interactions mainly cancel, leaving only relatively small corrections. [Although small in magnitude, the great density dependence of $\bar{\delta}\epsilon$, Eq. (22), makes it important in the nuclear saturation.] The relativistic approach introduces considerable formalism to handle these small corrections. On the other hand, nonrelativistic many-body theory has been developed much further than the relativistic one. At best, the latter has reached the level of relativistic G -matrix calculations,¹⁹⁻²¹ whereas the nonrelativistic theory goes far beyond this in its treatment of many-body correlations.¹³

As we noted from Eq. (31), level spacings in the relativistic theory go as $[\mu(1 + F_1/3)]^{-1} \cong [\mu(1 - (g_v^2/m_v^2) \rho/M)]^{-1}$ where we use the lowest-order expression (37) for F_1 . (A similar result emerges in a nonrelativistic Brueckner Hartree-Fock calculation where the level spacings go as M^{*-1} .) Such a level spacing is substantially larger than that observed near the Fermi surface. Detailed analyses^{22,23} show that in heavy nuclei the level spacing in fact corresponds to an effective mass near unity;

$$\left(\frac{M^*}{M}\right)_{\text{Fermi surface}} = 1.15 \pm 0.10 \quad (40)$$

is deduced for ^{208}Pb . This larger effective mass near the Fermi surface can be understood²³ in terms of the coupling of quasiparticles near the Fermi surface to collective excitations which increase M^*/M and thus reduce the level spacing substantially.

MAGNETIC MOMENTS OF VALENCE NUCLEONS

Magnetic moments are a good example of how, by ignoring well-established consistency requirements, one can be led to “large” relativistic effects at the level of Hartree theory. The clearest example is the isoscalar Dirac magnetic moment, which involves the expectation value of $\boldsymbol{\alpha} \times \mathbf{r}$. As discussed just below Eq. (16), the expectation value of $\boldsymbol{\alpha}$ in the exact Dirac state equals \mathbf{p}/\bar{M} . One might therefore expect that the moment would be enhanced²⁴ by a factor $\bar{M} \cong 1.7$. Indeed, adding the first-order relativistic correction, Fig. 6, to the free Dirac moment, the nuclear magneton $e/2M$ is multiplied by a factor $(1 - U_s/M)$ [see Eq. (15) and following discussion]. Carried out to all orders, this would replace $e/2M$ by $e/2\bar{M}$. This effect is given a serious discussion at several places in the literature, although it clearly destroys our understanding of Schmidt lines and well-controlled corrections to these. The point is that the vacuum polarization effect involving $N\bar{N}$ pairs should be included [see Fig. 5 and discussion surrounding Eqs. (32) and (36)] and this cancels all but a small binding-energy correction from the M/\bar{M} “enhancement.”²⁵ The corrections to the isovector magnetic moments do not cancel as completely as for the isoscalar moments, as we see in Eq. (33), since $F'_1 \neq F_1$. The correction term to the orbital angular momentum part of the proton moment, from (34), is

$$(\delta g_l)_{\text{proton}} = \frac{1}{2} \frac{M}{\mu} \frac{(F'_1 - F_1)/3}{1 + F_1/3}. \quad (41)$$

After correction downwards for effects of high-energy virtual admixtures, chiefly from the two-body tensor interaction, this agrees well with experiment.²⁶

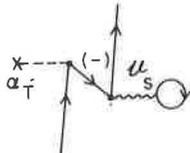


FIGURE 6 Renormalization of the transverse velocity α_T in nuclear matter.

With the exception of the factor M/μ this expression is identical to the nonrelativistic correction in Migdal's book.¹⁷ (A factor of 2 enters in Migdal's expression because of a different definition of the density of states.) As the relativistic and nonrelativistic formulas are essentially the same, the correction term Eq. (41) depends on the magnitude of the Landau-parameters F_1 and F_1' . In the relativistic mean field as well as in the nonrelativistic Brueckner Hartree–Fock calculations, the F_1 derived from the effective mass is very large and negative ($F_1' \approx -(1/2)F_1^{27}$) which is in disagreement with the observed density of states at the Fermi surface of heavy mass nuclei. Higher-order corrections which are not considered in these calculations increase the effective mass appreciably. If second-order corrections are included, Fantoni *et al.*²⁸ found that the effective mass in nuclear matter increased from 0.6 to 0.82. In finite nuclei, surface vibrations will further increase the effective mass. As long as these effects are not included in the relativistic mean field and nonrelativistic Brueckner Hartree–Fock calculations, a reliable estimate of δg_l from Eq. (41) is not possible.

Brown and Rho,²⁹ beginning with the nonrelativistic form of (34), found, from π - and ρ -exchange, Fock terms $(\delta g_l)_{\text{proton}} = 0.22$. However, this model underestimates the magnitude of F_1 as well as F_1' . Fujita and Hirata³⁰ have connected δg_l with the enhancement K' of the nuclear dipole sum rule in the giant resonance region:

$$K' \cong 2 (\delta g_l)_{\text{proton}}. \quad (42)$$

A recent accurate measurement³¹ for ²⁰⁹Bi gives $K' = 0.46 \pm 0.05$ in close agreement with the suggested δg_l .

THE SPIN-ORBIT INTERACTION

The primary success of Dirac phenomenology has been in predicting the very detailed structure in the spin-rotation parameter measured in high energy proton–nucleus scattering,³² once the strengths and shapes of the scalar and vector potentials have been chosen so as to reproduce the elastic scattering and polarization. Polarization and spin rotation in scattering of high energy nucleons arise because of the spin-orbit interaction, through a rather com-

plicated interference between waves from different paths of the nucleon through the nucleus. The density dependence of the spin-orbit interaction introduced through relativistic effects changes the interference patterns dramatically, improving greatly³² the fits to the data. Spin observables are the obvious domain for applications of relativistic mean field theory, since coupling to the spin emerges in a natural way in Dirac theory.

These successes are related to genuine relativistic effects in the spin-orbit interaction. The spin-orbit interaction results from the small components of the Dirac wave functions. To derive the spin-orbit interaction we first eliminate the small components from the Dirac equation (5), and find that the large components ψ (a two component spinor) obey the second-order equation:

$$(\mathbf{p}^2 + (M + U_s)^2 + W_{s.o.} + W_{r,p})\psi(\mathbf{r}) = (E - U_v)^2\psi(\mathbf{r}) \quad (43)$$

where the spin-orbit term is

$$W_{s.o.} = \frac{1}{(E + M + U_s - U_v)} \frac{\boldsymbol{\sigma} \cdot \mathbf{l}}{r} \frac{d}{dr} (U_v - U_s) \quad (44)$$

and

$$W_{r,p} = \frac{-i}{(E + M + U_s - U_v)} \frac{1}{r} \frac{d}{dr} (U_v - U_s) \mathbf{r} \cdot \mathbf{p}. \quad (45)$$

Equation (43) lends itself to scattering calculations since it has the same structure as a nonrelativistic Schrödinger equation for scattering, with a spin-orbit term $W_{s.o.}/2M$.³³ However, if we take the nonrelativistic limit of (43), we find the Pauli equation

$$\left[\frac{1}{2M(\mathbf{r})} \mathbf{p}^2 + U_s(\mathbf{r}) + U_v(\mathbf{r}) + V_{s.o.} + V_{r,p} \right] \psi(r) = (E - M)\psi(r) \quad (46)$$

where now

$$V_{s.o.} = \frac{\mathbf{l} \cdot \boldsymbol{\sigma}}{4M^2} \frac{1}{r} \frac{d}{dr} (U_v(r) - U_s(r)). \quad (47)$$

Since the observable nonrelativistic single-particle energy occurs on the right-hand side of the equation, we identify $V_{s.o.}$ as the spin-orbit potential. We note also that in deriving equivalent potentials to use in the Schrödinger equation from the relativistic theory, one should in fact keep terms quadratic in U_v and U_s , which reduce the magnitude of the central potential in (46) from $U_v + U_s \simeq -100$ MeV.

The process leading to the spin-orbit interaction can be described diagrammatically as in Fig. 7(a). As noted earlier, use of the exact Dirac spinor (10, 11 with \bar{M}) or in diagrammatic language, the admixture of negative-energy states by the process, Fig. 7(b), changes M^{-1} in Fig. 7(a) into \bar{M}^{-1} , resulting in a large enhancement of the spin-orbit potential at nuclear matter density. This enhancement, which is less significant in the surface region of finite nuclei, has been the main success of Dirac phenomenology in describing the behavior of spin observables.

In nuclear matter one should take into account Fermi liquid corrections to $U_{s.o.}$. In an infinite system, the screening of the scalar and fourth-component vector interaction, with the approximation that the scalar density $\rho_s \cong \rho$, would introduce a factor $(1 + F_0)^{-1}$. One cannot use this factor in finite nuclei in the local-density approximation because this factor blows up¹⁰ for $\rho \sim 2/3 \rho_0$. The fact that the simple screening correction blows up in local density approximation indicates that it may be appreciable in finite nuclei. In finite nuclei, one should rather introduce the induced coupling of the two nucleons via exchange of the giant monopole resonance.

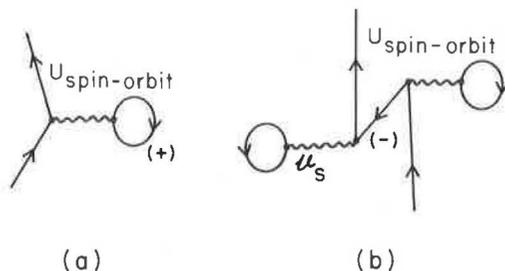


FIGURE 7 The spin-orbit interaction (47) results from the small components of the Dirac wave functions in the process (a). Coupling to negative-energy states (b) enhances $U_{s.o.}$.

These effects from interaction via vibrations are “shaken off” in high energy scattering and can be neglected there. Detailed calculations have yet to be carried out.

TIME COMPONENT OF THE NUCLEAR AXIAL CURRENT

The time component of the axial current $A_\mu(x)$ has recently received some interest³⁴ as an operator that couples to negative-energy states and is possibly strongly renormalized. As we argue, in the long wavelength limit the time component is insensitive to relativistic effects.

The axial current of a nucleon moving in Dirac scalar and vector mean fields, taking a pseudo-vector coupling to the pion field, is

$$A_\mu(q) = g_A \bar{\psi} \left(\gamma_\mu \gamma_5 - \frac{q_\nu q_\mu \gamma^\nu \gamma_5}{q^2 - m_\pi^2} \right) \psi, \quad (48)$$

where g_A is the axial current renormalization constant (and we suppress form factors and the isospin). Using the Dirac equation (5), we may write its divergence as

$$\begin{aligned} \partial^\mu A_\mu &= - \frac{im_\pi^2 g_A}{q^2 - m_\pi^2} \bar{\psi} \{ \gamma_5, \tilde{M} + \beta U_v \}_+ \psi \\ &\simeq - \frac{2ig_A m_\pi^2}{q^2 - m_\pi^2} (M + U_s) \bar{\psi} \gamma_5 \psi \end{aligned} \quad (49)$$

This equation, in the form of a Ward identity, connects the axial vector coupling vertex with the potential experienced by the nucleon. In (49), the vector part βU_v of the potential anticommutes with γ_5 and drops out; the scalar field breaks chirality, however.

In the limit in which momentum transfers are small ($|\mathbf{q}| \rightarrow 0$), as is appropriate for β decay, the space-like part $\nabla \cdot \mathbf{A} = i\mathbf{q} \cdot \mathbf{A}$

of the axial current divergence is small, and we may write Eq. (49) as

$$\dot{A}_0 = 2ig_A \frac{m_\pi^2}{q^2 - m_\pi^2} (M + U_s) \bar{\psi} \gamma_5 \psi. \quad (50)$$

The time derivative is replaced by a factor $-i\omega$ in nuclear matrix elements, where ω is the energy transferred to the nucleus.

We see now that in this limit A_0 is insensitive to relativistic effects, since the factor $(M + U_s)$ in Eq. (50) precisely cancels the characteristic enhancement factor $(M + U_s)^{-1}$ which arises from the coupling by γ_5 to the small components of the Dirac wave function. The energy transfer ω , on the other side of the equation, just reflects the nuclear excitation spectrum, which is well described by nonrelativistic theory. Hence as $|\mathbf{q}| \rightarrow 0$ the time-like axial current A_0 is not sensitive to relativistic effects. We should note, however, that at large momentum transfers, as in muon capture, the space-like term in (49) cannot be neglected and one cannot conclude that the relativistic corrections are small. Indeed the spatial component \mathbf{A} in (48) contributes significantly in muon capture.³⁵

CONCLUSIONS

The two important effects arising from the relativistic theory, as we have seen, are:

(1) A repulsive term in the energy per particle which varies with a high power of the density. Although the repulsive term is small in magnitude, it has an extremely strong density dependence, which is important for saturation. Since relativistic effects add only repulsion and since good two-body calculations underbind at nuclear matter density, clearly substantial attractive three-body forces must enter. Much of this contribution might be describable in terms of the underlying three-body force in the chiral Lagrangian.

(2) The spin-orbit interaction is enhanced, essentially by the inverse squared of the density dependent effective mass \bar{M} . This leads to the effects which are the main successes of Dirac phenomenology, which we have referred to only sketchily in this Comment.

APPENDIX: CONTACT WITH REALITY

The relativistic mean field model of Walecka and Serot¹ is a toy model, far from reality in many respects. The relationship between the Walecka–Serot model and realistic calculations have been made by Celenza and Shakin²; we sketch the most important points here.

First, what is the σ meson in the mean field theory? In nature there is no low-mass elementary σ . Attraction in this channel comes from the exchange of correlated two-pion systems, coupled to $J = 0, I = 0$. From the point of view of the chiral Lagrangian, the σ -meson starts out with a mass $m_\sigma \cong M$, if one is to understand the smoothness of soft-pion extrapolations, which go as q_π^2/m_σ^2 . However, this elementary σ couples to pions, as shown in Fig. 8, increasing the range of the σ -exchange interaction considerably beyond $\hbar m_\sigma c$; most of the range comes from the two-pion loops.³⁶ From the usual uncertainty principle argument applied to virtual states with two pions, one finds a range $\sim \hbar/2 \sqrt{m_\pi^2 + k^2}$ where k is a typical virtual pion momentum. Kinetic energies are usually at least of order of the pion rest mass, so this range can be estimated to be $\sim \hbar/4m_\pi$.

In the Bonn potential,³⁷ exchange of correlated two-pion systems produce the attraction, as above. However, in a rough approximation their exchange can be replaced by the exchange of an effective scalar meson with a mass $m_\sigma = 550$ MeV, not far from $4m_\pi$. This is the scalar meson in the Walecka model, and the one we have been discussing in the text.

Celenza and Shakin² have shown that with introduction of anti-symmetry, additional effective scalar interactions come from exchange terms in ω, π , etc., exchange. They have also shown that short-range correlations are quantitatively very important in the strongly repulsive vector-meson exchange.

The pion is neglected in the Walecka–Serot model; however, while the lowest-order pion exchange goes out with averages over

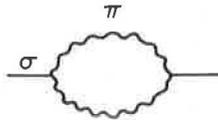


FIGURE 8 A typical modification in σ -exchange when coupling to pions is included.

spin and isospin in a Hartree mean-field theory, the second- and higher-order pion exchange terms do not. A rough but instructive way to include these higher-order interactions is as a second-order interaction with an average energy denominator³⁶

$$V_{\text{eff}}(r) = - \frac{V_{\text{tensor}}^2}{\bar{E}}, \quad (\text{A1})$$

where V_{tensor} is the tensor force from pion exchange (which gives most of the contribution) and³⁶

$$\bar{E} \cong \frac{3}{2} m_{\pi^*} \quad (\text{A2})$$

Note that the tensor interaction acts only in $S = 1$ states, and hence $V_{\text{eff}}(r)$ of (A1) cannot be summarized as an effective scalar interaction. Because of Pauli blocking in intermediate states and other effects, V_{eff} tends to decrease rapidly in magnitude with increasing density. This behavior, rather than repulsion from vector-meson exchange, is chiefly responsible for saturation in nuclear matter calculations. The vector-meson interactions are too short in range compared with the average distance between nucleons (~ 2.5 fm) to have much effect on saturation. (Because short-range correlations in the wave functions are neglected in the Walecka model, vector mesons there provide ~ 2.5 times more repulsion than they should.) Second- and higher-order tensor as well as exchange terms due to antisymmetry are included in the Dirac–Brueckner approach, as first done by Celenza and Shakin, and more recently in Refs. 20, 21 and 38.

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