

Introduction to Mirror Symmetry in Aspects of Topological String Theory

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Abstract. Under the compactification by the Calabi-Yau threefold, the string theory shows there is duality called mirror symmetry, which implies there is an isomorphism between two string theories under the compactifications of two topologically different internal manifolds. By twisting the topological string theory in two methods, the twisted theories named A -model and B -model have an isomorphism to each other under the mirror symmetry.

Keywords: mirror symmetry.

1. Introduction

Superstring theory predicts that our universe have 10 dimensions in spacetime and there should be 6 extra spatial dimensions compactified in a very small scale, and the corresponding compact space is called a Calabi-Yau manifold. Calabi in 1957 first conjectured the existence of such kind of manifolds [1] and then it was proved by Yau in 1977 [2][3]. There is a symmetry relation among the Calabi-Yau manifolds, which is called the mirror symmetry [4]. It implies a duality between the string theories with two topologically different Calabi-Yau manifolds. Such duality relation would play an important role in the topological string theory. In topological string theory, we can twist the theory in two different ways to obtain two theories, A -model and B -model. In 1991, Witten found that the mirror symmetry can also make the duality between two twisted models [5], and it links the Kähler structure on A -model to the complex structure on B -model.

2. Calabi-Yau Compactification

There are some requirements on the compact 6-dimensional spatial manifold M_6 . Firstly, the manifold should be a vacuum solution of the Einstein's field equation, which also means it should be Ricci-flat. On the other hand, the manifold is also required to preserve some supersymmetries rather than break them all, so it implies that the manifold needs to be a Kähler manifold [6][7]. All the requirements leads that manifold M_6 should be a compact Ricci-flat 6-dimensional Kähler manifold, that is to say, we need a Calabi-Yau three-fold for the extra spatial dimensions.

2.1. Calabi-Yau Manifold

A m -dimensional Calabi-Yau manifold is defined to be a compact, complex, Kähler manifold which has $SU(m)$ holonomy, where m is the complex dimension of the manifold [2]. The $SU(m)$ holonomy implies the corresponding Kähler manifold should be Ricci-flat, and therefore it has a vanishing first



Chern class, i.e. $c_1(M) = tr i\mathcal{R} / 2\pi$ is equal to $[\mathfrak{R} \setminus 2\pi]$. Furthermore, due to the Ricci-flat condition, there is a unique nowhere vanishing holomorphic $(m, 0)$ -form on the Calabi-Yau manifold which implies the Hodge number $h^{m,0} = 1$. Also, by using this non-vanishing $(m, 0)$ -form, one can make a isomorphism mapping $(0, p)$ -forms to $(0, m - p)$ -forms, where $1 \leq p \leq m$. Then the $SU(m)$ holonomy group does not allow a $(0, p)$ -form to be harmonic, which means the corresponding cohomology is trivial, $h^{0,p} = 0$. It makes the Hodge diamond of a Calabi-Yau three-fold even simpler, one can see:

$$\begin{array}{ccccc}
 & & & 1 & \\
 & & & 0 & 0 \\
 & 0 & & h^{1,1} & 0 \\
 1 & & h^{2,1} & & h^{1,2} & 1 \\
 & 0 & & h^{2,2} & 0 & \\
 & & & 0 & 0 & \\
 & & & 1 & &
 \end{array} \tag{2.1.1}$$

where there is only two independent Hodge number in the diamond ,i.e. $h^{1,1}$ and $h^{2,1}$, due to the vertical and horizontal symmetry relations.

2.2. Moduli Spaces of Calabi-Yau Manifolds

According to the Yau’s theorem[2], The Calabi-Yau manifold is a Ricci-flat Kähler manifold. Thus it seems that one can perturb the metric $g_{\mu\nu}$ of the manifold to a new metric $g_{\mu\nu} + \delta g_{\mu\nu}$ without changing the Ricci flat condition[8][9],

$$R_{\mu\nu}(g) = 0 \implies R_{\mu\nu}(g + \delta g) = 0. \tag{2.2.1}$$

One can impose a coordinate condition to the system to fix the choice of coordinate, and the condition can be written as

$$\nabla^\mu \delta g_{\mu\nu} = 0, \tag{2.2.2}$$

which is pretty similar to fixing a gauge. Then one can re-express the perturbation (2.2.1) together with the coordinate condition as:

$$\nabla^\rho \nabla_\rho \delta g_{\mu\nu} - 2R_{\mu\nu}^{\rho\sigma} \delta g_{\rho\sigma} = 0 \tag{2.2.3}$$

By using the (anti-)holomorphic coordinate system, it shows that the perturbation can be applied by $\delta g_{i\bar{j}}$ in pure indices and $\delta g_{i\bar{j}}$ in mixed indices, because of the index structure of the metric and the Riemann tensor of the Kähler manifold.

Under the metric perturbation with mixed indices, it remains the original index structure of g and keeps the metric still Hermitian. The condition in (2.2.3) with $\delta g_{i\bar{j}}$ is equivalent to $(\Delta \delta g)_{i\bar{j}} = 0$, and $\delta g_{i\bar{j}}$ can be the components of a $(1,1)$ -form. Such $(1,1)$ -forms also correspond to the changes of the Kähler form Ω and therefore the Kähler class $[\Omega]$ of the manifold. Because the $(1,1)$ -form is also harmonic, it is uniquely associated to an element of $H_{\bar{\partial}}^{1,1}(M)$. Hence one can expand $\delta g_{i\bar{j}}$ in the basis of real $(1,1)$ -forms with dimension $h^{1,1}$:

$$\delta g_{i\bar{j}} = \sum_{\alpha=1}^{h^{1,1}} \tilde{t}^\alpha b_{i\bar{j}}^\alpha \quad \tilde{t}^\alpha \in \mathbb{R} \tag{2.2.4}$$

According to Yau’s theorem, there is a Ricci-flat Kähler metric for any $[\Omega + \delta\Omega]$ [2]. Hence, the perturbation with mixed indices can make the new metric still be Kähler metric and also deform the Kähler structure (which is parameterized by the Kähler class) at the same time. Therefore, it means the perturbation expressed in (2.2.4) is the Kähler moduli space of the Calabi-Yau manifold.

Under the metric perturbation with pure indices, it will not keep the Hermitian structure in the manifold and the Kähler metric is also not preserved. The condition (2.2.2) with δg_{ij} is equivalent to

$$\Delta_{\bar{\partial}} \delta g^i = (\bar{\partial} \bar{\partial}^\dagger + \bar{\partial}^\dagger \bar{\partial}) \delta g^i \quad (2.2.5)$$

where $\delta g^i = \delta g^i_j d\bar{z}^j = g^{i\bar{k}} \delta g_{\bar{k}j} d\bar{z}^j$. It is a harmonic (1,0)-form associated to an element of cohomology group $H_{\bar{\partial}}^{1,0}(M)$. As the transformation makes the metric not Kähler any more, it is needed to change the coordinate system so that the metric becomes Kähler again. However, one cannot use a holomorphic transformation to remove the pure-index perturbation, because such transformation does not change the type of the indices. Then there is a unique non-vanishing holomorphic (3,0)-form Ω in the Calabi-Yau manifold, and one can use it to define an isomorphism between $H_{\bar{\partial}}^{1,0}(M)$ and $H^{2,1}(M)$ by defining the complex (2,1)-forms

$$\Omega_{ijk} \delta g_l^k dz^i \wedge dz^j \wedge d\bar{z}^l \quad (2.2.6)$$

which is still harmonic. Then one can expand the (2,1)-form in a basis with dimension $h^{2,1}$, such that

$$\Omega_{ijk} \delta g_l^k = \sum_{\alpha=1}^{h^{2,1}} t^\alpha b_{ij\bar{l}}^\alpha \quad t^\alpha \in \mathbb{C} \quad (2.2.7)$$

which is called the complex structure moduli space.

3. Topological String Theory

Topological String Theory is a theory which only discusses how the topological properties affect the theory without caring the exact form of metric of the manifolds. Mostly the Calabi-Yau manifold has both Kähler structure and complex structure. One can twist the theory in two different ways to obtained two models called *A*-model and *B*-model, and these models will depend on the Kähler structure and the complex structure of the target Calabi-Yau space respectively. It may link two models to the two moduli spaces separately.

3.1. Supersymmetric Nonlinear Sigma Model

The Supersymmetric nonlinear sigma model with $N = (2,2)$ in two dimensions maps a Riemann surface Σ , i.e.the worldsheet, to Riemannian manifold M , i.e. the target space, with the metric g such that $\Phi: \Sigma \rightarrow M$. As the target space M is a complex manifold, one needs to use the coordinate z, \bar{z} on the worldsheet Σ and (anti-)holomorphic coordinate $\phi^i = \bar{\phi}^{\bar{i}}$ on M , where locally it can describe Φ in a function $\phi^i(z, \bar{z})$. Then Let K and \bar{K} be the canonical and anti-canonical line bundles on the Riemann surface Σ . Let TM be the complexified tangent bundle of M , and it can be decomposed as $TM = TM^+ \oplus TM^-$. Then the fermi fields of the model are ψ_+ and ψ_- , where ψ_+ can be projected in $K^{1/2} \otimes \Phi^*(TM^+)$ and $K^{1/2} \otimes \Phi^*(TM^-)$ denoted as ψ_+^i and $\psi_+^{\bar{i}}$ respectively and ψ_- can be projected in $\bar{K}^{1/2} \otimes \Phi^*(TM^+)$ and $\bar{K}^{1/2} \otimes \Phi^*(TM^-)$ denoted as ψ_-^i and $\psi_-^{\bar{i}}$ respectively. Then the action is:

$$S = 2t \int_{\Sigma} d^2z \left(\frac{1}{2} g_{IJ} \partial_z \phi^I \partial_{\bar{z}} \phi^J + i\psi_-^{\bar{i}} D_z \psi_-^i g_{i\bar{i}} + i\psi_+^{\bar{i}} D_{\bar{z}} \psi_+^i g_{i\bar{i}} + R_{i\bar{i}j\bar{j}} \psi_+^i \psi_+^{\bar{i}} \psi_-^j \psi_-^{\bar{j}} \right) \quad (3.1.1)$$

where d^2z is $-idz \wedge d\bar{z}$, t is the coupling constant, I and J are the real indices, $R_{i\bar{i}j\bar{j}} = g_{i\bar{k}} R^{\bar{k}}_{\bar{i}j\bar{j}}$ is the Riemann tensor of the target space M and $D_{\bar{z}}$ is the $\bar{\partial}$ operator on $K^{1/2} \otimes \Phi^*(TM)$ constructed by using the pullback of the Levi-Civita connection on TM . It can be expressed as

$$D_z \psi_+^i = \frac{\partial}{\partial z} \psi_+^i + \frac{\partial \phi^j}{\partial z} \Gamma_{jk}^i \psi_+^k \tag{3.1.2}$$

where Γ_{jk}^i is the affine connection of the target space M in holomorphic indices. Then it is similar for $D_{\bar{z}}$ (∂ operator on $\bar{K}^{1/2} \otimes \Phi^*(TM)$ with Levi-Civita connection $\Gamma_{\bar{j}\bar{k}}^{\bar{i}}$ on M).

3.2. *A-model*

In the *A-model*, it regards ψ_-^i and $\psi_+^{\bar{i}}$ as the sections of $\Phi^*(TM^+)$ and $\Phi^*(TM^-)$ and regards the ψ_+^i and $\psi_-^{\bar{i}}$ as the section of $K \otimes \Phi^*(TM^+)$ and $\bar{K} \otimes \Phi^*(TM^-)$, and the scalar fields ϕ^i and $\phi^{\bar{i}}$ remain the same. To classify the twisted fermionic fields, one can express the field in a new way:

$$\psi_+^i \equiv \psi_z^i \in K \otimes \Phi^*(TM^+) \tag{3.2.1.a}$$

$$\psi_-^i \equiv \chi^i \in \Phi^*(TM^+) \tag{3.2.1.b}$$

$$\psi_+^{\bar{i}} \equiv \chi^{\bar{i}} \in \Phi^*(TM^-) \tag{3.2.1.c}$$

$$\psi_-^{\bar{i}} \equiv \psi_{\bar{z}}^{\bar{i}} \in \bar{K} \otimes \Phi^*(TM^-) \tag{3.2.1.d}$$

In term of these renamed variables the action is

$$S = 2t \int_{\Sigma} d^2z \left(\frac{1}{2} g_{IJ} \partial_z \phi^I \partial_{\bar{z}} \phi^J + i \psi_{\bar{z}}^{\bar{i}} D_z \chi^i g_{i\bar{i}} + i \psi_z^i D_{\bar{z}} \chi^{\bar{i}} g_{i\bar{i}} + R_{i\bar{i}j\bar{j}} \psi_z^i \chi^{\bar{i}} \psi_{\bar{z}}^{\bar{j}} \chi^j \psi_{\bar{z}}^{\bar{j}} \right). \tag{3.2.2}$$

A-model is a topological cohomological theory[5][10], so one can write the Lagrangean in the form of $\{Q_A, V\}$ and the theory is also independent of t . Then the action of the theory is rewritten as

$$S' = it \int_{\Sigma} d^2z \{Q_A, V\} + t \int_{\Sigma} \Phi^*(\Omega) \tag{3.2.3}$$

where

$$V = g_{i\bar{i}} (\psi_z^i \partial_{\bar{z}} \phi^{\bar{i}} + \partial_z \phi^i \psi_{\bar{z}}^{\bar{i}}) \tag{3.2.4}$$

and Ω is the Kähler form of the target space, i.e. $\Omega = -i g_{i\bar{j}} dz^i \wedge d\bar{z}^{\bar{j}}$.

$$t \int_{\Sigma} \Phi^*(\Omega) = 2t g_{i\bar{i}} (\partial_z \phi^i \partial_{\bar{z}} \phi^{\bar{i}} - \partial_{\bar{z}} \phi^i \partial_z \phi^{\bar{i}}) \tag{3.2.5}$$

The Lagrangean can be partly written in the Q -exact form, and the other term is related to the Kähler structure of the target manifold. On the second term of the action, one can make the pullback Φ^* on the Kähler form Ω act on the worldsheet, and it becomes important that Ω is a closed form so that the integral will only depend on the cohomology class of $\Phi(\Sigma)$. We denote the cohomology class as $\beta \in H_2(M)$, and the integral of the Kähler form over it is written as $\Omega \cdot \beta$. Then it shows that

$$\int_{\Sigma} \Phi^*(\Omega) = \int_{\Phi(\Sigma)} \Omega = \Omega \cdot \beta \geq 0. \tag{3.2.6}$$

Then one can find the correlation relations of the *A-model* can be written as

$$\langle O_1 \dots O_n \rangle = e^{-it \Omega \cdot \beta} \int_M D\phi D\chi D\psi e^{-it \{Q_A, \int V\}} O_1 \dots O_n \tag{3.2.7}$$

where $D\phi$, $D\chi$ and $D\psi$ are the corresponding measures of the path integral. The correlation function does not have dependence of the metric on the term V , but it only depends on the metric of target space M via the Kähler form Ω from the term $e^{-it\Omega\beta}$. It means the theory does not depend on any structure which appears only in V , but it depend on the structure in rest term on the action. Therefore the A -model is independent of the choice of the complex structure, but it clearly depends on the choice of the Kähler class of the target space. It means that the theory is “half-topological” with respect to the target space, as it depends on “half” of the moduli of the Calabi-Yau manifold. In conclusion, the A -model theory only topologically depends on the Kähler moduli of the Calabi-Yau target space.

3.3. B -model

In the B -model, it will do the similar method as for the A -model. it twists ψ_+^i and ψ_-^i to be the section of $\bar{K} \otimes \Phi^*(TM^+)$ and $K \otimes \Phi^*(TM^-)$ respectively, and it also regards both $\psi_+^{\bar{i}}$ and $\psi_-^{\bar{i}}$ as the section of $\Phi^*(TM^-)$. Then the field ϕ^i and $\phi^{\bar{i}}$ remain the same as before. It shows the new twisted scalar fields $\psi_{\pm}^{\bar{i}}$ are both the space-time $(1,0)$ -forms, which is slightly different from the case in the A -model. Therefore, the new scalar field can be chosen in a more convenient way, and the renamed fields are:

$$\eta^{\bar{i}} = \psi_+^{\bar{i}} + \psi_-^{\bar{i}} \quad (3.3.1.a)$$

$$\theta_i = g_{i\bar{j}}(\psi_+^{\bar{j}} - \psi_-^{\bar{j}}) \quad (3.3.1.b)$$

$$\rho_z^i = \psi_+^i \quad (3.3.1.c)$$

$$\rho_{\bar{z}}^i = \psi_-^i \quad (3.3.1.d)$$

One of the reason that we introduce θ field with lower indices is that it may lead to some simpler expressions, e.g. $\{Q_B, \theta_i\} = 0$, but $\{Q_B, \theta^{\bar{i}}\} = -2\Gamma^{\bar{i}}_{\bar{j}k}\eta^{\bar{j}}\theta^{\bar{k}}$. The new action of the B -model is:

$$S = t \int_{\Sigma} d^2z \left(\frac{1}{2} g_{I\bar{J}} \partial_z \phi^I \partial_{\bar{z}} \phi^{\bar{J}} + i\eta^{\bar{i}} (D_z \rho_z^i + D_{\bar{z}} \rho_{\bar{z}}^i) g_{i\bar{i}} + i\theta_i (D_{\bar{z}} \rho_z^i - D_z \rho_{\bar{z}}^i) + R_{i\bar{i}j\bar{j}} \rho_z^i \rho_{\bar{z}}^{\bar{j}} \eta^{\bar{i}} \theta_k g^{k\bar{j}} \right). \quad (3.3.2)$$

The action of the B -model can also be rewritten partly in the form of $\{Q_B, V\}$ which is in the same way as what was done in the A -model section.

$$S = t \int_{\Sigma} \{Q_B, V\} + tW \quad (3.3.3)$$

where

$$V = g_{i\bar{j}}(\rho_z^i \partial_{\bar{z}} \phi^{\bar{j}} + \rho_{\bar{z}}^i \partial_z \phi^{\bar{j}}) \quad (3.3.4)$$

and

$$W = \int_{\Sigma} \left(-\theta_i D \rho^i - \frac{i}{2} R_{i\bar{i}j\bar{j}} \rho^i \wedge \rho^{\bar{j}} \eta^{\bar{i}} \theta_k g^{k\bar{j}} \right) \quad (3.3.5)$$

where D is the exterior derivative on the worldsheet Σ . The term W in the action is anti-symmetric in the exchange of the holomorphic and anti-holomorphic z indices, and it can be written as a differential $(1,1)$ -form. Then its integral over the two-dimensional worldsheet Σ is independent of the metric, so the only metric dependence of the action is in the term $\{Q_B, V\}$. Furthermore the variations of W with respect to the cohomology class of the Kähler form Ω on the target space M are Q_B -exact, so the metric dependence on the Kähler structure of the Calabi-Yau target space is also vanishing. However,

by looking through the anti-commutation relations between the B -model scalar supercharge operator and the untwisted original scalar field ϕ , one can find the asymmetry in the relation,

$$\{Q_B, \phi^i\} = 0 \quad \{Q_B, \phi^{\bar{i}}\} = -\eta^{\bar{i}}, \quad (3.3.6)$$

so that it implies the theory depends on the choice of the complex structure of the target space [11]. It shows that the B -model is also “half-topological” with respect to the target space, and the theory only topologically depends on the complex structure moduli of the Calabi-Yau target space.

4. Mirror Symmetry

Mirror symmetry can make two string theories isomorphic to each other by exchanging the two topological structures. This kind of duality relation will make it possible to calculate one theory by doing the calculation on the other. Such duality relation can make some tough problems more solvable by just find the answer in the dual case. The relation between two internal spaces may lead a new understanding of the string theory to the physicists nowadays.

Mirror symmetry is a conjecture that there are pairs of Calabi-Yau manifolds with different topological properties that imply the same superconformal field theory, and we call such pair of Calabi-Yau manifold a mirror pair. In the topological string theory, the mirror symmetry implies that the A -model in some target space is isomorphic to the B -model in the mirror pair.

In the aspect of Hodge numbers, the mirror symmetry implies an extra symmetry relation on the Hodge diamond for Calabi-Yau manifolds such that

$$h^{r,s}(X) = h^{n-r,s}(\hat{X}) \quad (4.1)$$

where X and \hat{X} are the mirror pair manifold, n is the complex dimension of the Calabi-Yau manifolds, $0 \leq r, s \leq n$. For the $n = 3$ case, one can find the mirror manifold through relation, but for the manifold that $h^{2,1} = 0$, there is not mirror symmetry since that the Hodge number $h^{1,1}$ of a Calabi-Yau manifold must be positive integer number[12]. By looking at the Hodge diamond of the Calabi-Yau threefold, one can find the symmetry switches the only two independent Hodge numbers to each other, i.e. $h^{1,1}(X) = h^{2,1}(\hat{X})$ and $h^{2,1}(X) = h^{1,1}(\hat{X})$. As the mirror symmetry exchange the dimensions of two cohomology groups, it would lead the isomorphism that $H^{1,1}(X) \simeq H^{2,1}(\hat{X})$ and $H^{2,1}(X) \simeq H^{1,1}(\hat{X})$. One can find that the Kähler moduli space and the complex structure moduli space are exchanged under the mirror symmetry. Such moduli exchange can be understood in a different direction. In the topological string theory, the A -model has the topological dependence of the only Kähler moduli on the target space, and the B -model only has the topological dependence of the complex structure moduli of the target space. Then the mirror symmetry exchanges the two moduli to lead the equivalence between two models.

$$\begin{array}{ccc} A\text{-model on } X & \leftrightarrow & \text{Kähler moduli of } X \\ & & \text{Mirror symmetry } \updownarrow (X, \hat{X}) \\ B\text{-model on } \hat{X} & \leftrightarrow & \text{complex moduli of } \hat{X} \end{array} \quad (4.2)$$

Consequently, the triple products of the cohomological fields of each model will also have an isomorphism under the mirror symmetry, and that will lead a kind of Yukawa coupling equivalence relation between two theories. It leads an isomorphism between $H^{1,1}$ and $H^{2,1}$ for the triple products. In the A -model the Yukawa three-point correlation function is computed as

$$\langle h_a, h_b, h_c \rangle = \kappa_{abc}^0(X) + \sum_{\beta \neq 0} n_\beta \int_\beta h_a \int_\beta h_b \int_\beta h_c \frac{e^{2\pi i \int_\beta \Omega^c}}{1 - e^{2\pi i \int_\beta \Omega^c}} \quad (4.3)$$

where n_β is the instanton number and $\beta \in H_2(X; \mathbb{Z})$ and $\Omega^{\mathbb{C}}$ is the complexified Kähler form. Then the corresponding dual case in B -model have the three point correlation defined as

$$\langle b_\alpha, b_\beta, b_\gamma \rangle = \int_X \Omega \wedge (\nabla_{b_\alpha} \nabla_{b_\beta} \nabla_{b_\gamma} \Omega) \quad (4.4)$$

where Ω is still the holomorphic 3-form on X and ∇_{b_α} is the Gauss-Manin connection taking a (r, s) class to a $(r + 1, s - 1)$ class.

5. Conclusion

It has been discussed how the compactification of string theory required the internal space to be 3-dimensional Calabi-Yau manifold and the two moduli spaces of such manifolds. Furthermore the two twisted $N = (2,2)$ topological string theories, A -model and B -model were also introduced and it showed that they are totally determined by the Kähler structure and the complex structure of the target space respectively. Then, the mirror symmetry conjecture were illustrated and it makes the mirror pairs by exchanging the two topological structures and makes an isomorphism between two string theories, including two twisted models.

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