ISBN: 978-3-95450-247-9 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2024-THPC59

INVESTIGATION OF TRANSVERSE NARROW-BAND IMPEDANCE BY COUPLED-BUNCH INSTABILITY MEASUREMENT IN CIRCULAR ACCELERATOR*

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Abstract

The transverse narrow-band impedance makes a major contribution to the transverse coupled-bunch instability, which may deteriorate the beam quality in multi-bunch, high-intensity circular accelerators. Thus, strict restriction on the transverse narrow-band impedance are implemented during the initial accelerator design phase. However, slight component structure deviations during the construction of accelerators and component modifications during the subsequent operation may lead to impedance difference from the design value. It is therefore more meaningful to obtain the impedance parameters of circular accelerators by beam experimental measurement during the machine operation. In this paper, by mode distribution of coupled-bunch instability and its growth rate, a method was proposed to obtain the transverse narrow-band impedance which is represented with an LRC resonator. In order to verify the effectiveness of the method, the numerical calculation with three known LRC resonators was used to check their difference and the fitted LRC resonator parameters are in good agreement with the setting values.

INTRODUCTION

To reduce the impact of transverse coupled-bunch instabilities and ensure the high-quality performance of beam, it is necessary to assess the impedance for each component in advance. Based on the analysis for beam instability threshold, the impedance restriction for each accelerator component was set and the global impedance model was estimated for the ring. However, the construction and upgrade of accelerators is a practical engineering project. During the decades of machine operation, many old components in accelerators will be replaced and new devices will be installed to upgrade its performance. Thus, it is necessary to study the global impedance of accelerators based on the beam instability measurement during the machine operation [1]. Since the coupled-bunch instability is driven by narrow-band impedances, it could be an effective method to obtain the impedance model from the coupled-bunch mode distribution and their growth rates.

THEORETICAL ANALYSIS

In accelerators, due to the discontinuities of beam pipe, the leading bunch interacts with vacuum components and produces an electromagnetic field, known as the wakefield. If this residual wakefield does not decay to zero before the arrival of subsequent bunches, it can lead to coupled oscillation between multi-bunches and thus drive coupled-bunch instabilities. Assuming M uniformly-filled bunches, each bunch with N_b particles, and defining the first-order transverse wakefield function as $W_1(z)$, the transverse linearized Vlasov equation cab be expressed as [2]

$$\begin{aligned}&\left(\Omega^{(\mu l)}-\omega_\beta-l\omega_s\right)\alpha_l R_l(r)=-i\frac{M\pi r_0\omega_s}{\gamma T_0^2\omega_\beta\eta}g_0(r)\\&\sum_{l'=-\infty}^{\infty}\int_0^\infty r'dr'\alpha_{l'}R_{l'}(r')\,i^{l-l'}\times\\&\sum_{q=-\infty}^{\infty}Z_1^\perp(\omega_q)J_l\left(\frac{\omega_q-\omega_\xi}{c}r\right)J_{l'}\left(\frac{\omega_q-\omega_\xi}{c}r'\right),\\&\omega_\xi=\frac{\omega_\beta\xi}{\eta},\end{aligned}\tag{1}$$

where ω_β is the transverse betatron frequency, ω_s is the longitudinal synchrotron frequency, R_l is the radial distribution of the l-th order perturbation, α_l is the corresponding coefficient of R_l, M is the bunch number, r_0 is the classical radius, η is the slippage factor, T_0 is the revolution period, g_0 is the normalized radial distribution function in the longitudinal phase space at the initial moment, $\omega_q=\omega_\beta+\mu\omega_0+pM\omega_0$ is the sampling frequency, ξ is the chromaticity, and J_l is the l-th order Bessel function.

The air-bag model is chosen as the longitudinal bunch particle distribution,

$$g_0(r)=\frac{N\eta c}{2\pi\omega_s\hat{z}}\delta(r-\hat{z}),\tag{2}$$

$$\rho_0(z)=\frac{N}{\pi\sqrt{\hat{z}^2-z^2}}\quad for\ \ z<|\hat{z}|,\tag{3}$$

where $\hat{z}$ is the maximum position of the longitudinal bunch particle distribution. Substituting this distribution into Eq. (1), the formula is extended as

$$\begin{aligned}&\left(\Omega^{(\mu l)}-\omega_\beta-l\omega_s\right)\alpha_l=-i\frac{Ic}{4\pi\nu_\beta(E_0/e)}\sum_{l'=-\infty}^{\infty}\alpha_{l'}i^{l-l'}\\&\times\sum_{q=-\infty}^{\infty}Z_1^\perp(\omega_q)J_l\left(\frac{\omega_q-\omega_\xi}{c}\hat{z}\right)J_{l'}\left(\frac{\omega_q-\omega_\xi}{c}\hat{z}\right).\end{aligned}\tag{4}$$

* Work supported by the National Foundation of Natural Sciences (Grant No. U1832132 and Grant No.12175249).
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ISBN: 978-3-95450-247-9 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2024-THPC59

When the beam intensity is not very high, the coupling between modes can be neglected and Eq. (4) is simplified to

$$\Omega^{(l)} - \omega_\beta - l\omega_s = -i\frac{Ic}{4\pi\nu_\beta (E_0/e)} \sum_{q=-\infty}^{\infty} Z_1^{\perp}(\omega_q) J_l^2\left(\frac{\omega_q - \omega_\xi}{c}\hat{z}\right). \tag{5}$$

Since the lowest mode, $l = 0$, plays a dominant role in the instability, the complex frequency shift is

$$\Omega - \omega_\beta = -i\frac{Ic}{4\pi\nu_\beta (E_0/e)} \sum_{q=-\infty}^{\infty} Z_1^{\perp}(\omega_q) h_0(\omega_q - \omega_\xi). \tag{6}$$

$$h_0(\omega) = J_0^2\left(\frac{\omega}{c}\hat{z}\right), \tag{7}$$

$$\omega_q = \omega_\beta + \mu\omega_0 + pM\omega_0. \tag{8}$$

Thus, the growth rate of transverse coupled-bunch mode is

$$\frac{1}{\tau} = -\frac{Ic}{4\pi\nu_\beta (E_0/e)} \sum_{q=-\infty}^{\infty} Re\, Z_1^{\perp}(\omega_q) h_0(\omega_q - \omega_\xi). \tag{9}$$

Equation (9) shows the impact of the energy spectrum $h_0(\omega)$ on the mode growth rate. When $h_0(\omega)$ is not considered, the growth rate is simply proportional to the current. However, since $h_0(\omega)$ contains the current-related term $\hat{z}$, the growth rate will not be linearized with the beam current.

METHOD OF OBTAINING TRANSVERSE NARROW-BAND IMPEDANCE FROM COUPLED-BUNCH MODE GROWTH RATE

Equation (9) indicates some methods to obtain the narrow-band impedances from coupled-bunch instabilities: (1) fill bunches uniformly in the ring with different bunch numbers. If M_1 and M_2 bunches filling pattern cause the same mode unstable, then the corresponding narrow-band impedance which drives this mode unstable is definitely at $\omega_\beta + \mu\omega_0 + pM_1M_2\omega_0$. Because $pM_1M_2\omega_0$ is greatly larger than $pM_1\omega_0$ and $pM_2\omega_0$, the position of the narrow-band impedance is restricted to a much fewer possible frequencies. This method is easy to operate but the accurate position of the narrow-band impedance cannot be obtained; (2) keep the current and lattice parameters stable and change the chromaticity so that ω_ξ equals ω_r of a certain narrow-band impedance. Then the peaks of $h_0(\omega)$ and the narrow-band impedance are at the same position and the growth rate of the unstable mode driven by this narrow-band impedance reaches its maximum. Therefore, by finding the value of ξ which makes the mode most unstable, we can obtain the position of the narrow-band impedance with $\omega_r = \omega_\xi$. This method is feasible theoretically, but it is difficult to maintain the beam stable when widely changing the chromaticity in an accelerator; (3) keep the current and lattice parameters stable, only change the slippage factor so that ω_ξ equals ω_r of a certain narrow-band impedance. This method is similar to the second method; (4) keep the chromaticity constant, change the current and measure mode growth rates, then use the nonlinear relationship shown in Eq. (9) to obtain the narrow-band impedances. This paper will explore on the last method.

The transverse narrow-band impedance is usually presented by an LRC resonator [2]

$$Z_1^{\perp} = \frac{c}{\omega}\frac{R_s}{1 + iQ\left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r}\right)}, \tag{10}$$

where R_s is the shunt impedance which represents the impedance magnitude, ω_r is the resonant frequency which represents the position of the resonance peak, and Q is the quality factor which represents the width of the resonance peak.

From Eq. (9), if one of the sampling frequencies for a certain mode equals or is close to the resonant frequency of the narrow-band impedance, ω_r, this mode will be unstable. The frequency relationship satisfies

$$\omega_\beta + \mu\omega_0 + pM\omega_0 \approx -\omega_r. \tag{11}$$

Thus, only the sampling frequency $-\omega_r$ in Eq. (9) is remained and Eq. (9) is simplified to

$$\frac{1/\tau}{I} = \frac{c^2}{4\pi\nu_\beta (E_0/e)}\frac{R_s}{\omega_r} J_0^2\left[\left(\omega_r + \omega_\xi\right)\hat{z}/c\right]. \tag{12}$$

By measuring the growth rates of the most unstable mode and bunch lengths in different currents, a series of results about $\frac{1/\tau}{I} \sim \hat{z}$ can be obtained. By fitting these measurements with the zeroth-order Bessel function in Eq. (9), we can obtain the narrow-band impedance model. Since the quality factor Q in the LRC resonator represents the width of the resonance peak, the value of Q maybe be acquired by considering the growth rates of modes nearby.

NUMERICAL VERIFICATION

In order to verify the method effectiveness, a numerical calculation is conducted with BEPCII parameters [3]. Assuming there are three vertical LRC resonators in the ring which represent three narrow band impedances with same shunt impedance and quality factor, $R_s = 250\,\mathrm{M\Omega\,m^{-2}}$, $Q = 1000$, and different resonant frequencies ω_r , $2\pi \times 1.473\,66\,\mathrm{GHz}$, $2\pi \times 3.768\,88\,\mathrm{GHz}$, and $2\pi \times 7.115\,60\,\mathrm{GHz}$, respectively, which are listed in Table 1.

During this verification, the bunch length in different current was used, which was due to the longitudinal broadband effective impedance [3].

Based on the fitted bunch lengthening relation, the growth rates of the most unstable modes driven by these three

15th International Particle Accelerator Conference,Nashville, TN JACoW Publishing
ISBN: 978-3-95450-247-9 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2024-THPC59

Table 1: Setting Value, Fitted Value, and Error of Resonator Parameters

$\omega_r/2\pi$(GHz)			Q			R_s(MΩ m^{-2})		
Setting	Fitted	Error	Setting	Fitted	Error	Setting	Fitted	Error
1.47366	1.473668347	0.00057 %	1000	1009.3	0.93 %	250	252.14525	0.85810 %
3.76888	3.768866612	−0.00036 %	1000	1000.8	0.08 %	250	250.07658	0.03063 %
7.11560	7.115627666	0.00039 %	1000	1001.8	0.18 %	250	249.90719	−0.03712 %

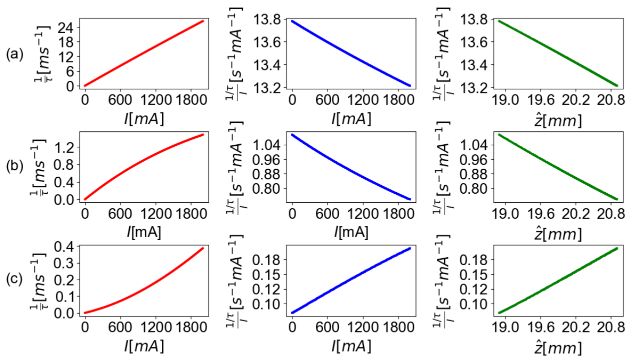

Figure 1: The growth rates of the most unstable modes versus current (red lines), the ratio of growth rate to beam current versus current (blue lines), and the ratio versus bunch length (green lines): (a) mode 15 with $\omega_r = 2\pi \times 1.473\,66$ GHz; (b) mode 43 with $\omega_r = 2\pi \times 3.768\,88$ GHz; (c) mode 35 with $\omega_r = 2\pi \times 7.115\,60$ GHz.

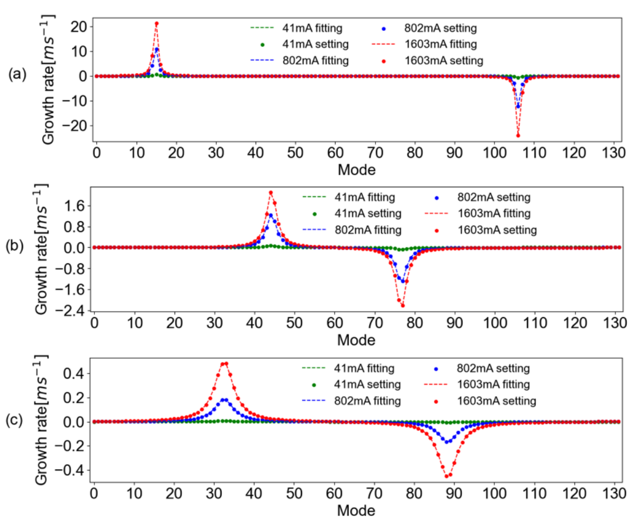

Figure 2: The mode growth rate and distribution for fitted impedance model (dash lines) and setting model (dotted lines): (a) mode 15 with $\omega_r = 2\pi \times 1.473\,66$ GHz; (b) mode 43 with $\omega_r = 2\pi \times 3.768\,88$ GHz; (c) mode 35 with $\omega_r = 2\pi \times 7.115\,60$ GHz.

LRC resonators in different currents can be calculated using Eq. (12), shown in Fig. 1 (red lines). The mode growth rate and mode distributions in three different currents are shown in Fig. 2 (dotted lines, 41 mA, 802 mA, 1603 mA as example). Since the resonant frequencies ω_r of these three LRC resonators are different, the most unstable mode position is also different, corresponding the mode number 15, 43, and 35, respectively. The ratio of growth rate to beam current versus beam current and bunch length $\hat{z}$ can also be calculated, as shown in Fig. 1 (blue and green lines).

Assuming the relation of mode growth rates with bunch length in different beam currents (mode 15 & 43 & 35 in Fig. 1) are known, the three LRC resonator parameters (ω_r & Q & R_s) can be fitted out with Eq. (12) and then compared with the setting values, shown in Table 1. With the fitted impedance model, the mode distribution and its growth rate are calculated and shown in Fig. 2 (dash lines). The result in Table 1 and Fig. 2 show a good consistency between the fitted results and setting values. These numerical calculations verified the method to obtain the impedance model from coupled-bunch mode distribution and growth rates. Usually, the bunch-by-bunch feedback system can be used to obtain the instability growth rates in different beam currents [1, 4], so the accurate transverse narrow-band impedance model can be presented.

CONCLUSION

By theoretical analysis, the transverse coupled-bunch instability including the longitudinal bunch particle oscillation and distribution was discussed, which shows the growth rate can be modulated by chromaticity and bunch length. From the expression on growth rate, Eq. (9) ~ Eq. (12), a method to get transverse narrow-band impedance model by instabilities measurements was proposed and its availability was verified by simulation. The verification shows that the error in ω_r is less than 0.001 % and the errors in R_s and Q are both less than 1 %.

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