A NEW METHOD FOR DETECTING BARYON ACOUSTIC OSCILLATIONS

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Baryon Acoustic Oscillations (BAOs) are a feature imprinted in the galaxy distribution by acoustic waves traveling in the plasma of the early universe. Their detection at the expected scale in large-scale structures strongly supports current cosmological models with a nearly linear evolution from redshift $z \approx 1000$ and the existence of dark energy. Besides, BAOs provide a standard ruler for studying cosmic expansion. We study BAO detection methods using the correlation function measurement $\hat{\xi}$ which can be formulated as an hypothesis test between $\mathcal{H}_0$ (no-BAO hypothesis) and $\mathcal{H}_1$ (BAO hypothesis). We describe problems with the classical method based on the $\Delta\chi^2$ statistic and we propose a new method, the Δl method, to overcome these difficulties.

1 BAOs in the correlation function

BAOs are relic of acoustic waves which traveled in the plasma before recombination. They stopped to propagate around the time of recombination, leaving a small excess of power at the sound horizon scale ($r_s \approx 150$ Mpc) in the matter distribution. As a consequence, BAOs can be detected as a peak around 150 Mpc in the correlation function of the galaxy distribution. Note that the BAO peak in the correlation function and its whole shape has a dependence on cosmological parameters. It is also possible to construct correlation functions without BAOs, by artificially erasing the BAO peak using a 'no wiggles' form[2] or setting a baryon density $\Omega_b h^2 = 0$. In order to estimate the correlation function we will use the Landy-Szalay[5] estimator $\hat{\xi}$.

2 A new method for BAO detection

BAO detection can be formulated as an hypothesis test

$$\mathcal{H}_0 \quad : \quad \exists\theta \in \Theta \text{ s.t. } \hat{\xi} \sim \mathcal{N}\left(\xi_{noBAO,\theta}, C_{noBAO,\theta}\right) \qquad (1)$$

$$\mathcal{H}_1 \quad : \quad \exists\theta \in \Theta \text{ s.t. } \hat{\xi} \sim \mathcal{N}\left(\xi_{BAO,\theta}, C_{BAO,\theta}\right) \qquad (2)$$

The classical BAO detection method makes the assumption that covariance matrices are constant ($C_{noBAO,\theta} = C_{BAO,\theta} = C$). It is based on the $\Delta\chi^2$ statistic, which can be thought as a generalized likelihood ratio (the optimal statistic to test between simple hypotheses is the likelihood ratio according to the Neyman-Pearson lemma)

$$\Delta\chi^2 = \min_\theta \chi^2_{noBAO,\theta}(\hat{\xi}) - \min_\theta \chi^2_{BAO,\theta}(\hat{\xi}) \qquad (3)$$

$$= -2\log\left[\frac{\max_\theta \mathcal{L}_{noBAO,\theta}(\hat{\xi})}{\max_\theta \mathcal{L}_{BAO,\theta}(\hat{\xi})}\right] \qquad (4)$$

Table 1: Expected significance averaged over all $\mathcal{H}_1$ models in the two different cases of constant C and model-dependent C_θ.

	Classical $\sqrt{\Delta\chi^2}.\sigma$ (wrong)	$\Delta\chi^2$ with correct significance using Eq. 5	Δl method
Constant C	2.21σ	2.0σ	2.0σ
Model-dependent C_θ	2.32σ	1.59σ	1.96σ

Given some regularity assumptions, one can show[4] that the BAO detection significance (i.e. the rejection of the $\mathcal{H}_0$ hypothesis) can be estimated as $\sqrt{\Delta\chi^2}.\sigma$. However these assumptions are usually wrong, so that the classical method can overestimate the detection significance. Another problem is that it only works for hypotheses with a constant covariance matrix which can be a poor estimation of the real hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$ of Eq. (1) and (2).

For these reasons we propose a new method, that we call the Δl method. First we modify the procedure to obtain the significance of the BAO detection, in order to make it rigorous. For a constant covariance matrix, the significance of the detection as a p-value $p(x)$ is computed as the p-value of $\Delta\chi^2 = x$ in the 'worst-case' $\mathcal{H}_0$ model

$$p(x) = \max_{\theta\in\Theta} P(\Delta\chi^2 \geq x \,|\, \mathcal{H}_0, \theta) \quad (5)$$

The second modification that we propose is to extend the statistic $\Delta\chi^2$ in the case of varying covariance matrices $C_{noBAO,\theta}$ and $C_{BAO,\theta}$ in (1) and (2). We call the new statistic the Δl statistic, as it is a difference of generalized log-likelihoods

$$\Delta l = -2\left[\max_\theta \log \mathcal{L}_{noBAO,\theta}(\hat{\xi}) - \max_\theta \log \mathcal{L}_{BAO,\theta}(\hat{\xi})\right] \quad (6)$$

3 Results

We test our new method and compare it to the classical method using lognormal simulations of the Luminous Red Galaxies sample of the Sloan Digital Sky Survey Data Release 7[4]. We estimate a covariance matrix C from 2000 simulations with realistic parameters. As a toy example we only take into account a model-dependent covariance matrix of the form $C_\theta = b^4 C$ (i.e. we only take into account a approximate dependence on b).

We show in Table 1 the average significances under $\mathcal{H}_1$ in the 2 different cases of constant C and model-dependent C_θ for different methods.

We obtain the following results:

- $\sqrt{\Delta\chi^2}.\sigma$ slightly overestimates the significance for hypotheses with constant C
- $\sqrt{\Delta\chi^2}.\sigma$ grossly overestimates the significance for hypotheses with model-dependent C_θ
- Δl largely outperforms $\Delta\chi^2$ for hypotheses with model-dependent C_θ

References

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