

APPROVED BY

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Fixed Point Newton-Cotes Quadrature, MU NCI2

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PURPOSE

To approximate the value of an integral $\int_a^b zy^2 dx$ by means of a Newton-Cotes Quadrature formula with all additions done in double precision. The variables z and y are both functions of x .

RESTRICTIONS

Two auxiliary subroutines are required whose location symbols (DPA2 and FACT) must be defined at assembly time. MU DPA2, MURA Fixed Point Double Precision Addition routine is necessary or possibly another equivalent routine may be used provided it is labeled DPA2. In the latter case special precaution must be observed in placing the double-length sum in COMMON and COMMON + 1 as well as in the AC-MQ. A second subroutine labeled FACT is also required. This subroutine shall serve the sole purpose of providing the quantity z and placing it in the accumulator. Its return must be 1 location beyond the transfer instruction which entered it initially.

No internal checks are made on overflow, since it is left to the user to ensure that all numbers remain in the range $-(1-2^{-35})$ to $+(1-2^{-35})$.

METHOD

The Newton-Cotes formula used is:

$$\int_{x_0}^{x_4} zy^2 dx = \frac{1}{45N} (14z_0y_0^2 + 64z_1y_1^2 + 24z_2y_2^2 + 64z_3y_3^2 + 14z_4y_4^2)$$

where N is the total number of subdivisions in the entire integration. N is always a multiple of 4 and the condition $N \geq 4$ must also hold.

The $(N + 1)$ ordinates $y_0, y_1, \dots, y_N$ are assumed to be stored in the core memory as 35 bit numbers beginning at location B which is specified in the

calling sequence. The ordinates are to be stored in sequence with d words separating each ordinate. The parameter d is also specified in the calling sequence and can assume any value from 0 to 7.

Range: $|y| < 1$
 $|z| < 1$

Accuracy: An approximation to the truncation error is given by $\frac{8 y^{(6)} h^7}{945}$,

where $y^{(6)}$ is the value of the sixth derivative at some point between x_0 and x_4 and h is precisely $1/N$.

(Cf. Milne, Numerical Calculus, The Princeton University Press, New Jersey, 1955, Chapter IV, pp. 122-124.)

USAGE

Calling Sequence:

UA SAP		MURASS	
Loc.	Instruction	Loc.	Instruction
α	TSX NCI2, 4	α	B TSX A 4
$\alpha + 1$	(prefix) B, O, N	$\alpha + 1$	C d B O N

where (prefix) $\equiv d$ is number of words separating each ordinate, y ; B is the address of the first ordinate; and N is the total number of subdivisions in the integral. Note that some caution must be observed in the writing of the FACT subroutine regarding the term z_0 on integration steps other than the initial one. In these instances the z_0 is always the same quantity as the previous z_4 .

The routine is left with double-precision sum in the AC-MQ and also in COMMON + 2 and COMMON + 3 respectively.

Space Required:

77 words program at
 11 words temporary at

UA SAP

NCI 2
 COMMON

MURASS

A
 T

For an assembly with MURASS the location symbols Y and Z must be defined in place of DPA2 and FACT.

Relocatable Cards: The nominal addresses in the relocatable program deck have been defined as follows:

Symbolic address	Nominal address
NC12	0
DPA2	3000 ₈
FACT	4000 ₈
COMMON	2000 ₈

CODING INFORMATION

Timing: $(3.47N + 1.5)$ milliseconds exclusive of the time spent in the auxiliary subroutines DPA2 and FACT.

12	REM NCI2 NEWTON-COTES QUADRATURE	MU	NCI2
	SXD COMMON+4,1	NCI2	0001
	SAVE INDEX REGISTERS	NCI2	0002
	SXD COMMON+5,2	NCI2	0003
	SXD COMMON+6,4	NCI2	0004
	CAL 1,4	NCI2	0005
	PARAMETER WORD IN AC	NCI2	0006
	STA NCI2+34	NCI2	0007
	PLACE ADDRESSES	NCI2	0008
	STA NCI2+35	NCI2	0009
	PDX 0,4	NCI2	0010
	PLACE N IN X4	NCI2	0011
	ARS 15	NCI2	0012
	COM	NCI2	0013
	COMPLEMENT D	NCI2	0014
	STD NCI2+5,2	NCI2	0015
	D TO TXI INSTRUCTION	NCI2	0016
	PXD 5,4	NCI2	0017
	N INTO AC	NCI2	0018
	LXA NCI2+6,2	NCI2	0019
	M=0 TO X2	NCI2	0020
	CAS NCI2+76	NCI2	0021
	COMPARE N WITH 5	NCI2	0022
	ARS 1	NCI2	0023
	HALVE N	NCI2	0024
	TXI NCI2+12,2,1	NCI2	0025
	ADD 1 TO M	NCI2	0026
	STO COMMON+7	NCI2	0027
	STORE SCALED N	NCI2	0028
	ALS 2	NCI2	0029
	ADD COMMON+7	NCI2	0030
	5N	NCI2	0031
	STO COMMON+7	NCI2	0032
	ALS 3	NCI2	0033
	40N	NCI2	0034
	ADD COMMON+7	NCI2	0035
	45N	NCI2	0036
	ALS 9	NCI2	0037
	POSTITION 45N	NCI2	0038
	STO COMMON+7	NCI2	0039
	STORE TO (C+7)	NCI2	0040
	PXD 0,2	NCI2	0041
	M TO AC	NCI2	0042
	ARS 18	NCI2	0043
	M TO ADDRESS POSTITION	NCI2	0044
	PLACE M IN SHIFT ORDERS	NCI2	0045
	STA NCI2+45	NCI2	0046
	STA NCI2+51	NCI2	0047
	STZ COMMON+2	NCI2	0048
	CLEAR SUMS	NCI2	0049
	STZ COMMON+3	NCI2	0050
	LXA NCI2+6,2	NCI2	0051
	J=0 TO X2	NCI2	0052
	LXA NCI2+10,1	NCI2	0053
	I=5 TO X1	NCI2	0054
	SXD COMMON+8,4	NCI2	0055
	SAVE X4	NCI2	0056
	TSX FACT,4	NCI2	0057
	SUBROUTINE TO SUPPLY Z	NCI2	0058
	STO COMMON+9	NCI2	0059
	STORE Z	NCI2	0060
	LDQ **,2	NCI2	0061
	Y	NCI2	0062
	MPY **,2	NCI2	0063
	Y SQUARED	NCI2	0064
	STO COMMON+10	NCI2	0065
	SOTRE MSP OF Y SQUARED	NCI2	0066
	MPR COMMON+9	NCI2	0067
	MULTIPLY LSP BY Z	NCI2	0068
	LRS 35	NCI2	0069
	PREPARE FOR DPA2	NCI2	0070
	STO COMMON	NCI2	0071
	STQ COMMON+1	NCI2	0072
	MSP	NCI2	0073
	LDQ COMMON+10	NCI2	0074
	MULTIPLY BY Z	NCI2	0075
	MPY COMMON+9	NCI2	0076
	TSX DPA2,4	NCI2	0077
	MULTIPLY ZY SQUARED BY A	NCI2	0078
	MPY NCI2+76,1	NCI2	0079
	LRS **	NCI2	0080
	RND	NCI2	0081
	ROUND LSP	NCI2	0082
	LDQ COMMON	NCI2	0083
	STO COMMON+1	NCI2	0084
	STZ COMMON	NCI2	0085
	MSP=0	NCI2	0086
	MPY NCI2+76,1	NCI2	0087
	LRS **	NCI2	0088
	TSX DPA2,4	NCI2	0089

CLA COMMON+2
LDQ COMMON+3
TSX DPA2,4
STO COMMON+2
STQ COMMON+3
TNX NCI2+60,1,1
TXI NCI2+32,2,**
LXD COMMON+8,4
TIX NCI2+30,4,4
DVH COMMON+7
STQ COMMON+2
DVH COMMON+7
STQ COMMON+3
CLA COMMON+2
LXD COMMON+4,1
LXD COMMON+5,2
LXD COMMON+6,4
TRA 2,4
OCT 016000000000
OCT 100000000000
OCT 030000000000
OCT 100000000000
OCT 016000000000
OCT 000005000000

FIRST HALF SUM
SECOND HALF SUM
ACCUMULATE

OUT AFTER 5 PASSES
DECREMENT D
RESTORE X4
OUT AFTER N/4 SUMMATIONS
DIVIDE BY 45N

DIVIDE AGAIN
RESULT IN (C+2) AND (C+3)
ALSO IN AC-MQ
RESTORE ALL INDEX REGISTERS

NCI2 0054
NCI2 0055
NCI2 0056
NCI2 0057
NCI2 0058
NCI2 0059
NCI2 0060
NCI2 0061
NCI2 0062
NCI2 0063
NCI2 0064
NCI2 0065
NCI2 0066
NCI2 0067
NCI2 0068
NCI2 0069
NCI2 0070
NCI2 0071
NCI2 0072
NCI2 0073
NCI2 0074
NCI2 0075
NCI2 0076
NCI2 0077