Applied Physics Research; Vol. 17, No. 1; 2025
ISSN 1916-9639 E-ISSN 1916-9647
Published by Canadian Center of Science and Education

Interpretation of Gravity and Interactions by Entropy

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Received: March 20, 2025 Accepted: April 24, 2025 Online Published: May 1, 2025
doi:10.5539/apr.v17n1p168 URL: https://doi.org/10.5539/apr.v17n1p168

Abstract

The purpose of this research is to provide models that interpret gravity and interactions by entropy. We introduce D_+-division partial entropy by applying ideas of the logistic function to its entropy and define the inverse of D_+-division partial entropy as the gravitational potential. By applying these concepts, we attempt to explain as follows: 1) Gravity becomes a constant value within small distances under specific conditions. Gravity may have 5-states within small enough distances. There may exist anti-gravity which is the opposite of Newton's gravity among 5-states. Furthermore, the gravitational potential and the Coulomb potential can be treated similarly within small distances. 2) The rotation speed of the galaxy does not depend on its radius if the radius is within the size level of the universe. 3) The gravitational acceleration toward the center may fluctuate at long distance compared to Newton's gravity. These become the extension of Newton's gravity and suggest that there may exist constants that control gravity and the speed of galaxies. Besides, it describes the relationship between the Yukawa type potential and D_--division partial entropy with negative. We attempt to suggest 11-types interactions including the gravitational acceleration g and compare the ratios of fundamental 4-interactions in nature using suggested equations. Furthermore, we mention that the relationship with entropic gravity, that the possibility of existence new interactions, and that the gravitational constant G can fluctuate if entropy changes. Gravity and electromagnetic and quantum may unify through entropy.

Keywords: entropy, gravity, galaxy rotation curve, MOND, Planck's law, dynamical systems, inverse square law, logistic function, Yukawa potential, entropic gravity, unification theory, theory of everything

1. Introduction

1. First, to make it easier the understanding, let us introduce the Boltzmann principle and the Planck distribution function. By applying Planck's ideas (Planck,1906), we define D_+-division entropy $S_{D_+}(x, k)$ and D_+-division partial entropy $S_{D_+}(x)$ divided by the division function $D_+(x)$, and introduce acceleration of partial entropy $S''_{D_+}(x)$ and the positive function $Q_{D_+}(x)$ as satisfied $Q_{D_+}(x) = \xi x/D_+(x)$, where x is a positive variable and ξ is a positive constant.

2. Second, by applying the idea of logistic function (May,1976; Wiki/Logistic_function,2025) to D_+-division entropy, we derive the function $Q_{D_+}(x)$ that defines the division function $D_+(x)$. Moreover, we assume that D_+-division entropy $S_{D_+}(x, k)$ approximate by a second-degree polynomial, that is, the formula $\lambda_2 x^2 + \lambda_1 x$. In other words, we assume that the second derivative of $S_{D_+}(x, k)$ is a constant $\lambda_2/2$.

3. Third, the inverse of D_+-division partial entropy $S_{D_+}(x)$ is defined as potential $V_{D_+}(x, k)$, and the first derivative of potential $V_{D_+}(x, k)$ is defined as acceleration $V'_{D_+}(x, k)$. The $D_\pm$-division partial entropy $S_{D_\pm}(x)$ is inversely related to $V_{D_+}(x, k)$. Namely, it assumes that potential and acceleration are derived from $D_\pm$-division partial entropy $S_{D_\pm}(x)$.

4. Fourth, we consider the application of the above assumptions to the theory of gravity. Namely, we interpret the inverse $1/\lambda_2$ as mass m, the constant k as the gravitational constant G, and the variable x as distance R. Thereby, potential $V_{D_+}(x, k)$ and acceleration $V'_{D_+}(x, k)$ are interpreted as the gravitational potential $V_{D_+}(R, G)$ and the gravitational acceleration $\bar{g}_\pm = -V'_{D_+}(R, G)$ (Feynman,1995; Wiki/Gravity,2025). Therefore, we show and suggest some conclusions as follows:

   (a) If distance R is small enough, then gravity becomes a constant value independent of R under specific conditions. Gravity may have 5-states within distance R is small enough. Among 5-states, there may exist anti-gravity which is the opposite of Newton's gravity. Furthermore, within small distances, we show the gravitational potential and the Coulomb potential can be treated in the same way.

   (b) If distance R is large enough within the size of the universe, then the rotation speed of the galaxy v follows the gravitational constant G, mass m and specific constants on the adjusted gravitational $\bar{g}_\pm$, not depend on the galaxy radius R (the galaxy rotation curve problem). Furthermore, the comparison between MOND and $\bar{g}_{\pm}$ is mentioned (Milgrom,1983; Moffat,2005; Wiki/Modified_Newtonian_dynamics,2025).

(c) If distance R is large enough within the size of the universe, then by comparing to conventional gravity g, the adjusted gravitational acceleration $\tilde{g}_{\pm}$ towards the center of rotation becomes slightly weaker or stronger. This means that the gravitational acceleration towards the center of a rotating object can fluctuate slightly with distance (The pioneer anomaly), (Masreliez,2005; Wiki/Pioneer_anomaly,2025).

The adjusted gravitational acceleration $V'_{D_+}(R,G)$ can be shown to be an expansion of Newton's gravity. Therefore, if the assumptions are true, there may exist specific constants which control gravity and the rotation speed of galaxies.

5. Fifth, we similarly introduce D_--division entropy $S_{D_-}(x,k)$ and potential $V_{D_-}(R,G)$. We attempt to explain the relationship between the Yukawa type potential and D_--division partial entropy with negative. Besides, we define that strong proximity acceleration(interaction) $g^{sp}_{\pm}$, weak proximity acceleration(interaction) $g^{wp}_{\pm}$, adjusted gravity $\tilde{g}_{\pm}$ and adjusted electromagnetic force $\hat{E}_{\pm}$ and $\bar{E}_{\pm}$. We attempt to suggest 11-types interactions (accelerations) and compare the size of these interactions. Moreover, let strong proximity acceleration(+) g^{sp}_{+} considered as strong interaction, weak proximity acceleration(−) g^{wp}_{-} as weak interaction, the adjusted gravity $\tilde{g}_{\pm}$ and $\hat{g}_{\pm}$ as gravity, and adjusted electromagnetic force $\hat{E}_{+}$ or $\bar{E}_{-}$ as electromagnetic force. Thereby, we attempt to compare the ratios of the fundamental 4-interactions in nature (strong interaction, electromagnetic force, weak interaction, and gravity) are 1, $1E$-2, $1E$-5 and $1E$-39, respectively if the strong interaction set to 1.

6. Sixth, we consider the relationship between entropic gravity F_R (Verlinde,2011) and $V_{D_{\pm}}(R,G)$. The entropy S_R in entropic gravity F_R is shown to be proportional to the potential $V_{D_{\pm}}(R,G)$ and to be inversely proportional to the $S_{D_{\pm}}(R)$ in this paper. Besides, if it assumes $S_R = S_{D_{\pm}}(R)$, then it is derived that S_R and $S_{D_{\pm}}(R,G)$ can be only expressed as mass M and temperature T, and entropic force becomes $F_R = 0$.

7. Finally, it suggests that there may exist new accelerations(interactions), that mass m may represent by entropy and that the gravitational constant G can fluctuate if entropy changes. Thermodynamics, quantum, gravity, electromagnetic and ecology may unify through entropy.

8. Issues: The existence of these interactions and constants needs to be verified, and numerical verification of models in this paper are future challenges. Moreover, the relationship between the gravity of relativity and the contents of this paper does not well explain. We would like to consider these points as future issues.

2. The Boltzmann Principle and the Planck Distribution Function

2.1 Introduction for Entropy S and the Planck Distribution Function

To make it easier the understanding, let us introduce the Boltzmann principle and the Planck distribution function.

Definition 2.1. *We define symbols using on this article as follows:*

$$\begin{aligned} &P : the\ number\ of\ particles, && N : the\ number\ of\ resonators, \\ &U : average\ energy\ per\ resonator, && U_N : total\ energy, \\ &\varepsilon : an\ element\ of\ energy, && \nu : frequency, \\ &T : temperature, && k_B : the\ Boltzmann\ constant, \\ &h : the\ Planck\ constant, && \beta : inverse\ temperature. \end{aligned} \tag{1}$$

□

Using the definitions above, the following equations are satisfied:

$$U_N = NU = P\varepsilon, \quad that\ is, \quad \frac{P}{N} = \frac{U}{\varepsilon}, \tag{2}$$

$$\beta = \frac{1}{k_B T}, \tag{3}$$

where the inequality $P > N$ is satisfied.

The number of states $W_{N,P}$ is defined as the number of particles P is divided by the number of partitions $N-1$ and Entropy S (the Boltzmann Principle) is defined as the logarithm of the number of states W, where P and N can be regarded positive integer numbers as follows:

Definition 2.2. *Let the number of particles P and the number of resonators N be positive integers. The number of states and the Boltzmann Principle are defined as follows:*

$$W_{N,P} = \frac{(N+P-1)!}{(N-1)!P!}, \quad (the\ number\ of\ states,\ combination), \tag{4}$$

$$S_{N,P} = k_B \log W_{N,P}, \quad (Boltzmann\ Principle), \tag{5}$$

$$S = \frac{S_{N,P}}{N}, \quad (the\ average\ of\ S_{N,P},\ the\ partial\ entropy\ of\ S_{N,P}). \tag{6}$$

□

Using Stirling's formula, for sufficiently large P and N, the following formulas are satisfied:

$$W_{N,P} = \frac{(N+P-1)!}{(N-1)!P!} \approx \frac{(N+P)^{N+P}}{N^N P^P}. \tag{7}$$

Using the Boltzmann principle above, for sufficiently large particles P and resonators N, we can obtain the following equations:

$$\begin{aligned} S_{N,P} &= k_B \log W_{N,P} \\ &= k_B\{(N+P)\log(N+P) - \log N^N - \log P^P\} \\ &= k_B N\{(1+\frac{P}{N})\log(1+\frac{P}{N}) - \frac{P}{N}\log\frac{P}{N}\}. \end{aligned} \tag{8}$$

Using the definition the equality(3) and (6) above, the equality(8) is satisfied as follows:

$$S = k_B\Big\{(1+\frac{U}{\varepsilon})\log(1+\frac{U}{\varepsilon}) - \frac{U}{\varepsilon}\log\frac{U}{\varepsilon}\Big\}. \tag{9}$$

Differentiate both sides of the equation(9) above with respect to average energy per resonator U. Hence, the following equation is satisfied:

$$\frac{dS}{dU} = \frac{k_B}{\varepsilon}\Big\{\log(1+\frac{U}{\varepsilon}) - \log\frac{U}{\varepsilon}\Big\}. \tag{10}$$

Furthermore, differentiate both sides of the equation(10) with respect to average energy per resonator U, the following equation is satisfied:

$$\frac{d^2S}{dU^2} = \frac{-k_B}{U(\varepsilon+U)}. \tag{11}$$

The change of entropy by energy U ,that is dS/dU, is equal to the inverse of temperature T. Namely, the following equation is satisfied between Entropy S, average energy per resonator U and temperature T:

$$\frac{dS}{dU} = \frac{1}{T}. \tag{12}$$

Thus, using the equation(11) and (12), the following relation is satisfied:

$$\frac{d}{dU}\Big(\frac{1}{T}\Big) = \frac{-k_B}{U(\varepsilon+U)}. \tag{13}$$

Integrating both sides of the equation(13) with respect to average energy per resonator U, the following relation is satisfied:

$$U = \frac{\varepsilon}{\exp(\frac{\varepsilon}{k_B T}) - 1}. \tag{14}$$

Here, put ε as follows:

$$\varepsilon = h\nu. \tag{15}$$

Therefore, the following equations are obtained:

$$U = \frac{h\nu}{\exp(\frac{h\nu}{k_B T}) - 1} = \frac{h\nu}{\exp(h\nu\beta) - 1}. \quad (Planck's\ law) \tag{16}$$

The equation above(16) is determined by the expression for the average energy of particles in a single mode of frequency ν in thermal equilibrium T, that is, named Planck's law.

(Note): In this paper, Planck's radiation law refers to the following expression:

$$U_p = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{\exp(h\nu\beta)-1}. \quad (Planck's\ Radiation\ law) \tag{17}$$

where the constant c is the speed of light. (End of Note)

3. D_+-Division Entropy and Application to Dynamical Systems

We continue the discussion with reference to ideas in the above subsection 2.1. We consider that the number of particles P is replaced to the positive real variable x, and the number of resonators N is replaced to the number of division function $D_+(x)$. We introduce D_+-division entropy, D_+-division partial entropy and D_+-division acceleration entropy and define D_+-division entropy as follows. In this paper, the logarithm log represents the natural logarithm $\log_e$.

3.1 D_+-Division Partial Entropy $S_{D_+}(x)$ That Divided x by $D_+(x)$

We first define D_+-division partial entropy $S_{D_+}(x)$ and the number of states $W_{D_+}(x)$. D_+-division partial entropy $S_{D_+}(x)$ under $W_{D_+}(x)$ is defined by the number of states $W_{D_+}(x)$, (Fujino,2024) for the source of these ideas.

Definition 3.1. *We define that the number of states $W_{D_+}(x)$ that divided x by $D_+(x)$, and D_+-division partial entropy $S_{D_+}(x)$ as follows:*

$$W_{D_+}(x) = \frac{(D_+(x)+x)^{D_+(x)+x}}{D_+(x)^{D_+(x)}x^x}, \tag{18}$$

$$S_{D_+}(x) = \log W_{D_+}(x), \tag{19}$$

where x is a positive real variable, and $D_+(x)$ is a positive real valued function that divides x. If $x = 0$, then it defines as $D_+(0) = 1$, $W_{D_+}(0) = 1$ and $S_{D_+}(0) = 0$. □

According to the above definition 3.1, entropy S of the definition(6) can be regarded as ε-division partial entropy.

(Note): Originally, the number of states should be defined as below equation(20). However, the equation(20) cannot define on real values well as follows:

$$W_{D_+}(x) = \frac{(D_+(x)+x-1)!}{(D_+(x)-1)!x!}. \tag{20}$$

Therefore, we adopt the definition of equation(18) by applying Stirling's approximation to the above equation(20). The division function $D_+(x)$ can be considered as the quantization $D_+(x)$. (End of Note)

3.2 D_+-division Entropy $S_{D_+}(x,k)$ and D_+-division partial entropy $S_{D_+}(x)$.

The relationship between D_+-division entropy $S_{D_+}(x,k)$ and D_+-division partial entropy $S_{D_+}(x)$ is defined as follows:

Definition 3.2. *D_+-division entropy $S_{D_+}(x,k)$ and D_+-division partial entropy $S_{D_+}(x)$.*
Let $x > 0$ be a real variable, $k \gtrnsim 0$ and $\xi \gtrnsim 0$ be real constants. Let $D_+(x) > 0$ be a positive real valued function that divides x. $S_{D_+}(x,k)$, $S_{D_+}(x)$ and $Q_{D_+}(x)$ are defined as follows:

$$S_{D_+}(x,k) = kD_+(x)S_{D_+}(x), \tag{21}$$

$$Q_{D_+}(x) = \frac{\xi x}{D_+(x)}, \tag{22}$$

$$\begin{aligned} S_{D_+}(x) &= (1+\frac{x}{D_+(x)})\log(1+\frac{x}{D_+(x)}) - \frac{x}{D_+(x)}\log(\frac{x}{D_+(x)}) \\ &= (1+\frac{Q_{D_+}(x)}{\xi})\log(1+\frac{Q_{D_+}(x)}{\xi}) - \frac{Q_{D_+}(x)}{\xi}\log(\frac{Q_{D_+}(x)}{\xi}), \end{aligned} \tag{23}$$

where for any positive variable $x > 0$, the function Q_{D_+} is satisfied $Q_{D_+} \gtrnsim 0$ and $Q'_{D_+} \gtrnsim 0$. □

The above equation(23) can be obtain by applying the definition3.1 and Stirling's approximation. By the above definition, $S'_{D_+}(x)$ and $S''_{D_+}(x)$ are represented as follows:

$$S'_{D_+}(x) = \frac{Q'_{D_+}(x)}{\xi}\Big(\log(1+\frac{Q_{D_+}(x)}{\xi}) - \log(\frac{Q_{D_+}(x)}{\xi})\Big), \tag{24}$$

$$S''_{D_+}(x) = \frac{Q'_{D_+}(x)}{\xi}\Big(\frac{Q'_{D_+}(x)}{\xi+Q_{D_+}(x)} - \frac{Q'_{D_+}(x)}{Q_{D_+}(x)}\Big) + \frac{Q''_{D_+}(x)}{\xi}\Big(\log(1+\frac{Q_{D_+}(x)}{\xi}) - \log(\frac{Q_{D_+}(x)}{\xi})\Big). \tag{25}$$

Let $S'_{D_+}(x)$ be D_+-division entropy generation (velocity) of $S_{D_+}(x)$ (Nicolis,Prigogine,1989,1997) and $S''_{D_+}(x)$ be D_+-division entropy acceleration of $S_{D_+}(x)$. The function $Q_{D_+}(x)$ can be regarded as the position divided a real value ξx by $Q_{D_+}(x)$. The first order derivative of $Q_{D_+}(x)$, that is, $Q'_{D_+}(x)$ can be regarded as the change of the position by x and ξ (Fujino,2023,2024) for details on how to derive D_+-division entropy, entropy acceleration and its partial entropy).

3.3 The Function $Q_{D_+}(x)$ and Approximation of D_+-Division Entropy $S_{D_+}(x,k)$

Next, we find the function $Q_{D_+}(x)$ using the ideas behind Planck's law and the logistic function for dynamical systems. Put the part of partial entropy $S''_{D_+}(x)$ as follows:

$$\frac{Q'_{D_+}(x)}{\xi}\Big(\frac{1}{\xi + Q_{D_+}(x)} - \frac{1}{Q_{D_+}(x)}\Big) = -\mu(x), \tag{26}$$

where $\mu(x) > 0$ is a positive real function. The left side of above equation(26) looks like spectra divided by $\xi x/Q_{D_+}(x)$ and the right side of (26) becomes an approximation by the function $\mu(x)$. We consider $Q'_{D_+}(x)$ as follows:

$$Q'_{D_+}(x) = \frac{dQ_{D_+}}{dx}. \tag{27}$$

By transforming according to the equation(27), we can represent as follows:

$$dQ_{D_+}\Big(\frac{1}{\xi + Q_{D_+}(x)} - \frac{1}{Q_{D_+}(x)}\Big) = -\xi\mu(x)dx. \tag{28}$$

Integrating both sides gives as follows:

$$\log(\xi + Q_{D_+}(x)) - \log(Q_{D_+}(x)) = -\xi\int\mu(x)dx \pm \mu_1, \tag{29}$$

where $\mu_1 \geq 0$. The left side of the above equation is positive number, $\xi > 0$ and $\mu_1 > 0$, therefore, the right side is also positive as follows:

$$-\xi\int\mu(x)dx \pm \mu_1 > 0. \tag{30}$$

Therefore, we consider only the case sign of μ_1 is positive as follows:

$$\log(1 + \frac{\xi}{Q_{D_+}(x)}) = -\xi\int\mu(x)dx + \mu_1. \tag{31}$$

By transforming the above equation, it is satisfied as follows:

$$1 + \frac{\xi}{Q_{D_+}(x)} = \exp(-\xi\int\mu(x)dx + \mu_1). \tag{32}$$

Therefore, the function $Q_{D_+}(x)$ represents as follows:

$$Q_{D_+}(x) = \frac{\xi}{\exp(-\xi\int\mu(x)dx + \mu_1) - 1}. \tag{33}$$

$Q_{D_+}(x)$ becomes the distribution function of the position which divided the real value ξx by $D_+(x)$. Let the distribution function $Q_{D_+}(x)$ be the Planck type distribution function. The equation(26) also looks like spectra divided by $\xi x/Q_{D_+}(x)$. If we take $Q_{D_+}(x)$ to $\log(x)$, then we obtain the equation(33) like an expansion of the Planck distribution function (Planck,1906; Fujino,2024) Besides, the Planck type distribution function $Q_{D_+}(x)$ is thought of the expansion of the Bose-Einstein distribution function.

Next, we make assumption about approximation of D_+-division entropy $S_{D_+}(x,k)$.

Assumption 3.3. *Assume D_+-division entropy $S_{D_+}(x,k)$ can approximate by a second-degree polynomial. Hence, set as follows:*

$$S_{D_+}(x,k) = \lambda_2 x^2 \pm \lambda_1 x, \tag{34}$$

where $\lambda_2 \gneqq 0$ $\lambda_1 \geq 0$ are real numbers, and $S_{D_+}(0,k) = 0$. □

Hence, the first derivative $S'_{D_+}(x,k)$ represents a first-degree polynomial as follows:

$$S'_{D_+}(x,k) = 2\lambda_2 x \pm \lambda_1. \tag{35}$$

Besides, the second derivative $S''_{D_+}(x,k)$ is constant. Namely, it is satisfied as follows:

$$S''_{D_+}(x,k) = 2\lambda_2. \tag{36}$$

In other words, we assume that the second derivative of $S_{D_+}(x,k)$ is a constant.

3.4 The Inverse of D_+-Division Partial Entropy $S_{D_+}(x)$ and Potential $V_{D_+}(x,k)$

Next, we focus on the inverse of D_+-division partial entropy $S_{D_+}(x)$ as follows:

$$\frac{1}{S_{D_+}(x)} = k\frac{\xi x}{Q_{D_+}(x)}\frac{1}{\lambda_2 x^2 \pm \lambda_1 x}. \tag{37}$$

By the equation(33), we can represent as follows:

$$\frac{1}{S_{D_+}(x)} = k\frac{1}{\lambda_2 x \pm \lambda_1}(\exp(-\xi\int\mu(x)dx + \mu_1) - 1). \tag{38}$$

We define the inverse of $S_{D_+}(x)$ as potential $V_{D_+}(x,k)$:

$$V_{D_+}(x,k) = -k\frac{\frac{1}{\lambda_2}}{x \pm \frac{\lambda_1}{\lambda_2}}(1 - \exp(-\xi\int\mu(x)dx + \mu_1)). \tag{39}$$

In other words, the above potential $V_{D_+}(x,k)$ can be defined as the product of a constant k, the di$D_+(x) = \xi/Q_{D_+}(x)$, and the inverse of D_+-division partial entropy $S_{D_+}(x)$. Here, let us reorganize the above, that is, we assume as follows:

$$S_{D_+}(x,k) = \lambda_2 x^2 \pm \lambda_1 x, \tag{40}$$

$$\frac{Q'_{D_+}(x)}{\xi}\Big(\frac{1}{\xi + Q_{D_+}(x)} - \frac{1}{Q_{D_+}(x)}\Big) = -\mu(x), \tag{41}$$

where $\lambda_2 \gneq 0$ is a positive real number and $\mu(x) \gneq 0$ is a positive real function. Therefore, we define the inverse of $S_{D_+}(x)$ as potential $V_{D_+}(x,k)$:

$$V_{D_+}(x,k) = -k\frac{\frac{1}{\lambda_2}}{x \pm \frac{\lambda_1}{\lambda_2}}(1 - \exp(-\xi\int\mu(x)dx + \mu_1)), \tag{42}$$

where $\xi \gneq 0$, $\lambda_2 \gneq 0$, $\lambda_1 \geq 0, \mu_1 \geq 0$ are real numbers and $\mu(x) \gneq 0$ is a positive real function. The first derivative $V'_{D_+}(x,k)$ is satisfied as follows:

$$V'_{D_+}(x,k) = \frac{k\frac{1}{\lambda_2}}{(x \pm \frac{\lambda_1}{\lambda_2})^2}(1 - \exp(-\xi\int\mu(x)dx + \mu_1)) - \frac{k\frac{1}{\lambda_2}\xi\mu(x)}{x \pm \frac{\lambda_1}{\lambda_2}}\exp(-\xi\int\mu(x)dx + \mu_1). \tag{43}$$

Let $V_{D_+}(x,k)$ be potential of $S_{D_+}(x)$, and $V'_{D_+}(x,k)$ be acceleration of $S_{D_+}(x)$. We define as follows:

Assumption 3.4. *Potential $V_{D_+}(x,k)$ is defined the inverse of D_+-division partial entropy $S_{D_+}(x)$. Acceleration $V'_{D_+}(x,k)$ is defined as the first derivative of $V_{D_+}(x,k)$.* □

Namely, it assumes as follows:

1. If low entropy means low order and a high possibility of emergence, then potential is high.
2. If high entropy means high order and a low possibility of emergence, then potential is low.

Therefore, entropy and potential have the inverse relationship as follows:

$$V_{D_+}(x,k) = \frac{1}{S_{D_+}(x)}. \tag{44}$$

In the next chapter, we will describe applications of $V_{D_+}(x,k)$ and $V'_{D_+}(x,k)$ to gravity.

4. Application of Potentials $V_{D_+}(x,k)$ to Gravity

The constants, variables and functions in the above equations(42) and (43) can be selected arbitrarily within specific conditions. Therefore, we attempt to interpret these constants, variables and functions as theory of gravity. Namely, we attempt to interpret $V_{D_+}(x,k)$ as the gravitational potential and $V'_{D_+}(x,k)$ as the gravitational acceleration.

4.1 Interpretation to $V_{D_+}(R,G)$.

We consider the interpretation of equation $V_{D_+}(x,k)$ as follows:

$$
\begin{aligned}
&x := R \geq 0, && R \textit{ is distance},\\
&\frac{1}{\lambda_2} := m \gneqq 0, && m \textit{ is mass within } R,\\
&k := G, && G \textit{ is the gravitational constant},\\
&\xi := \xi^g, && \xi^g \textit{ is a constant},\\
&\mu(x) := \mu_2^g \gneqq 0, && \mu_2^g \textit{ is a positive real constant},\\
&\mu_1 := \mu_1^g \geq 0, && \mu_1^g \textit{ is a real constant},\\
&\lambda_1 := \lambda_1^g \geq 0, && \lambda_1^g \textit{ is a real constant},
\end{aligned}
\tag{45}
$$

where the symbol g in the upper right corner of the alphabet means g of gravity. We assume as follows: The direction under smaller R is defined as the central direction, that is, the direction towards the center is negative. The gravitational potential increases away from the center and decreases toward the center. However, if $R = 0$, then $V_{D_+}(R,G) = 0$. Moreover, it assumes the constant $1/\lambda_2$ is equal to mass m within R.

Assumption 4.1. *It assumes the constant $1/\lambda_2$ is equal to mass m within R. Assume that 2-times the inverse of entropy acceleration, $2/S''_{D_+}(R,G)$, that is, $1/\lambda_2$ is equal to mass m within R. In other words, mass m within R is defined as the inverse of second order term of $S_{D_+}(R,G)$, that is, $1/\lambda_2$.* □

According to the assumption 4.1 above, the relationship between mass m and entropy $S_{D_+}(R,G)$ can be thought of as follows:

1. if $S''_{D_+}(R,G)$ is large, then mass m becomes small,
2. if $S''_{D_+}(R,G)$ is small, then mass m becomes large.

Is this assumption reasonable? This assumption will be re-stated in the following subsection 7.1.

Next, we define $V_{D_+}(R,G)$ as the gravitational potential of G as follows:

$$
\begin{aligned}
V_{D_+}(R,G) &= -\frac{Gm}{R \pm \lambda_1^g m}(1 - \exp(-\xi^g \mu_2^g \int dR + \mu_1^g))\\
&= -\frac{Gm}{R \pm \lambda_1^g m}(1 - \exp(-\xi^g \mu_2^g R + \mu_1^g)).
\end{aligned}
\tag{46}
$$

The first derivative $V'_{D_+}(R,G)$ of $V_{D_+}(R,G)$ is satisfied as follows:

$$
V'_{D_+}(R,G) = \frac{Gm}{(R \pm \lambda_1^g m)^2}(1 - \exp(-\xi^g \mu_2^g R + \mu_1^g)) - \frac{Gm\xi^g \mu_2^g}{R \pm \lambda_1^g m}\exp(-\xi^g \mu_2^g R + \mu_1^g),
\tag{47}
$$

where $\xi^g \gneqq 0$, $\mu_2^g \gneqq 0$, $\mu_1^g \geq 0$ and $\lambda_1^g \geq 0$.

Definition 4.2. *The Planck type adjusted gravitational acceleration $\bar{g}_\pm = g(R,G)$ is defined as $-V'_{D_+}(R,G)$. Namely, it is satisfied as follows:*

$$
\bar{g}_\pm = g(R,G) = -V'_{D_+}(R,G).
\tag{48}
$$

□

The above equation(47) becomes an expansion of the gravitational acceleration g (Feynman,1995; Wiki/Gravity,2025) The second term in brackets of equation(46) becomes like the Yukawa potential (Feynman,1963,1995; Yukawa,1935;

Fujii,1971; Murata,Tanaka,2014; Ourabah,2023). On section 5 later, we will be the differences between the Planck type potential $V_{D_+}(R,G)$ and the Yukawa type potential $V_{D_-}(R,G)$.

(Note): Let M be mass located in range R of mass m. Potential energies $U_{\pm}(R,G)$ of potentials $V_{D_{\pm}}(R,G)$ is represented as follows (Feynman,1995; Wiki/Gravity,2025)

$$U_{\pm}(R,G) = V_{D_{\pm}}(R,G)M, \tag{49}$$

and forces $F_{\pm}(R,G)$ of accelerations $V'_{D_{\pm}}(R,G)$ is represented as follows:

$$F_{+}(R,G) = \bar{g}_{\pm}M = -V'_{D_+}(R,G)M, \tag{50}$$

$$F_{-}(R,G) = \hat{g}_{\pm}M = -V'_{D_-}(R,G)M. \tag{51}$$

The relationship between the gravitational acceleration g and the gravitational potential are as follows (Feynman,1995; Wiki/Gravity,2025):

$$F_R(G) = -M\nabla\Phi(R) = -gM. \tag{52}$$

where $\Phi(R)$ is gravitational potential. Therefore, the above equations (50) and (51) can be considered as the extension of (52). Force (interaction) is also acceleration multiplied by mass, so we will treat it in the same way. The gravitational acceleration $\hat{g}_{\pm}$ and potential V_{D_-} are define later subsection 5.4.

Moreover, $Q_{D_+}(R)$ becomes like spectra within R that is independent of G and depends on ξ, μ_2^g, μ_1^g and R, and can be represented as follows:

$$Q_{D_+}(R) = \frac{\xi}{\exp(-\xi\mu_2^g R + \mu_1^g) - 1}. \tag{53}$$

Furthermore, the equation $\bar{g}_{\pm}$, which contains $Q_{D_+}(R)$, becomes itself an equation regarding as describe wave distributions. If the definition (assumption) of $\bar{g}_{\pm}$ is valid, isn't it possible to think that the equation $\bar{g}_{\pm}$ itself represents the distribution of waves influenced by gravity, that is the gravity waves? (End of Note)

The solution of equation(47) for μ_1^g is satisfied as follows:

$$\mu_1^g = \log\left(\frac{1}{1 + (R \pm \lambda_1^g m)\xi^g\mu_2^g}\right) + \xi^g\mu_2^g R, \quad \mu_1^g \geq 0. \tag{54}$$

Because the following conditions are needed to satisfy:

$$\exp(-\xi^g\mu_2^g R + \mu_1^g) = \frac{1}{1 + (R \pm \lambda_1^g m)\xi^g\mu_2^g} \geq 0, \tag{55}$$

hence, it is satisfied as follows:

$$1 + (R \pm \lambda_1^g m)\xi^g\mu_2^g \geq 0. \tag{56}$$

Therefore, we suggest as follows:

Suggestion 4.3. *The classification of $V'_{D_+}(R,G)$. According to the values $\xi^g \gtrless 0, \mu_2^g \gtrless 0$, $\lambda_1^g \geq 0$ and $\mu_1^g \geq 0$, the equation $V'_{D_+}(R,G)$, (47) can be classified as follows:*

1) *If the constant μ_1^g is satisfied as follows:*

$$\mu_1^g > \log\left(\frac{1}{1 + (R \pm \lambda_1^g m)\xi^g\mu_2^g}\right) + \xi^g\mu_2^g R, \tag{57}$$

then the above equation $V'_{D_+}(R,G)$, (47) becomes negative, that is, it is satisfied as follows:

$$V'_{D_+}(R,G) < 0. \tag{58}$$

(Note): The right side of inequality(57) can become positive or negative.(End of Note)

2) *If the constant μ_1^g is satisfied as follows:*

$$\mu_1^g < \log(\frac{1}{1+(R \pm \lambda_1^g m)\xi^g \mu_2^g}) + \xi^g \mu_2^g R, \tag{59}$$

then the above equation $V'_{D_+}(R,G)$, (47) becomes positive, that is, it is satisfied as follows:

$$V'_{D_+}(R,G) \geq 0. \tag{60}$$

3) *If the constant $\mu_1^g \to 0$, then the following equation is satisfied:*

$$V'_{D_+}(R,G) = \frac{Gm}{(R \pm \lambda_1^g m)^2}(1-\exp(-\xi^g \mu_2^g R)) - \frac{Gm\xi^g \mu_2^g}{R \pm \lambda_1^g m}\exp(-\xi^g \mu_2^g R). \tag{61}$$

□

4.2 When Distance R Is Small Enough

If distance R is small enough, that is, because distance R approaches 0, hence the value $\exp(-\xi^g \mu_2^g R)$ approaches 1 infinitely. Therefore, the equation $V'_{D_+}(R,G)$, (47) is satisfied as follows:

$$V'_{D_+}(R,G) \simeq \frac{G}{(\pm\lambda_1^g)^2 m}(1-\exp(\mu_1^g)) - \frac{G\xi^g \mu_2^g}{\pm\lambda_1^g}\exp(\mu_1^g), \quad (\because \exp(-\xi^g \mu_2^g R) \to 1). \tag{62}$$

If distance $R \neq 0$ and $\lambda_1^g = 0$, then it is satisfied as follows:

$$V'_{D_+}(R,G) = \frac{Gm}{R^2}(1-\exp(-\xi^g \mu_2^g R + \mu_1^g)) - \frac{Gm\xi^g \mu_2^g}{R}\exp(-\xi^g \mu_2^g R + \mu_1^g). \tag{63}$$

The case of the above equation(63), if $R \to 0$, then it becomes $V'_{D_+}(R,G) \to \infty$. Hence, we consider that it makes $\lambda_1^g \neq 0$ and R is small enough. Later we consider the case $\lambda_1^g = 0$. Therefore, if distance R is small enough, then acceleration $V'_{D_+}(R,G)$ approximate by a constant value. The following expressions are satisfied:

Suggestion 4.4. *Acceleration $V'_{D_+}(R,G)$ becomes a constant value under small enough R. Let m be a positive real number (mass). For sufficiently small distances $R > 0$, the following equation is satisfied: Acceleration $V'_{D_+}(R,G)$ becomes a constant value, that is,*

$$V'_{D_+}(R,G) \simeq \frac{G}{(\pm\lambda_1^g)^2 m}(1-\exp(\mu_1^g)) - \frac{G\xi^g \mu_2^g}{\pm\lambda_1^g}\exp(\mu_1^g), \tag{64}$$

where $\xi^g \gneq 0, \mu_2^g \gneq 0$, $\lambda_1^g \geq 0$ and $\mu_1^g \geq 0$. □

The solution of equation(64) for μ_1^g is satisfied as follows:

$$\mu_1^g = \log(\frac{1}{1 \pm \lambda_1^g \mu_2^g \xi^g m}), \qquad \mu_1^g \geq 0. \tag{65}$$

Therefore, the following equation is satisfied:

$$\exp(\mu_1^g) = \frac{1}{1 \pm \lambda_1^g \mu_2^g \xi^g m} \geq 0. \tag{66}$$

Because $\xi^g \gneq 0, \mu_2^g \gneq 0$, $m \gneq 0$ and $\lambda_1^g \geq 0$, it is satisfied as follows:

$$1 \pm \lambda_1^g \mu_2^g \xi^g m > 0. \tag{67}$$

Therefore, it is satisfied as follows:

$$m < \frac{1}{\lambda_1^g \mu_2^g \xi^g} \quad \left(and \quad \frac{-1}{\lambda_1^g \mu_2^g \xi^g} < m\right). \tag{68}$$

According to the sign plus(+) or minus(−) of λ_1^g, the value of equation $V'_{D_+}(R,G)$, (64), and its solution for μ_1^g can be classified as finite as follows:

1) if $V'_{D_+}(R,G) \neq 0$;

$$V'_{D_+}(R,G) \simeq \frac{G}{(\lambda_1^g)^2 m}(1-\exp(\mu_1^g)) - \frac{G\xi^g\mu_2^g}{\pm\lambda_1^g}\exp(\mu_1^g), \tag{69}$$

2) if $V'_{D_+}(R,G) \neq 0$ and $\mu_1^g \to 0$;

$$V'_{D_+}(R,G) \simeq -\frac{G\xi^g\mu_2^g}{\pm\lambda_1^g}, \qquad (\because \exp(\mu_1^g) \to 1), \tag{70}$$

3) if $\mu_1^g = \log(\frac{1}{1\pm\lambda_1^g\mu_2^g\xi^g m})$;

$$V'_{D_+}(R,G) = 0. \tag{71}$$

Therefore, we suggest as follows:

Suggestion 4.5. *The classified $V'_{D_+}(R,G)$ under small enough R. According to the values $\xi^g \gneq 0, \mu_2^g \gneq 0$ $\lambda_1^g \geq 0$ and $\mu_1^g \geq 0$, the equation $V'_{D_+}(R,G)$, (64), can be classified as follows:*

1) If the constant μ_1^g is satisfied as follows:

$$\mu_1^g > \log(\frac{1}{1\pm\lambda_1^g\mu_2^g\xi^g m}), \tag{72}$$

then the above equation(64) becomes negative, that is, it is satisfied as follows:

$$V'_{D_+}(R,G) < 0. \tag{73}$$

(Note): The right side of inequality(72) can become positive or negative. (End of Note)

2) If the constant μ_1^g is satisfied as follows:

$$\mu_1^g \leq \log(\frac{1}{1\pm\lambda_1^g\mu_2^g\xi^g m}), \tag{74}$$

then the above equation $V'_{D_+}(R,G)$, (64), becomes positive, that is, it is satisfied as follows:

$$V'_{D_+}(R,G) \geq 0. \tag{75}$$

3) If the constant $\mu_1^g \to 0$, then the following expressions are satisfied:

$$V'_{D_+}(R,G) \simeq -\frac{G\xi^g\mu_2^g}{\pm\lambda_1^g}, \qquad (\because \exp(\mu_1^g) \to 1). \tag{76}$$

□

4.2.1 Summarize the Gravitational Acceleration for Small Enough R

We summarize the gravitational acceleration $\bar{g}_\pm = -V'_{D_+}(R,G)$. According to the symbol of plus or minus rule for values λ_1^g, it defines $\bar{g}_\pm$ as the adjusted gravitational acceleration with ξ^g and μ_2^g, that is, $\bar{g}_{\pm\lambda_1^g\pm\mu_1^g}$ become as follows:

$$\bar{g}_{\pm\lambda_1^g+\mu_1^g} = -\frac{Gm}{(R\pm\lambda_1^g m)^2}(1-\exp(-\xi^g\mu_2^g R+\mu_1^g)) + \frac{Gm\xi\mu_2^g}{R\pm\lambda_1^g m}\exp(-\xi^g\mu_2^g R+\mu_1^g), \tag{77}$$

If the constant $\lambda_1^g = 0$, then the adjusted gravitational acceleration with ξ^g, μ_2^g and $\lambda_1^g = 0$ become as follows:

$$\bar{g}_{\pm 0+\mu_1^g} = -\frac{Gm}{R^2}(1-\exp(-\xi^g\mu_2^g R+\mu_1^g)) + \frac{Gm\xi^g\mu_2^g}{R}\exp(-\xi^g\mu_2^g R+\mu_1^g), \tag{78}$$

If distance $R \to 0$, then the adjusted gravitational acceleration with ξ^g and $R \to 0$ become as follows:

$$\bar{g}_{\pm\lambda_1^g+\mu_1^g} = -\frac{G}{(\pm\lambda_1^g)^2 m}(1-\exp(\mu_1^g)) + \frac{G\xi^g\mu_2^g}{\pm\lambda_1^g}\exp(\mu_1^g), \tag{79}$$

Therefore, if distance $R \to 0$, then it is satisfied as follows:

$$\begin{aligned}
&\bar{g}_{\pm\lambda_1^g+\mu_1^g} = \lim_{R\to 0}\bar{g}_{\lambda_1^g+\mu_1^g\pm},\\
&\lim_{\mu_1^g\to 0}\bar{g}_{\pm\lambda_1^g+\mu_1^g} = \frac{G\xi^g\mu_2^g}{\pm\lambda_1^g},\\
&\lim_{\mu_1^g\to 0,\lambda_1^g\to 0}\bar{g}_{\pm\lambda_1^g+\mu_1^g} = \pm\infty,\\
&\lim_{\mu_1^g\to 0,\lambda_1^g\to\infty}\bar{g}_{\pm\lambda_1^g+\mu_1^g} = 0,\\
&\lim_{\mu_1^g\to\infty}\bar{g}_{+\lambda_1^g+\mu_1^g} = \infty, \quad \because)\ \frac{-1}{\pm\lambda_1^g m} + \mu_2^g\xi^g > 0,\\
&\lim_{\mu_1^g\to\infty}\bar{g}_{-\lambda_1^g+\mu_1^g} = -\infty \quad \because)\ \frac{-1}{\pm\lambda_1^g m} + \mu_2^g\xi^g < 0.
\end{aligned} \tag{80}$$

According to the above discussion, we suggest as follows:

Suggestion 4.6. *Within small enough distances R, it is possible that the Planck type adjusted gravity $\bar{g}_\pm$ has 5-states such that finite 2-states $\frac{G\xi^g\mu_2^g}{\pm\lambda_1^g}$, that infinite 2-states $\pm\infty$, and that zero 1-state* 0. □

The values $-G\xi^g\mu_2^g/\lambda_1^g$ and $\bar{g}_\pm = -V'_{D_+}(R,G) < 0$ have the same direction as Newton's gravity, where the direction towards the center is negative. However, the values $G\xi^g\mu_2^g/\lambda_1^g$ and $\bar{g}_\pm = -V'_{D_+}(R,G) > 0$ have the opposite direction as Newton's gravity. This means that it may represent the existence of anti-gravity. If distance R is small enough, then the acceleration $V'_{D_+}(R,G)$ becomes a finite constant depend on constants $\xi^g, \lambda_1^g, \mu_1^g$ and μ_2^g, not infinite. However, if the constant λ_1^g or μ_1^g approach 0 or ∞, then the acceleration $V'_{D_+}(R,G)$ becomes ∞ or 0. Depending on the value of μ_1 and λ_1^g, the value $V'_{D_+}(R,G)$ can be positive or negative. When the value $V'_{D_+}(R,G)$ is negative, deceleration acts toward the center. These constants are dependent on D_+-division entropy and the part of D_+-division partial entropy. Namely, acceleration depends on D_+-division partial entropy. Therefore, $\bar{g}_\pm$ exists 5-states within small enough distances R.

The above description can be applied to Coulomb's law (electric field). By adjusting the values μ_1, μ_2, λ_1, $m = 1/\lambda_2$ and ξ, it may be possible to make the argument by replacing the gravitational constant G to Coulomb's constant k_e. (In this paper, Coulomb's constant is defined as k_e.) We will describe this possibility next.

4.2.2 Compare $V_{D_+}(R,G)$ and $V_{D_+}(R,k_e)$ for Small R

We attempt to compare $V_{D_+}(R,G)$ and $V_{D_+}(R,k_e)$. Similarly, the gravitational potential $V_{D_+}(R,G)$, we define the Coulomb potential $V_{D_+}(R,k_e)$ as follows:

$$V_{D_+}(R,k_e) = -\frac{k_e e_q}{R \pm \lambda_1^c e_q}(1-\exp(-\xi^c\mu_2^c R + \mu_1^c)), \tag{81}$$

where $e_q > 0$ is the elementary charge, and $\xi^c \gneq 0, \mu_2^c \gneq 0, \mu_1^c \geq 0$ and $\lambda_1^c \geq 0$, and the symbol c in the upper right corner of the alphabet means c of Coulomb. For example, we set constants as follows:

$$\begin{aligned}
&G := 6.674E\text{-}11, && G \textit{ is the gravitational constant},\\
&\xi^g = \xi^c := h = 6.626E\text{-}34, && h \textit{ is Planck's constant},\\
&k_e := 8.987E\text{+}9, && k_e \textit{ is Coulomb's constant},\\
&e_q := 1.604E\text{-}19, && e_q \textit{ is the elementary charge},\\
&m := m_p = 2.176E\text{-}8, && m_p \textit{ is the Planck mass}(kg),\\
&\mu_2^g := 1, && \mu_2^g \textit{ is a real constant},\\
&\mu_2^c := 1, && \mu_2^c \textit{ is a real constant},\\
&\lambda_1^g := 1, && \lambda_1^g \textit{ is a real constant},\\
&\lambda_1^c := 1, && \lambda_1^c \textit{ is a real constant}.
\end{aligned} \tag{82}$$

Using the above constants, the gravitational potential $V_{D_+}(R, G)$ is satisfied as follows:

$$V_{D_+}(R, G) = 2.442E\text{-}12, \quad if\ \mu_1^g = 1, \tag{83}$$
$$V_{D_+}(R, G) = 2.480E\text{-}3, \quad if\ \mu_1^g = 21.28, \tag{84}$$

where $R := 1.000E$-6 meter and the Planck mass m_p is used instead of mass m. Let the sign of μ_1^g and λ_1^g be plus such as $+\mu_1^g$ and $+\lambda_1^g$. Similarly, the Coulomb potential $V_{D_+}(R, k_e)$ is satisfied as follows:

$$V_{D_+}(R, k_e) = 2.474E\text{-}3, \quad if\ \mu_1^c = 1, \tag{85}$$
$$V_{D_+}(R, k_e) = 2.512E\text{+}6, \quad if\ \mu_1^c = 21.28, \tag{86}$$

where $R := 1.000E$-6 meter, the elementary charge e_q is used instead of mass m and Coulomb's constant k_e is used instead of G. The signs of μ_1^c and λ_1^c are $+\mu_1^c$ and $+\lambda_1^c$. The values $V_{D_+}(R, G)$ and $V_{D_+}(R, k_e)$ change depending on how the constants μ_1^g and $\mu 1^c$ are selected. The above values (84) and (85) is close. Therefore, for small distances R, $\mu_1^g = 21.28$ and $\mu_1^c = 1$, it is satisfied $V_{D_+}(R, G) \simeq V_{D_+}(R, k_e)$. In consequence, it is satisfied as follows:

Suggestion 4.7. *Let m_p be the Planck mass, e_q be the elementary charge, G be the gravitational constant and k_e be Coulomb's constant. For small distances $R > 0$, there exist specific constants ξ^g, ξ^c, μ_2^g, μ_2^c, μ_1^g, μ_1^c, λ_1^g and λ_1^c such that the following equation is satisfied:*

$$V_{D_+}(R, G) \simeq V_{D_+}(R, k_e), \tag{87}$$

where $\xi^g, \xi^c, \mu_2^g, \mu_2^c \gneqq 0$ and $\lambda_1^g, \lambda_1^c, \mu_1^g, \mu_1^c \geq 0$. □

Because, if it is satisfied as follows:

$$\begin{aligned} V_{D_+}(R, G) &= -\frac{Gm_p}{R \pm \lambda_1^g m_p}(1 - \exp(-\xi^g \mu_2^g R + \mu_1^g)) \\ &= -\frac{k_e e_q}{R \pm \lambda_1^c e_q}(1 - \exp(-\xi^c \mu_2^c R + \mu_1^c)) \\ &= V_{D_+}(R, k_e), \end{aligned} \tag{88}$$

then transforming the above equation, it becomes as follows:

$$\exp(-\xi^g \mu_2^g R + \mu_1^g) = 1 + \frac{R \pm \lambda_1^g m_p}{Gm_p}\frac{k_e e_q}{R \pm \lambda_1^c e_q}(\exp(-\xi^c \mu_2^c R + \mu_1^c) - 1). \tag{89}$$

Therefore, if the value μ_2^c is given, the value μ_2^g can be found as follows:

$$\mu_2^g = \frac{1}{\xi^g R}\Big[\mu_1^g - \log\Big(1 + (\frac{R \pm \lambda_1^g m_p}{R \pm \lambda_1^c e_q})(\frac{k_e e_q}{Gm_p})(\exp(-\xi^c \mu_2^c R + \mu_1^c) - 1)\Big)\Big]. \tag{90}$$

Namely, using the equation for potential derived from entropy, within small distances, it may be possible to treat the gravitational potential and the Coulomb potential in the same way by appropriately selecting specific constants. In same way, applying the gravitational acceleration $V'_{D_+}(R, G)$ and Coulomb's law (electric field) $V'_{D_+}(R, k_e)$, we can obtain a suggestion as follows:

Suggestion 4.8. *Let m_p be the Planck mass, e_q be the elementary charge, G be the gravitational constant and k_e be Coulomb's constant. For small distances $R > 0$, there exist specific constants ξ^g, ξ^c, μ_2^g μ_2^c, μ_1^g μ_1^c λ_1^g and λ_1^c such that the following equation is satisfied:*

$$V'_{D_+}(R, G) \simeq V'_{D_+}(R, k_e), \tag{91}$$

where $\xi^g, \xi^c, \mu_2^g, \mu_2^c \gneqq 0$ and $\lambda_1^g, \lambda_1^c, \mu_1^g, \mu_1^c \geq 0$. □

4.3 When Distance R Is Large, However, ξ Is Small Enough

Assuming distance R is large and the constant ξ^g is small like Planck's constant, that is, $\xi^g \sim h$. The constant h is Planck's constant, $6.626E\text{-}34 J \cdot s$ and the constant $\mu_2^g = 1$. Assume that R is the radius of the universe within 46.5 billion light years ($4.65E$+10). Because one light year is approximately $9.461E$+15 meter, we assume that the radius of the universe $R \simeq 4.399E$+26 meter. Therefore, the following condition is satisfied:

$$\xi^g \mu_2^g R \simeq 2.915E\text{-}7 \ll 1. \tag{92}$$

The function $\exp(-\xi^g \mu_2^g R)$ is approximately equal to 1, that is, it is satisfied as follows:

$$\exp(-\xi^g \mu_2^g R) \simeq 1. \tag{93}$$

Therefore, the following expressions are satisfied:

$$\begin{aligned}\bar{g}_\pm &= -\frac{Gm}{(R \pm \lambda_1^g m)^2}(1-\exp(-\xi^g \mu_2^g R+\mu_1^g))+\frac{Gm\xi^g \mu_2^g}{(R \pm \lambda_1^g m)}\exp(-\xi^g \mu_2^g R+\mu_1^g)\\ &\simeq -\frac{Gm}{(R \pm \lambda_1^g m)^2}(1-\exp(\mu_1^g))+\frac{Gm\xi^g \mu_2^g}{(R \pm \lambda_1^g m)}\exp(\mu_1^g), \quad (\because \exp(-\xi^g \mu_2^g R) \simeq 1).\end{aligned} \tag{94}$$

When the condition $\xi^g \mu_2^g R \ll 1$ is satisfied, applying to mass M in circular orbit around mass m, the following equation is satisfied:

$$-\frac{GmM}{(R \pm \lambda_1^g m)^2}(1-\exp(\mu_1^g))+\frac{GmM\xi^g \mu_2^g}{(R \pm \lambda_1^g m)}\exp(\mu_1^g) = M\frac{v^2}{R}, \tag{95}$$

where m is mass within radius R and v is the rotation speed of mass M on radius R. The right side of equation(95) becomes centrifugal acceleration of mass M. Hence the following expressions satisfied:

$$\begin{aligned}v &= \sqrt{\frac{-GmR}{(R \pm \lambda_1^g m)^2}(1-\exp(\mu_1^g))+\frac{Gm\xi^g \mu_2^g}{(1 \pm \frac{\lambda_1^g m}{R})}\exp(\mu_1^g)}\\ &= \sqrt{\frac{-Gm}{(R \pm \lambda_1^g m)(1 \pm \frac{\lambda_1^g m}{R})}(1-\exp(\mu_1^g))+\frac{Gm\xi^g \mu_2^g}{(1 \pm \frac{\lambda_1^g m}{R})}\exp(\mu_1^g)}\\ &\simeq \sqrt{Gm\xi^g \mu_2^g \exp(\mu_1^g)}, \qquad (\because R \textit{ is large enough and } (1+\frac{\lambda_1^g m}{R}) \to 1).\end{aligned} \tag{96}$$

Therefore, we suggest that the following is satisfied:

Suggestion 4.9. *Let $m > 0$ (mass) and v (the rotation speed) be positive real numbers. For large distances $R > 0$ within $4.399E$+26, the following condition is satisfied:*

$$v \simeq \sqrt{Gm\xi^g \mu_2^g \exp(\mu_1^g)}, \tag{97}$$

where $\xi^g, \mu_2^g \gneqq 0$ and $\lambda_1^g, \mu_1^g \geq 0$. As a result, the rotation speed v at radius R approximate by a constant value $\sqrt{Gm\xi^g \mu_2^g \exp(\mu_1^g)}$, not depend on radius R. □

Milgrom have suggested the following equations for modified Newtonian dynamics (MOND), (Milgrom,1983; Moffat,2005; Wiki/Modified_Newtonian_dynamics,2025)

$$v \simeq \sqrt[4]{Gma_0}, \tag{98}$$

where a_0 is a new fundamental constant of Milgrom. Comparing equations (97) and (98), the following relationship holds:

$$\xi^g \mu_2^g \exp(\mu_1^g) = \sqrt{\frac{a_0}{Gm}}. \tag{99}$$

Namely, the equation (97) contains the equation (98).

According to equation (97), the rotation speed v depends on the constants G, m, ξ^g, μ_1^g and μ_2^g, not depend on radius R. It is noticed that these constants are decided by D_+-division entropy $S_{D_+}(x,k)$ and the distribution function $Q_{D_+}(x)$. According to the suggestion 4.9, let m be equal to mass of the Milky Way Galaxy, that is, $m \simeq 1.989E$+30$\times 2.0E$+12 kg, where mass of the sun is $1.989E{+}30$ and the sun count in the Milky Way Galaxy is $2.0E{+}12$. Therefore, if setting $\xi = 1E\text{-}34 \sim h$ (Planck's constant) and $\mu_2^g = 1$, then the rotation speed is satisfied depending on the constant μ_1^g as follows:

$$v \simeq 4.194E\text{-}1 \sqrt{\exp(\mu_1^g)}\ m/s. \tag{100}$$

For example, let $\mu_1^g = 26.36$, the rotation speed v became as follows:

$$v \simeq 2.222E{+}5\ m/s. \tag{101}$$

The speed of (101) is close to the rotation speed of the Milky Way Galaxy, that is, approximately $2.200E{+}5 \sim 2.400E{+}5$ m/s. Even without assuming dark matters, the galaxy rotation problem can be explained by the concept of entropy. This does not mean denying dark matters, however, new constants μ_1^g and μ_2^g may represent some kind of dark or virtual mass.

We have been discussing the rotation speed of galaxy using the Planck-type potential, but similar discussion can be made for the Yukawa type potential (refer to section 5 later). Furthermore, Napolitano et al. (Napolitano,2012) have analyzed the extended stellar kinematics of elliptical galaxies using the model with the Yukawa type gravitational potential. They have considered as a substitute for dark matters using the Yukawa correction to the Newton gravitational potential .

4.4 When Distance R Is Large Enough

If distance R is large enough, the equation(47) is satisfied as follows:

$$\tilde{g}_\pm = -V'_{D_+}(R, G) = -\frac{Gm}{(R \pm \lambda_1^g m)^2}, \quad (\because \exp(-\xi^g \mu_2^g R + \mu_1^g) \to 0). \tag{102}$$

Therefore, the following conditions are satisfied:

$$-\frac{Gm}{(R - \lambda_1^g m)^2} \lesssim -\frac{Gm}{R^2} \lesssim -\frac{Gm}{(R + \lambda_1^g m)^2}. \tag{103}$$

If distance R is large enough and the constant λ_1^g is small enough, that is, $\lambda_1^g \to 0$, then the gravitational acceleration $V'_{D_+}(R, G)$ becomes Newton's gravity.

4.4.1 Summarize the Gravitational Acceleration for Large Distances R

We summarize the above gravitational acceleration $V'_D(R, G)$ as follows:

1. The adjusted gravitational acceleration, R is large enough:

$$\tilde{g}_\pm = -\frac{Gm}{(R \pm \lambda_1^g m)^2}, \tag{104}$$

2. The original gravitational acceleration, R is large enough and $\lambda_1^g \to 0$:

$$g = -\frac{Gm}{R^2}, \tag{105}$$

where R is large enough. Newton's gravity is satisfied when R is large enough and $\lambda_1^g \to 0$. According to g of (105) and $\tilde{g}_\pm$ of (104) in the above equation, we suggest as follows:

Suggestion 4.10. *Gravity fluctuates depending on the value λ_1^g of D_+-division entropy coefficient. Let $m > 0$ be a real number (mass). For large $R > 1$, the following conditions are satisfied:*

$$\tilde{g}_- = -\frac{Gm}{(R - \lambda_1^g m)^2} \lesssim g \lesssim -\frac{Gm}{(R + \lambda_1^g m)^2} = \tilde{g}_+, \tag{106}$$

where $\lambda_1^g \geq 0$ is a real constant. □

The adjusted gravitational acceleration $\tilde{g}_\pm$ and $\bar{g}_\pm$ suggested above(4.10) is thought to be extensions of Newton's gravity. Besides, for large distances R, it is possible that the adjusted gravity $\tilde{g}_-$ or $\tilde{g}_+$ are smaller or larger towards the center than Newton's gravity g. Namely, the gravitational acceleration towards the center of a rotating object may fluctuate slightly with sufficient large distances. It is possible that $\tilde{g}_\pm$ and $\bar{g}_\pm$ can be given new model. Besides, the verification of existence such constants λ_1^g, λ_1^g and μ_2^g remains a topic for future issues.

5. The Yukawa Type Potential and Entropy, Comparison of Accelerations

5.1 Relationship With the Yukawa Type Potential and Potential $V_{D_+}(R,G)$

In this section, we consider the Yukawa type potentials, which represent the potential of elementary particles. The Yukawa type potential for gravitational acceleration has been studied to be related to near-field gravity theory (Yukawa,1935; Murata,Tanaka,2014; Ourabah,2023). Potential $V_{D_+}(R,G)$ of (46) contains an equation like the Yukawa potential. If we omit the first term in the equation(46), we can obtain as follows:

$$V_{D_+}(R,G)_{omit} = \frac{Gm}{R \pm \lambda_1^g m} \exp(-\xi^g \mu_2^g R \pm \mu_1^g). \tag{107}$$

We consider that by substituting constants as follows:

$$Gm := g_y^2, \quad \xi^g \mu_2^g := \lambda, \quad \mu_1^g := 0, \quad \lambda_1^g := 0, \tag{108}$$

where g_y is Yukawa's constant and $\lambda = mc/\hbar$. Thereby, we can obtain the Yukawa potential (Yukawa,1935) as follows:

$$V_{yukawa1}(R) = \pm g_y^2 \frac{\exp(-\lambda R)}{R}. \tag{109}$$

Therefore, it may also have applications in particle theory and other potential theory. Moreover, we also consider that by substituting constants into the equation(46) as follows:

$$\xi^g \mu_2^g := \lambda, \quad \exp(\mu_1^g) := \alpha, \quad \lambda_1^g := 0, \tag{110}$$

where $\lambda = mc/\hbar$ (Yukawa,1935; Fujii,1971; Murata,Tanaka,2014; Ourabah,2023). Thereby, we can obtain the Planck type potential as follows:

$$V_\alpha(R) = -G\frac{m}{R}(1 - \alpha \exp(-\lambda R)). \tag{111}$$

However, the Yukawa type potential suggested as follows (Yukawa,1935; Fujii,1971; Murata,Tanaka,2014; Ourabah,2023).

$$V_{yukawa2}(R) = -G\frac{m}{R}(1 + \alpha \exp(-\lambda R)). \tag{112}$$

The difference between (111) and (112) is the sign plus(+) and minus(−) of the second term in parentheses, that is, the sign of $\alpha \exp(-\lambda R)$. Therefore, the above equation(111) and (112) are incompatible, but similar. It seems necessary to consider another way to integrate two equations. Namely, we need to change the definition3.2 and the assumption3.4. These issues are described in the next subsection.

5.2 D_--Division Partial Entropy With Negative

Same as section 3, we define D_--division entropy $S_{D_-}(x,k)$, D_--division partial entropy $S_{D_-}(x)$, and the number of states $W_{D_-}(x)$. D_--division partial entropy $S_{D_-}(x)$ under $W_{D_-}(x)$ is defined by the number of states $W_{D_-}(x)$ (Fujino,2024).

Definition 5.1. *D_--division partial entropy $S_{D_-}(x)$. Let $x > 0$ be a real variable and $\xi \gneq 0$ be real constants. Let $D_-(x)$ be a negative real valued function that divides x. We define that the number of states $W_{D_-}(x)$ that divided x by $D_-(x)$, D_--division partial entropy $S_{D_-}(x)$ and a function $Q_{D_-}(x)$ as follows:*

$$W_{D_-}(x) = \frac{(D_-(x)+x)^{D_-(x)+x}}{D_-(x)^{D_-(x)} x^x}, \tag{113}$$

$$Q_{D_-}(x) = \frac{-\xi x}{D_-(x)} \gneq 0, \tag{114}$$

$$S_{D_-}(x) = \log W_{D_-}(x). \tag{115}$$

If $x = 0$, then it defines as $D_-(0) = 1$, $W_{D_-}(0) = 1$ and $S_{D_-}(0) = 0$. For any x, $Q_{D_-}(x)$ is satisfied as follows:

$$Q_{D_-}(x) \gneq 0, \quad Q'_{D_-}(x) \gneq 0, \quad \xi > Q_{D_-}(x). \tag{116}$$

However, because $D_-(x)$ is a negative real value, hence the part of $D_-(x)^{D_-(x)}$ and log *with a negative argument become a complex number. Therefore, $W_{D_-}(x)$ and $S_{D_-}(x)$ become a complex number. Therefore, it needs to be extended its to* *complex numbers as follows:*

$$D_{-}(x) = e^{2(n+1)\pi i} e^{\log(-D_{-}(x))}, \tag{117}$$

$$\begin{aligned} W_{D_{-}(x)} &= \frac{e^{2(n+1)\pi i(D_{-}(x)+x)} e^{(D_{-}(x)+x)\log(-(D_{-}(x)+x))}}{e^{2(n+1)\pi i D_{-}(x)} e^{D_{-}(x)\log(-D_{-}(x))} x^x} \\ &= e^{2(n+1)\pi i x}(1 + \frac{x}{D_{-}(x)})^{D_{-}(x)}(1 + \frac{D_{-}(x)}{x})^x, \end{aligned} \tag{118}$$

$$S_{D_{-}}(x) = (1 + \frac{Q_{D_{-}}(x)}{-\xi})\log(1 + \frac{Q_{D_{-}}(x)}{-\xi}) - \frac{Q_{D_{-}}(x)}{-\xi}\Big(\log(\frac{Q_{D_{-}}(x)}{\xi}) + \log(-1)\Big), \tag{119}$$

where $\log(-1) = 2(n+1)\pi i$*,* $n \geq 0$*, and* i *is an imaginary number. Treating the above equation*(119)*,* $S'_{D_{-}}(x)$ *and* $S''_{D_{-}}(x)$ *can be obtained as follows:*

$$S'_{D_{-}}(x) = \frac{Q'_{D_{-}}(x)}{-\xi}\Big(\log(1 + \frac{Q_{D_{-}}(x)}{-\xi}) - \log(\frac{Q_{D_{-}}(x)}{\xi})\Big) - \frac{Q'_{D_{-}}(x)}{-\xi}\log(-1), \tag{120}$$

$$S''_{D_{-}}(x) = \frac{Q'_{D_{-}}(x)}{-\xi}\Big(\frac{Q'_{D_{-}}(x)}{-\xi + Q_{D_{-}}(x)} - \frac{Q'_{D_{-}}(x)}{Q_{D_{-}}(x)}\Big) + \frac{Q''_{D_{-}}(x)}{-\xi}\Big(\log(1 + \frac{Q_{D_{-}}(x)}{-\xi}) - \log(\frac{Q_{D_{-}}(x)}{\xi})\Big) - \frac{Q''_{D_{-}}(x)}{-\xi}\log(-1). \tag{121}$$

The real value of D_{-}*-division partial entropy* $Re[S_{D_{-}}(x)]$ *becomes a negative value, where the function* $Re[x]$ *denotes the real value* x *for any positive variable* $x > 0$*.* □

Namely, this means that the above definition assumes the existence of negative partial entropy.

Definition 5.2. *D_{-}-division entropy $S_{D_{-}}(x, k)$. Let $x > 0$ be a real variable, $k \gneq 0$ and $D_{-}(x)$ be a negative real valued function that divides x. $S_{D_{-}}(x, k)$ is defined as follows:*

$$S_{D_{-}}(x, k) = kD_{-}(x)Re\Big[S_{D_{-}}(x)\Big] > 0, \tag{122}$$

It defines an approximation of $S_{D_{-}}(x, k)$ *and obtains* $Re[S_{D_{-}}(x)]$ *as follows:*

$$S_{D_{-}}(x, k) = \lambda_2 x^2 \pm \lambda_1 x, \tag{123}$$

$$Re\Big[S_{D_{-}}(x)\Big] = \frac{1}{k}\frac{Q_{D_{-}}(x)}{-\xi x}S_{D_{-}}(x, k) \leq 0, \tag{124}$$

where $Q_{D_{-}}(x)$ *is discussed again in the following subsection* 5.3. □

(Note): The parts (that is, D_{-}-division partial entropy) are negative, however, the whole (that is, D_{-}-division entropy) is positive. It can be thought of the division function $D_{-}(x)$ and the number of states $W_{D_{-}(x)}$ become a kind of wave function. Therefore, D_{-}-division partial entropy can be thought of as information in the form of the logarithm of a wave $W_{D_{-}(x)}$, which is the state of complex number. We would further research the meaning of negative divisions. (End of Note).

5.3 The Function $Q_{D_{-}}(x)$ for the Yukawa Type Potential

We find the function $Q_{D_{-}}(x)$ using the idea behind Planck's law and the logistic function for dynamical systems. Put the part of partial entropy $S''_{D_{-}}(x)$ as follows:

$$\frac{Q'_{D_{-}}(x)}{-\xi}\Big(\frac{-1}{\xi - Q_{D_{-}}(x)} - \frac{1}{Q_{D_{-}}(x)}\Big) = \mu(x) \geq 0, \tag{125}$$

where $\mu(x) > 0$ is a positive real function and $\xi > Q_{D_{-}}(x)$. The above parts of $\frac{Q'_{D_{-}}(x)}{-\xi}$ is negative value and the above parts of $(\frac{-1}{\xi - Q_{D_{-}}(x)} - \frac{1}{Q_{D_{-}}(x)})$ is also negative. Therefore, the above equation(125) is positive. By transforming according to the equation(27), we can represent as follows:

$$dQ_{D_{-}}\Big(\frac{-1}{\xi - Q_{D_{-}}(x)} - \frac{1}{Q_{D_{-}}(x)}\Big) = -\xi\mu(x)dx. \tag{126}$$

Integrating both sides gives as follows:

$$\log(\xi - Q_{D_{-}}(x)) - \log(Q_{D_{-}}(x)) = -\xi\int \mu(x)dx \pm \mu_1, \tag{127}$$

where $\mu_1 > 0$ and $\xi > 0$. Therefore, the following equation is satisfied:

$$\log(\frac{\xi}{Q_{D_-}(x)} - 1) = -\xi \int \mu(x)dx \pm \mu_1. \tag{128}$$

By transforming the above equation, it is satisfied as follows:

$$\frac{\xi}{Q_{D_-}(x)} - 1 = \exp(-\xi \int \mu(x)dx \pm \mu_1). \tag{129}$$

Because the right side is positive, the left side must also be positive, thus, it needs to be satisfied $\xi > Q_{D_-}(x)$. Therefore, the function $Q_{D_-}(x)$ is represented as follows:

$$Q_{D_-}(x) = \frac{\xi}{\exp(-\xi \int \mu(x)dx \pm \mu_1) + 1}. \tag{130}$$

$Q_{D_-}(x)$ becomes the distribution function of the position which divided the real value $-\xi x$ by $D_-(x)$. Let the distribution function $Q_{D_-}(x)$ be the Yukawa type distribution function. Therefore, the above equation can be adopted as the definition of $Q_{D_-}(x)$. The Yukawa type distribution function $Q_{D_-}(R)$ is thought of the expansion of the Fermi-Dirac distribution function and the distribution of the Woods-Saxon potential.

5.4 The Inverse of $Re[S_{D_-}(x)]$ and Potential $V_{D_-}(x,k)$

We have defined the approximation of D_--division entropy $S_{D_-}(x,k)$ by the definition 5.2. Therefore, the inverse of $Re[S_{D_-}(x)]$ is obtained as follows:

$$\frac{1}{Re[S_{D_-}(x)]} = k\frac{-\xi x}{Q_{D_-}(x)}\frac{1}{\lambda_2 x^2 \pm \lambda_1 x}, \tag{131}$$

where because $Re[S_{D_-}(x)] \leq 0$ and $Q_{D_-}(x) \geq 0$, therefore $\lambda_2 x^2 \pm \lambda_1 x > 0$. By the equation(130), we can represent as follows:

$$\frac{1}{Re[S_{D_-}(x)]} = -k\frac{1}{\lambda_2 x \pm \lambda_1}(\exp(-\xi \int \mu(x)dx \pm \mu_1) + 1). \tag{132}$$

We define the inverse of $Re[S_{D_-}(x)]$ as potential $V_{D_-}(x,k)$:

$$V_{D_-}(x,k) = -k\frac{\frac{1}{\lambda_2}}{x \pm \frac{\lambda_1}{\lambda_2}}(1 + \exp(-\xi \int \mu(x)dx \pm \mu_1)). \tag{133}$$

In other words, the above potential $V_{D_-}(x,k)$ can be defined as the product of a constant k, the function $Q_{D_-}(x) = -\xi/D_-(x)$, and the inverse of real parts of D_+-division partial entropy $Re[S_{D_-}(x)] \leq 0$. The first derivative $V'_{D_-}(x,k)$ is satisfied as follows:

$$V'_{D_-}(x,k) = \frac{k\frac{1}{\lambda_2}}{(x \pm \frac{\lambda_1}{\lambda_2})^2}(1 + \exp(-\xi \int \mu(x)dx \pm \mu_1)) + \frac{k\frac{1}{\lambda_2}\xi\mu(x)}{x \pm \frac{\lambda_1}{\lambda_2}}\exp(-\xi \int \mu(x)dx \pm \mu_1). \tag{134}$$

Let $V_{D_-}(x,k)$ be potential of $S_{D_-}(x)$, and $V'_{D_-}(x,k)$ be acceleration of $S_{D_-}(x)$. Same as assumption3.4 and 4.1, the above description assumes the following assumption:

Assumption 5.3. *It assumes that potential $V_{D_-}(x,k)$ is defined the inverse of real parts of D_+-division partial entropy $Re[S_{D_-}(x)]$. Therefore, acceleration $V'_{D_-}(x,k)$ is defined the first derivative of $V_{D_-}(x,k)$.* □

Assumption 5.4. *It assumes the constant $1/\lambda_2$ is equal to mass m within R. Assume that 2-times the inverse of entropy acceleration, $2/S''_{D_-}(x,k)$, that is, $1/\lambda_2$ is equal to mass m within R. In other words, mass m within R is defined as the inverse of second order term of $S_{D_-}(x,k)$, that is, $1/\lambda_2$.* □

Therefore, same as the description of section4, we obtain the following results:

$$Q_{D_-}(R) = \frac{\xi^g}{\exp(-\xi^g\mu_2^g R \pm \mu_1^g) + 1}, \tag{135}$$

$$V_{D_-}(R,G) = -\frac{Gm}{R \pm \lambda_1^g m}(1 + \exp(-\xi^g\mu_2^g R \pm \mu_1^g)), \tag{136}$$

$$\begin{aligned}\hat{g}_\pm &= -V'_{D_-}(R,G)\\ &= -\frac{Gm}{(R \pm \lambda_1^g m)^2}(1+\exp(-\xi^g\mu_2^g R \pm\mu_1^g)) - \frac{Gm\xi^g\mu_2^g}{R \pm \lambda_1^g m}\exp(-\xi^g\mu_2^g R \pm\mu_1^g),\end{aligned} \tag{137}$$

$$\lim_{\mu_1^g\to 0} \hat{g}_{\pm\lambda_1^g+\mu_1^g} = -\frac{2G}{(\pm\lambda_1^g)^2 m} - \frac{G\xi^g\mu_2^g}{\pm\lambda_1^g}, \quad (\because R \to 0), \tag{138}$$

where $\xi^g \gneqq 0, \mu_2^g \gneqq 0, \mu_1^g \geq 0$ and $\lambda_1^g \geq 0$.

Let $V_{D_-}(R,G)$ be the Yukawa type potential(±) and $\hat{g}_\pm$ be the Yukawa type adjusted gravitational accelerations(±).

The above equation(135) resembles the distribution of nuclei model, and the negative of (135) resembles the Woods-Saxon potential. The above equation(136) and (112) are compatible because the sign of exp is same. Namely, if setting as follows:

$$\exp(\mu_1^g) := \alpha, \quad \xi^g\mu_2^g := \lambda, \quad \lambda_1^g := 0, \tag{139}$$

then the equation(136) becomes the Yukawa potential(112). The second term of (136) is same as negative expression of the Yukawa potential (109). Namely, by introducing partial entropy with negative, that is, D_--division partial entropy $Re[S_{D_-}(x)] \leq 0$, the Yukawa type potential can be explained. The existence of the Yukawa type potential is thought to indicate the existence of negative partial entropy.

For comparison, we describe results of section4 as follows:

$$Q_{D_+}(R) = \frac{\xi^g}{\exp(-\xi^g\mu_2^g R + \mu_1^g) - 1}, \tag{140}$$

$$V_{D_+}(R,G) = -\frac{Gm}{R \pm \lambda_1^g m}(1 - \exp(-\xi^g\mu_2^g R \pm \mu_1^g)), \tag{141}$$

$$\begin{aligned}\bar{g}_\pm &= -V'_{D_+}(R,G)\\ &= -\frac{Gm}{(R \pm \lambda_1^g m)^2}(1-\exp(-\xi^g\mu_2^g R \pm\mu_1^g)) + \frac{Gm\xi^g\mu_2^g}{R \pm \lambda_1^g m}\exp(-\xi^g\mu_2^g R \pm\mu_1^g),\end{aligned} \tag{142}$$

$$\lim_{\mu_1^g\to 0} \bar{g}_{\pm\lambda_1^g+\mu_1^g} = \frac{G\xi^g\mu_2^g}{\pm\lambda_1^g}, \quad (\because R \to 0), \tag{143}$$

where $\xi^g \gneqq 0, \mu_2^g \gneqq 0, \mu_1^g \geq 0$ and $\lambda_1^g \geq 0$.

Let $V_{D_+}(R,G)$ be the Planck type potential(±) and $\bar{g}_\pm$ be the Planck type adjusted gravitational accelerations(±).

The second term of equation(141) is same as the positive expression of the Yukawa type potential(109). As weak proximity accelerations(±), we put the equation(143) as follows:

$$g_\pm^{wp} = \lim_{\mu_1^g\to 0} \bar{g}_{\pm\lambda_1^g+\mu_1^g} = \frac{G\xi^g\mu_2^g}{\pm\lambda_1^g}, \qquad (\because R \to 0), \tag{144}$$

and as strong proximity accelerations(±), we put the equation(138) as follows:

$$g_\pm^{sp} = \lim_{\mu_1^g\to 0} \hat{g}_{\pm\lambda_1^g+\mu_1^g} = -\frac{2G}{(\pm\lambda_1^g)^2 m} - \frac{G\xi^g\mu_2^g}{\pm\lambda_1^g}, \quad (\because R \to 0). \tag{145}$$

The acceleration $g_\pm^{wp}$ has no effect of mass m instead it depends on ξ^g, μ_2^g and λ_1^g, and $g_\pm^{sp}$ has the effect of mass m. According to the above discussion, there may exist 11-types accelerations (interactions) related to g, such as $\bar{g}_\pm, \hat{g}_\pm, \tilde{g}_\pm, g_\pm^{sp}$, and $g_\pm^{wp}$, including the gravitational acceleration g. Next, we compare these accelerations.

5.5 Comparing Accelerations $\bar{g}_\pm, \hat{g}_\pm, \tilde{g}_\pm, g^{sp}_\pm$ and $g^{wp}_\pm$

Comparing the above (137), (142), (144) and (145), we obtain the following are relationships:

$$g^{sp}_+ < \hat{g}_+ < \tilde{g}_+ < \bar{g}_+ < 0 < g^{wp}_+, \tag{146}$$

$$\hat{g}_- < \tilde{g}_- < g^{sp}_- < g^{wp}_- < \bar{g}_-, \quad \because) \quad m < \frac{1}{\lambda^g_1 \xi^g \mu^g_2}, \tag{147}$$

$$\bar{g}_- < g^{wp}_- < g^{sp}_- < \tilde{g}_- < \hat{g}_-, \quad \because) \quad \frac{1}{\lambda^g_1 \xi^g \mu^g_2} < m, \tag{148}$$

and the followinga are satisfied:

$$\hat{g}_+ < \bar{g}_-, \qquad \tilde{g}_- < g < \tilde{g}_+, \qquad \bar{g}_+ < g < \hat{g}_- < 0. \tag{149}$$

It assumes that $g^{wp}_- < g^{sp}_-$, for all $R > 0$, it is satisfied as follows:

$$\frac{G\xi^g \mu^g_2}{-\lambda^g_1} < -\frac{2G}{(-\lambda^g_1)^2 m} - \frac{G\xi^g \mu^g_2}{-\lambda^g_1}. \tag{150}$$

Therefore, it is satisfied as follows:

$$\frac{1}{\lambda^g_1 \xi^g \mu^g_2} < m. \tag{151}$$

It assumes that $g^{sp}_- < \tilde{g}_-$, for all $R > 0$, it is satisfied as follows:

$$R - \lambda^g_1 m > \pm \lambda^g_1 m \sqrt{\frac{1}{2 - \lambda^g_1 m \xi^g \mu^g_2}}, \tag{152}$$

The above inequality(152) holds true even for small enough R. The following is satisfied:

$$-\lambda^g_1 m \geq -\lambda^g_1 m \sqrt{\frac{1}{2 - \lambda^g_1 m \xi^g \mu^g_2}}. \tag{153}$$

Therefore, it is satisfied as follows:

$$\frac{1}{\lambda^g_1 \xi^g \mu^g_2} \leq m. \tag{154}$$

Namely, because $R > 0$, we only consider that inequalities(148) on the condition $\frac{1}{\lambda^g_1 \xi^g \mu^g_2} < m$. Summarizing the above inequalities (146), (148) and (149)the following are satisfied:

$$g^{sp}_+ < \hat{g}_+ < \bar{g}_- < g^{wp}_- < g^{sp}_- < \tilde{g}_- < g < \tilde{g}_+ < \bar{g}_+ < \hat{g}_- < 0 < g^{wp}_+, \quad (\because \frac{1}{\lambda^g_1 \xi^g \mu^g_2} < m). \tag{155}$$

(Note): The direction toward the center is negative, therefore, the more negative value is smaller. (End of Note)

5.6 One Attempt to Compare the Ratios of 4-Interactions

For example, we consider the above inequalities(155). Set constants as follows:

$$\begin{aligned}
&G := 6.674E\text{-}11, && G \textit{ is the gravitational constant},\\
&\xi^g := h = 6.626E\text{-}34, && h \textit{ is Planck's constant},\\
&m := m_p = 2.176E\text{-}8, && m_p \textit{ is the Planck mass}(kg),\\
&\mu^g_1 \gneq 0, && \mu^g_1 \textit{ is a real constant},\\
&R \leq 1.305E{+}26, && R \textit{ is a radius within the Universe}(meter).
\end{aligned} \tag{156}$$

where according to case 2) of suggestion4.3 and of suggestion4.5, inequalities (68) and (154), the values λ^g_1 and μ^g_1 are satisfied as follows:

$$0 \leq 1 + (R \pm \lambda^g_1 m_p)\xi^g \mu^g_2, \quad \frac{-1}{\lambda^g_1 \xi^g \mu^g_2} < m_p. \tag{157}$$

(Note): The values μ_2^g in each equation $g_\pm^{sp}$ and g_-^{wp} take on different values $(\mu_2^g)_\pm^{sp}$ and $(\mu_2^g)_-^{wp}$, respectively. (End of Note)

On the above inequalities (155), the following are satisfied:

1. Compare $\hat{g}_-$ and g_-^{wp}; The ratio of the Yukawa type adjusted gravitational(−) $\hat{g}_-$ to weak proximity acceleration(−) g_-^{wp} is obtained as follows:

$$\left| \frac{\hat{g}_-}{g_-^{wp}} \right| \simeq \left| \frac{\frac{2Gm_p}{(R-((\hat{\lambda}_1^g)_-)m_p)^2}}{\frac{G\xi^g(\mu_2^g)_-^{wp}}{(\lambda_1^g)_-^{wp}}} + \frac{\frac{Gm_p\xi^g}{(R-((\hat{\lambda}_1^g)_-)m_p)}}{\frac{G\xi^g(\mu_2^g)_-^{wp}}{(\lambda_1^g)_-^{wp}}} \right| \simeq \left| \frac{2\cdot 10^{-8}(\lambda_1^g)_-^{wp}}{10^{2A-34}(\mu_2^g)_-^{wp}} + \frac{10^{-8}(\lambda_1^g)_-^{wp}}{10^{A-34}(\mu_2^g)_-^{wp}} \right| \simeq 1E\text{-}34, \tag{158}$$

where

$$\begin{aligned}
&(\mu_2^g)_-^{wp} := 1E{+}60, && (\mu_2^g)_-^{wp} \textit{ is the constant } \mu_2^g \textit{ of } g_-^{wp},\\
&(\lambda_1^g)_-^{wp} := 1E{+}A, && (\lambda_1^g)_-^{wp} \textit{ is the constant } \lambda_1^g \textit{ of } g_-^{wp},\\
&(\hat{\mu}_2^g)_- := 1E{+}A, && (\hat{\mu}_2^g)_- \textit{ is the constant } \mu_2^g \textit{ of } \hat{g}_-,\\
&(\hat{\lambda}_1^g)_- := 1E{+}A, && (\hat{\lambda}_1^g)_- \textit{ is the constant } \lambda_1^g \textit{ of } \hat{g}_-,\\
&(R \pm ((\hat{\lambda}_1^g)_-)m_p)^2 \simeq 1E{+}2A, && 0 \le A \le 26,\ A \textit{ is a constant},\\
&(\hat{\mu}_1^g)_- := \xi^g((\hat{\mu}_2^g)_-)R, && (\hat{\mu}_1^g)_- \textit{ is the constant } \mu_1^g \textit{ of } \hat{g}_-.
\end{aligned} \tag{159}$$

2. Compare $\bar{g}_+$ and g_-^{wp}; The ratio of the Planck type adjusted gravitational(+) $\bar{g}_+$ to weak proximity acceleration(−) g_-^{wp} is obtained as follows:

$$\left| \frac{\bar{g}_+}{g_-^{wp}} \right| \simeq \left| \frac{\frac{Gm_p\xi^g}{(R-((\bar{\lambda}_1^g)_-)m_p)}}{\frac{G\xi^g(\mu_2^g)_-^{wp}}{(\lambda_1^g)_-^{wp}}} \right| \simeq \left| \frac{10^{-8}(\lambda_1^g)_-^{wp}}{10^{A-34}(\mu_2^g)_-^{wp}} \right| \simeq 1E\text{-}34, \tag{160}$$

where

$$\begin{aligned}
&(\mu_2^g)_-^{wp} := 1E{+}60, && (\mu_2^g)_-^{wp} \textit{ is the constant } \mu_2^g \textit{ of } g_-^{wp},\\
&(\lambda_1^g)_-^{wp} := 1E{+}A, && (\lambda_1^g)_-^{wp} \textit{ is the constant } \lambda_1^g \textit{ of } g_-^{wp},\\
&(\bar{\mu}_2^g)_+ := 1E{+}A, && (\bar{\mu}_2^g)_+ \textit{ is the constant } \mu_2^g \textit{ of } \bar{g}_+,\\
&(\bar{\lambda}_1^g)_+ := 1E{+}A, && (\bar{\lambda}_1^g)_+ \textit{ is the constant } \lambda_1^g \textit{ of } \bar{g}_+,\\
&(R \pm ((\hat{\lambda}_1^g)_-)m_p)^2 \simeq 1E{+}2A, && 0 \le A \le 26,\ A \textit{ is a constant},\\
&(\bar{\mu}_1^g)_+ := \xi^g((\bar{\mu}_2^g)_+)R, && (\bar{\mu}_1^g)_+ \textit{ is the constant } \mu_1^g \textit{ of } \bar{g}_+.
\end{aligned} \tag{161}$$

(Note): The following are satisfied:

$$\exp(-\xi^g((\mu_2^g)_-)R + \mu_1^g) > \exp(0) = 1, \tag{162}$$

where μ_1^g is satisfied $-\xi^g((\mu_2^g)_-)R + \mu_1^g > 0$. Thus, if the case $\hat{g}_-$, then it is satisfied $\frac{1}{2} > Q_{D_-}(R) > 0$. Similarly, if the case $\bar{g}_+$, then it is satisfied $Q_{D_+}(R) > 0$. (End of Note)

3. Compare $\tilde{g}_\pm$ and g_-^{wp}; The ratio of the adjusted gravitational(±) $\tilde{g}_\pm$ to weak proximity acceleration(−) g_-^{wp} is obtained as follows:

$$\left| \frac{\tilde{g}_\pm}{g_-^{wp}} \right| = \left| \frac{\frac{Gm_p}{(R\pm((\tilde{\lambda}_1^g)_-)m_p)^2}}{\frac{G\xi^g(\mu_2^g)_-^{wp}}{(\lambda_1^g)_-^{wp}}} \right| = \left| \frac{10^{-8}(\lambda_1^g)_-^{wp}}{10^{2A-34}(\mu_2^g)_-^{wp}} \right| \simeq 1E\text{-}34, \tag{163}$$

where

$$\begin{aligned}
&(\mu_2^g)_-^{wp} := 1E{+}60, && (\mu_2^g)_-^{wp} \textit{ is the constant } \mu_2^g \textit{ of } g_-^{wp},\\
&(\lambda_1^g)_-^{wp} := 1E{+}2A, && (\lambda_1^g)_-^{wp} \textit{ is the constant } \lambda_1^g \textit{ of } g_-^{wp},\\
&(R \pm ((\tilde{\lambda}_1^g)_-)m_p)^2 \simeq 1E{+}2A, && 0 \le A \le 26,\ A \textit{ is a constant},\\
&(\tilde{\mu}_1^g)_- \ll \xi^g((\tilde{\mu}_2^g)_-)R, && (\tilde{\mu}_1^g)_- \textit{ is the constant } \mu_1^g \textit{ of } \tilde{g}_-.
\end{aligned} \tag{164}$$

4. Compare g_+^{sp} and g_-^{wp}; The ratio of strong proximity(+) g_+^{sp} to weak proximity acceleration(−) g_-^{wp} is obtained as follows:

$$\left| \frac{g_+^{sp}}{g_-^{wp}} \right| = \left| \frac{-\frac{2G}{(\lambda_1^g)^2 m_p} - \frac{G\xi^g(\mu_2^g)_+^{sp}}{+\lambda_1^g}}{\frac{G\xi^g(\mu_2^g)_-^{wp}}{-\lambda_1^g}} \right| = \left| \frac{2}{\lambda_1^g m_p \xi^g(\mu_2^g)_-^{wp}} + \frac{(\mu_2^g)_+^{sp}}{(\mu_2^g)_-^{wp}} \right| \simeq 1E{+}5, \tag{165}$$

where

$$\begin{aligned}
&(\mu_2^g)_+^{sp} := 1E{+}65, && (\mu_2^g)_+^{sp} \textit{ is the constant } \mu_2^g \textit{ of } g_+^{sp},\\
&(\mu_2^g)_-^{wp} := 1E{+}60, && (\mu_2^g)_-^{wp} \textit{ is the constant } \mu_2^g \textit{ of } g_-^{wp},\\
&\lambda_1^g := 1E\text{-}20, && \lambda_1^g \textit{ is the constant } \lambda_1^g \textit{ of } g_-^{wp} \textit{ and } g_+^{sp}.
\end{aligned} \tag{166}$$

5. Compare g_-^{sp} and g_-^{wp}; The ratio of strong proximity(−) g_-^{sp} to weak proximity acceleration(−) g_-^{wp} is obtained as follows:

$$\left|\frac{g_-^{sp}}{g_-^{wp}}\right| = \left|\frac{-\frac{2G}{(\lambda_1^g)^2 m_p} - \frac{G\xi^g(\mu_2^g)_-^{sp}}{+\lambda_1^g}}{\frac{G\xi^g(\mu_2^g)_-^{wp}}{-\lambda_1^g}}\right| = \left|\frac{2}{\lambda_1^g m_p \xi^g (\mu_2^g)_-^{wp}} - \frac{(\mu_2^g)_-^{sp}}{(\mu_2^g)_-^{wp}}\right| \simeq 1, \tag{167}$$

where

$$\begin{aligned} &(\mu_2^g)_-^{sp} := 1E\text{+}60, && (\mu_2^g)_-^{sp} \textit{ is the constant } \mu_2^g \textit{ of } g_-^{sp},\\ &(\mu_2^g)_-^{wp} := 1E\text{+}60, && (\mu_2^g)_-^{wp} \textit{ is the constant } \mu_2^g \textit{ of } g_-^{wp},\\ &\lambda_1^g := 1E\text{-}20, && \lambda_1^g \textit{ is the constant } \lambda_1^g \textit{ of } g_-^{wp} \textit{ and} g_-^{sp}. \end{aligned} \tag{168}$$

Let g_+^{sp} considered as the strong interaction, g_-^{wp} as the weak interaction and $\tilde{g}_\pm$ (or g) as gravity. If the strong interaction (strong proximity acceleration(+) g_+^{sp}) set to 1, then the ratios of the fundamental 3-interactions in nature (strong interaction, weak interaction and gravity) can be expressed as follows:

$$\begin{array}{ccc} \textit{strong}, & \textit{weak}, & \textit{gravity},\\ 1, & 1E\text{-}5, & 1E\text{-}39,\\ g_+^{sp}, & g_-^{wp} \textit{ or } g_-^{sp}, & \tilde{g}_\pm \textit{ or } \hat{g}_\pm\\ (\mu_2^g)_+^{sp}, & (\mu_2^g)_-^{wp} \textit{ or } (\mu_2^g)_-^{sp}, & \dfrac{m_p}{\xi^g},\\ 1E\text{+}65, & 1E\text{+}60, & 1E\text{+}26. \end{array} \tag{169}$$

The above ratios correspond to those of $(\mu_2^g)_+^{sp} \simeq 1E+65$, $(\mu_2^g)_-^{wp} \simeq 1E+60$, $(\mu_2^g)_-^{sp} \simeq 1E+60$ and $\frac{m_p}{\xi^g} \simeq 1E+26$.

Next, let us consider the relationship with electromagnetic forces. We interpret adjusted gravity $\bar{g}_\pm$ and $\hat{g}_\pm$ to adjusted electromagnetic force $\bar{E}_\pm$ and $\hat{E}_\pm$ as follows:

$$\begin{aligned} \bar{E}_\pm &:= -V'_{D_+}(R, k_e)\\ &= -\frac{k_e e_q}{(R \pm \lambda_1^c e_q)^2}(1 - \exp(-\xi^c \mu_2^c R \pm \mu_1^c)) + \frac{k_e e_q \xi^c \mu_2^c}{R \pm \lambda_1^c e_q}\exp(-\xi^c \mu_2^c R \pm \mu_1^c), \end{aligned} \tag{170}$$

$$\begin{aligned} \hat{E}_\pm &:= -V'_{D_-}(R, k_e)\\ &= -\frac{k_e e_q}{(R \pm \lambda_1^c e_q)^2}(1 + \exp(-\xi^c \mu_2^c R \pm \mu_1^c)) - \frac{k_e e_q \xi^c \mu_2^c}{R \pm \lambda_1^c e_q}\exp(-\xi^c \mu_2^c R \pm \mu_1^c), \end{aligned} \tag{171}$$

where $\xi^c \gneq 0$, $\mu_2^c \gneq 0$, $\mu_1^c \geq 0$ and $\lambda_1^c \geq 0$. Let $\bar{E}_\pm$ be the Planck type adjusted electromagnetic(±) and $\hat{E}_\pm$ be the Yukawa type adjusted electromagnetic(±). Similarly, we set as follows:

$$\begin{aligned} &G := 6.674E\text{-}11, && G \textit{ is the gravitational constant},\\ &\xi^g = \xi^c := h = 6.626E\text{-}34, && h \textit{ is Planck's constant},\\ &k_e := 8.987E\text{+}9, && k_e \textit{ is Coulomb's constant},\\ &e_q := 1.604E\text{-}19, && e_q \textit{ is the elementary charge},\\ &m := m_p = 2.176E\text{-}8, && m_p \textit{ is the Planck mass}(kg),\\ &R \textit{ is small enough}, && R \textit{ is distance}(unit : meter). \end{aligned} \tag{172}$$

where according to case 2) of suggestion 4.5, the values λ_1^g and μ_1^g are satisfied as follows:

$$0 < 1 \pm \lambda_1^g m_p \xi^g \mu_2^g, \quad 0 < 1 \pm \lambda_1^c e_q \xi^c \mu_2^c, \quad (R \to 0). \tag{173}$$

(Note): The values μ_2^c in each equation $\hat{E}_+$ and $\bar{E}_-$, take on different the values $\hat{\mu}_2^c$ and $\bar{\mu}_2^c$, respectively. (End of Note)

On the above inequalities(155), the following are satisfied:

1. Compare $\hat{E}_+$ and g_-^{wp}; The ratio of the Yukawa type adjusted electromagnetic(+) $\hat{E}_+$ to weak proximity acceleration(−) g_-^{wp} is obtained as follows:

$$\left|\frac{\hat{E}_+}{g_-^{wp}}\right| = \left|\frac{-\frac{2k_e}{\lambda_1^{c2} e_q}}{\frac{G\xi^g(\mu_2^g)_-^{wp}}{\lambda_1^g}} + \frac{-\frac{k_e \xi^c (\hat{\mu}_2^c)_+}{\lambda_1^c}}{\frac{G\xi^g(\mu_2^g)_-^{wp}}{\lambda_1^g}}\right| \simeq 1.347E\text{+}3 \ \simeq 1E\text{+}3, \tag{174}$$

where
$$
\begin{aligned}
&(\hat{\mu}_2^c)_+ := 1E{+}43, \quad (\hat{\mu}_2^c)_+ \textit{ is the constant } \mu_2^c \textit{ of } \hat{E}_+,\\
&(\mu_2^g)_-^{wp} := 1E{+}60, \quad (\mu_2^g)_-^{wp} \textit{ is the constant } \mu_2^g \textit{ of } g_-^{wp},\\
&\lambda_1^g = \lambda_1^c < 1E\text{-}20.
\end{aligned} \tag{175}
$$

2. Compare $\bar{E}_-$ and g_+^{wp}; The ratio of the Planck type adjusted electromagnetic$(-)$ $\bar{E}_-$ to weak proximity acceleration$(-)$ g_-^{wp} is obtained as follows:

$$
\left| \frac{\bar{E}_-}{g_-^{wp}} \right| = \frac{k_e \xi^c (\bar{\mu}_2^c)_- \lambda_1^g}{G \xi^g (\mu_2^g)_-^{wp} \lambda_1^c} \exp(\mu_1^c) \simeq 1.347E{+}3 \simeq 1E{+}3, \tag{176}
$$

where
$$
\begin{aligned}
&(\bar{\mu}_2^c)_- := 1E{+}43, \quad (\bar{\mu}_2^c)_- \textit{ is the constant } \mu_2^c \textit{ of } \bar{E}_-,\\
&(\mu_2^g)_-^{wp} := 1E{+}60, \quad (\mu_2^g)_-^{wp} \textit{ is the constant } \mu_2^g \textit{ of } g_-^{wp},\\
&\mu_1^c \to 0, \quad \lambda_1^g = \lambda_1^c < 1E\text{-}20.
\end{aligned} \tag{177}
$$

We interpret $\hat{E}_+$ and $\bar{E}_-$ as electromagnetic force. If the strong interaction (strong proximity acceleration(+) g_+^{sp}) set to 1, then by combine with the table(169), the ratios of the fundamental 4-interactions in nature (strong interaction, electromagnetic force, weak interaction and gravity) can be expressed as follows:

$$
\begin{array}{cccc}
\textit{strong}, & \textit{electromagnetic}, & \textit{weak}, & \textit{gravity},\\
1, & 1E\text{-}2, & 1E\text{-}5, & 1E\text{-}39,\\
g_+^{sp}, & \hat{E}_+ \textit{ or } \bar{E}_-, & g_-^{wp}, & \tilde{g}_\pm \textit{ or } \hat{g}_\pm,\\
(\mu_2^g)_+^{sp}, & \frac{k_e}{G}(\hat{\mu}_2^c)_+ \textit{ or } \frac{k_e}{G}(\bar{\mu}_2^c)_-, & (\mu_2^g)_-^{wp}, & \frac{m_p}{\xi^g},\\
1E{+}65, & 1E{+}63, & 1E{+}60, & 1E{+}26.
\end{array} \tag{178}
$$

The above ratios correspond to those of $(\mu_2^g)_+^{sp} \simeq 1E{+}65$, $\frac{k_e}{G}(\hat{\mu}_2^c)_+ \simeq \frac{k_e}{G}(\bar{\mu}_2^c)_- \simeq 1E{+}63$, $(\mu_2^g)_-^{wp} \simeq 1E{+}60$ and $\frac{m_p}{\xi^g} \simeq 1E{+}26$, and depend on the values m_p and ξ^g. Therefore, by considering strong proximity acceleration(+) g_+^{sp} is regarded as strong interaction, weak proximity acceleration$(-)$ g_-^{wp} as weak interaction, adjusted gravity $\tilde{g}_\pm$ as gravity and adjusted electromagnetic force $\hat{E}_+$ or $\bar{E}_-$ as electromagnetic force, it is possible to explain the ratios of the fundamental 4-interactions in nature.

Napolitano et al (Napolitano,2012) have analyzed the extended stellar kinematics of elliptical galaxies using the Yukawa-type gravitational potential. Furthermore, they have considered as one substitute for dark matters using the Yukawa correction to Newton's law. When comparing the ratios of the above accelerations, constants $(\mu_2^g)_\pm^{sp}$, $(\mu_2^g)_\pm^{wp}$, $(\hat{\mu}_2^c)_-$, $(\bar{\mu}_2^c)_-$, $(\hat{\lambda}_1^g)_-$ and $(\tilde{\lambda}_1^g)_-$ take extremely large numbers, and ξ^g, λ_1^g and λ_1^c extremely small numbers. Therefore, it is thought to be difficult to verify the existence of such numbers. However, just because a constant is too large or too small to be detected does not mean that the constant does not exist. These constants may be considered as one substitute for dark matters. These interactions and constants need to be verified, and numerical verification of the above model would be future issues. In the future, we would like to further examine the relationship with gravity and interactions through the Planck potential and the Yukawa potential.

5.7 Relationship Diagram

The contents of the discussion so far are shown in a diagram.

Relationship diagram:

$$
\begin{array}{ccc}
S_{D_+}(k,x) = k\frac{\xi}{Q_{D_+}(x)} S_{D_+}(x) > 0 & & S_{D_-}(k,x) = -k\frac{\xi}{Q_{D_-}(x)} S_{D_-}(x) > 0\\
S_{D_+}(x) > 0 & \xrightarrow{negative} & S_{D_-}(x) < 0\\
\textit{Partial Entropy} & & \textit{Negative Partial Entropy}\\
\downarrow & & \downarrow\\
Q_{D_+}(x) = \dfrac{\xi}{\exp(-\xi\int \mu(x)dx \pm \mu_1) - 1} & & Q_{D_-}(x) = \dfrac{\xi}{\exp(-\xi\int \mu(x)dx \pm \mu_1) + 1}\\
\textit{Planck type distribution} & & \textit{Yukawa type distribution}\\
(\textit{Bose-Einstein type}) & & (\textit{Fermi-Dirac type})\\
\downarrow & & \downarrow\\
V_{D_+}(R,G) = & & V_{D_-}(R,G) =\\
-\frac{Gm}{R \pm \lambda_1^g m}(1 - \exp(-\xi^g \mu_2^g R \pm \mu_1^g)) & & -\frac{Gm}{R \pm \lambda_1^g m}(1 + \exp(-\xi^g \mu_2^g R \pm \mu_1^g))\\
\textit{Planck type potential} & & \textit{Yukawa type potential}\\
{\scriptstyle \lambda^g \to 0,\ \xi^g \mu_2^g := \lambda,\ \exp(\mu_1^g) := \alpha} \downarrow & & {\scriptstyle \lambda^g \to 0,\ \xi^g \mu_2^g := \lambda,\ \exp(\mu_1^g) := \alpha} \downarrow
\end{array} \tag{179}
$$

$$
\begin{array}{ccccc}
V_\alpha(R) = -G\frac{m}{R}(1-\alpha\exp(-\lambda R)) & & & & V_{yukawa2}(R) = -G\frac{m}{R}(1+\alpha\exp(-\lambda R)) \\
\textit{Planck type potential} & & & & \textit{Yukawa type potential} \\
\downarrow{\scriptstyle R\to large} & & & & \downarrow{\scriptstyle R\to large} \\
V_\alpha(R) = -G\frac{m}{R} & & & & V_{yukawa2}(R) = -G\frac{m}{R} \\
\downarrow & & & & \downarrow \\
g = -V'_\alpha(R) = -G\frac{m}{R^2} & \rightarrow & g = -G\frac{m}{R^2} & \leftarrow & g = -V'_{yukawa2}(R) = -G\frac{m}{R^2} \\
\textit{Gravitational acceleration} & & & & \textit{Gravitational acceleration} \\
\downarrow{\scriptstyle R\to 0} & & & & \downarrow{\scriptstyle R\to 0} \\
\lim_{\mu_1^g\to 0}\bar{g}_{\pm\lambda_1^g+\mu_1^g} = \frac{G\xi^g\mu_2^g}{\pm\lambda_1^g} & & & & \lim_{\mu_1^g\to 0}\bar{g}_{\pm\lambda_1^g+\mu_1^g} = -\frac{2G}{(\pm\lambda_1^g)^2 m} - \frac{G\xi^g\mu_2^g}{\pm\lambda_1^g} \\
\textit{Weak proximity } g_\pm^{wp} & & & & \textit{Strong proximity } g_\pm^{sp}
\end{array}
$$

The Planck type distribution may be thought to be the wave distribution such that represents electromagnetic waves, light, thermal radiation, etc. The Yukawa type distribution may be thought to be the Nuclei distribution such that represents the atomic nucleus model, etc. Further research into these relationships is thought to be needed in the future.

6. Relationship Entropic Gravity and $D_\pm$-Division Potential $V_{D_\pm}(R,G)$

In this section, we consider the relationship between entropic gravity F_R and $D_\pm$-division potential $V_{D_\pm}(R,G)$. As described below, the relationship between entropic gravity S_R and $V_{D_\pm}(R,G)$ can be considered as the entropy S_R in entropic gravity F_R is considered to be proportional to the potential $V_{D_\pm}(R,G)$.

(Note): The descriptions of variables and constants are differed from those in the original paper (Verlinde,2011; An,2020; An,Cheng,2021). (End of Note)

6.1 Interpretation of Entropic Gravity by $V_{D_\pm}(R,G)$ (1)

Verkinde have suggested entropic forces (Verlinde,2011) Entropic forces suggest that the force F_x is generated by the variation of entropy S_x and temperature T, and that forces (gravity) emerges:

$$F_x = T\frac{\partial S_x}{\partial x}, \tag{180}$$

where x is a position (length). According to the equation(180), by set $x = R$, we can obtain as follows:

$$F_R = T\frac{\partial S_R}{\partial R}, \tag{181}$$

where entropy of entropic gravity S_R. The relationship between the gravitational acceleration and its potential are as follows (see the definition of gravity (Feynman,1995; Wiki/Gravity,2025) the above equations (50), (51) and (52)):

$$F_R(G) = -M\nabla\Phi(R) = -gM \simeq -MV'_{D_\pm}(R,G). \tag{182}$$

In other words, the force $F_R(G)$ is proportional to the change in potential $\Phi(R)$. Therefore,

$$F_R(G) \simeq -MV'_{D_\pm}(R,G) = -M\frac{\partial V_{D_\pm}(x,k)}{\partial x}, \tag{183}$$

According to the equation (44), that is $V_{D_\pm}(R,G) = 1/S_{D_\pm}(R)$, and put $x = R$, we obtain as follows:

$$F_R(G) \simeq -M\frac{\partial V_{D_\pm}(R,G)}{\partial R} = -M\frac{\partial}{\partial R}\Big(\frac{1}{S_{D_\pm}(R)}\Big). \tag{184}$$

Assume that $F_R = F_R(G)$. The following equation are obtained by (181) and (184):

$$T\frac{\partial S_R}{\partial R} \simeq -M\frac{\partial}{\partial R}\Big(\frac{1}{S_{D_\pm}(R)}\Big). \tag{185}$$

By integrating both sides of the above equation (185) with R, the following are satisfied:

$$TS_R = -M\frac{1}{S_{D_\pm}(R)} = -MV_{D_\pm}(R,G), \tag{186}$$

where T and M are constants, not the function of R. Because, by definitions of (46) and (136), the following equation is satisfied:

$$V_{D_\pm}(R,G) \simeq -\frac{Gm}{R}. \tag{187}$$

From the assumption $F_R = F_R(G)$, by the equation (186), we can obtain Newton's gravity:

$$F_R = T\frac{\partial S_R}{\partial R} = -MV'_{D_\pm}(R,G) = -\frac{GmM}{R^2}. \tag{188}$$

Namely, assuming $F_R = F_R(G)$ is equivalent to assuming $S_R = V_{D_\pm}(R,G)$, and the entropy S_R in entropic gravity F_R is proportional to the potential $V_{D_\pm}(R,G)$ in this paper.

Furthermore, by relating entropic force to Schwarzschild's solution, An,Y have derived the equation for the inertial force as follows (An,2020; An,Cheng, 2021):

$$F_R = -\frac{GmM}{R^2(1-\frac{2Gm}{R})}. \tag{189}$$

We compare the above equation (48) and (189). Put the exponential part of the adjusted gravity $\bar{g}_\pm$ (48) and $\hat{g}_\pm$ (137) as follows:

$$\exp(-\xi^g\mu_2^g R + \mu_1^g) = 0. \tag{190}$$

Namely, if μ_2^g set to be large enough and $\mu_2^g \gg \mu_1^g$ or R set large enough, then equations (48) and (189) can be considered to the same as follows:

$$F_R = -\frac{GmM}{R^2(1-\frac{2Gm}{R})} = -\frac{GmM}{(R\pm\lambda_1^g m)^2} \simeq F_\pm(R,G), \tag{191}$$

where $F_\pm(R,G) = \bar{g}_\pm M$, (50), $F_\pm(R,G) = \hat{g}_\pm M$, (51) and λ_1^g is satisfied as follows:

$$\lambda_1^g = \frac{R}{m}\Big(-1 \pm \sqrt{1-\frac{2Gm}{R}}\Big). \tag{192}$$

Namely, by selecting the constant λ_1^g, we can assume that $F_R \simeq F_\pm(R,G)$, that is, $F_R = F_R(G)$.

6.2 Interpretation of Entropic Gravity by $V_{D_\pm}(R,G)$ (2)

Next, if we assume entropy S_R of entropic gravity F_R and $D_\pm$-division partial entropy $S_{D_\pm}(R)$ are the same, that is we assume as follows:

$$S_R = S_{D_\pm}(R), \quad \frac{\partial S_R}{\partial R} = S'_{D_\pm}(R), \tag{193}$$

then, it is satisfied as follows:

$$F_R = T\frac{\partial S_R}{\partial R} = TS'_{D_\pm}(R). \tag{194}$$

$F_\pm(R,G)$ is satisfied as follows:

$$F_\pm(R,G) = \bar{g}_\pm M = -V'_{D_\pm}(R,G)M = -\Big(\frac{1}{S_{D_\pm}(R)}\Big)'M = \Big(\frac{S'_{D_\pm}(R)}{(S_{D_\pm}(R))^2}\Big)M. \tag{195}$$

Therefore, by comparing F_R, (194) and $F_\pm(R,G)$, (195), the following equation is satisfied:

$$S_R = S_{D_\pm}(R) = \Big(\frac{M}{T}\Big)^{\frac{1}{2}}. \tag{196}$$

Therefore, $S_{D_\pm}(R)$ depends on mass M and temperature T where mass M is located around mass m. if it assumes $S_R = S_{D_\pm}(R)$, then entropic gravity F_R can be interpreted as $S'_{D_\pm}(R)$, then $S_{D_\pm}(R)$ may depend on the mass M surrounding mass m and the temperature T of the environment. Furthermore, if it assumes $S_R = S_{D_\pm}(R)$, then S_R is a constant and $\partial S_R/\partial R = 0$, thus $F_R = 0$.

Consequently, if it assumes $F_R = F_R(G)$, then the entropy S_R in entropic gravity F_R is proportional to the potential $V_{D_\pm}(R,G)$ and to be inversely proportional to the $S_{D_\pm}(R)$ in this paper. Besides, if it assumes $S_R = S_{D_\pm}(R)$, entropic force becomes $F_R = 0$, which is a special case.

7. Possibility That Mass Generation by Entropy, Fluctuating of the Constant G and the Existence of New Accelerations

7.1 Possibility That Mass Generation by Entropy

The inverse of the second order part λ_2^g of the approximation of $D_\pm$-division entropy have assumed as mass m. Leave this the first order part λ_1^g as it is. The $D_\pm$-division entropy $S_{D_\pm}(R,G)$ is determined by mass m, distance R (the radius of range under consideration) and the correction factor λ_1^g. In other words, mass m is determined by $D_\pm$-division entropy $S_{D_\pm}(R,G)$, distance R and the factor λ_1^g.

According to the assumption 4.1 above, the relationship between mass m and entropy $S_{D_+}(R,G)$ can be thought of as follows:

1. if entropy acceleration $S''_{D_+}(R,G)$ is large, then mass m becomes small,
2. if entropy acceleration $S''_{D_+}(R,G)$ is small, then mass m becomes large.

Furthermore,

1. if mass m in the range R are small, entropy $S_{D_+}(R,G)$ becomes large,
2. if mass m in the range R are large, entropy $S_{D_+}(R,G)$ becomes small.

For example, Hydrogen molecules (light weight) in the same range R will be more disordered and have more entropy, but Iron atoms (heavy) in the same range R will be more ordered and have less entropy. Doesn't this relationship between entropy acceleration and mass seem intuitive?

By transforming the equations(34) and (123), mass m can be represented as follows:

$$m = \frac{R^2}{S_{D_\pm}(R,G) \mp \lambda_1^g R}. \tag{197}$$

By transforming the equations(37), (131) and $S_{D_\pm}(R,G) = G\frac{\xi^g R}{Q_{D_\pm}(R)} S_{D_\pm}(R)$, mass m can be represented as follows:

$$m = \frac{R}{(\exp(-\xi^g \mu_2^g R \pm \mu_1^g) \mp 1) G S_{D_\pm}(R) \mp \lambda_1^g}. \tag{198}$$

Namely, mass m can be represented as $D_\pm$-division partial entropy $S_{D_\pm}(R)$, distance R, correction factors λ_1^g, μ_1^g, μ_2^g and constants G, ξ^g. Mass also depends on entropy. Besides, if we consider $Q_{D_\pm}(R)$ to represent the spectrum (wave) distribution within the range of distance R, we can consider that mass depends on the partial entropy $S_{D_\pm}(R)$, the spectrum distribution $Q_{D_\pm}(R)$ (or division $D_\pm(R)$) within the range R, and constants G, ξ^g, and λ_1. In other words, mass m within R may depend on the partial entropy and spectrum (waves). Mass m may be generated depending on entropy and wave. This is natural since we are assuming that mass m is 2-times the inverse of entropy acceleration $2/S_\pm(R,G)$ on the assumption 4.1 above.

7.2 Possibility That Fluctuating of the Constant G

As mentioned above, if we consider that there exist many interactions, then we can assume that there will be many variations in constants. If there exists a constant change (difference) in a variable, the constant will appear to be fluctuating. Furthermore, the gravitational constant G can be considered as determined by $D_\pm$-division entropy $S_{D_\pm}(G,R)$, the division function $D_\pm(R)$ and the partial entropy $S_{D_\pm}(R)$ divided by $D_\pm(R)$ (or the distribution $\pm\xi R/Q_{D_\pm}(R)$). Namely, $D_\pm$-division entropy $S_{D_\pm}(G,R)$ is represented as follows:

$$S_{D_\pm}(R,G) = G \cdot D_\pm(R) \cdot S_{D_\pm}(R) = G\frac{\pm\xi R}{Q_{D_\pm}(R)} S_{D_\pm}(R). \tag{199}$$

Therefore, the gravitational constant G is represented as follows:

$$G = \frac{S_\pm(R,G)}{D_\pm(R) \cdot S_{D_\pm}(R)} = \frac{S_{D_\pm}(R,G)}{S_{D_\pm}(R)} \cdot \frac{Q_{D_\pm}(R)}{\pm\xi R}. \tag{200}$$

In other words, it is possible that the gravitational constant G can fluctuate if entropy changes.

7.3 Possibility That the Existence of New Accelerations

By appropriately selecting constants, the variables and the functions within the range of conditions, $V_{D_\pm}(x,k)$ is interpreted as the gravitational potential and $V'_{D_\pm}(x,k)$ is interpreted as the gravitational acceleration according to gravity theory. However, if we consider carefully, the constants, variables, and functions in the above equations can be arbitrary selected within the range of conditions, so these may be applicable to interactions other than gravity and the Coulomb force. By selecting the constants in the equation $V'_{D_\pm}(x,k)$ appropriately, it may be possible to represent the weak and the strong interaction. Furthermore, there exists the possibility of expansion and the existence of new interactions that are different from the conventional interactions (forces). Therefore, it is possible that there exist many new interactions. Namely, the following suggestion may be considered the possibility that there exist many new potentials and accelerations:

Suggestion 7.1. *Possibility that there exist many new accelerations of the Planck type (1). Let $n \geq 0$ be an integer, m be a weight (mass) and R be a relation (distance). There exist countable numbers of potential $V_{D_+}(R, G_n)$ and an acceleration $V'_{D_+}(R, G_n)$ such that the following conditions are satisfied: There exist constants G_n, ξ_n, μ_{1n}, λ_{1n} and a function $\mu_n(R)$ such that the following equations are satisfied:*

$$V_{D_+}(R, G_n) = -\frac{G_n m}{R \pm \lambda_{1n} m}(1 - \exp(-\xi_n \int \mu_n(R)dR \pm \mu_{1n})), \tag{201}$$

$$V'_{D_+}(R, G_n) = \frac{G_n m}{(R \pm \lambda_{1n} m)^2}(1-\exp(-\xi_n\int\mu_n(R)dR \pm\mu_{1n})) - \frac{G_n m \xi_n \mu_n(R)}{R \pm \lambda_{1n} m}\exp(-\xi_n\int\mu_n(R) \pm\mu_{1n}), \tag{202}$$

where $G_n \gtrapprox 0$, $\xi_n \gtrapprox 0$, $\mu_{1n} \geq 0$, $\lambda_{1n} \geq 0$, $m > 1$, $R > 1$ and $\mu_n(R) > 0$. Namely, it is possible that there exist many new accelerations(interactions). Note that these descriptions above assumed that assumptions 3.4 *and* 4.1. □

Similarly, for potentials $V_{D_-}(R, G_n)$ and accelerations $V'_{D_-}(R, G_n)$, we can describe as follows:

Suggestion 7.2. *Possibility that there exist many new accelerations of the Yukawa type (2). Let $n \geq 0$ be an integer, m be a weight (mass) and R be a relation (distance). There exist countable numbers of potential $V_{D_-}(R, G_n)$ and an acceleration $V'_{D_-}(R, G_n)$ such that the following conditions are satisfied: There exist constants G_n, ξ_n, μ_{1n}, λ_{1n} and a function $\mu_n(R)$ such that the following equations are satisfied:*

$$V_{D_-}(R, G_n) = -\frac{G_n m}{R \pm \lambda_{1n} m}(1 + \exp(-\xi_n \int \mu_n(R)dR \pm \mu_{1n})), \tag{203}$$

$$V'_{D_-}(R, G_n) = \frac{G_n m}{(R \pm \lambda_{1n} m)^2}(1+\exp(-\xi_n\int\mu_n(R)dR \pm\mu_{1n})) + \frac{G_n m \xi_n \mu_n(R)}{R \pm \lambda_{1n} m}\exp(-\xi_n\int\mu_n(R) \pm\mu_{1n}), \tag{204}$$

where $G_n \gtrapprox 0$, $\xi_n \gtrapprox 0$, $\mu_{1n} \geq 0$, $\lambda_{1n} \geq 0$, $m > 1$, $R > 1$ and $\mu_n(R) > 0$. Namely, it is possible that there exist many new accelerations(interactions). Note that these descriptions above assumed that the assumptions 5.3 *and* 5.4. □

The gravitational constant G and Coulomb's constant k_e may simply be coefficients related to interactions that humans can currently sense throughout the universe. Instead of asking why there exist 4-interactions, it might be better to ask why humans are primarily only able to sense 4-interactions.

8. Discussions, Conclusions and Issues

8.1 Possibility That Gravity Depending on Entropy

The ideas behind Planck's law are to apply the number of states of divided by resonators to entropy. These ideas are similar to the logistic function for dynamical systems (Planck, 1906; May, 1976; Yamaguchi, 1989; Wiki/Logistic_function, 2025; Fujino, 2024). Applying these, we have treated the division of entropy as a non-minimal and non-linear function $D_+(x)$, and derived potential $V_{D_+}(x,k)$ and acceleration $V'_{D_+}(x,k)$. Therefore, we have assumed that $D_\pm$-division entropy $S_{D_+}(x,k)$ can represent as a second-degree polynomial, and potential $V_{D_+}(x,k)$ is defined as the inverse of $S_{D_+}(x)$. As a result, each variable and constant used in potential $V_{D_+}(x,k)$ and acceleration $V'_{D_+}(x,k)$ are interpreted as the perspective of gravity, and mass is defined as the inverse of the quadratic coefficient term of $S_{D_+}(x,k)$, that is, $1/\lambda_2$. The constant λ_1 is the first order coefficient of approximated $D_\pm$-division entropy $\lambda_2 x^2 \pm \lambda_1 x$. In other words, the gravitational acceleration fluctuates depending on coefficients of approximated $D_\pm$-division entropy.

In addition, the constants μ_2 and μ_1 are defined as coefficients of $D_\pm$-division partial entropy or the distribution function $Q_\pm(x)$. In other words, the inverse of the second order part λ_2^g of the second order approximation of $D_\pm$-division entropy is considered as mass. The first order part λ_1^g is left unchanged. The $D_\pm$-division entropy $S_{D_\pm}(R,G)$ is determined using mass m, distance(radius) R of the range under consideration, and the correction factor λ_1^g. In this paper, it has assumed that the assumptions 3.4 and 4.1, and the existence of entropy dependent constants $\xi, \lambda_2, \lambda_1, \mu_2$, and μ_1. We have suggested the following conclusions:

1. If distance R is small enough, then the gravitational acceleration $V'_{D_+}(R,G)$ has 2-states with finite value depend on the constants $\xi, \lambda_1^g, \mu_1^g$ and μ_2^g. Depending on the values μ_1^g and λ_1^g, the value $V'_{D_+}(R,G)$ can be positive or negative. If the constant $\lambda_1^g \to 0$, then the gravitational acceleration $V'_{D_+}(R,G)$ becomes $\pm\infty$. If the constant $\lambda_1^g \to \infty$, then the gravitational acceleration $V'_{D_+}(R,G)$ becomes 0. Therefore, gravity may have 5-states within distance R is small enough. Among the 5-states, there may exist anti-gravity which is the opposite of Newton's gravity (Possibility existence of anti-gravity). Furthermore, within small distances, we have shown that it can treat the gravitational potential and the Coulomb potential in the same way by appropriately selecting specific constants. Similarly, the same can apply to the gravitational acceleration and Coulomb's law (electric field).

2. If distance R is large enough within the size of the universe, then gravity follows the adjusted inverse square law. Within this distance, the rotation speed of the galaxy v follows the gravitational constant G, mass $m = 1/\lambda_2^g$ and specific constants ξ^g, μ_2^g and μ_1^g which depend on the adjusted gravitational acceleration $\bar{g}_\pm$, and is independent of its radius R, (the galaxy rotation curve problem). Even without assuming dark matters, the problem of the rotation speed of the galaxy may be explained by the concept of entropy. This does not mean denying dark matters. The new constants μ_1^g and μ_2^g have suggested in this paper may represent some kind of dark matters.

3. If distance R is large enough within the size of the universe, then by comparing to conventional gravity g, the adjusted gravitational acceleration $\tilde{g}_\pm$ towards the center of rotation becomes slightly weaker or stronger. This means that gravitational acceleration towards the center of a rotating object can change slightly with distance (The Pioneer Anomaly).

From the above suggestions, there may exist specific constants $\xi, \lambda_2, \lambda_1, \mu_2$ and μ_1 which depend on entropy that control gravity and the speed of the galaxy.

8.2 Interpretation of the Yukawa Type Potential by D_--Division Partial Entropy With Negative

Same as $V_{D_+}(x,k)$, by defining $V_{D_-}(x,k)$ and acceleration $V'_{D_-}(x,k)$, it is considered relating to the Yukawa type potential and D_--division partial entropy with negative. Namely, this assumes the existence of negative partial entropy, and in fact, negative partial entropy is thought to exist. Besides, the Yukawa type potential is related to particle physics, hence particle physics may be also related to entropy.

From the discussion so far, it may be suggested that there exist 11-types accelerations (interactions) including the gravitational acceleration g, such as $\bar{g}_\pm, \hat{g}_\pm, \tilde{g}_\pm, g_\pm^{sp}$ and $g_\pm^{wp}$ related to g. Using these interactions, we attempted to compare the ratios of the fundamental 4-interactions in nature (strong interaction, electromagnetic force, weak interaction and gravity) are 1, 1E-2, 1E-5 and 1E-39, respectively if the strong interaction (strong proximity acceleration(+) g_+^{sp}) set to 1. We have shown that strong proximity acceleration(+) g_+^{sp} can be regarded as strong interaction, weak proximity acceleration(−) g_-^{wp} as weak interaction, adjusted gravity(±) $\tilde{g}_\pm$ as gravity and adjusted electromagnetic force $\hat{E}_+$ or $\bar{E}_-$ as electromagnetic force.

8.3 Relationship Entropic Gravity and $V_{D_\pm}(R)$

About the relationship between entropic gravity F_R and $F_R(G)$ by $V_{D_\pm}(R,G)$, if it assumes $F_R = F_R(G)$, then the entropy S_R in entropic gravity F_R is considered to be proportional to the potential $V_{D_\pm}(R,G)$ and to be inversely proportional to the $S_{D_\pm}(R)$. Besides, if it assumes $S_R = S_{D_\pm}(R)$, then entropy of entropic gravity S_R and $S_{D_\pm}(R)$ becomes a constant $(M/T)^{\frac{1}{2}}$ and entropic force becomes $F_R = 0$. Therefore, the entropy of entropic gravity S_R is proportional to the potential $V_{D_\pm}(R,G)$. It seems that the discussion of the relationship between entropic gravity and $V_{D_\pm}(R,G)$ are insufficient. We would like to make this difference a topic for further research in the future.

8.4 Mass Generation, the Existence of New Forces and the Fluctuation of Gravitational Constant G

We have interpreted the gravitational potential as the inverse of $D_\pm$-division partial entropy and $1/\lambda_2$ as mass m. This means that mass can be expressed by entropy. In other words, this suggests that mass is generated by entropy. Besides, we have seen that the inverse of $D_\pm$-division partial entropy can be interpreted as the gravitational potential and Coulomb potential. By considering that the constants in these equations are arbitrary, there can exist many potentials and accelerations (forces) exist. Furthermore, the gravitational constant G can be expressed by entropy. This suggests that the gravitational constant G may fluctuate depending on entropy. We would like to consider the possibility that the existence of new potentials and that the fluctuation of the gravitational constant with entropy.

8.5 Integration of Thermodynamics, Quantum, Gravity and Ecology by Entropy

By combining concepts of the logistic function for dynamical systems, Boltzmann's entropy and Planck's quantum, we have obtained adjusted gravity. Namely, by developing the concept of logistic function and combining it with entropy and

Planck's ideas, we have derived potentials $V_{D_\pm}(x,k)$ and accelerations $V'_{D_\pm}(x,k)$. From these equations, we have proposed the extensions of Newton's law and have shown that it is possible to approximate Newton's law.

Moreover, the Planck type distribution function $Q_{D_+}(R)$ is thought of the expansion of the Bose-Einstein distribution function and the Planck distribution function (Planck's law). The Yukawa type distribution function $Q_{D_-}(R)$ is thought of the expansion of the Fermi-Dirac distribution function and the distribution of nuclei model. This suggests that gravity and quantum mechanics may be linked through entropy.

The concept of logistic function is applied to ecology like population theory and the evolution of life. It is also known that the concept of entropy was established by Clausius and related to quantum theory by Planck (Planck,1906). Besides, Boltzmann's concept of division by the number of states can be seen as quantization. Therefore, gravity can be quantized by interpreting it as entropy, and gravity might be thought of as generating entropy.

Furthermore, it is considered that entropy is related to the concept of logistic function (Fujino,2024). The concept of logistic function is applied to population theory, the evolution of life and ecology. Therefore, entropy is related to ecology (Yamaguchi,1989; Fujino,2024).

In this paper, we have discussed that the concept of entropy is related to gravity theory. These findings suggest that by combining the concepts of entropy and the logistic function, it may be possible to understand the evolution of the universe in the same way as the evolution of life. Entropy can also be thought of as information, as can gravity, quantum, and ecology. It is therefore believed that thermodynamics, quantum (particle), gravity, electromagnetism, and ecology (biology) can be unified through entropy.

$$
\begin{array}{ccc}
Thermodynamics & Information & Ecology(Biology) \\
\searrow\nwarrow & \downarrow\uparrow & \swarrow\nearrow \\
 & Entropy & \\
\swarrow\nearrow & \downarrow\uparrow & \searrow\nwarrow \\
(Particle)Quantum & Gravity & Electromagnetic
\end{array}
$$

8.6 Issues

In the above discussion, we have suggested models of the Planck and the Yukawa type potentials and interactions (acceleration) based on entropy. The existence of these interactions and constants needs to be verified, and numerical verification of the model in this paper is a future challenge. Besides, the content presented in this paper is related to the theory of relativity through classical gravity, but the relationships do not seem to be well explained. It is believed that further research is needed on the Planck type and the Yukawa type distribution functions proposed in this paper. We hope that entropy will explain more and provide new perspectives and insights.

Acknowledgments

We would like to thank everyone who supported this challenge and deeply respect the ideas they gave us. Special thank to William Chen (Assistant Editor, Applied Physics Research) for identifying our paper and inviting us to submit it to APR.

Funding

This research received no external funding.

Contributions

The original contributions presented in the study are included in the article.

Conflicts

The author declares no conflicts of interest.

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