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Search for Charged Higgs Decays of the Top Quark Using Hadronic Tau Decays

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Abstract

We present the result of a search for charged Higgs decays of the top quark, produced in $p\bar{p}$ collisions at $\sqrt{s} =$ 1.8 TeV. When the charged Higgs is heavy and decays to a tau lepton, which subsequently decays hadronically, the resulting events have a unique signature: large missing transverse energy and the low-charged-multiplicity tau. Data collected in the period 1992-1993 at the Collider Detector at Fermilab, corresponding to 18.7 ± 0.7 pb^{-1}, exclude new regions of combined top quark and charged Higgs mass, in extensions to the standard model with two Higgs doublets.

I. INTRODUCTION

We have conducted a search for decays of the top quark to a charged Higgs boson, using the Higgs decays to hadronically decaying tau leptons. The results presented here come from data collected during the years 1992–1993 at the Collider Detector at Fermilab, corresponding to an integrated luminosity of 18.7 ± 0.7 pb^{-1}. A charged Higgs arises in extensions to the Standard Model with two Higgs doublets [1]. If the charged Higgs exists in such a model and is lighter than the top quark, then two competing channels are possible: $t{\rightarrow}H^+b$ and $t{\rightarrow}W^+b$. The charged Higgs can decay either to $\tau\bar{\nu}$ or to $c\bar{s}$. The branching ratios of these processes depend on the top quark and charged Higgs masses, and on $\tan\beta$, the ratio of the vacuum expectation values of the two Higgs doublets in the model.[1] We consider here only the kinematically allowed cases where $m_{\mathrm{top}} > m_H + m_b$ and $m_{\mathrm{top}} > m_W + m_b$. In these cases three decay modes of the quark pairs are possible: $t\bar{t}{\rightarrow}H^+H^-b\bar{b}$, $t\bar{t}{\rightarrow}H^{\pm}W^{\mp}b\bar{b}$, and $t\bar{t}{\rightarrow}W^+W^-b\bar{b}$.

This analysis of hadronic decays of the τ-lepton uses a method similar to that used previously [2] but with four times more integrated luminosity and an event selection designed for larger top and charged Higgs masses. Our most recent limit [3] uses the leptonic decays of the tau using the data set collected in the same period as in this paper, i.e. 1992–1993. Charged Higgs masses from 45 GeV/c^2 to 110 GeV/c^2, and top quark masses from 90 GeV/c^2 to 110 GeV/c^2 were excluded at a 95% CL, as shown in the lower hatched part of the plot in Fig. 5. Experiments at LEP exclude a charged Higgs with mass less than 45 GeV/c^2 [4].

In the present analysis a more stringent limit results from the 64% hadronic branching ratio of the tau, compared with the 36% leptonic branching ratio. However, the larger expected background, mainly hadronic processes, must be well modelled. The analysis presented here addresses top masses in the range extending from the limits of previous searches [2, 3], about 100 GeV/c^2, up to the mass range which has been measured, $176\pm8(\mathrm{stat})\pm10(\mathrm{syst})$ GeV/c^2 [5]. The analysis excludes by direct search a top or top-like object decaying via a charged Higgs in this region.

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[1] We consider here only models in which one Higgs doublet couples to the up-type quarks, and the other doublet couples to the down-type quarks and the leptons.

Top quark pair events with one or two charged Higgs decays should contain energetic jets which come from b quarks and the decays of the taus. Each top quark leads to the production of two energetic neutrinos, leading in turn to a large missing transverse energy, denoted $\not{E}_T$.[2] The first neutrino is emitted in the charged Higgs or W boson decay, and the second results from the tau decay. The signature of an hadronically decaying tau is a narrow jet with either one or three associated charged particles. Thus, top quark pair events with charged Higgs decays to tau leptons can be found by looking for an excess of events with narrow energetic jets with one or three charged particles coming from the tau, along with the presence of other energetic hadronic jets and neutrinos. This signature differs from that of standard model top events due to the higher probability of decays to tau leptons, and larger missing E_T.

II.   CDF DETECTOR AND TRIGGER

The CDF detector is described in detail elsewhere [6]. The most important components of the CDF detector for this analysis are the tracking chambers and calorimeters. The relevant tracking chambers are the vertex time projection chamber (VTX) and the central tracking chamber (CTC), which is a large cylindrical drift chamber surrounding the VTX. Both are located inside a superconducting solenoid magnet generating a 1.4 T field. The VTX provides z-vertex reconstruction and $r - z$ tracking over the pseudorapidity range $|\eta| < 3.25$ [7], where the z axis is the proton direction along the beam line and r refers to the radial coordinate transverse to the beam line. The momenta of charged particles are measured in the CTC. The solenoid and the tracking volume of CDF lie inside electromagnetic and hadronic calorimeters which cover 2π in azimuth and up to $|\eta| = 4.2$. The calorimeters are segmented in azimuth and pseudorapidity to form a tower geometry which points back to the nominal interaction point $z = 0$.

The "trigger" decision as to whether or not the data from a particular interaction should be recorded

[2]For a calorimeter energy deposit, assuming the particles came from some point along the beam axis, a direction in space is defined. The transverse energy E_T is the component of the energy vector in the plane perpendicular to the beam axis. The $\not{E}_T$ is defined as the magnitude of the vector sum of the transverse energy E_T of each calorimeter energy deposit.

depends on the particular pattern of energy deposited in the calorimeters, the presence of charged tracks in the CTC, and the presence of penetrating charged particles in the muon chambers which surround the calorimeter. This analysis relies in particular on a trigger which uses analog sums of the calorimeter energy deposits to determine the missing transverse energy. Since the charged Higgs events sought in this analysis generally have large missing transverse energy, this analysis uses only those events which satisfy a trigger requirement of at least 35 GeV of missing transverse energy.

III.   EVENT SELECTION

The criteria to reject background and to select the charged Higgs signal were determined using a Monte Carlo simulation based on top quark and charged Higgs masses just beyond those excluded in previous analyses, namely $m_{\mathrm{top}} = 120$ GeV$/c^2$, and $m_{\mathrm{Higgs}} = 100$ GeV$/c^2$. A version of the Monte Carlo program ISAJET [8], modified to correctly model the polarization of the taus, generated events which were then passed through the CDF detector simulation.

The selection criteria aim to select events with large missing transverse energy due to the neutrinos, the presence of a hadronically decaying tau lepton, and at least one other jet due either to another tau or to one of the jets from the top quark. Each event must have

- $\not{E}_T > 40$ GeV,

- $S(\not{E}_T) > 4$ GeV$^{1/2}$,

- a tau lepton identified as discussed below, with

  - $E_T > 30$ GeV,
  - $|\eta| < 1$,

- a jet as defined below, with

  - $E_T > 20$ GeV,
  - $|\eta| < 2$,

- $\Delta\phi_{\tau-jet} < 140°$, and

- scalar $\Sigma|E_T| > 100$ GeV,

where we use the definition $S(\not{E}_T) \equiv \not{E}_T / \sqrt{\Sigma|E_T|}$ for the "significance" of the missing E_T.
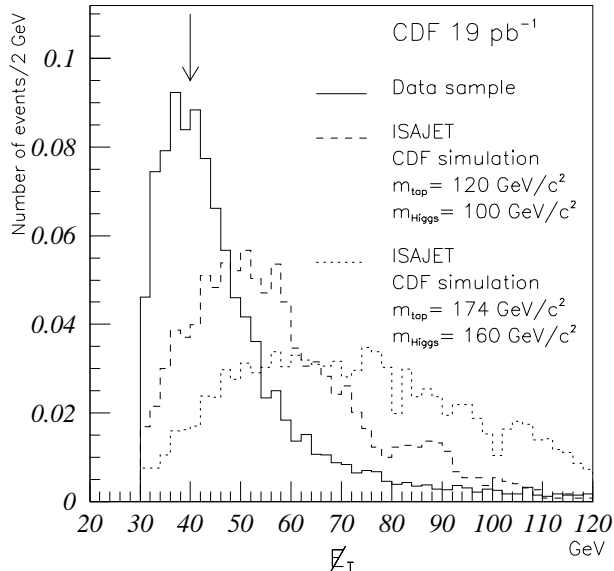
Figure 1: Distribution of $\not{E}_T$ for the data sample (solid line) and for the Monte Carlo simulation (dashed and dotted line). Events to the left of the arrow are removed by the cuts on the data sample.

Figure 2: Distribution of $S(\not{E}_T)$ for the data sample (solid line) and for the Monte Carlo simulations (dashed and dotted line). Events to the left of the arrow are removed by the cut.

The number of events which satisfy each criterion are listed in Table I. The relative efficiencies between consecutive cuts for the ISAJET Monte Carlo simulation with $m_{\rm top} = 120$ GeV/c^2, and $m_{\rm Higgs} = 100$ GeV/c^2 are also shown in Table I.

The scalar nature of the charged Higgs implies that the two neutrinos produced in the decay chain tend to travel in the same direction, resulting in a large $\not{E}_T$. Furthermore, the charged Higgs decays mainly to a tau for large values of the parameter $\tan\beta$. For smaller $\tan\beta$ values, the probability for the top quark to decay to a W boson increases, and the charged Higgs decays more often to a quark-antiquark pair. In this case the average $\not{E}_T$ consequently becomes smaller. Thus, the trigger and selection requirements on $\not{E}_T$ enhance the acceptance in the case of large values of $\tan\beta$.

The criteria on the missing transverse energy and significance reinforce the trigger requirements and select events with energetic neutrinos. The distributions of $\not{E}_T$, and $S(\not{E}_T)$ are shown in Fig. 1 and Fig. 2, for the data sample and for the Monte Carlo simulation of the signal.

When a top quark decays to a charged Higgs, a large fraction of its energy goes into creation of the charged Higgs. The smaller remaining energy for the b quark produces jets of lower E_T. Since the charged Higgs carries a large energy, its decay prod-
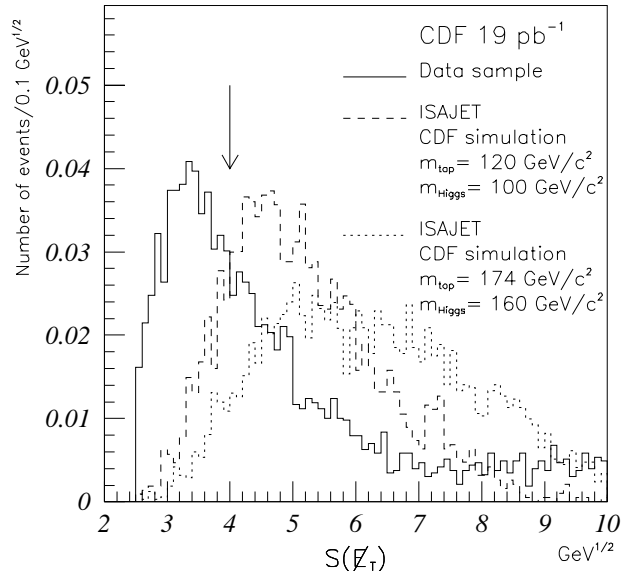
ucts receive a strong boost. In particular, the taus which come from the charged Higgs have very large E_T, resulting in a large-transverse-momentum (p_T) associated charged particle near the jet axis. The average p_T increases with increasing charged Higgs mass. A charged particle is associated with a jet if its initial direction points within a cone of radius $\Delta R = \sqrt{\Delta\eta^2 + \Delta\phi^2} = 0.4$ of the jet direction, where ϕ is the azimuthal angle in the plane perpendicular to the beam axis.

The event selection requires the presence of at least two jets, formed from calorimeter energy deposits in a cone of radius $\Delta R = 0.4$. The first jet must have $E_T > 30$ GeV, lie in the region $|\eta| < 1$, and have an associated charged particle with $p_T > 4$ GeV/c. If more than one jet satisfies these criteria, the jet with the largest E_T is chosen.

The requirements on the second jet are less stringent. The Monte Carlo simulation shows that the second jet has a smaller E_T, and is less often in the central region of the detector, $|\eta| < 1$. The second jet must have $E_T > 20$ GeV, $|\eta| < 2$, and an associated charged particle.

Requiring that the z-intercept of the largest-p_T charged particle associated with each jet be within 5 cm of the primary z-vertex of the event rejects jets from additional interactions in the event.

Subsequent criteria to identify hadronically decay-

Cuts	Remaining Events	Relative Efficiency
Initial selection	7109	$22.2\pm0.4\%$
$\not{E}_T > 40$ GeV	4766	$85.8\pm0.8\%$
Jet 1 $E_T > 30$ GeV	2579	$83.5\pm0.8\%$
Jet 2 $E_T > 20$ GeV	1601	$81.6\pm1.0\%$
Azimuthal angle between jets	1579	$100.0^{+0.0}_{-0.1}\%$
$S(\not{E}_T) > 4$ GeV$^{1/2}$	659	$79.7\pm1.1\%$
Isolation	193	$48.9\pm1.6\%$
Electron/jet rejection	104	$93.4\pm1.1\%$
Δz vertex	81	$100.0^{+0.0}_{-0.2}\%$
$\Sigma|E_T| > 100$ GeV	74	$93.3\pm1.2\%$
Charged multiplicity	19	$88.6\pm1.5\%$
Total efficiency	-	$3.9\pm0.2\%$

Table I: Number of events selected and signal efficiency for each selection criterion. The efficiencies represent the successive effect of each criterion, for events in a Monte Carlo simulation with $m_{\mathrm{top}} = 120$ GeV$/c^2$ and $m_{\mathrm{Higgs}} = 100$ GeV$/c^2$.

ing taus assign one of the two jets as the "tau." Since the charged particles in hadronically decaying energetic taus must lie in a narrow cone around the calorimeter energy deposit, the tau must satisfy an isolation criterion in which there must be no associated charged particle with $p_T > 1$ GeV/c found between cones of 10° and 30° defined around the direction of the associated track with the largest p_T. If the first jet fails the cut, the algorithm applies the cut to the second jet which in addition must then pass the stricter jet E_T, associated charged particle p_T, and η cuts of the first jet. The jet which passes the isolation cut is called the tau candidate.

A small fraction of electrons and single hadrons or low-multiplicity hadronic jets also satisfy the tau selection criteria. To reject electrons, the tau candidate must satisfy $1 - (10 E_T/\Sigma|p_T| - 1)^{-1} > f_{EM}$, where f_{EM} is the fraction of the total energy deposited in the electromagnetic calorimeter and $\Sigma|p_T|$ is the sum of the magnitudes of the transverse momenta of charged particles in the 10° cone around the jet axis. A fraction of the hadronic background is rejected by a similar cut: $1 - (bE_T/\Sigma p_T - 1)^{-1} < f_{EM}$, where the factor b has been optimized as a function of the E_T of the tau candidate: $b = 0.815$ for 30 GeV$\leq E_T < 45$ GeV, $b = 0.995$ for 45 GeV$\leq E_T < 69$ GeV, and $b = 0.860$ for $E_T \geq 69$ GeV.

Lastly, the requirement that the scalar sum of transverse energy exceeds 100 GeV removes back-

ground from W + jets events in which the W decays leptonically.

IV. TAU SIGNAL IN HADRONIC EVENTS

In order to demonstrate that the criteria select hadronic tau decays in a process known to contain tau leptons, one can extract the tau signal from the process $p\bar{p} \to W + $ jets, $W \to \tau\nu$ by selecting a sample of events with a tau, a jet and missing transverse energy. Employing less stringent cuts on the $\not{E}_T$, $S(\not{E}_T)$, E_T and p_T of the charged particles associated with the jets, and then requiring $\Sigma|E_T| < 85$ GeV and a tight cut on the width of the energy deposit if the jets, results in the multiplicity distribution shown in Fig. 3. The plot shows a clear excess of one and three charged particles in the distribution of the associated charged particle multiplicity distribution in the 10° cone, attributed to W plus jets events. The data (points) are compared to a background estimate (cross-hatched histogram), and a HERWIG Monte Carlo simulation [9] of $W \to \tau\nu$ (open histogram), normalized according to the HERWIG predicted cross section.

V. MONTE CARLO SIMULATION AND TRIGGER EFFICIENCY

The simulation of charged Higgs events uses the physics generator ISAJET and the CDF detector
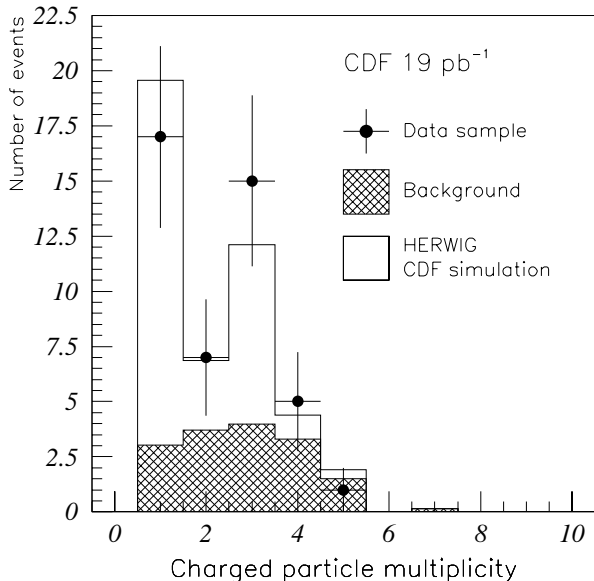
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Figure 3: The tau signal found in the data sample using a similar algorithm with less stringent cuts. The data (points) are compared with an estimate of jet background (cross-hatched), and with a Monte Carlo simulation of W+jets events, with $W \rightarrow \tau\nu$ (open histogram).

simulation. In order to compute the acceptance at any point in the top quark versus charged Higgs mass plane, it is necessary to make a mixture of events from the simulation of $t\bar{t} \rightarrow H^+ H^- b\bar{b}$, $t\bar{t} \rightarrow H^\pm W^\mp b\bar{b}$, and $t\bar{t} \rightarrow W^+ W^- b\bar{b}$ processes for charged Higgs masses in the range of 50 GeV/c^2–160 GeV/c^2 and for top quark masses in the range of 100 GeV/c^2– 174 GeV/c^2. The $t\bar{t}$ production cross-section is taken from a next-to-leading order theoretical calculation [10]. The simulation of the effect of the $\not{E}_T$ trigger efficiency comes from a measurement of the efficiency as a function of the $\not{E}_T$ in events which triggered on the presence of jets.

In the Monte Carlo simulation, more than 95% of tau candidates found by the algorithm correspond in spatial direction to the actual taus generated in the event.

VI.   BACKGROUNDS

The dominant backgrounds to charged Higgs events in the selected event sample are hadronic processes, and processes in which a Z or W is produced, possibly accompanied by jets. In almost all of the background, an hadronic jet fluctuates to have low charged particle multiplicity and satisfies the tau criteria. A small additional contribution to the back-

ground comes from W and Z events where the tau jet comes from a tau or mismeasured electron from the boson decay. In this case the tau jet typically has one or three associated charged tracks.

A combination of events satisfying the various jet energy triggers in the experiment models the hadronic background well. The background normalization is computed as a function of the E_T and charged multiplicity of the tau. The normalization equalizes the number of events of any charged multiplicity except 1 or 3, in three ranges of E_T. The Monte Carlo simulation shows that real taus contribute less than a few percent to these bins in multiplicity. An estimated total of 17.4±2.5 events come from processes where the tau jet came from an hadronic jet; the error is statistical only.

The estimate of the non-hadronic-jet contribution to the background comes from Monte Carlo simulation of the various processes. Of these, only the contribution from $Z \rightarrow \tau^+ \tau^-$ remains non-negligible after all cuts. Using a total of 30,000 events generated with ISAJET and then passed through the CDF detector simulation and analysis, we expect 1.1±0.4 (stat) events with 1 or 3 associated charged particles. The production cross section comes from the measured Z cross section, assuming lepton universality: $\sigma(p\bar{p} \rightarrow ZX; Z \rightarrow e^+ e^- X) = 0.209 \pm 0.013(\text{stat}) \pm 0.017(\text{syst})$ nb [11]. This background is small for several reasons: the process has a small cross-section, the two outgoing taus are azimuthally back-to-back, and the $\not{E}_T$ is typically not large since the tau decay neutrinos are back-to-back.

The predicted background from W+jets events, with $W \rightarrow \tau\nu$, comes from 40,000 HERWIG events which were passed through the CDF detector simulation, including the relevant trigger efficiencies. Again assuming lepton universality the production cross section $\sigma(p\bar{p} \rightarrow WX; W \rightarrow e\nu) = 2.19 \pm 0.04(\text{stat}) \pm 0.21$ (syst) nb [11] is used for normalization. Most of these events are rejected by the cuts on $\not{E}_T$, $S(\not{E}_T)$, and $\Sigma|E_T|$. No event passed the selection criteria.

The other processes involving W and Z bosons result in background taken into account by the hadronic jet sample, and contribute negligibly to the non-hadronic-jet component.

There is a small acceptance for events from standard model top quark pair production; for a top quark with mass of 176 GeV/c^2 one expects 0.2±0.1 (stat) events. This acceptance affects the number of

expected events in the signal, and does not enter the background estimate.

VII.   SYSTEMATIC UNCERTAINTIES

Systematic effects which can lead to uncertainty in the final result can be classified into those which affect the background estimate and those which affect the number of expected events. Many of the systematic uncertainties affecting the number of expected events depend on the top quark mass. Table II lists the different estimated systematic uncertainties. For the cases where there exists a top quark mass dependence, the extreme values appear in the table.

Various effects can bias the background estimation, such as the binning of the E_T distribution and the normalization method. Dividing the E_T distribution into smaller bins and following the same normalization method leads to a negligibly small difference in the expected number of hadronic background events. The normalization method is based on jet trigger events, but one can check for a trigger bias by removing the jet which was responsible for the trigger, and no significant effect appears. The total hadronic background is conservatively estimated to be 17.4 ± 2.5(stat)±0.6(sys) events, based on these cross checks.

We have compared the number of expected events for different masses of the top quark and the charged Higgs with and without initial-state gluon radiation in ISAJET. Half the difference between these numbers was taken as the systematic uncertainty. The mean value between the number of expected events with and without initial-state gluon radiation was taken as the number of expected events. The isolation cut is the criterion most affected by initial-state gluon radiation; the number of jets is smaller with no initial-state gluon radiation. The probability to have associated charged particles of the tau candidate mixed with a particle of another jet becomes smaller and the efficiency of the cut increases. This effect also depends on the top quark mass, since for a heavy top quark there is less energy to produce jets as a result of the initial-state gluon radiation.

The systematic uncertainty on the trigger efficiency was estimated by varying each point of the measured trigger efficiency by its uncertainty. The relative uncertainty on the number of expected events due to the systematic uncertainty in the trig-
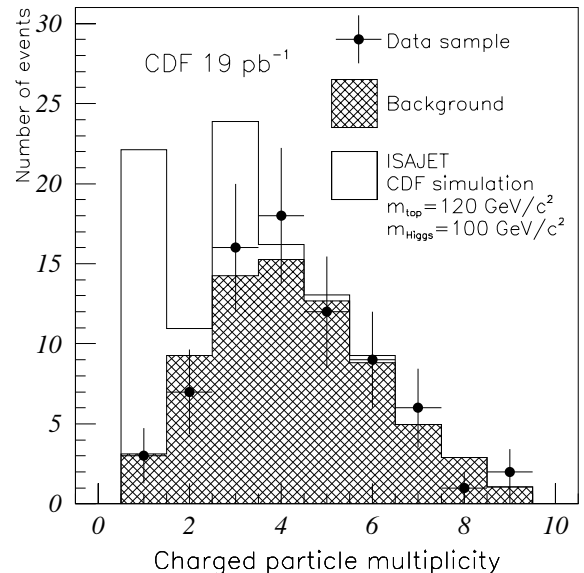

Figure 4: Charged particle multiplicity distribution in the 10° cone after all the cuts, for the data sample (points), for the background normalized to the data (cross-hatched histogram), and for the expected signal from the normalized Monte Carlo simulations (open histogram) added to the background.

ger efficiency is conservatively estimated to be 5.5%. In this calculation, $m_{\mathrm{top}} = 120$ GeV/c^2 and $m_{\mathrm{Higgs}} = 100$ GeV/c^2.

The absolute energy scale uncertainty varies from $\pm10\%$ at 8 GeV to $\pm3\%$ at 100 GeV. In the Monte Carlo simulations, we shifted the jet energy scale by these values, and repeated the analysis, reconstructing the E_T of each jet and other relevant event parameters. We used the mean relative difference of the change in the number of expected events when the energy scale is shifted as the systematic uncertainty on the energy scale.

VIII.   RESULTS

After selection, there remain 74 events from the data sample, of which a total of 19 events have a tau candidate with either 1 or 3 associated charged particles. Fig. 4 shows the multiplicity distribution for the data sample and the hadronic background normalized to the data. For comparison the plot shows the distribution from the Monte Carlo signal simulation normalized to the total integrated luminosity and added to the hadronic background estimation. The estimated total number of events is 18.5 ± 2.6, where the error comes from adding in quadrature the systematic and statistical uncertainties.

Uncertainty	Value
Top quark cross section	30–10%
ISAJET gluon radiation	16–1.3%
Integrated luminosity	3.7%
Trigger efficiency	5.5%
MC statistics	10–4%
Energy scale effect	32–7.5%
Background estimation	14%

Table II: Sources and magnitudes of the systematic uncertainties in the analysis. The values are the relative uncertainties in the number of expected events, and represent the extremes for the top mass range 100-174 GeV/c^2.

The mass limits must take into account the uncertainties, both statistical and systematic, on the number of expected background and signal events. For a given mass point, a simple Monte Carlo generates a large ensemble of trials with the numbers of expected signal and background events varying in a Gaussian fashion about the mean. In each trial it generated a number of observed events from a Poisson distribution with a mean equal to the number of signal plus background. The standard deviations of the Gaussians are the combined statistical and systematic uncertainties. The mass point can be excluded with 95% confidence if in 95% or more of the trials the total number of events exceeds the 19 events actually observed.

Fig. 5 shows the resulting limit for large values of the parameter $\tan\beta$, for which the branching ratios of $t \rightarrow Hb$ and $H \rightarrow \tau\nu$ approach unity. Fig. 6 shows the limit for $\tan\beta = 50, 100$, and 500 in the plane of the top quark mass versus the charged Higgs mass. As the charged Higgs mass decreases, the missing transverse energy decreases on average, reducing the efficiency of the selection. Also, as the top mass increases, its production cross section decreases, and the number of expected events decreases. As the parameter $\tan\beta$ increases, the branching ratios of the top to charged Higgs and charged Higgs to tau both increase, allowing a better limit.

The event selection used is well optimized for large masses of the top quark and the charged Higgs. The present statistics exclude the region for large $\tan\beta$, extending from the limit of the previous analyses
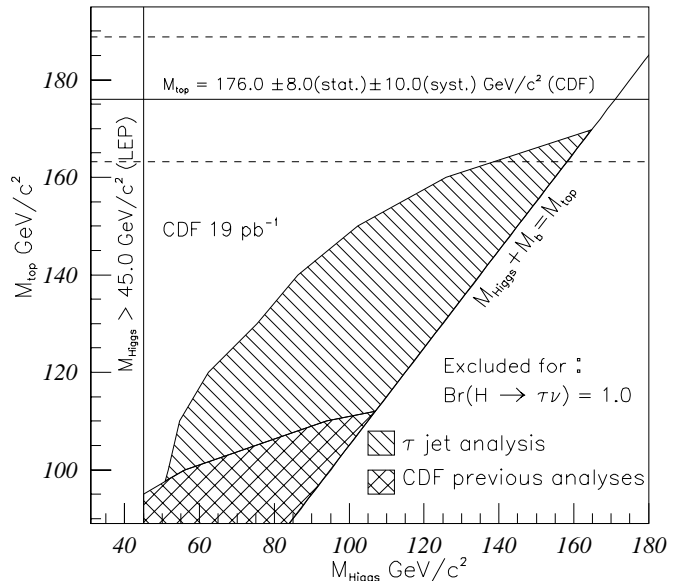

Figure 5: Regions of the $(m_{\text{top}}, m_{\text{Higgs}})$ plane excluded at 95% CL for a 100% branching ratio of $H^{\pm} \rightarrow \tau\nu$. The plot also shows the limit from the previous analyses [2, 3].

just up to the region where the top mass has been measured.

IX. ACKNOWLEDGEMENTS

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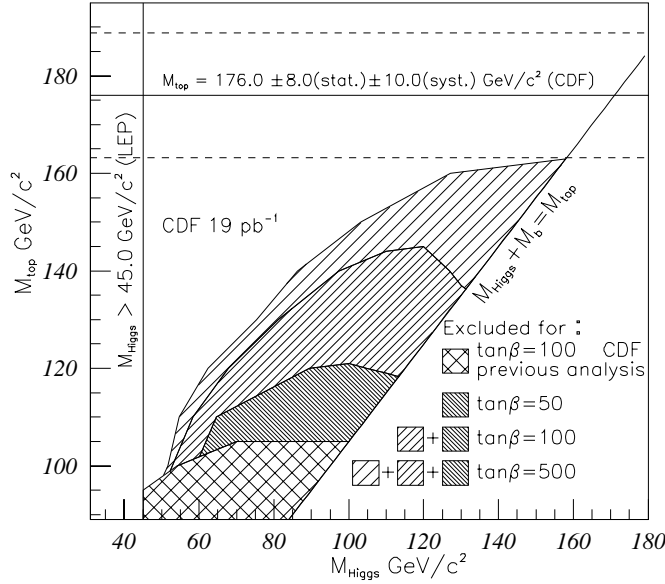
Figure 6: Regions of the $(m_{\text{top}}, m_{\text{Higgs}})$ plane excluded at 95% CL for different values of $\tan\beta$. The plot also shows the limit from the previous analyses [2, 3].

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