# Alternative scale-invariant Higgs mass generation using hidden sector SU($N_c$)xU(1)$^3$

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**Abstract.** Spontaneous Symmetry Breaking (SSB) is a method to generate mass for all elementary particles in the Standard Model (SM). Even so, it turns out that the Higgs mass term in the Higgs potential, which responsible for the SSB, does not scale invariant. Based on this problem, instead of using SSB from the SM to explain the origin of mass, we erase Higgs's mass term from the SM Lagrangean and introduce a hidden sector Lagrangean to produce an alternative way to generate Higgs mass. We use Scalar Bilinear Condensate ineffective theory to obtain Higgs mass via the Higgs portal and generate the SSB. The hidden sector in this alternative model is described by an $SU(N_C A)$ gauge theory with $U(1)^3$ flavor. In the calculation, we use the Mean-Field Approximation Lagrangean in obtaining the vacuum of the potential to get a Higgs mass term in this model.

## 1. Introduction

Standard Model (SM) in particle physics has been a reference for a model of matter in nature. It started in 1961 when Sheldon Glashow [1] proposed to unify the electromagnetic and weak interactions. His project was completed by Weinberg in 1967 and by Abdus Salam independently in 1968 [2,3]. Combined with Quantum Chromodynamics (QCD) for strong interaction, this model has been accepted as the SM in particle physics. Since the discovery of the Higgs particle in 2012 [4,5], SM has become more established. Nevertheless, there are several shortcomings of this model and it is also widely admitted that SM is not the ultimate model for elementary particles. Several shortcomings of this model are: it does not allow for any finite neutrino mass, it does not have any dark matter candidate, it does not provide an explanation for the baryon number asymmetry in the Universe. There are also theoretical shortcomings of the SM: It does not include gravity, it can not explain why the electroweak scale is seventeen orders of magnitude smaller than the Planck scale, it contains an unnatural Higgs mass term which is put by hand in the SM Lagrangian. Moreover, the Higgs mass term is the only term in the SM that breaks scale-invariant.

Considering the last problem mentioned above, in this paper we try to discuss an alternative way to construct Higgs mass without using a mass term in the SM Lagrangian, thus keeping the theory scale-invariant. There are basically two types of scenarios to proceed with this problem. The first one relies on the Coleman-Weinberg (CW) potential [6] which have been studied thoroughly in several papers [7-

10], and the second one base on non-perturbative effects in a non-abelian group such as dynamical chiral symmetry breaking [11-13] and gauge-invariant scalar bilinear condensation. In this paper, we will discuss the second one that uses gauge-invariant scalar bilinear condensation in a strongly interacting hidden sector to generate Higgs mass term via the Higgs portal term. More precisely, this model assumes that below a certain energy scale the scalar fields condensate in the form of a scalar bilinear by a non-perturbative effect of a non-abelian hidden sector. This condensation turns the Higgs portal term into a Higgs mass term. There have been several comprehensive studies in this approach before [14-17].

We start by introducing the Lagrangian of this extension model and then we will use a self-consistent mean-field approximation method [18] to get the effective Lagrangian around the vacuum structure of this theory. The Higgs potential in the effective Lagrangian will contain a Higgs mass term from the Higgs portal interaction with the hidden sector.

## 2. Lagrangian of the model

The Lagrangian for the hidden sector of this model-based is assumed to be invariant under the $SU(N_c)$ gauge theory, given by

$$\begin{aligned}\mathcal{L}_{\rm H} =& -\frac{1}{2}{\rm Tr}F^2 + \left([D^\mu S_i]^\dagger D_\mu S_i\right) - \hat{\lambda}_{S_{ij}}(S_i^\dagger S_i)(S_j^\dagger S_j) \\ & - \hat{\lambda}'_{S_{ij}}(S_i^\dagger S_j)(S_j^\dagger S_i) + \hat{\lambda}_{HS_i}(S_i^\dagger S_i)H^\dagger H - \lambda_H(H^\dagger H)^2\end{aligned} \tag{1}$$

where the scalar fields $S_i^a$ ($i = 1,2,\cdots,N_f$ and $a = 1,2,\cdots,N_c$) are in the fundamental representation of $SU(N_c)$. $D_\mu S_i = \partial S_i - ig_H G_\mu S_i$ is the covariant derivative with $G_\mu$ is the matrix-valued gauge field. The total Lagrangian of this model is

$$\mathcal{L}_{\rm T} = \mathcal{L}_{\rm H} + \mathcal{L}'_{\rm SM} \tag{2}$$

The second term on the right-hand side is the SM Lagrangian with the absence of the Higgs mass term, therefore the potential term have form $V(\phi) = \lambda(\phi^\dagger\phi)$. Because of this absence, now the total Lagrangian is scaled invariant. Below certain energy, we assume that the gauge coupling becomes so strong that the invariant scalar bilinear forms $U(1)^{N_f}$ invariant condensate

$$\langle (S_i^\dagger S_j)\rangle = \langle \sum_{a=1}^{N_c} S_i^\dagger S_j^a \rangle = f_{ij} \tag{3}$$

This non-perturbative condensate breaks scale invariance by scale anomaly, not by order parameter like in the SM. In this paper, we specify $N_f = 3$, and for this particular case the effective Lagrangian can be written as

$$\begin{aligned}\mathcal{L}_{\rm eff} =& [\partial^\mu S_i]^\dagger \partial_\mu S_i - \lambda_1(S_1^\dagger S_1)(S_1^\dagger S_1) - \lambda_2(S_2^\dagger S_2)(S_2^\dagger S_2) - \lambda_3(S_3^\dagger S_3)(S_3^\dagger S_3) \\ & - \lambda_{12}(S_1^\dagger S_1)(S_2^\dagger S_2) - \lambda_{13}(S_1^\dagger S_1)(S_3^\dagger S_3) - \lambda_{23}(S_2^\dagger S_2)(S_3^\dagger S_3) \\ & - \lambda'_{12}(S_1^\dagger S_2)(S_2^\dagger S_1) - \lambda'_{13}(S_1^\dagger S_3)(S_3^\dagger S_1) - \lambda'_{23}(S_2^\dagger S_3)(S_3^\dagger S_2) \\ & + \lambda_{HS_i}(S_i^\dagger S_i)H^\dagger H - \lambda_H(H^\dagger H)^2\end{aligned} \tag{4}$$

It is important to note that the effective Lagrangian is defined above the energy before condensation takes place. To investigate the vacuum state of this effective Lagrangian, we employ a Self-Consistent Mean-Field approximation [18]. First, we split the effective Lagrangian into two terms

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm MFA} + \mathcal{L}_I \tag{5}$$

where $\mathcal{L}_{\mathrm{MFA}}$ describe mean-field dynamics and $\mathcal{L}_I$ is normal ordered which is vanishing in the vacuum (i.e. $\langle 0|\mathcal{L}_I|0\rangle$=0). To obtain the mean-field approximation for this Lagrangian, we define the mean-field $f$ and $\phi$ as

$$\langle 0|S_i^\dagger S_j|0\rangle = \overbracket{S_i^\dagger S_j} = \langle f_{ij}\rangle + t^a_{ji}\phi \tag{6}$$

with $t^a$ are generators of $SU(3)$ ($a = 1,\cdots,8$) in the hermitian matrix representation and $\phi$ introduced as the excitation of condensate $\langle f_{ij}\rangle$ (where $\langle f_{ij}\rangle = \delta_{ij} f$ and $\langle \phi^a\rangle = 0$). Using Wick theorem

$$S^\dagger S = :S^\dagger S: + \overbracket{S^\dagger S}, \tag{7}$$

we obtain

$$\begin{aligned}
\mathcal{L}_{\mathrm{MFA}} \;=\;& ([\partial^\mu S_i]^\dagger \partial_\mu S_i) - M_{10}^2(S_1^\dagger S_1) - M_{20}^2(S_2^\dagger S_2) - M_{30}^2(S_3^\dagger S_3)\\
&+\lambda_1 f_1^2 + \lambda_2 f_2^2 + \lambda_3 f_3^2 + \lambda_{12} f_1 f_2 + \lambda_{13} f_1 f_3 + \lambda_{23} f_2 f_3\\
&-\lambda'_{12}\left(\frac{1}{\sqrt{2}}\phi_1^- S_2^\dagger S_1 + \frac{1}{\sqrt{2}}\phi_1^+ S_1^\dagger S_2 - \frac{1}{2}\phi_1^-\phi_1^+\right)\\
&-\lambda'_{13}\left(\frac{1}{\sqrt{2}}\phi_2^- S_3^\dagger S_1 + \frac{1}{\sqrt{2}}\phi_2^+ S_1^\dagger S_3 - \frac{1}{2}\phi_2^-\phi_2^+\right)\\
&-\lambda'_{23}\left(\frac{1}{\sqrt{2}}\phi_3^- S_3^\dagger S_2 + \frac{1}{\sqrt{2}}\phi_3^+ S_2^\dagger S_3 - \frac{1}{2}\phi_3^-\phi_3^+\right) - \lambda_H (H^\dagger H)^2
\end{aligned} \tag{8}$$

with

$$\phi_1^\pm = \frac{1}{\sqrt{2}}(\phi^1 \pm \phi^2), \quad \phi_2^\pm = \frac{1}{\sqrt{2}}(\phi^4 \pm \phi^5), \quad \phi_3^\pm = \frac{1}{\sqrt{2}}(\phi^6 \pm \phi^7) \tag{9}$$

and

$$\begin{aligned}
M_{10}^2 &= 2\lambda_1 f_1 + \lambda_{12} f_2 + \lambda_{13} f_3 - \lambda_{HS1} H^\dagger H\\
M_{20}^2 &= 2\lambda_2 f_2 + \lambda_{12} f_1 + \lambda_{23} f_3 - \lambda_{HS2} H^\dagger H\\
M_{30}^2 &= 2\lambda_3 f_3 + \lambda_{13} f_1 + \lambda_{23} f_2 - \lambda_{HS3} H^\dagger H.
\end{aligned} \tag{10}$$

## 3. Potential and Higgs mass

In calculating the effective potential, we have assumed that non-perturbative effect of the gauge theory will not break the hidden color symmetry and flavor symmetry, which means

$$\langle S_i\rangle = 0 \tag{11}$$

and

$$\begin{aligned}
\langle S_1^\dagger S_2\rangle &= \langle \phi_1^-\rangle/\sqrt{2} = \langle S_2^\dagger S_1\rangle = \langle \phi_1^+\rangle/\sqrt{2} = 0\\
\langle S_1^\dagger S_3\rangle &= \langle \phi_2^-\rangle/\sqrt{2} = \langle S_3^\dagger S_1\rangle = \langle \phi_2^+\rangle/\sqrt{2} = 0\\
\langle S_2^\dagger S_3\rangle &= \langle \phi_3^-\rangle/\sqrt{2} = \langle S_3^\dagger S_2\rangle = \langle \phi_3^+\rangle/\sqrt{2} = 0
\end{aligned} \tag{12}$$

We also expand $S_i$ around the homogenous background, $S_i \longrightarrow \bar{S}_i + \eta_i$. As well as we integrate out $S$ around its background field $\bar{S}$

$$\begin{aligned}\int \mathcal{D}f_i\mathcal{D}\eta_i^\dagger\mathcal{D}\eta_i \exp\left[\mathcal{L}_{\text{MFA}}\right] = \int \mathcal{D}f_i\mathcal{D}\eta_i^\dagger\mathcal{D}\eta_i \exp\Big[&\left([\partial^\mu S_i]^\dagger\partial_\mu S_i\right) - M_{i0}^2(S_i^\dagger S_i)\\ &+\lambda_i f_i^2 + \lambda_{12}f_1f_2 + \lambda_{13}f_1f_3 + \lambda_{23}f_2f_3\\ &-\lambda_{12}'\left(\frac{1}{\sqrt{2}}\phi_1^- S_2^\dagger S_1 + \frac{1}{\sqrt{2}}\phi_1^+ S_1^\dagger S_2 - \frac{1}{2}\phi_1^-\phi_1^+\right)\\ &-\lambda_{13}'\left(\frac{1}{\sqrt{2}}\phi_2^- S_3^\dagger S_1 + \frac{1}{\sqrt{2}}\phi_2^+ S_1^\dagger S_3 - \frac{1}{2}\phi_2^-\phi_2^+\right)\\ &-\lambda_{23}'\left(\frac{1}{\sqrt{2}}\phi_3^- S_3^\dagger S_2 + \frac{1}{\sqrt{2}}\phi_3^+ S_2^\dagger S_3 - \frac{1}{2}\phi_3^-\phi_3^+\right)\\ &-\lambda_H(H^\dagger H)^2\Big]\end{aligned} \tag{13}$$

In the first term, we have

$$\begin{aligned}[\partial^\mu S_i]^\dagger(\partial_\mu S_i) =&[\partial^\mu(\bar{S}_i+\eta_i)]^\dagger(\partial_\mu(\bar{S}_i+\eta_i))\\ =&[\partial^\mu\eta_i]^\dagger(\partial_\mu\eta_i)\\ =&\eta_i^\dagger(\partial_\mu\eta_i)|_{\text{boundary}} - \eta_i^\dagger\partial^2\eta_i\\ =&-\eta_i^\dagger\partial^2\eta_i.\end{aligned} \tag{14}$$

and from the second term

$$-M_{i0}^2(S_i^\dagger S_i) = -M_{i0}^2\left(\bar{S}_i^\dagger\bar{S}_i + \eta_i^\dagger\bar{S}_i + \eta_i\bar{S}_i^\dagger + \eta_i^2\right) \tag{15}$$

Third, fourth, fifth, sixth, and tenth terms are still the same, but seventh, eighth, and ninth are vanish because of the assumption that we have made before. In this integral, terms that are linear in $S$, vanish (tadpole diagram) and term that has squared of $S$ are Gaussian integrals so we can integrate it out and it will not contribute again over Lagrangian. Thus, we arrived at

$$\begin{aligned}&\int \mathcal{D}f_i\mathcal{D}\eta_i^\dagger\mathcal{D}\eta_i \exp\left[-\eta_i^\dagger\partial_\mu\eta_i - M_{i0}^2(\eta_i^\dagger\eta_i) + \lambda_i f_i^2\right.\\ &\left.+\lambda_{12}f_1f_2 + \lambda_{13}f_1f_3 + \lambda_{23}f_2f_3 - \lambda_H(H^\dagger H)^2\right]\end{aligned} \tag{16}$$

and using functional determinant, we have

$$\begin{aligned}Z = \int \mathcal{D}f_i \exp&\left[\lambda_i f_i^2 + \lambda_{12}f_1f_2 + \lambda_{13}f_1f_3 + \lambda_{23}f_2f_3 - \lambda_H(H^\dagger H)^2\right.\\ &\left.+N_c\ln\det\left[\partial^2 + M_{i0}^2\right]\right].\end{aligned} \tag{17}$$

To solve this, we use minimal subtraction scheme in the last term

$$\ln\det[\partial^2 + M^2] = VT\frac{M^4}{2(4\pi)^2}\left(\frac{1}{\bar{\epsilon}} - \ln(M^2) + \frac{3}{2}\right) \tag{18}$$

and then we obtain

$$\begin{aligned}\frac{\Gamma[\bar{S},f,H]}{VT} =& - M_{i0}^2\bar{S}_i^\dagger\bar{S}_i + \lambda_i f_i^2 + \lambda_{12}f_1f_2 + \lambda_{13}f_1f_3 + \lambda_{23}f_2f_3 - \lambda_H(H^\dagger H)^2\\ &+\frac{N_cM_{i0}^4}{2(4\pi)^2}\left(\frac{1}{\bar{\epsilon}} - \ln(M_{i0}^2) + \frac{3}{2}\right)\end{aligned} \tag{19}$$

which give

$$V_{\rm MFA}(f_i,H)=-\frac{\Gamma[\bar{S},f,H]}{VT}$$
$$=-\lambda_i f_i^2-\lambda_{12}f_1f_2-\lambda_{13}f_1f_3-\lambda_{23}f_2f_3+\lambda_H(H^\dagger H)^2+\frac{N_cM_i^4}{32\pi^2}\ln\left(\frac{M_{i0}^2}{\Lambda_H^2}\right) \tag{20}$$

where the ultraviolet divergence subtracted by Modified Minimal Subtraction ($\overline{\rm MS}$) scheme and $\Lambda_H=\mu e^{-3/4}$. After we got the mean-field approximation potential, the next step is to look for the minimum value of the potential. To obtain a minimum value of the potential, we look for the solution from the gap equation

$$0=\frac{\partial}{\partial f_i}V_{\rm MFA}=\frac{\partial}{\partial H}V_{\rm MFA} \tag{21}$$

From the derivative of $f_i$, we obtain

$$16\pi^2\langle f_i\rangle=N_c\langle M_{i0}^2\rangle\left(\frac{1}{2}+\ln\frac{\langle M_{i0}^2\rangle}{\Lambda_H^2}\right). \tag{22}$$

with the assumption that $\lambda_i\neq 0$, $\lambda_{12}\neq 0$, $\lambda_{13}\neq 0$, and $\lambda_{23}\neq 0$. From this equation, we can see that if $\langle f_i\rangle=0$, then $\langle M_{i0}^2\rangle=0$, and we also can not have a negative value on term inside parentheses. Next, from a derivative of $v_H$, where $H^\dagger H=\frac{1}{2}v_H^2$, we obtain

$$2\lambda_H\langle H^\dagger H\rangle=\lambda_{HS_i}\langle f_i\rangle. \tag{23}$$

By substituting this two-equation to mean-field approximation potential in eq.(21), we obtain the vacuum equation of potential

$$\langle V_{\rm MFA}(f_1,f_2,f_3,H)\rangle=-\frac{N_c}{64\pi^2}\langle M_{i0}^2\rangle^2 \tag{24}$$

From the mean-field approximation potential, Higgs mass term can be obtained by taking derivatives twice with respect to $v_H$:

$$m_h^2=\frac{\partial^2V_{\rm MFA}}{\partial v_h^2}$$
$$=6\lambda_H\langle H^\dagger H\rangle+\frac{N_c}{8\pi^2}\lambda_{HS_i}^2\langle H^\dagger H\rangle+2\lambda_{HS_i}^2\langle H^\dagger H\rangle\frac{\langle f_i\rangle}{M_i^2}-\lambda_{HS_i}\langle f_i\rangle. \tag{25}$$

To obtain Higgs mass value, we must set parameters like $\lambda_i$, $\lambda_{12}$, $\lambda_{13}$, $\lambda_{23}$, $\lambda_H$, $\lambda_{HS1}$, $\lambda_{HS2}$, and $N_c$. From these parameter, we can get minimum value of $f_1$, $f_2$, $f_3$, $v_H$,and Higgs mass (around 125 GeV).

## 4. Summary

We have considered SM with a scale-invariant scale term, with no Higgs mass term. The hidden sector was introduced in the gauge group $SU(N_c)\times U(1)^{N_f}$, with $N_f=3$. It is shown that the hidden sector generates a scale in a strongly interacting gauge sector and transmitted to the SM sector via Higgs portal coupling and create a Higgs mass term via bilinear condensation.

## Acknowledgments

The work of QMBS, MA, and VR are supported by research grant Mathematical and Science Faculty, Diponegoro University.

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