On accelerated expansion in string theory

Dissertation

zur Erlangung des Doktorgrades
an der Fakultät für Mathematik,
Informatik und Naturwissenschaften

Fachbereich Physik
der Universität Hamburg

vorgelegt von

Jakob Moritz

Hamburg

2019

Gutachter der Dissertation:	Dr. Alexander Westphal Prof. Dr. Marco Zagermann
Zusammensetzung der Prüfungskommission:	Prof. Dr. Günter Sigl Dr. Elli Pomoni Prof. Dr. Dieter Horns Prof. Dr. Marco Zagermann Dr. Alexander Westphal
Vorsitzender der Prüfungskommission:	Prof. Dr. Günter Sigl
Datum der Disputation:	03.07.2019
Vorsitzender Fach-Promotionsausschusses PHYSIK:	Prof. Dr. Michael Potthoff
Leiter des Fachbereichs PHYSIK:	Prof. Dr. Wolfgang Hansen
Dekan der Fakultät MIN:	Prof. Dr. Heinrich Graener

Abstract

In the first part of this thesis we investigate the viability of a class of models for de Sitter (dS) vacua in string theory, due to Kachru, Kallosh, Linde and Trivedi (KKLT). We start by explaining why the success or failure of such models is sensitive to a large number of Planck suppressed operators, and collect circumstantial evidence that this *UV sensitivity* cannot be evaded with delicately engineered models such as the Kallosh-Linde (KL) racetrack due to a generic conflict with the weak gravity conjecture (WGC) for axions. We take this as motivation to study the KKLT mechanism from a ten-dimensional point of view. Building on earlier work we show that the form of the four-dimensional supergravity model as proposed by KKLT is remarkably consistent with the ten-dimensional perspective with respect to supersymmetry breaking as well as consistency requirements such as tadpole cancellation conditions. Nevertheless we point out a generic loss of parametric control over the ten-dimensional geometry in the interesting regime of 4d parameter space where an uplift to dS is believed to occur.

In the second part we argue for the existence of a new type of ultralight axion in the type IIB flux landscape of string theory. This axion can be thought of as the integral of the Ramond-Ramond (RR) two form over a certain two-sphere which is trivial in homology, and it arises when fluxes stabilize a Calabi-Yau (CY) threefold near a conifold transition locus in complex structure moduli space. The axion receives a non-trivial but strongly suppressed potential because its field excursion weakly twists two or more Klebanov-Strassler (KS) throats against each other. This can be understood as a purely geometric effect in 10d, but also as misalignment of gaugino condensates in different field theory sectors that are dual descriptions of the ten-dimensional throat system. The scalar potential turns out to be periodic while its periodicity can be enhanced with respect to the natural axion periodicity by a finite monodromy factor allowing its decay constant to become parametrically super Planckian in many cases. While our model does *not* obey the strong form of the weak gravity conjecture we identify an alternative bound that enforces the generic presence of dominant sub-Planckian harmonics, thus preventing us from building models of natural inflation using this construction.

Zusammenfassung

Im ersten Teil dieser Arbeit untersuchen wir die Realisierbarkeit einer Klasse von Modellen für de Sitter (dS) Vakua in der Stringtheorie, vorgeschlagen von Kachru, Kallosh, Linde und Trivedi (KKLT). Wir beginnen damit, zu erklären, warum der Erfolg oder Misserfolg solcher Modelle von einer großen Anzahl von Planck-unterdrückten Operatoren abhängt, und wir finden Indizien dafür, dass diese *UV-Empfindlichkeit* nicht mit technisch ausgereifteren Modellen wie dem Kallosh-Linde (KL) *racetrack* Szenario beseitigt werden kann, aufgrund eines generischen Konflikts mit der *Vermutung über die Gravitation als schwächste Kraft* (WGC). Wir betrachten dies als Motivation, den KKLT-Mechanismus aus zehndimensionaler Sicht zu untersuchen. Aufbauend auf früheren Arbeiten zeigen wir, dass die von KKLT vorgeschlagene Form des vierdimensionalen Supergravitationsmodells in bemerkenswerter Weise mit der zehndimensionalen Perspektive übereinstimmt, in Bezug auf das Brechen der Supersymmetrie, sowie den Anforderungen an Widerspruchsfreiheit wie *tadpole-cancellation*-Bedingungen. Trotzdem weisen wir auf einen generischen Verlust der parametrischen Kontrolle über die zehndimensionale Geometrie in dem interessanten Bereich des 4d-Parameterraums hin, wo angenommen wird, dass ein Anstieg nach dS auftritt.

Im zweiten Teil argumentieren wir für die Existenz eines neuen Typs ultraleichter Axionen in der Typ IIB Flusslandschaft der Stringtheorie. Man kann sich dieses Axion als das Integral der Ramond-Ramond (RR) Zwei-Form über eine bestimmte, in der Homologie triviale Zwei-Sphäre vorstellen, und es entsteht, wenn Flüsse eine Calabi-Yau (CY) Mannigfaltigkeit in der Nähe eines Konifoldübergangslokus im Modulraum der komplexen Strukturen stabilisieren. Das Axion erhält ein nicht-triviales, aber stark unterdrücktes Potential, da seine Feldauslenkung zwei oder mehr Klebanov-Strassler (KS) *throats* gegeneinander verdreht. Dies kann als ein rein geometrischer Effekt in 10d verstanden werden, aber auch als Fehlausrichtung von Gaugino-Kondensaten in verschiedenen Feldtheoriesektoren, die eine duale Beschreibung des zehndimensionalen *throat*-Systems sind. Das skalare Potential erweist sich als periodisch, während seine Periodizität in Bezug auf die natürliche Axionperiodizität durch einen endlichen Monodromiefaktor erhöht werden kann, wodurch seine Zerfallskonstante in vielen Fällen parametrisch super-planckisch wird. Während unser Modell der starken Form der WGC nicht gehorcht, identifizieren wir eine Bedingung, die die generische Präsenz dominanter subplanckischer Modulierungen erzwingt, sodass wir mit dieser Konstruktion keine Modelle natürlicher Inflation bauen können.

This thesis is based on the publications:

Gaugino condensation and small uplifts in KKLT [1]
F. Carta, J. Moritz, A. Westphal
arXiv:1902.01412 [hep-th]

Thraxions: ultralight throat axions [2]
A. Hebecker, S. Leonhardt, J. Moritz, A. Westphal
JHEP 1904 (2019) 158, arXiv:1812.03999 [hep-th]
We would like to emphasize that this paper is the result of shared work in particular with A. Hebecker's PhD student S. Leonhardt.

On uplifts by warped anti-D3-branes [3]
J. Moritz, A. Retolaza, A. Westphal
Fortschr. Phys. 2018, 1800098, arXiv:1809.06618 [hep-th]

Racing through the swampland: de Sitter uplift vs weak gravity [4]
J. Moritz, Thomas Van Riet
JHEP 1809 (2018) 099, arXiv:1805.00944 [hep-th]

Toward de Sitter space from ten dimensions [5]
J. Moritz, A. Retolaza, A. Westphal
Phys.Rev. D97 (2018) no.4, 046010, arXiv:1707.08678 [hep-th]

Für Lisa

Contents

Chapter 1

Introduction

1.1 The search for new physics

Symmetries and the quest for unification have perhaps been the most important organizing principles of elementary particle physics in the twentieth century. All the particles that have so far been identified have taken their place in the standard model (SM) of particle physics. The interactions among them are determined by their charges under the gauge symmetry group $G_{\rm SM} = SU(3)\times SU(2)\times U(1)$, the gauge coupling constants, the Higgs potential, and the Yukawa couplings. Its remarkable consistency and completeness as a quantum field theory (QFT) means both a triumph for its inventors and a challenge for those in search of beyond the SM (BSM) Physics. The discovery of the Higgs boson [6, 7] marks the end of a long era that perhaps started with the Fermi theory of the weak interaction. While remarkably consistent at low energies, the Fermi theory predicts its own breakdown at energies of order 100 GeV. This was correctly interpreted as a signal for new physics at this mass-scale, and led to the advent of the Glashow-Salam-Weinberg model of the electro-weak interaction [8–10] and eventually to the formulation and experimental confirmation of the SM. Unfortunately in practical terms this series of guaranteed discoveries at energy scales just around the corner has terminated. The SM contains no coupling constants with negative mass dimension (it is *renormalizable* [11]). As a consequence it predicts its own breakdown only at an energy scale that is fantastically high. The perhaps strongest indication for new physics beyond the SM (BSM) comes from neutrino oscillation experiments [12–14] indicating a non-vanishing coefficient of the *non-renormalizable* Weinberg operator, and from the fact that the gauge couplings of the standard model meet roughly at the scale of

$$\Lambda_{\rm GUT} \sim 10^{16}{\rm GeV}\,. \tag{1.1}$$

This suggests that the SM gauge group could in the end be embedded into a unified gauge group (GUT) that breaks spontaneously in the vacuum [15].[1]

However, so far we have not mentioned gravity. Coupling gravity to the SM yields an effective field theory (EFT) that is *not* renormalizable just as Fermi's theory of the weak interaction is not. So gravity seems to come to our rescue: The standard interpretation of an effective field theorist would be that the SM model coupled to Einstein gravity breaks down at the Planck scale

$$M_P \equiv (8\pi G_N)^{-1/2} = 2.4 \cdot 10^{18}\text{GeV}\,. \tag{1.2}$$

At this scale, the scale of *quantum gravity*, the effective coupling constant that determines the scattering amplitude between gravitons becomes of order unity. This is *not* a scale that we will reach with collider experiments in the near future. Nevertheless it is the lowest scale we know at which (radically) new physical phenomena are guaranteed to become relevant. So, while of course there are still many reasons to expect that new physical degrees of freedom are hidden just around the corner at scales accessible to future collider experiments, it is justified to ask what kinds of physical phenomena we can access today or in the near future that are directly or indirectly tied to the scale of quantum gravity. More so, we will now outline reasons why we should not take the high scale of quantum gravity as a reason to despair but as an encouragement to work at the interface between particle physics, cosmology and a beautiful theory called string theory.

1.2 The expanding universe

Since the time of Edwin Hubble we know that our universe is expanding [16]. In fact, since only rather recently we know that the expansion is *accelerating* [17, 18], and the history of our universe is extremely well described by the standard model of cosmology called ΛCDM. It consists of the standard model of particle physics coupled to Einstein gravity and assumes only two further (though very much mysterious) ingredients: The first is *cold dark matter* (CDM), a yet unknown type of non-relativistic matter that couples to ordinary matter only extremely weakly and accounts for about 25% of the energy of our universe. The second is the notorious *cosmological constant* (Λ), a homogeneous fluid with peculiar equation of state making up 70%. It is a curious fact that the baryonic matter that we are made of makes up only about 5% of the energy budget

[1] A single generation of the standard model fermions is obtained from the $10 \oplus \bar{5}$ of $SU(5)$ under the breaking pattern $SU(5) \longrightarrow G_{SM}$. Even better, the 16 of $SO(10)$ decomposes into the $10 \oplus \bar{5}$ of $SU(5)$ plus one singlet under $SO(10) \longrightarrow SU(5)$. The singlet is a natural candidate for a right handed neutrino required for the see-saw mechanism to generate neutrino masses in a renormalizable fashion.

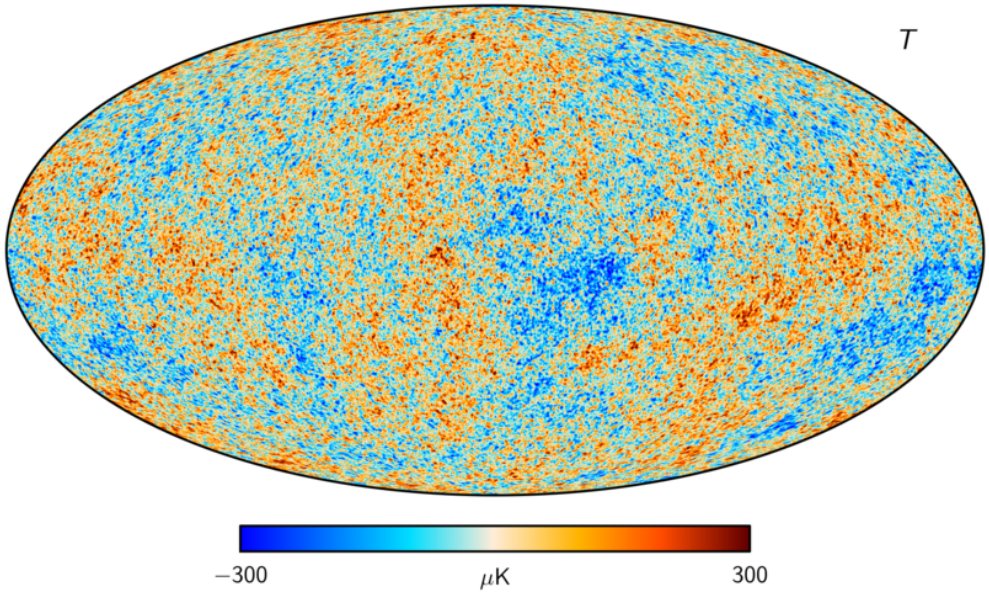

Figure 1.1: The temperature fluctuations in the CMB as seen by Planck 2018. Figure taken from [19].

of our current universe. With this set of minimal ingredients the ΛCDM model gives rise to a remarkably plausible history of our universe starting with an extremely hot and dense phase (the Big Bang) about 13.8 billion years ago. It successfully describes many eras of the history of the universe: in the very first minutes after the Big Bang the process of *Big Bang nucleosynthesis* led to the formation of the light nuclei. Subsequently the universe expanded and cooled down until the temperature was low enough that neutral hydrogen could form efficiently about $380,000$ years after the Big Bang. This event is called *recombination* and marks the time at which the universe became transparent to light. Today we observe the light that was released at this time as the *cosmic microwave background* (CMB) (see Figure 1.1) which gives us a (red-shifted) snapshot of a very young version of our universe. Furthermore, the formation of large scale structure can be understood to arise from the growth of tiny inhomogeneities in the early universe, leading to the distribution of galaxies we find today. Finally, the accelerated expansion of the current universe is attributed to the cosmological constant.

Fortunately for us this picture is hardly complete. The CMB provides us with a detailed temperature map of a large patch of the universe when it was much smaller and much hotter. This patch was in fact so large that it contained a large number of causally disconnected regions. The time since the Big Bang simply did not suffice to bring them into causal contact. Nevertheless we observe that the temperature was the same everywhere to one part in 10^5 and the tiny temperature fluctuations were correlated on all scales we can observe. This is called the *horizon problem*. Declaring

this to be a coincidence is extremely implausible. As the formation of our universe is not an experiment that we can repeat, implausibility of a proposed cosmological history is perhaps the best indication for a seriously incomplete understanding that one will ever get.

A remedy for this serious problem is offered by the theory of *inflation*. As proposed long ago by A. Guth, A. Linde, A. Albrecht, P. Steinhardt and others [20–22] our universe underwent an era of exponential expansion before it was populated by the matter we know (and the one we don't know). During this time the energy budget of our universe was dominated by a slowly evolving *dark energy* that drove the exponential expansion much in the same way that a cosmological constant might drive today's accelerated expansion. A crucial ingredient is a new (scalar) degree of freedom called the inflaton that serves as a 'clock' for the progress of inflation. Its tiny (and correlated) quantum fluctuations were stretched to very large scales during the time of inflation and translated into tiny density fluctuations in the early universe as we know it. These in turn translated into the temperature fluctuations at the time of recombination that we can observe in the CMB, and served as the seeds for large scale structure formation. Crucially these fluctuations are tiny and naturally correlated on what appear to be super-horizon scales at the time of recombination. This beautiful idea not only solves the problem of implausibility but it also *predicts* the precise form of the CMB power spectrum using only two a priori undetermined constants to be fixed by observational data (or a specific inflationary model).

This simple and successful idea begs the question what were the relevant degrees of freedom whose quantum fluctuations shaped the form of the CMB? One of the most interesting questions is whether also gravitational quantum fluctuations played an important role. Crucially the answer to this question can in principle be inferred from the *polarization* (so called B-modes) of the CMB [23–25]. In other words, it is possible that the quantum fluctuations of the gravitational field are detected in the near future. Beside this obvious reason for excitement it turns out that such a detection would teach us quite a lot about the inflationary era itself [26]. First, the *scale* of inflation would have to be of order of the scale of grand unification. Second, we would learn that during the era of inflation the inflaton traversed a *distance* in field space that is larger than the Planck scale. From the point of view of effective field theory (EFT) there is nothing obviously wrong with this. As long as the scalar potential stays small in Planck units the use of Einstein gravity as an effective field theory is valid. In fact the types of successful models that a (naive) effective field theorist would write down would all share the feature of inflationary field excursions *much bigger* than the Planck scale. It is also easy to ensure that the flatness of the scalar potential over super-Planckian field ranges

is not endangered by large quantum corrections. There are really only two problems with these types of models. First, they are (almost) ruled out by experiment [27]. Second, the predictions of such models are sensitive to a large (or even infinite) number of *Wilson coefficients* [28]. These are the coefficients of the expansion of the effective Lagrangian in terms of operators of increasing mass dimension. As a consequence, bottom up EFT models of large field inflation are usually not very predictive[2]: The number of Wilson coefficients that specify an inflationary model exceed by far the number of observables that we can extract from CMB data. At least, in writing down a bottom up model of large field inflation one is far from being agnostic about physics at the Planck scale. As a consequence, making a choice of EFT model can really lead to meaningful predictions only if one is able to *predict* infinitely many relations among the Wilson coefficients from more fundamental principles. So here we have arrived at a remarkable and encouraging result: If gravitational wave fluctuations generated during inflation, called *primordial tensor modes*, are detected, an infinite number of Wilson coefficients is constrained. This is getting us rather close to *testing physics at the Planck scale*. Conversely, if a theory of quantum gravity *predicts* that models of large field inflation cannot exist, we can in principle falsify it observationally via a detection of primordial tensor modes using CMB data.[3]

1.3 String theory and the swampland

This is only one of many ideas how a full theory of quantum gravity may severely constrain the set of low energy EFTs and in particular the set of low energy observables, thereby acquiring the status of a *falsifiable* theory. We call the set of EFTs including gravity that can be realized as a low energy limit of the full quantum gravity the *landscape* while the ones that cannot form the complementary set that we call the *swampland* [29, 30]. For this concept (the idea of the swampland) to be a useful one, one has to show that there exist clear boundaries in the space of low energy observables that divide regions belonging to the landscape from those that belong to the swampland (see Figure 1.2). We will come back to what types of criteria have been proposed that could distinguish the landscape from the swampland in section 3.8. While some criteria (henceforth called *lore quantum gravity statements*) can be motivated more or less clearly from (say) black hole physics [31, 32] it is clearly of great interest to have in hand an actual candidate theory of quantum gravity in order to 1) give further evidence for or even prove lore quantum gravity statements and 2) collect further constraints

[2]This does not mean that the theory of inflation is not predictive (it certainly is!).

[3]The question to what extend such a theory would actually be *confirmed* by a null detection is a more subtle one because we do not have so many quantitative theories of quantum gravity to compare.

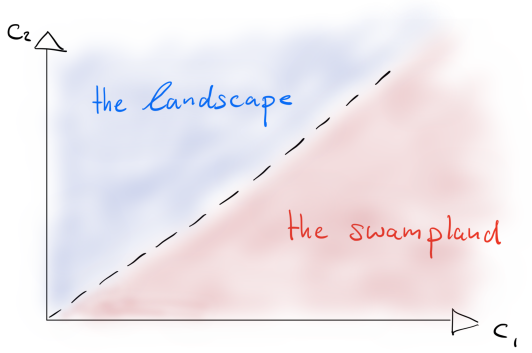

Figure 1.2: A cartoon of a 2d subspace of the total parameter space of EFTs coupled to gravity parameterized by two Wilson coefficients (c_1, c_2). The landscape (blue) is hoped to be well separated from the swampland (red).

that cannot be motivated as easily from the *bottom-up*. The only theory of quantum gravity that is developed to a sufficient degree that we may begin to address such questions in principle to a satisfactory quantitative degree is *string theory*.

String theory is believed to be a unique theory of quantum gravity although to date there exists no formulation that would make this manifest. In contrast, there exist five perturbative string theories, that all have *ten* dimensions of spacetime.[4] In each perturbative string theory the light degrees of freedom can be thought of as vibrating one-dimensional objects called strings (but there are also heavy solitonic membranes [34]). Crucially, in the 1990s (the *second superstring revolution*) it was realized that the different perturbative string theories should be understood as different weak coupling limits of the same underlying theory (sometimes called M-theory).

The formulation of these theories and (some of) the relations among them are standard textbook material found e.g. in [35]. For us it will (mostly) suffice to consider their ten-dimensional low energy limits. These are ten-dimensional *supergravity* theories, and in fact they are the *only* consistent ten-dimensional ones [36] (at two-derivative level).[5] These ten-dimensional theories allow non-trivial gravitational backgrounds where the total spacetime is a product of a non-compact four-dimensional spacetime and a compact (also called *internal*) six-dimensional space

$$M_{9,1} = M_{3,1} \times M_6 \,, \tag{1.3}$$

[4]To be precise these are the only *supersymmetric* string theories in ten dimensions. There is also a somewhat less studied non-supersymmetric one [33].

[5]In ten dimensions we know for a fact that *all* consistent low energy effective supergravity theories arise as a low energy limit of string theory. In other words, if we lived in a supersymmetric ten-dimensional world we would know almost for certain that string theory is *the* theory of quantum gravity, without the need to access high energies.

so that at low energies (at wavelengths larger than the size of M_6) the EFT of perturbations around the background solution is a four-dimensional QFT coupled to gravity. Obtaining lower-dimensional theories from higher-dimensional ones in this way is called Kaluza-Klein (KK) reduction, *compactification*, or *compactifying* the higher-dimensional theory, and goes back to early ideas by Kaluza and Klein [37, 38]. We will consider the set of EFTs that arise via compactification of ten-dimensional string theories.

More precisely we will focus on the so-called flux landscape of type IIB string theory [39, 40]. It arises by compactifying type IIB string theory on a so called Calabi-Yau (CY) three-fold[6] and turning on higher-dimensional analogues of electric and magnetic fields (called *fluxes*) along the internal directions. We focus on this weakly coupled corner because there it is best understood how a truly enormous set of four-dimensional low energy EFTs can arise from discrete ten-dimensional data (see Section 3.6). This degeneracy is in fact so large that some EFT parameters are believed to be tunable to almost arbitrary precision [41–43]. Due to this existence of an almost continuous set of four-dimensional EFTs the distinction between the landscape and the swampland is perhaps most easily addressed. Moreover, the ability to tune many parameters makes it a particularly interesting arena for model building.

Having stated what is our starting point, let us return to the swampland idea in general, and accelerated expansion in particular. There is an ongoing community effort to construct models of (single field) large field inflation in string theory. Despite the emergence of a set of promising ideas [44–49] so far no model has been established beyond any reasonable doubt. In fact some authors have taken the persistent difficulties that appear in construction attempts as evidence that large field inflation is in fact impossible in string theory [50, 51]. While this may well be true, so far we do not understand *why*. We find it useful to simplify the problem by restricting to the perhaps most promising inflaton candidates. In our opinion these are *axions*. We will discuss in more detail what we mean by an axion but for now let us work with the simple minded definition that these are (pseudo-)scalar fields $a(x)$ that in some well understood limit develop a *shift-symmetry*

$$a(x) \longrightarrow a(x) + c\,, \quad c \in \mathbb{R}\,, \tag{1.4}$$

valid to all orders in the perturbative expansion.

In other words all non-derivative interactions can be suppressed in a very natural way. It is then feasible in principle that a small computable axion potential can be generated that could drive the inflationary expansion. Such potentials can be di-

[6] To be precise half of the spectrum has to be projected out in a way that produces a so called *orientifold*.

vided into two classes. The first ones arise naturally (really unavoidably) from non-perturbative effects such as instantons. Such contributions to the potential preserve a discrete (gauged) shift symmetry $a \longrightarrow a + 2\pi f_a$. For such potentials to host large field inflation, the *axion decay constant* f_a must be bigger than the Planck scale. However, the basic string theory axions seem to all have *small* decay constants [52]. In fact there is a class of *swampland conjectures* that constrains axion decay constants. These are called the *weak gravity conjecture* (WGC) in various versions [31].[7] In its simplest form (which has nothing to do with axions) it states that in every consistent low energy EFT of gravity and electromagnetism (and whatever else) there should exist at least one light charged particle with mass m and $U(1)$ charge e such that $m \lesssim eM_P$. This conjecture has been extrapolated [31] (and in some cases been shown to extend [56, 58]) to versions that constrain non-perturbative axion potentials (see Section 3.8.2). A strong form of this conjecture states that in controlled regimes the most dominant contributions to the axion potential have small periodicities in Planck units. If this conjecture is true (in the appropriate strong sense), it implies that large field axion inflation using non-perturbative axion potentials is impossible.

In principle one might also be able to engineer contributions to the scalar potential that break the shift symmetry completely but still by a controllable small amount. The weak gravity conjecture does not readily apply to this case.[8] This idea is called *axion monodromy* [46, 73] and will play a key role in what follows.

1.4 Outline of thesis

Let us give a brief outline of this thesis. In chapter 2 we introduce some of the relevant concepts of cosmology surrounding accelerated expansion. Then, in chapter 3 we introduce some of the concepts and technology of string compactifications that will become relevant later on. We will outline the idea of the swampland of effective field theories in section 3.8 and explain the basics of stringy moduli stablization, in particular the famous KKLT model, in section 3.7.

The bulk of this thesis is divided into three chapters: chapter 6 is mostly based on [2], and devoted to the top-down construction of a class of models of ultralight axions, with potentially interesting applications to large field inflation and the weak gravity conjecture for axions. Chapters 4 and 5 are about the question whether there exist solutions of string theory with a positive cosmological constant, and are based on [1, 3–5]. We now outline their content in some detail.

[7] It is by far not clear which version should hold (if any), in particular when many light axions are present. For discussions about this issue see e.g. [2, 50, 53–71].

[8] This is not strictly true as a *magnetic* version does apply but does not preclude large field inflation [72].

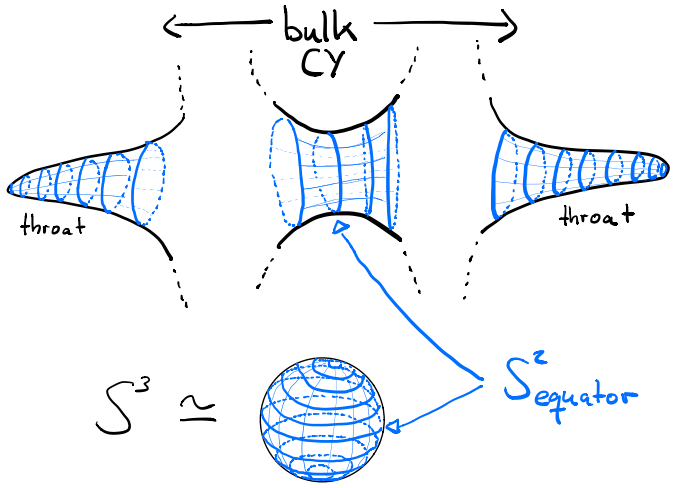

Figure 1.3: The two-throat system embedded into a larger bulk CY. The bulk S^2 used to define the axion field excursion can be thought of as the equator of a three-sphere that reaches into the two throats.

1.4.1 Is large field inflation possible in string theory?

In chapter 6, following ref. [2], we build on and develop further an idea of [74] in constructing explicit models of axion monodromy. We work in the flux landscape of type IIB string theory, in a regime where fluxes stabilize the shape of the CY (orientifold) near a so called conifold transition locus in moduli space [40, 41, 75], as reviewed in section 3.6. Such a locus can be thought of as a shared singular locus in the moduli spaces of two topologically *distinct* CY manifolds [76, 77], in particular the light spectrum changes across such loci (see section 3.4 for details). Moreover, multiple *exponentially* red shifted regions called warped throats [78, 78] are known to develop in this regime due to backreaction by fluxes [40], as explained in section 3.6.2.

We will argue that the light spectrum on *one side* of the transition locus contains light axionic degrees of freedom that we can associate with the light axions *on the other side* of the transition locus. The axion mass is a measure of proximity to the transition locus. Thus, in many cases the EFT arising from such a compactification contains as light degrees of freedom the moduli of *two* distinct CY three-folds which might come as a surprise.

The axion can be thought of as the integral of the Ramond-Ramond (RR) two-form C_2 axion over a bulk CY representative two-sphere of one of the resolution two-cycles on the *resolved side* of the conifold transition. As fluxes stabilize the complex structure moduli onto the *deformed side* of the transition this sphere is trivial in homology and best thought of as an *equatorial* two-sphere of a non-trivial three-sphere that stretches down into two or more throats, see figure 1.3 for an illustration. By Stokes theorem, a

non-trivial field excursion of the axion leads to the creation of flux/anti-flux pairs at the IR ends of the throats [74]. We will explain that a ten-dimensional backreaction effect sets in that drives the geometry away from complex structure moduli space but allows the local throats to restore supersymmetry locally. The remaining vacuum energy is mainly due to the 'twisting' of the throats against each other and turns out to return to a *periodic one*, but with non-trivial *finite* axion monodromy.

So, somewhat unexpectedly, we are in the position to apply the WGC for axions to a model of axion monodromy. In fact we find that the conjecture is challenged by our model. The scalar potential is simply much smaller than one would predict by naively applying the conjecture. Moreover, the axion decay constant can in many cases be parametrically super Planckian. Nevertheless, in the cases that we have analyzed the form of the scalar potential does not admit large field inflation due to generically dominant short wavelength harmonics in the potential.[9]

We summarize that due to backreaction effects that are truly ten-dimensional in that they can *not* be associated with backreaction on CY moduli, a worked out example of axion monodromy is presented that returns the scalar potential to a periodic form. At least generically it is compatible with the general expectation that single field large field inflation is impossible in string theory, while the standard weak gravity conjecture is violated parametrically.

1.4.2 Do de Sitter vacua exist in string theory?

In chapters 4 and 5 we consider the present time expansion of the universe. In ΛCDM it is sourced by a positive cosmological constant, leading to a universe that asymptotes to a *de Sitter* (dS) universe, as discussed in section 2.3. However, in string theory it is notoriously difficult to realize this. It is so difficult that another (perhaps the most dramatic) swampland conjecture has been put forward, the *no-dS* conjecture [79–81]. It implies that string theory solutions cannot have a positive cosmological constant. Depending on what precise form of the conjecture holds [82–88] (if any) this could lead to an in principle observationally testable prediction of a non-trivial equation of state of dark energy. But as a first step it is of great interest to settle the question whether string theory possesses vacua with positive cosmological constant, in other words dS vacua. Again we focus on the flux landscape of type IIB string theory where some of the most (but not yet fully) convincing arguments for the existence of de Sitter vacua in string theory have been made. These are called the Kachru-Kallosh-Linde-Trivedi (KKLT) [41] mechanism and the large volume scenario (LVS) [75]. Both incorporate perturbative and non-perturbative corrections to the tree level action in an arguably

[9] Similar effects have been observed in [71].

self-consistent way. We proceed as follows.

In chapter 4 we explain, following [3, 5] that due to the universal existence of light moduli fields the question of existence of 4d de Sitter vacua is *UV sensitive* in a way that is very much analogous to the UV sensitivity of large field inflation: It is usually not a question that can be answered from knowledge of only a few Wilson coefficients in a 4d effective supergravity theory of the light moduli coupled to sectors with SUSY breaking states such as gauge theories, but rather requires knowledge of a large or even infinite number of coefficients of Planck-suppressed operators. Furthermore, we outline a rather generic way how standard ways of *uplifting* controlled AdS vacua to de Sitter vacua, i.e. perturbing the former in a controlled way to produce the latter, *could* (and in many instances do) fail: As an effective order parameter for supersymmetry breaking (the uplift potential) is dialed up, backreaction on the light moduli becomes increasingly severe, lowering their mass-scale. As the effective 4d vacuum energy approaches zero from below, the light moduli are destabilized. We call this phenomenon *uplift-flattening*. We relate this behavior with well-known no-go theorems against dS solutions that can be derived in various classical corners of string theory [89–103] from so-called tadpole cancellation conditions. We also argue that a class of models that could in principle suppress the UV sensitivity of uplifts parametrically [104] is in generic conflict with the weak gravity conjecture for axions [4], indicating that the question of de Sitter uplifts is naturally addressed from a top-down perspective. We take this as motivation to study in detail the KKLT mechanism from a ten-dimensional perspective.

In chapter 5 we explain how the KKLT mechanism, originally proposed from a four-dimensional perspective, is lifted to ten-dimensional solutions, following [1, 5, 105–110]. All no-go theorems against the existence of de Sitter vacua in type IIB string theory that we are aware of are evaded *in principle*. More so, we argue that the original no-go statements even can be turned around to *confirm* the form of the four-dimensional KKLT model in a non-trivial way [1, 110].[10] We take this as evidence that uplift flattening is suppressed efficiently in this particular model, enforcing its status as one of the leading candidates for controlled dS vacua in string theory.

In contrast, following [1], we point out that nevertheless the so called *warped uplifts* employed in KKLT can at best work marginally due to the generic loss of parametric control over the *ten-dimensional* supergravity approximation in the regime of 4d parameter space where an uplift to dS is believed to occur. This is *not* related with uplift flattening, but rather due to parametric control problems encountered already in the supersymmetric KKLT vacua once the 10d flux geometry has been engineered to allow for sufficiently small warped uplifts. In the non-marginal regime where the 10d

[10] Note that the authors of [111] come to the opposite conclusion. We will outline the discrepancy in chapter 5.

geometry is parametrically controlled they would lead to run-away solutions only.

It is not obvious if the last point carries over to other *uplifting mechanisms*. Assuming it does, we speculate on the physical meaning. One may for instance take it as evidence for the no-dS conjecture, but there are many other possibilities: For instance, de Sitter vacua might be marginal phenomena that cannot be obtained from supersymmetric AdS vacua by a small perturbation, or they might just lie *near the interior of moduli space*.

Despite the open questions that remain we are confident that in the near future at least the viability of the KKLT mechanism for generating de Sitter vacua in string theory can be settled to a satisfactory degree.

1.5 Conventions

Throughout this thesis we work in units $\hbar = c = k_B = 1$, and with 'mostly plus' metric conventions $\eta = (-, +, \cdots, +)$. Ten-dimensional indices are capital roman $M, N, ... = 0, ..., 9$, four-dimensional ones are greek $\mu, \nu = 0, ..., 3$, and *internal* indices are lower case roman $i, j = 4, ..., 9$. Moreover we will often choose to express four-dimensional quantities in units $M_P = 1$, while ten-dimensional quantities are expressed in units $l_s = 1$, using the 10d *Einstein frame* metric. We will sometimes make exceptions of the latter rule by using the *string frame* metric instead. The two are related by a dilaton dependent Weyl rescaling,

$$G_{MN}|_{\text{Einstein}} = e^{-\frac{\phi}{2}} G_{MN}|_{\text{string}} \,. \tag{1.5}$$

The norm of a (complex) p-form F_p with indices $F_{M_1,...,M_p}$ is defined as

$$|F_p|^2 \equiv \frac{1}{p!} \overline{F_{M_1,...,M_p}} F^{M_1,...,M_p} \,. \tag{1.6}$$

Chapter 2

Inflation and dark energy

In this section we will give a slightly more detailed account of what we know about dark energy and the theory of inflation. We will focus on the aspects that will become relevant in this thesis. As we observe our universe to be homogeneous and isotropic on very large scales it is appropriate to describe the geometry of the universe with a Friedmann-Lemaitre-Robertson-Walker (FLRW) metric with line element

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2_{S^2} \right) , \tag{2.1}$$

where $a(t)$ is the scale factor that encodes the growth of the spatial slices with time and is conveniently set to one when evaluated today. By isotropy and homogeneity the spatial slices are fixed to flat space $\mathbb{R}^3$ ($k = 0$), a three-dimensional sphere ($k > 0$), or three-dimensional hyperbolic space ($k < 0$), and k encodes the spatial curvature. With this ansatz, and in the presence of a homogeneous and isotropic fluid with energy density ρ and pressure P, Einstein's equations are solved provided Friedmann's equations are solved,

$$H(t)^2 \equiv \left(\frac{\dot{a}}{a} \right)^2 = \frac{\rho}{3M_P^2} - \frac{k}{a^2} , \qquad \frac{\ddot{a}}{a} = -\frac{\rho + 3P}{6M_P^2} , \tag{2.2}$$

where we have introduced the Hubble parameter $H(t)$. It is useful to divide the cosmological fluid into four components. On the one hand, non-relativistic matter is essentially pressure-less $P^{(m)} = 0$, and dilutes with the volume growth of the spatial slices, i.e. $\rho^{(m)} = \rho_0^m a^{-3}$. On the other hand, relativistic matter and radiation (in short radiation) have an approximately trace-less stress energy tensor, so $\rho^{(r)} = 3P^{(r)}$. On top of the volume dilution relativistic energy is *red-shifted* by a scale factor, so $\rho^{(r)} = \rho_0^{(r)} a^{-4}$. Moreover, spatial curvature appears in Friedmann's equations as a fluid

with pressure $P^{(k)} = -\frac{1}{3}\rho^{(k)}$, and

$$\rho^{(k)} \equiv -\frac{3M_P^2 k}{a^2} \equiv \rho_0^{(k)} a^{-2} \,. \tag{2.3}$$

Finally, a cosmological constant (cc) has a stress energy tensor proportional to the metric, so in particular its energy and pressure are constant over time and satisfy $\rho^{(cc)} = -P^{(cc)}$. For a positive cosmological constant, the energy density is positive while the pressure is negative. We can now write Friedmann's first equation as

$$H(t)^2 = H_0^2 \left(\Omega^r a^{-4} + \Omega^m a^{-3} + \Omega^k a^{-2} + \Omega^{cc}\right) \,, \tag{2.4}$$

where $H_0 \approx 70 \frac{\text{km}}{\text{s}} \text{Mpc}^{-1}$ is the *Hubble constant* we measure today[1] [112], and $\Omega^{(r,m,k,cc)}$ is the fraction of energy density supplied by a given component today. It is a crucial observational fact [112] that today our universe is approximately flat $\Omega^k < 0.2\%$, and a positive cosmological constant (or maybe a slightly different version of dark energy) plays an important but not yet completely dominating role, $\Omega^{cc} \sim 70\%$. This is interesting because due to the different scaling behaviors of the cosmological components it means that dark energy will dominate in the future while matter and radiation have dominated in the past. Curvature however never did and never will play an important role. This is easy to see: during matter domination which started near the time of recombination ($a^{rec} \sim 10^{-3}$), the relative contribution of curvature to the energy budget of the universe decreased linearly in the scale factor, while in the preceding radiation dominated era it even decreased quadratically. So we get that at scale factors $a_* < a^{rec}$,

$$\Omega_*^k \lesssim 10^{-2} \frac{a^{rec}}{a_0} \left(\frac{a_*}{a^{rec}}\right)^2 \sim 10 \left(\frac{T_0}{T_*}\right)^2 \sim \begin{cases} 10^{-30} & T_* = \text{TeV} \\ 10^{-56} & T_* = \Lambda_{\text{GUT}} \,, \end{cases} \tag{2.5}$$

using $T_0 \approx 2.7K \sim 2.3 \times 10^{-4}\text{eV}$. So, in the early universe its contribution to the energy budget must have been truly minuscule. This is the *flatness* puzzle of the standard Big Bang theory. In the early times of the universe radiation dominated the energy budget of the universe, until it was succeeded by matter at the time of recombination. Only now, dark energy is taking over. During these three phases the scale factor evolved as

$$a(t) \propto \begin{cases} t^{1/2} & \text{during radiation domination} \,, \\ t^{2/3} & \text{during matter domination} \,, \\ e^{Ht} \,, H = const. & \text{during domination by a cosmological constant} \,. \end{cases} \tag{2.6}$$

[1] There is actually a $> 3\sigma$ tension between the one inferred from CMB data [112] and the one from local redshift measurements [113], see e.g. [114] for a discussion.

2.1 The horizon problem

An even more severe problem is the *horizon problem*. This arises as follows. The maximum co-moving distance that a signal can traverse (*the co-moving horizon*) in a time interval $[t_i, t_f]$ is given by

$$\max(\Delta x)|_{t_i}^{t_f} = \int_{t_i}^{t_f} \frac{dt}{a(t)} = \int_{\log a(t_i)}^{\log a(t_f)} d(\log a)\,(aH)^{-1}\,, \tag{2.7}$$

which is the integral of the *co-moving Hubble radius* $(aH)^{-1}$ over the logarithm of the scale factor. $(aH)^{-1}$ quantifies the co-moving distance that can be traveled within an e-fold of cosmological expansion. The co-moving horizon gives us the maximal size of a causally connected region of space at a given time.[2] From Friedmann's second equation it is apparent that if all sources satisfy the *strong energy condition* $\rho + 3P \geq 0$, one has that

$$\frac{d}{dt}(aH)^{-1} > 0\,. \tag{2.8}$$

While a cosmological constant violates the strong energy condition it has only become relevant rather recently. So within most of the standard cosmological history the co-moving Hubble radius has grown. One might ask, what is the maximal co-moving distance that a signal could have traversed between the initial singularity and a given time t^*, say the time of recombination. It is easy to convince oneself that in the standard Big Bang cosmology with only standard matter and radiation sources, the integral is dominated by late times, so that the co-moving horizon at the time of recombination is of order the co-moving Hubble radius at that time. This is a problem because the CMB that we measure today is a picture of a large (of the order $(aH)^{-1}|_{\text{now}}$) patch of the universe at the time of recombination.[3] We can compare the co-moving Hubble radius today with the one at the time of recombination (at red-shift $z_{rec} \equiv a_{rec}^{-1} - 1 \sim 1100$). It is bigger by a factor of order $a_{rec}^{1/2} \sim 33$ so one finds that the part of the universe we observe through the CMB should have consisted of order $a_{rec}^{3/2} \sim 3 \times 10^4$ causally disconnected regions at the time of recombination. In other words we would expect that the signals we receive from two such regions (which correspond to regions in the sky that are separated by more than a degree) have nothing in common whatsoever. We observe quite the contrary: On *all* scales that we can observe the temperature of the CMB is the same to one part in 10^5 (see again Figure 1.1). Moreover, the tiny temperature fluctuations are correlated on all scales. This mystery is called the *horizon*

[2] I.e. all pairs of points in the interior have overlapping past light cones.

[3] This is because when neutral hydrogen atoms formed electric charges where screened and photons could freely propagate.

problem and it is far worse than the flatness problem. Declaring it to be a coincidence amounts to finely tuning a huge number of relations among the CMB observables while the flatness problem only means tuning one number small.

2.2 Inflation

Inflation solves this problem by postulating an era before radiation domination where the universe was dominated by an energy that is much like the cosmological constant today. This idea goes back to A. Guth [20] but its modern version that we will explain was invented by A. Linde [21], A. Albrecht and P. Steinhardt [22]. Crucially, it is an era with *shrinking* Hubble sphere,

$$\frac{d}{dt}(aH)^{-1} < 0\,, \quad \text{i.e.} \quad \ddot{a} > 0\,. \tag{2.9}$$

Such an era of *accelerated* expansion (provided it lasts long enough) allows us to extend the time between the initial singularity and radiation domination to make all past light-cones emanating from the CMB patch at the time of recombination intersect *before* they hit the initial singularity. One may take eq. (2.9) as the definition of inflation. Note that it implies that the Hubble rate of expansion varies slowly over a Hubble time,

$$\epsilon \equiv -\frac{\dot{H}}{H^2} < 1\,, \tag{2.10}$$

the $\epsilon \equiv 0$ limit corresponding precisely to the *exponential* expansion as sourced by a cosmological constant. Thus, one might say we are entering a new era of inflation just now. However, during the early epoch of inflation ϵ could not have been truly zero as inflation must have ended after a finite number of e-foldings.

The simplest way to implement this explicitly is to postulate the existence of a further scalar degree of freedom $\phi(x)$ called the *inflaton*, minimally coupled to gravity. The relevant terms in the action are

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right)\,, \tag{2.11}$$

with Ricci scalar R and inflaton potential $V(\phi)$. The stress-energy associated with the scalar field enters Friedman's equations with energy density and pressure

$$\rho^{inf} = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \quad P^{inf} = \frac{1}{2}\dot{\phi}^2 - V(\phi)\,. \tag{2.12}$$

It is apparent that in the limit of vanishing kinetic energy $\frac{1}{2}\dot{\phi}^2 \ll V(\phi)$ the scalar field

sector contributes just like a cosmological constant, so this is the $\epsilon = 0$ limit. Since one would like inflation to end after a finite time while giving lots of expansion before, one is drawn to the case $\epsilon \ll 1$. Whether or not such a limit can be taken (and sustained for a sufficient period of time) is of course dictated by the dynamics of the scalar field, via its equation of motion

$$\ddot{\phi} + 3H\dot{\phi} = -V'(\phi)\,. \tag{2.13}$$

Using this one computes

$$\epsilon = \frac{\frac{1}{2}\dot{\phi}^2}{H^2 M_P^2}\,, \quad \text{and} \quad \eta \equiv \frac{\dot{\epsilon}}{H\epsilon} = 2\left(\epsilon + \frac{\ddot{\phi}}{H\dot{\phi}}\right)\,, \tag{2.14}$$

where η measures the relative change in ϵ over a Hubble time. As expected one has $\epsilon \ll 1$ if the kinetic energy is negligible in comparison with the potential. η is small if the second derivative term in the scalar equations of motion can be neglected in comparison with the first derivative term. We really should require it to be small as otherwise inflation is not prolonged. In this case, the second order differential equation collapses to a first order one

$$\dot{\phi} = -\frac{M_P}{\sqrt{3}}\frac{V'(\phi)}{\sqrt{V(\phi)}}\,, \quad \Longrightarrow \quad \epsilon \approx \frac{1}{2}M_P^2\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \equiv \epsilon_V\,. \tag{2.15}$$

Clearly we need that the scalar potential satisfies the so-called *first slow roll condition* $\epsilon_V \ll 1$. But in order for $|\eta| \ll 1$ to hold as well, we also need that

$$\frac{\ddot{\phi}}{H\dot{\phi}} \approx -\eta_V + \epsilon_V \ll 1\,, \quad \eta_V \equiv M_P^2\frac{V''(\phi)}{V(\phi)}\,, \tag{2.16}$$

and hence the scalar potential must also satisfy the *second slow roll condition* $|\eta_V| \ll 1$. We conclude that if at some point ϕ_0 in field space the two slow roll conditions are satisfied the expansion of the universe will be nearly exponential with in particular a shrinking Hubble sphere. Such an era of inflationary expansion is called *slow-roll inflation*. Such a regime is particularly simple to handle computationally because the scalar field evolution is effectively determined by a first order differential equation. We can go on and compute the number of e-folds of slow roll inflation that occur in an interval of field space $[\phi_1, \phi_2]$

$$N([\phi_1, \phi_2]) = \int dt H = \int_{\phi_1}^{\phi_2}\frac{d\phi}{M_P}\frac{1}{\sqrt{2\epsilon}}\,. \tag{2.17}$$

Crucially, since $\epsilon \approx \epsilon_V$ during slow-roll, the number of e-folds of inflation can be

determined directly from knowledge of the scalar potential. Near a minimum of the scalar potential, the slow roll conditions are violated and the field starts to oscillate. Coupling the inflaton to standard model fields will lead to particle production, called *re-heating*, thus initiating the standard Big Bang history.

Usually, it is assumed that the *scale of inflation* $E_{inf} \equiv (3H^2 M_P^2)^{1/4}$ lies between the TeV and the GUT scale, so that the universe is reheated to sufficiently high temperatures so that e.g. baryogenesis can proceed.[4]

In order to solve the horizon problem, the number of e-folds of inflation has to be large enough. It is simple to estimate this: As (by definition) the co-moving Hubble radius shrinks during inflation (2.9) the co-moving horizon (2.7) is dominated by the co-moving Hubble radius at the beginning of inflation. So, the latter must be of order the co-moving Hubble radius of today so that all the patches of the early universe that we observe in the CMB today can be causally connected. The number of e-folds of inflation relate this to the Hubble radius at the end of inflation,

$$N = \log\left(\frac{a_{end}}{a_{begin}}\right) = \log\left(\frac{(a_{begin}H_{inf})^{-1}}{(a_{end}H_{inf})^{-1}}\right) > \log\left(\frac{(a_0 H_0)^{-1}}{(a_{end}H_{inf})^{-1}}\right) \tag{2.18}$$

Assuming for simplicity that between the end of inflation and today the universe was dominated by radiation $a(t) \sim t^{1/2}$, one has $H \sim 1/t \sim a^{-2}$, so we need

$$N > \log\left(a_{end}^{-1}\right) = \log\left(\frac{E_{inf}}{T_0}\right) \sim \begin{cases} 40 & E_{inf} \sim \text{TeV} \\ 60 & E_{inf} \sim \Lambda_{\text{GUT}}\,, \end{cases} \tag{2.19}$$

in order to solve the horizon problem. As during inflation the relative importance of curvature drops as e^{-2N}, by comparison with eq. (2.5) one notices that inflation solves the horizon and flatness problems simultaneously. But, the theory of inflation not only solves these puzzles but it actually *predicts* the precise form of the CMB power spectrum. In order to explain this, we need to consider quantum fluctuations around the inflationary background solution. In other words, we consider both metric and scalar perturbations around the FRLW metric and scalar field solution,

$$g_{\mu\nu}(x) = g^0_{\mu\nu}(t) + \delta g_{\mu\nu}(x)\,, \quad \phi(x) = \phi^0(t) + \delta\phi(x)\,. \tag{2.20}$$

This parametrization contains a lot of redundancies that should be eliminated by an appropriate gauge fixing. Intuitively, this goes as follows. First, the value of the scalar

[4] We do not know how baryogenesis worked but typical models require temperatures at least of order the TeV scale to operate. See e.g. [115] for a review.

field[5] serves as a measure of how far inflation has progressed. As such, perturbations $\delta\phi(x)$ can always be gauged away by choice of an appropriately adapted spatial slicing. In other words, $\phi(x)$ is the Goldstone boson of spontaneously broken time translation invariance [116]. It is 'eaten' by the metric via a variant of the Higgs mechanism. Setting $\delta\phi = 0$ amounts to going to unitary gauge. The physical perturbations are then all encoded in the metric,

$$ds^2 = -dt^2 + a(t)^2 e^{2\mathcal{R}(t,x)}(\delta_{ij} + \delta g_{ij}^{TT}(t,x))dx^i dx^j \,, \tag{2.21}$$

where $\mathcal{R}(t,x)$ parametrizes the scalar curvature perturbation, while $\delta g_{ij}^{TT}(t,x)$ encodes the tensor modes.[6] Plugging this into the action and expanding to 2nd order, one obtains a scalar kinetic term

$$S \supset M_P^2 \int d^4x\, a^3\, \frac{\epsilon}{c_s^2}\left(\dot{\mathcal{R}}^2 - \frac{c_s^2}{a^2}(\partial_i \mathcal{R})^2\right) , \tag{2.22}$$

where c_s is the speed of sound which is trivial in slow-roll inflation, $c_s = 1$. Note that the curvature perturbation is *massless*. As ϵ is approximately constant one may absorb all pre-factors into the definition of a canonically normalized field $v(t,x)$ and proceed with standard canonical quantization. As usual, one Fourier expands in terms of spatial harmonics and obtains time dependent mode functions $v_k(t)$ that satisfy the (classical) *Mukhanov-Sasaki* equation [117–119]

$$\ddot{v_k} + 3H\dot{v_k} + \frac{k^2}{a^2}v_k = 0 \,, \tag{2.23}$$

where $\vec{k}$ is a co-moving wave vector.[7] The physical wave vector $\vec{k}/a$ is time dependent due to the spatial expansion. This is a classical damped oscillator equation with (time dependent) frequency $\omega_k^2 = k^2/a^2$, and *Hubble-friction* $3H$. During slow-roll inflation $H \approx const$ while the oscillator frequency drops according to the usual red-shifting. For the short wavelength modes with $k/a \gg H$ the friction term is irrelevant so they will evolve according to the un-damped oscillator equation. In canonical quantization the mode functions are promoted to operators $\hat{v}_k$ and in the oscillator ground state the two-point function is

$$\langle \hat{v}_k \hat{v}_{k'} \rangle = (2\pi)^3 |v_k|^2 \delta^3(k + k') \,, \quad \text{with} \quad |v_k|^2 \equiv a^{-3}\frac{1}{2\omega_k} \,. \tag{2.24}$$

[5] We assume that there is only a single one.
[6] It is trace-less ($g_{ii}^{TT} = 0$) and transverse ($\partial_i g_{ij}^{TT} = 0$).
[7] Here we are neglecting the time dependence of ϵ.

For wave vectors $k/a \gg H$ the scale factor varies very little over the oscillation period of the oscillator so the expansion of the universe does not disturb its ground state (i.e. the ground state evolves *adiabatically*). Due to the inflationary expansion the physical wavelength of a given mode is stretched until *horizon crossing*,

$$k/a = H\,. \tag{2.25}$$

Once the wavelength is stretched to super horizon scales the amplitude is *frozen* at the value it took at horizon crossing,

$$|v_k|^2 = \frac{1}{2}\frac{H^2}{k^3}\,. \tag{2.26}$$

By a change in normalization this corresponds to the power spectrum of the curvature perturbation $\mathcal{R}(t,x)$,

$$\langle \mathcal{R}_k \mathcal{R}_{k'} \rangle = (2\pi)^3 |\mathcal{R}_k|^2 \delta^3(k+k')\,, \quad \text{with} \quad |\mathcal{R}_k|^2 = \frac{1}{4k^3}\frac{H^4}{M_P^2|\dot{H}|}\,. \tag{2.27}$$

The dimensionless power spectrum is conveniently defined as

$$\Delta^2_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}|\mathcal{R}_k|^2 = \frac{1}{8\pi^2}\frac{H^4}{M_P^2|\dot{H}|} = \frac{1}{8\pi^2}\frac{H^2}{M_P^2}\frac{1}{\epsilon}\,. \tag{2.28}$$

In slow roll inflation, all quantities on the r.h. side change slowly over an e-fold of inflation so the power spectrum is roughly *scale-invariant*. It is easy to quantify the departure from exact scale invariance to leading order in the slow roll approximation from

$$n_s - 1 \equiv \frac{\partial \log \Delta^2_{\mathcal{R}}}{\partial \log k} = -2\epsilon - \eta + ... = 2\eta_V - 6\epsilon_V + ...\,, \tag{2.29}$$

where $n_s \approx 1$ is the *spectral tilt*, and we have expressed the r.h. side in terms of the potential slow roll parameters (ϵ_V, η_V). Crucially, after reheating the curvature perturbation translated into the tiny temperature perturbations that we observe in the CMB, and served as the seeds for structure formation. Thus, we actually know it very well (see Figure 2.1)! It is indeed very well described by an almost scale invariant spectrum with

$$\Delta^2_{\mathcal{R}}(k_*) = (2.10 \pm 0.03) \times 10^{-9}\,, \quad n_s = 0.965 \pm 0.004\,, \tag{2.30}$$

at 68% confidence [27]. Here, k_* is a representative scale (called the Pivot scale) that is accessed by the Planck satellite. Note that the angular power spectrum does not

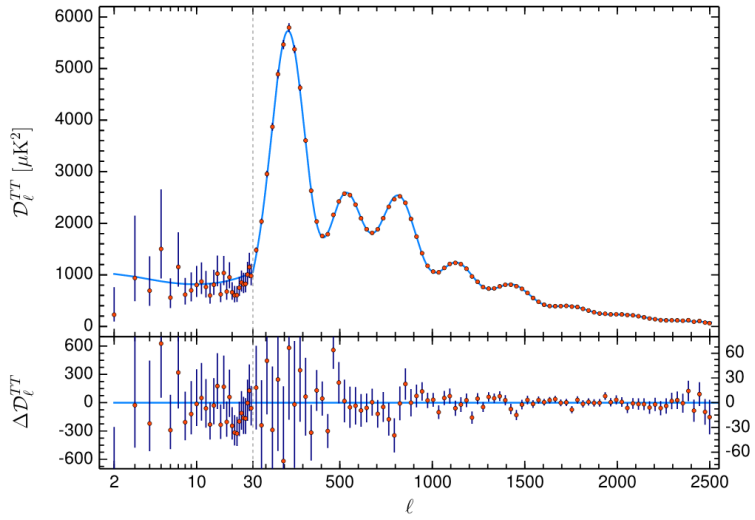

Figure 2.1: The angular power spectrum of the CMB. One notices an overall small tilt, distorted by acoustic peaks. The blue curve is the theoretical prediction while the red dots are the measured data points with tiny error bars, and yet remarkably compatible with theory.

look scale invariant at all. The peaks that distort the otherwise almost scale invariant spectrum are actually predicted by inflation [120]: After the end of inflation the Hubble sphere started to grow again and one after the other modes that had left the horizon during the inflationary expansion reentered the horizon and started to oscillate. As all modes with the same magnitude of the co-moving wave vector started oscillating at the same time they lead to *coherent* oscillations in the baryon photon plasma of the early universe. This effect is *predicted* to lead precisely to the peaks that we observe which is the perhaps greatest triumph of the theory of inflation. We should emphasize that with current CMB measurements we really only probe about 2500 multipole moments. In other words we observe the earliest about $\log(2500) \approx 8$ e-folds of inflation of the ones we can observe in principle (say, the last $40 - 60$ e-folds). This is due to the fact that modes that had left the horizon earlier have not yet reentered the horizon while the ones that left the horizon later we cannot resolve (yet).

Next, it is a straightforward exercise to obtain the power spectrum of the tensor modes. The result is

$$\Delta_h^2(k) = \frac{2}{\pi^2}\frac{H^2}{M_P^2}\,. \tag{2.31}$$

Interestingly, the tensor mode power spectrum is a direct measure for the inflationary Hubble scale H. Moreover, the relative strength of tensor modes is conveniently written

in terms of the *tensor-to-scalar ratio*

$$r(k) \equiv \frac{\Delta_h^2(k)}{\Delta_{\mathcal{R}}^2(k)} = 16\epsilon\,. \tag{2.32}$$

So far, from measurements of the CMB *polarization*, $r = r(k_*)$ has been bounded to be [121]

$$r \leq 0.06\,, \tag{2.33}$$

at 95% confidence. As the scalar power spectrum has been measured we can express the inflationary Hubble scale and the energy scale of inflation in terms of r,

$$H = 3 \times 10^{-5}\sqrt{\frac{r}{0.1}} M_P, , \quad E_{inf} = 8 \times 10^{-3}\left(\frac{r}{0.1}\right)^{1/4} M_P\,. \tag{2.34}$$

Thus, for tensor modes to be detectable in principle, inflation must have occured at the GUT scale, $E_{inf} \sim \Lambda_{\rm GUT}$, and $H \sim 10^{-5} M_P$ [26]. Assuming single field slow roll inflation it turns out that *if* tensor modes can be detected we learn about the *field range* traversed during the inflationary history. This is due to the famous *Lyth bound* [26] which is easily derived: The field range in Planck units traversed in the last N_* e-folds of inflation is given by

$$\frac{\Delta\phi}{M_P} = \int_0^{N_*} dN\sqrt{2\epsilon} = \int_0^{N_*} dN\sqrt{\frac{r(N)}{8}}\,. \tag{2.35}$$

If a non-vanishing tensor-to-scalar ratio was observed, it would have to satisfy $r \gtrsim 10^{-3}$. Assuming slow-roll only in the observed window of ~ 8 e-folds of inflation, the traversed field distance would be constrained as

$$\frac{\Delta\phi}{M_P} \gtrsim 8\sqrt{\frac{r}{8}} \sim 10^{-1}\,. \tag{2.36}$$

This is an extremely conservative bound as we have only used the e-folds of inflation that we really have observed. This is an important insight:

> ***Lyth bound [26]:*** *Models of single field inflation that predict observable tensor modes in the CMB feature Planckian field excursions traversed during inflation.*

So under what circumstances can the slow roll conditions be satisfied? Broadly speaking there two categories of models. The large field models feature *simple* scalar

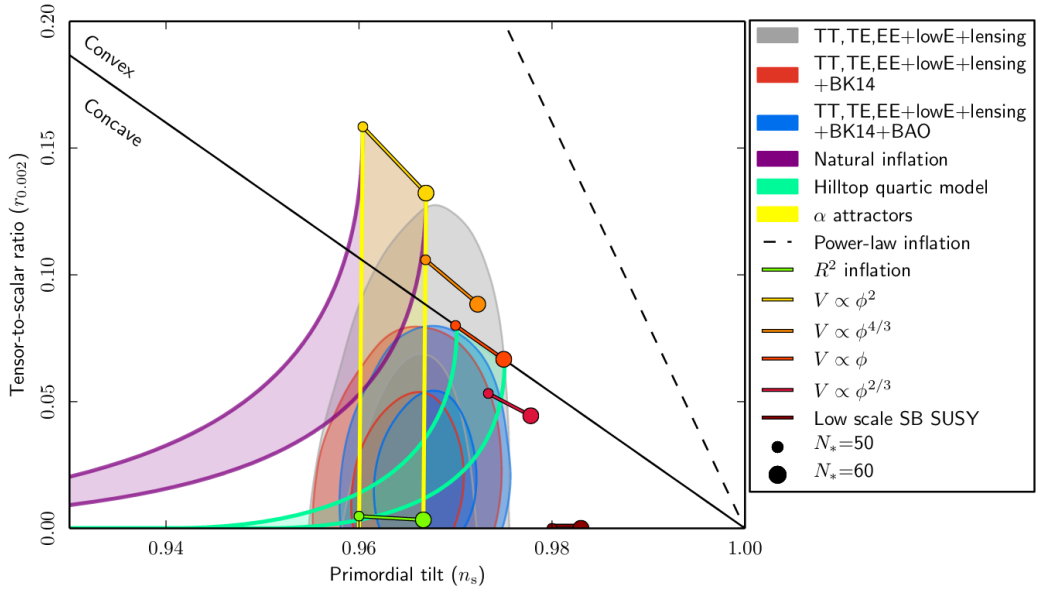

Figure 2.2: Constraints in the $n_s - r$ plane with 1σ and 2σ contours according to combined Planck [27], BICEP+Keck [121] analysis. Figure taken from [27].

potentials. As an example we take the so-called *chaotic inflation* models

$$V(\phi) \overset{\phi \gg M_P}{\longrightarrow} V_0 \cdot \left(\frac{\phi}{M_P}\right)^p , \tag{2.37}$$

for some $\mathcal{O}(1)$ asymptotic power p. Although such potentials do not look very flat, due to the large Hubble friction the slow roll conditions are satisfied at *large field values* $\phi > M_P$,

$$\epsilon_V = \frac{p^2}{2}\left(\frac{M_P}{\phi}\right)^2 , \quad \eta_V = p(p-1)\left(\frac{M_P}{\phi}\right)^2 . \tag{2.38}$$

In order to realize at least 60 e-folds of inflation, the initial field excursion must be bigger than $\sim \mathcal{O}(10)\sqrt{p}$, so these models really feature *super-Planckian* field excursions $\phi \gg M_P$. Moreover they predict

$$n_s - 1 \sim -(2+p)/120 , \quad r \sim p/15 . \tag{2.39}$$

This also means that most of them are ruled out at (more than) 95% confidence by experiment [27, 121] (see Figure 2.2). In other words, although slow roll inflation is remarkably consistent with experiment, the types of potentials that one would naively right down are mostly ruled out experimentally. In contrast, in *small field* models most of the inflationary history occurs near a special point in field space around which the form of the scalar potential has to be finely tuned in order to produce prolonged

inflation. These types of models typically predict unobservable tensor modes. Finally, we would like to highlight again the main point:

Theories that cannot host large field inflation are falsifiable. They can be ruled out by a detection of primordial tensor modes in the CMB.

2.3 Dark energy (the cosmological constant?)

It is the perhaps most shocking experimental result of recent history that a big part of the energy budget of the universe is supplied by *dark energy* with an equation of state compatible with that of a positive cosmological constant [17, 18]. Thus, the expansion of our universe is *accelerating* (again). In fact, the mysterious dark energy that seems to drive this has just about now started to become the dominant contribution to the energy budget of the universe. The simplest form of dark energy in an effective field theory of gravity is a *positive* cosmological constant (cc) Λ. This simply corresponds to a positive scalar potential $V_0 \equiv M_P^2\Lambda$. If dominant, it sources an exponential expansion

$$a(t) \propto \exp(H_0 t)\,, \quad H_0^2 \equiv \Lambda/3\,. \tag{2.40}$$

The FLRW universe with such a scale factor is actually a patch of so-called *de-Sitter space* (dS), which is one of only three *maximally-symmetric* space-times.[8]

Roughly speaking there are two ways to infer the existence of dark energy. Historically, it was first inferred from the distance to red-shift relation of so called type IA supernovae [17, 18] (see Figure 2.3 for the historic data and a modern version). Let us briefly explain how this works: Measuring the red-shift to distance relation of distant objects allows us to reconstruct the evolution of the scale factor in the past which can be compared with theory. However, the distance of a generic source is in general hard to determine. For so called *standard candles* the distance can be determined as a function of spatical curvature because (by definition) they emit their light at a known luminosity.[9] Type IA supernovae are believed to be such standard candles.

[8] The other two options are anti-de-Sitter space (AdS), corresponding to a *negative* cc, and Minkowski space $\mathbb{R}^{1,3}$ with vanishing cc. They are all isotropic in that their isometry algebras contain $so(1,3)$, completed by four additional generators that locally look like translations in space and time.

[9] The energy flux from a distant source is given by

$$F = \frac{L}{4\pi d_l^2}\,, \quad \text{with} \quad d_l = a(t_*)\cdot \begin{cases} \sin(\sqrt{k}d_m)/\sqrt{k} & k>0 \\ r_m & k=0 \\ \sinh(\sqrt{|k|}d_m)/\sqrt{|k|} & k<0\,, \end{cases} \tag{2.41}$$

where L is the luminosity, and the *metric distance* d_m is the distance from the source as measured with the

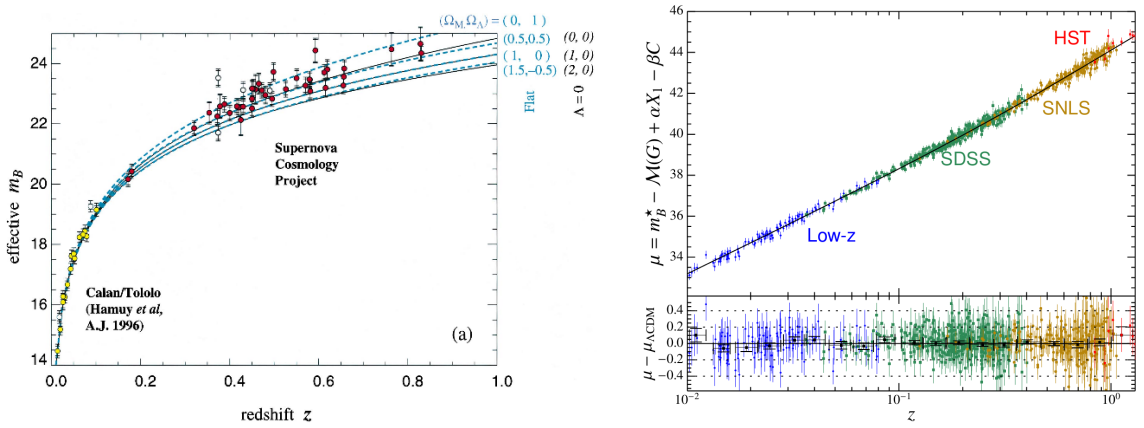

Figure 2.3: Left: The historical distance to red-shift relation of type IA supernovae adopted from Perlmutter et al [17]. The results indicated for the first time a positive cosmological constant. Plotted is the apparent magnitude versus red-shift. Right: Combined modern data as depicted in [122] (on log scale).

Up until 1998 it would have seemed possible that all of the energy budget of the universe is supplied by (negative) spatial curvature. As a sizable amount of curvature is incompatible with the theory of inflation, the detection of dark energy is also a triumph for inflation. Today, the most accurate measurements of dark energy come from CMB measurements combined with galaxy surveys[10] and supernova data. If one is just interested in the value of the cosmological constant, assuming validity of ΛCDM, the CMB gives the answer right away: The relevant scale in the problem is the *sound horizon* within the baryon photon fluid around the time of recombination which is computed from standard model physics. At this time the baryon acoustic oscillations were frozen (because photons decoupled) and imprinted a distortion into both the CMB as well as matter density. Then, from the position of the first baryon acoustic peak in the CMB we learn the angular scale associated to the BAO scale which allows us to conclude that our universe is spatially flat, i.e. $k = 0$. We learn that about 70% of the energy budget must be filled up with whatever is not matter, radiation or curvature, i.e. by assumption a positive cosmological constant. According to Planck 2018 [112],

$$\Omega_{cc} = 0.685 \pm 0.007\,, \tag{2.42}$$

at 68% confidence. Using the information of spatial flatness the supernova data becomes

three-metric $\frac{dr^2}{1-kr^2} + r^2 d\Omega_{S^2}^2$, and d_l is called the *luminosity distance*. The two powers of the scale factor (evaluated at emission time t_*) come from the red-shifting of the photon energy and the red-shifted rate of emission. By measuring F and $a(t_*)$ while knowing L from the theory of supernova explosions one learns about the distance as a function of spatial curvature k.

[10] such as the Sloan Digital Sky Survey (SDSS).

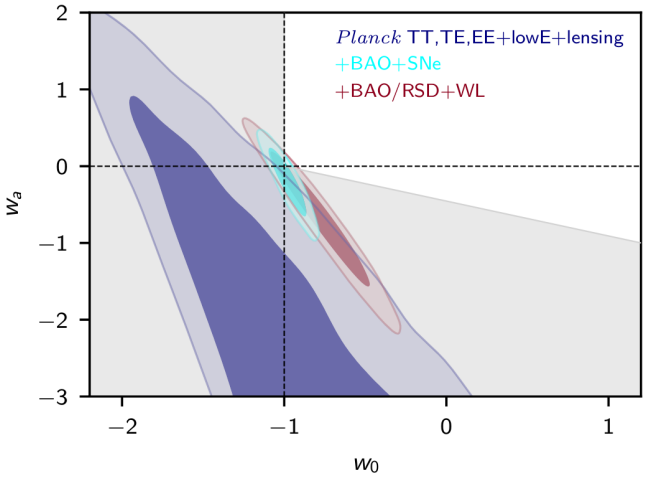

Figure 2.4: Constraints on alternative models of late time expansion using an ansatz $w(t) = w_0 + (1 - a(t))w_a$, as depicted in [112]. CMB Physics alone does not constrain the nature of dark energy very much (purple contours). Only by combining it with BAO and supernovae data (turquoise contours) the tight bounds can be obtained.

a very powerful tool as it can be used to uniquely reconstruct the scale factor. Even more data is supplied by galaxy surveys: The imprint of the scale of BAO can be recovered in sufficiently large samples of galaxies of equal redshift. As this scale is known (it is our so called *standard ruler*) we can determine the physical distance of galaxies of any given redshift, so again we can reconstruct the scale factor.

Putting everything together, alternative models of time-varying dark energy are constrained (see Figure 2.4). For a time independent equation of state parameter $w^{de} = \frac{P^{de}}{\rho^{de}}$ we learn that

$$w = -1.03 \pm 0.03\,, \tag{2.43}$$

at 68% confidence level. This is of course compatible with a cosmological constant (which has $w^{cc} = -1$). It is useful to *define* a cosmological constant as a time independent fluid with equation of state parameter -1. Dark energy in principle gives us another handle on constraining theories of quantum gravity. If de Sitter vacua do not exist in a candidate theory of quantum gravity, the equation of state parameter would have to deviate from -1 in all its solutions. If it can be derived by how much it has to deviate the theory is again falsifiable. How far the bounds have to be tightened of course depends on the precise properties of the candidate theory of quantum gravity.

Chapter 3

String theory and the landscape

The basic idea of string theory is to resolve the point-like 'fundamental' objects of QFT in terms of extended vibrating one-dimensional objects called strings. Historically, it was discovered almost by accident: First introduced as a theory for the strong interaction [123],[1] it was soon realized that it was something profoundly unexpected: A theory of quantum gravity [124]. We will sketch some of its properties now.[2]

Scattering between strings is described by the smooth splitting and joining of strings as depicted in Figure 3.1. As such it is naturally UV-finite due to the delocalized nature of the interaction 'vertices'.[3] Moreover, a propagating massless spin-2 *graviton* is a unavoidably part of the spectrum.

The perhaps most fundamental starting point that we have is a definition of string perturbation theory as a summing prescription over all intermediate worldsheet geometries that connect the in with the out state,

$$\langle \text{out}|\text{in}\rangle \text{"} = \text{"} \int_{\text{in}}^{\text{out}} \frac{\mathcal{D}h\,\mathcal{D}X\cdots}{\text{Vol}(G)} e^{iS[h,X,...]}\,, \tag{3.1}$$

where $X_M(\xi^a)$ ($M = 0, ..., D-1$, $a = 0, 1$) denotes the ambient space embedding of the worldsheet and $h(\xi^a)$ is an auxiliary worldsheet metric that upon classical integrating out becomes the *induced* worldsheet metric, and $S[h, X, ...]$ is a local worldsheet action. G denotes the group of local worldsheet symmetries that have to be modded out. The worldsheet action always contains the bosonic Polyakov action

$$S_P = -\frac{T_s}{2}\int_\Sigma d^2\sigma\sqrt{-h}\,\eta_{MN}h^{ab}\partial_a X^M\partial_b X^N\,. \tag{3.2}$$

[1] A meson looks like a spinning (open) string after all: The endpoints are what we now know to be a quark and anti-quark while the interior of the effective string is the confining flux tube connecting the pair.

[2] For self-contained introductions to string theory, and string phenomenology see e.g. [28, 35, 125].

[3] UV finiteness has been shown explicitly to two-loop order [126].

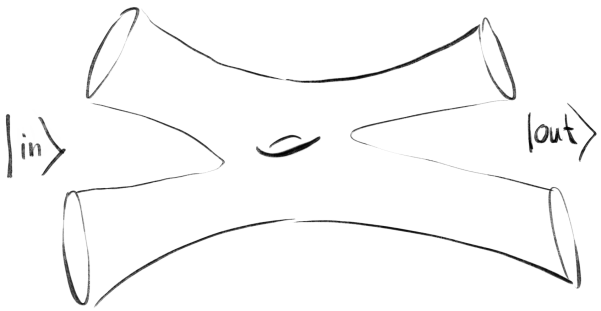

Figure 3.1: A two-to-two string scattering process at one-loop level. The point-like field theory scattering vertex is effectively smeared out which is key to the UV finiteness of string theory.

but also fermionic terms. Here,

$$T_s \equiv \frac{1}{2\pi\alpha'} \equiv \frac{2\pi}{l_s^2}\,, \tag{3.3}$$

is the *string tension*, and l_s is called the *string length*.

The only worldsheet theories we know that give rise to consistent (i.e. tachyon-free) spacetime theories possess some amount of (local) worldsheet supersymmetry. Then, G is a $2d$ (local) superconformal group. Turning this intuitive definition into a well defined one requires gauge fixing à la Faddeev-Popov. After gauge fixing the worldsheet theory becomes a superconformal field theory (SCFT).

Due to the operator state correspondence the stringy in/out states are created via the insertion of vertex operators $\mathcal{V}_i$ so we can define a perturbative expansion of an n-point string scattering amplitude $\mathcal{A}_n$ as

$$\mathcal{A}_n = \sum_{g=0}^{\infty} \left(e^{\phi}\right)^{-\chi} \int (DX\cdots)\,\mathcal{V}_1\cdots\mathcal{V}_n\, e^{-S_g[X,\ldots]}|_{\Sigma=\mathcal{M}_{g,n}}\,, \tag{3.4}$$

where $e^{\phi} \equiv g_s$ is called the string coupling that controls the perturbation theory, $S_g[X,...]$ is the gauge fixed worldsheet action, and at each level in perturbation theory the worldsheet is fixed to be a Riemann surface with g handles, and n holes, equipped with an arbitrarily gauge fixed metric h^0. $\chi = 2-2g-n$ is the Euler number. From the spacetime perspective g is the number of loops.

For consistency of this prescription the CFT should make sense when placed on arbitrary Riemann surfaces. This gives rise to a number of requirements. First, on a generic Riemann surface the conformal symmetry is anomalous (the theory is inconsistent) *unless* the dimension of spacetime is *critical*, $D = 10$. Additional consistency requirements arise at tree-level and at one-loop level: Tree-level corresponds to a CFT

	NS-NS	R-R	Yang-Mills sector
type IIA	G_{MN}, B_{MN}, e^{ϕ}	$(C_1)_M, (C_3)_{MNP}$	-
type IIB	G_{MN}, B_{MN}, e^{ϕ}	$C_0, (C_2)_{MN}, (C_4)_{MNPQ}$	-
type I	G_{MN}, e^{ϕ}	$(C_2)_{MN}$	A_M in vector of SO(32)
heterotic SO(32)	G_{MN}, B_{MN}, e^{ϕ}	-	A_M in vector of SO(32)
heterotic $E_8 \times E_8$	G_{MN}, B_{MN}, e^{ϕ}	-	A_M in adjoint of $E_8 \times E_8$

Figure 3.2: The ten-dimensional bosonic massless spectrum of the five string theories. G_{MN} is the metric, e^{ϕ} is the dilaton, and B_{MN} is a two form gauge field called the Kalb-Ramond or NS two form. The fields C_p are the RR p-form gauge potentials (C_4 is constrained to have self-dual field strength), and A_M denotes the Yang-Mills vector fields.

on the sphere, and the operator product expansion is generically not well-defined due to monodromies that arise when two operators encircle each other. The space-time partition function at one-loop level involves summation over all inequivalent worldsheet tori characterized by its complex structure τ modulo $Sl(2,\mathbb{Z})$. For consistency the result should not depend on our choice of fundamental domain of $Sl(2,\mathbb{Z})$. This requirement is called *modular invariance*.

Finally, there should not exist a spacetime tachyon because otherwise the ten-dimensional spacetime 'vacuum' decays immediately. All these constraints are satisfied if the spectrum is truncated by the so-called Gliozzi-Scherk-Olive (GSO) projection [127]. This leaves us with six ten-dimensional string theories. Two of these are $\mathcal{N} = 2$ supersymmetric in ten dimensions and descend from the same $\mathcal{N} = (1,1)$ worldsheet theory but with different GSO projection. They are called type IIA and type IIB. Another one is obtained by further projecting the type IIB theory and allowing also for *open strings*. It is an $\mathcal{N} = 1$ supersymmetric theory in ten dimensions called the type I theory and contains an open string Yang-Mills sector with gauge group SO(32). Three *heterotic* theories are obtained by starting from a worldsheet theory with $\mathcal{N} = (1,0)$ supersymmetry [128]. These give rise to the two ten-dimensional theories with $\mathcal{N} = 1$ supersymmetry, with gauge groups SO(32) and $E_8 \times E_8$ respectively. Finally there is a non-supersymmetric one with gauge group O(16)×O(16) [33]. From the low energy limit of string scattering amplitudes with only massless in/out states one can deduce the effective $10d$ spacetime actions of the perturbative string theories. These are precisely all five ten-dimensional supergravity theories which go by the same names, and a non-supersymmetric one with large ten-dimensional scalar potential. In Figure 3.2 we have listed the massless bosonic spectrum of the five supersymmetric string theories.

It is important to note that the ten-dimensional theories enjoy (local) p-form gauge symmetries associated with the form potentials B and C_p. The 'fundamental' string is electrically charged with respect to the NS two form B, while at the perturbative level nothing is charged under the RR gauge symmetries.

3.1 String dualities

All ten-dimensional string theories come equipped with a string coupling $g_s = e^{\phi}$ that controls the string loop expansion we started with. It is worthwhile noting that we have not said what we are really expanding but have simply written the prescription to sum over an infinite number of terms. Clearly, this makes sense only when $g_s < 1$. But even at small string coupling the expansion is not convergent just as the perturbative expansion in QFT is at most an *asymptotic* one. So really, we have defined approximation schemes rather than full theories. It is crucial to appreciate that expectation values of the ten-dimensional spacetime fields can be thought of as coherent superpositions of the massless string modes. At least for non-trivial vev's of the NS-NS fields we can even write down a generalization of the Polyakov action (called the non-linear σ-model) that makes this manifest. In particular, the string coupling e^{ϕ} is really the expectation value of the dilaton we have listed in Figure 3.2.

As the string coupling is the vev of a $10d$ field one should be able change its value, and in particular make it large. But what does this mean? This question led to the discovery of string dualities in the 1990s that is now called the second superstring revolution. At the ten-dimensional level there are two strong/weak coupling dualities, so called *S-dualities*. First, the type IIB theory is actually *self-dual* with respect to the duality group $SL(2,\mathbb{Z})$ which contains S-dualities [129]. Second, the type I theory is S-dual to the heterotic SO(32) theory [130]. The strong coupling limits of the $E_8 \times E_8$ heterotic string and the type IIA theory are something much more radical and seem to have nothing to do with strings [130–132]: They can both be thought of as compactifications of an *eleven*-dimensional theory called M-theory that has $11d$ supergravity as its low-energy limit. The heterotic $E_8 \times E_8$ arises via compactification on an interval $S^1/\mathbb{Z}_2$, while the type IIA theory corresponds to an ordinary circle compactification. The strong coupling limits are therefore limits of *decompactification*. In all cases, one should not consider $g_s > 1$ as there exists a dual description with string coupling $\tilde{g}_s \sim 1/g_s < 1$.

More dualities arise by compactifying the ten-dimensional theories to lower dimensions. A circle compactification is sufficient to convince oneself that all five string theories are dual to each other in one way or another: The type IIA theory compactified on a circle of circumference L_{IIA} is *equivalent* to the type IIB theory on a circle of circumference $L_{IIB} \equiv l_s^2/L_{IIA}$ [133, 134]. This is called *T-duality* and translates a type IIA string with KK momentum n_{KK} that winds around the circle w times into a type IIB string with the roles of n_{KK} and w interchanged, i.e.

$$(n_{KK}, w_{\text{winding}})|_{IIA} = (w_{\text{winding}}, n_{KK})|_{IIB}\,. \tag{3.5}$$

So there is really just one nine-dimensional theory rather than two. For small circle radii it is more appropriate to change the duality frame so that the light states have a geometrical meaning rather than a non-local one. This is the first hint that in string theory it really does not make sense to consider geometrical distances smaller than a string length just as it does not make sense to consider large string coupling. In fact, in string theory there are two perturbative expansions, one of which we have hidden so far. On top of the string loop expansion in g_s we also expand in powers of

$$\alpha'/R_k^2\,, \tag{3.6}$$

where R_k is a typical curvature length-scale of the background. Roughly speaking this is because we do not know in general how to determine the effective spacetime action even at lowest order in the string loop expansion. We can do so order by order in the α'-*expansion*. For instance one may expand the non-linear σ model generalization of the Polyakov action

$$S_\sigma = \frac{T_s}{2}\int d^2\sigma\sqrt{-h}\,G_{MN}(X)h^{ab}\partial_a X^M\partial_b X^N + iT_s\int B(X) + \frac{\phi(X)}{4\pi}\int d^2\sigma\sqrt{-h}\,\mathcal{R}(h)\,, \tag{3.7}$$

around the point $X=0$ by choosing Riemann normal coordinates such that

$$G_{MN} = \eta_{MN} - \frac{1}{3}R_{MPNQ}|_{X=0}X^P X^Q + \dots\,. \tag{3.8}$$

The terms in the expansion give rise to interaction terms on the worldsheet that generically have non-vanishing β functions. These can be evaluated order by order in worldsheet perturbation theory. The dimensionless coupling constant in this case is $\alpha' R_{MNPQ}$, i.e. curvature in string units. For conformal invariance of the worldsheet theory all β functions are required to vanish which leads to the *spacetime* equations of motion, and hence the effective spacetime action. So the α' expansion is related to the perturbative expansion of the worldsheet CFT. Thus, when curvatures are of order the string scale, the worldsheet sigma model is strongly coupled. This is by definition the regime where the α' expansion breaks down. For the simple circle compactification of the type II string we had at our disposal *two* manifestly equivalent and weakly coupled (even free) worldsheet descriptions, one for each T-duality frame. This luxury is lost once we consider compactification manifolds with non-trivial Riemann tensor: At small compactification volumes of (say) the type IIA string, we do not know how to make predictions using the type IIA theory but rather should expect that a weakly coupled type IIB description takes over. For a class of compactifications called Calabi-Yau (CY) manifolds this phenomenon is rather well understood and is called *mirror symmetry*

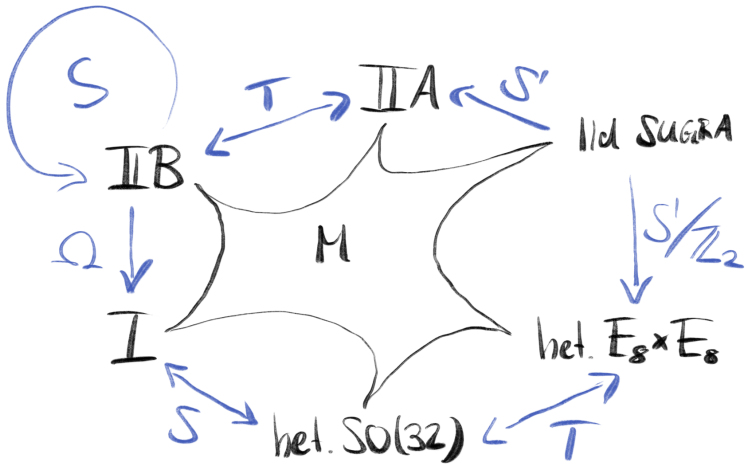

Figure 3.3: The M-theory star. Type IIB is $Sl(2,\mathbb{Z})$ self-dual, and T-dual to type IIA. Type I arises via an orientifolding of type IIB and is S-dual to the SO(32) heterotic string, which in turn is T-dual to the $E_8 \times E_8$ heterotic string. Its low-energy limit can be viewed as the compactification of 11d supergravity on an interval. Circle compactification of the same, gives the low energy limit of type IIA.

[135, 136].

Similarly, T-duality connects also the two heterotic theories upon compactification on a circle (and turning on Wilson lines that break both gauge groups down to $SO(16)\times SO(16)$) [137, 138]. These duality transformations are commonly expressed via the *M-theory star* (see Figure 3.3). It is clear that we should think about the five string theories as different weak coupling limits of one mother (M-)theory that is yet to be defined.

Finally, there exist non-perturbative (in g_s) $(p+1)$-dimensional *membranes* (solitonic objects) called Dp-branes which are the electric and magnetic charges associated with the p-form gauge symmetries. At the level of perturbative string theory these are the rigid objects on which open strings can end.[4] Crucially, one may stack N of these objects on top of each other to obtain non-abelian gauge sectors that live on the stacks. Loosely speaking this is because there are N^2 possibilities to let the two ends of an open string end on pairs of branes, and each possibility gives rise to one of the vectors of the adjoint representation of $U(N)$. With intersecting branes one even obtains bi-fundamental matter localized on the intersection locus. In type IIA there are Dp branes with p even while in type IIB p is odd. In the type I theory, there are only $D1$ and $D5$ branes, while in the heterotic theories there are no Dp branes.[5] It was famously shown by J. Polchinski that these must be viewed as dynamical gravitating objects in

[4] Their tension blows up as $g_s \longrightarrow 0$.

[5] Note that this matches perfectly with the spectrum of RR form potentials in the five theories.

their own right [34]. To lowest order in the α' expansion the effective action of a single p-brane is

$$S_p = -\frac{2\pi}{l_s^{p+1}} \int d^{p+1}x\, e^{-\phi} \sqrt{-(g_{mn} + 2\pi\alpha' \mathcal{F}_{mn})} + \frac{2\pi}{l_s^{p+1}} \int C_{p+1}\,, \tag{3.9}$$

where $2\pi\alpha'\mathcal{F}_{mn} \equiv B_{mn} + 2\pi\alpha' F_{mn}$, and (g_{mn}, B_{mn}) are the pullback of the ten-dimensional metric and NS two-form onto the brane, and $F = dA$ is the gauge field strength that lives on the brane. The second term is called the Chern-Simons (CS) term and encodes the fact that Dp branes are electric/magnetic charges with respect to the RR form gauge symmetries.[6] When placed on non-trivial backgrounds Dp branes also carry induced charges with respect to C_{q+1} with $q < p$, due to α'-corrections of the CS term [139, 140].

3.2 The $10d$ supergravity approximation

In this thesis we will focus on the type IIB corner of string theory, so let us consider its ten-dimensional supergravity approximation. The low energy effective action (in Einstein frame) reads[7]

$$\begin{aligned} S_{IIB} =& \frac{2\pi}{l_s^8} \int_{M_{10}} d^{10}x \sqrt{-G} \left(R - \frac{1}{2} \left| \frac{\partial \tau}{\mathrm{Im}(\tau)} \right|^2 - \frac{|G_3|^2}{2\mathrm{Im}(\tau)} - \frac{1}{4}|F_5|^2 \right) \\ & - \frac{2\pi}{l_s^8} \int_{M_{10}} C_4 \wedge \frac{iG_3 \wedge \overline{G_3}}{2\mathrm{Im}(\tau)} + \text{fermions}\,, \end{aligned} \tag{3.10}$$

to lowest order in the α' expansion. Here, G_3 is the complex three-form $G_3 \equiv F_3 - \tau H_3$, and τ is the axio dilaton $\tau \equiv C_0 + ie^{-\phi}$. The real field-strengths are defined in terms of the gauge potentials as

$$H_3 = dB_2\,, \quad F_3 = dC_2\,, \quad F_5 = dC_4 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3\,. \tag{3.11}$$

Finally, the equations of motion have to be supplemented by the self-duality constraint $F_5 = *F_5$. The theory enjoys $\mathcal{N} = (2,0)$ (local and chiral) supersymmetry, and p-form

[6] For $p > 4$ the coupling is understood as a *magnetic* charge w.r.t. C_{7-p}.

[7] Note that the mass scale $1/l_s$ as measured with the Einstein frame metric G_{MN} corresponds to the ten-dimensional Planck scale, and *not* to the string scale.

gauge invariance

$$B_2 \longrightarrow B_2 + d\Lambda_1^{NS}\,, \quad C_2 \longrightarrow C_2 + d\Lambda_1^{RR}\,,$$
$$C_4 \longrightarrow C_4 + d\Lambda_3^{RR} + \frac{1}{2}\Lambda_1^{RR}\wedge H_3 - \frac{1}{2}\Lambda_1^{NS}\wedge F_3\,. \tag{3.12}$$

Moreover, at the classical level it is invariant under the *global* symmetry $Sl(2,\mathbb{R})$,

$$\begin{pmatrix} B_2 \\ C_2 \end{pmatrix} \longrightarrow \Lambda \begin{pmatrix} B_2 \\ C_2 \end{pmatrix}\,, \quad \tau \longrightarrow \frac{a\tau + b}{c\tau + d}\,, \quad \text{with} \quad \Lambda = \begin{pmatrix} d & c \\ b & a \end{pmatrix} \in Sl(2,\mathbb{R})\,. \tag{3.13}$$

At the quantum level, $Sl(2,\mathbb{R})$ must be broken to $Sl(2,\mathbb{Z})$ as can be seen as follows: Consider for simplicity a ten-dimensional manifold $T^2\times S^1\times M_7$ where M_7 is an arbitrary Spin(7) manifold. We may choose a background with non-vanishing field strength

$$G_3 = (M - \tau K) l_s^2\, \frac{d\phi'\wedge d\phi''}{(2\pi)^2}\wedge\frac{d\phi}{2\pi}\,, \tag{3.14}$$

where ϕ is the S^1 angle and (ϕ',ϕ'') are the T^2 angles. This is consistent with the classical equations of motion and Bianchi identities of the three-forms, and integrates to $l_s^2(M-\tau K)$ over $T^2\times S^1$. The real numbers (M,K) denote the *fluxes* on the *three-cycle* $T^2\times S^1$. Now we can consider wrapping a fundamental string (or a $D1$) world sheet on the T^2 and place at a point in $S^1\times M_7$. We can ask whether the path integral of the worldsheet theory is well-defined on such a background, in particular if it is a single valued function of the S^1-position. Due to the CS term in the worldsheet action the path integral picks up a phase $e^{2\pi i K}$ as $\phi\longrightarrow\phi+2\pi$, so it is well-defined if $K\in\mathbb{Z}$. The same is true for the RR flux number M.[8] This argument generalizes to the case of RR and NS fluxes on general *cycles*, and is really just the Dirac quantization condition of monopole charges.

But a generic $Sl(2,\mathbb{R})$ transformation takes a consistently quantized pair of flux numbers to an inconsistent one! Thus, at the quantum level $Sl(2,\mathbb{R})$ must be broken (at least) down to $Sl(2,\mathbb{Z})$. As $Sl(2,\mathbb{Z})$ is generated by $T=\begin{pmatrix}1&0\\1&1\end{pmatrix}$ and $S=\begin{pmatrix}0&1\\-1&0\end{pmatrix}$, we see that for $C_0=0$, the S-transformation is a strong/weak coupling transformation that sends g_s to g_s^{-1}. As a consequence we should really view $Sl(2,\mathbb{Z})$ as a discrete *gauged* symmetry group, or a *duality group* [142]. Two field configurations related to one another by such a transformation are just different descriptions of the same physics.

[8] This argument does not always go through. In $8k+3$ dimensions the fermion measure in the path integral sometimes has a sign ambiguity that must be canceled by $\mathbb{Z}+1/2$ valued flux quanta [141]. But whenever the configuration without any fluxes is consistent, the fluxes have to be integers.

It is interesting to note that the theory contains an *axion* C_0 which transforms like a Goldstone boson under the one-parameter subgroup $\begin{pmatrix} 1 & 0 \\ c & 1 \end{pmatrix} \subset Sl(2,\mathbb{R})$. This is our first encounter with a *shift symmetry*. Another way to see that $Sl(2,\mathbb{R})$ is broken to $Sl(2,\mathbb{Z})$ is to note that there exists a $D(-1)$ brane which is an instanton. In the euclidean path integral it contributes as $e^{-S_{-1}} = e^{2\pi i\tau}$ which breaks the axion shift symmetry down to a discrete one $C_0 \longrightarrow C_0 + 1$. Viewing it as a gauge symmetry is necessary because in type IIB there exist $D7$ branes which are magnetically charged under C_0. This means that upon encircling it, the field configuration undergoes a monodromy $M = T$. If $Sl(2,\mathbb{Z})$ were not gauged, a $D7$ brane would have no right to exist! At the non-perturbative level, this means that not only must there exist fundamental strings and D-branes but there must exist (p,q) seven-branes five-branes and strings. These are simply all the objects that are generated via $Sl(2,\mathbb{Z})$ acting on the 'known' objects. This includes the $(0,1)$ five-brane, the NS5 brane, that so far we have swept under the rug.[9] It is the object magnetically charged under B_2. The $D3$ brane and the $D9$ brane are the only ones that come in just one version.

Finally it will be useful to keep in mind that the *magnetically* charged objects under B_2/C_2, the NS5 and $D5$ branes, can play the role of *domain walls* in effective four-dimensional theories. Namely, consider $M_{10} = \mathbb{R}^{1,3} \times T^3_A \times T^3_B$ with a $D5$ brane wrapped on T^3_A and spatially extended in the (x^1, x^2) plane in $\mathbb{R}^3$. The Bianchi identity of C_2 reads

$$dF_3 = l_s^2 \delta(x_3 - x_3^0) dx^3 \wedge \delta^3(\phi_B - \phi_B^0) d\phi_1^B \wedge d\phi_2^B \wedge d\phi_3^B \,, \tag{3.15}$$

so the RR flux number associated with T^3_B changes by one unit across the $4d$ domain wall. In general the D5 wrapped on a three-cycle is a domain wall across which the flux quantum of F_3 on the *dual* three-cycle changes by one.[10]

3.3 Calabi-Yau compactifications

We have repeatedly stated that string theory is ten-dimensional, while we have never observed anything but four dimensions of spacetime. As string theory is a gravitational theory, this is no need to worry. One may simply try to find non-trivial backgrounds with $10d$ spacetimes that factor as $M_{10} = M_4 \times M_6$, i.e. into a four-dimensional one M_4 (say a FRWL universe), times a (possibly time dependent) internal six-dimensional space M_6. The simplest such solutions are torus compactifications $M_{10} = \mathbb{R}^{1,3} \times T^6$,

[9] There is also an NS5 brane in the heterotic theories.

[10] There is a natural duality map between homology classes of p-cycles and $d-p$ cycles due to the non-degenerate pairing giving by the intersection map $H_p(X,\mathbb{Z}) \times H_{d-p}(X,\mathbb{Z}) \longrightarrow \mathbb{Z}$, where d is the dimension of the compact manifold, see e.g. chapter three of [143].

with neither field strengths nor curvature. If we consider energies far below the inverse torus size, all physics is four-dimensional. While this suffices to establish the existence of four-dimensional string vacua, these certainly have almost nothing to do with the world we live in. Toroidal compactification preserves *all* the 32 supercharges we started with, so the low energy effective theory is $\mathcal{N} = 8$ supergravity. In order to reduce the amount of supersymmetry we should try to find less trivial solutions. As a first step, we should find solutions with non-trivial Riemann tensor, that nevertheless solve Einsteins vacuum equations $R_{MN} = 0$. It is actually surprisingly hard to find metrics that do, which are not tori (in fact no analytical metric is known to date). However, a lot of progress can be made with an indirect approach. Namely one can try to find manifolds that are known to *admit* a Ricci-flat metric, although the explicit metric remains elusive.[11] Such examples are indeed known and they go by the name of Calabi-Yau (CY) threefolds. First, such manifolds are of course orientable Riemannian manifolds. They are also *complex* which means that in each coordinate patch U_α one can find three complex coordinates $z^1_{(\alpha)}, z^2_{(\alpha)}, z^3_{(\alpha)}$ that parametrize the patch such that on all overlaps $U_\alpha \cap U_\beta$ the transition functions are holomorphic. Moreover, they come equipped with a *hermitian* metric $g_{\alpha\bar{\beta}}$ which means that the line element can be brought to the form

$$ds^2 = 2g_{a\bar{b}}dz^a d\bar{z}^{\bar{b}}\,. \tag{3.16}$$

From this we can define the *Kähler form* J as

$$J \equiv -2ig_{a\bar{b}}dz^a \wedge d\bar{z}^{\bar{b}}\,, \tag{3.17}$$

which is everywhere non-vanishing. If this form is closed, $dJ = 0$, we call our manifold a *Kähler manifold*, and locally the metric is determined from a real *Kähler potential*[12],

$$g_{a\bar{b}} = \partial_a \bar{\partial}_{\bar{b}} K(z, \bar{z})\,. \tag{3.18}$$

Kähler manifolds have many nice properties[13], in particular their curvature form is a $(1,1)$-form valued map $TX \times \overline{TX} \longrightarrow TX \times \overline{TX}$ that does not mix holomorphic with anti-holomorphic components. Therefore, the *holonomy group* of Kähler manifolds is contained in $U(n) \subset SO(2n)$.

Now we wanted to find solutions with vanishing Ricci tensor, or in other words

[11] See however ref. [144] for recent progress.

[12] A Kähler potential is *not* a function on the manifold. On overlaps of different patches it transforms according to $K \longrightarrow K + f(z) + \bar{f}(\bar{z})$.

[13] For a nice introduction to complex geometry see chapter 13 of [145], and for many more details especially in the context of CY manifolds see [143].

globally defined metrics with trace-less curvature form

$$\mathrm{Tr}\,(R) = R_{ij}dx^i \wedge dx^j = -2iR_{a\bar{b}}dz^a \wedge d\bar{z}^{\bar{b}} = 0\,. \tag{3.19}$$

A necessary topological condition for this to be possible is that $\mathrm{Tr}\,(R)$ vanishes in cohomology, or in other words the *first Chern class* of the holomorphic tangent bundle[14] TX vanishes in cohomology, $[c_1(X)] = 0$. CY manifolds are Kähler manifolds that satisfy this:

Definition: *A Calabi-Yau threefold X is a complex three-dimensional Kähler manifold with vanishing first Chern class, $[c_1(X)] = 0$.*

Given such a manifold with some metric it does not take too much fantasy to imagine that under continuous changes of the metric a Ricci flat metric can be obtained. At least there seems to be no topological obstruction against it. The famous Calabi conjecture [146], proven by Yau [147], asserts that this intuition is correct:

Theorem (Calabi-Yau): *Given a Kähler manifold X with vanishing first Chern class and Kähler form J', there exists a unique Kähler form J in the same cohomology class, $[J] = [J']$, such that the associated hermitian metric is Ricci flat.*

It is important to note that a CY manifold equipped with its Ricci flat metric has holonomy group contained in $SU(n) \subset U(n)$ because the trace of the curvature form vanishes. This means that we can globally define a covariantly constant spinor,[15]

$$\nabla\eta = 0\,. \tag{3.20}$$

The relevance of this is that when we compactify (say) type IIB supergravity on a CY three-fold the background is left invariant under SUSY transformations

$$\epsilon_{1,2}^{10d} = (P_L\epsilon_{1,2}^{4d}) \otimes (P_L\eta) + c.c.\,, \tag{3.21}$$

for two arbitrary four-dimensional Majorana spinors $\epsilon_{1,2}^{4d}$. So, eight real supercharges are preserved corresponding to $\mathcal{N} = 2$ (local) supersymmetry in four dimensions. Moreover, the spinor can be used to construct an everywhere non-vanishing closed $(3,0)$ three-form,

$$\Omega_{ijk} \equiv \eta^T\Gamma_{ijk}\eta\,, \quad d\Omega = 0. \tag{3.22}$$

[14] For a holomorphic vector bundle E with connection A, the *total Chern class* is defined as $c(E) = \det\left(1 + \frac{iF}{2\pi}\right) \equiv 1 + \sum_{i=1}^{n} c_i(E)$, understood as a formal sum over forms of different degree.

[15] A spinor transforms in the 4 of $SU(4) = \mathrm{Spin}(6)$. To construct the covariantly constant spinor simply start at some point p and identify the unique one-dimensional subspace that is left invariant upon parallel transport around any loop, i.e. under $SU(3) \subset SU(4)$. Then, extend the definition of this to the whole threefold via parallel transport.

The existence of this form of course does not depend on whether we choose the Ricci flat metric or not, but only on $c_1(\Lambda^3 T^*X) \propto c_1(TX) = 0$. In other words, CY manifolds are Kähler manifolds with trivial bundle of holomorphic three-forms (the *canonical bundle*).[16] It is useful to ask that the holonomy group actually *is* $SU(3)$ in order to exclude products of lower-dimensional CYs.[17]

Unfortunately Yau's proof of the Calabi-conjecture does not teach us how to find the Ricci-flat metric. This is not as bad as it sounds. Using the tools of algebraic geometry, many physical properties of CY compactifications can be inferred even without knowing the explicit metric. Here is an example called the *quintic threefold*,

$$Q(P_5) \equiv \left\{ [X_0 : X_1 : X_2 : X_3 : X_4] \in \mathbb{P}^4 : \; P_5(X_i) = 0 \right\} , \tag{3.23}$$

where P_5 is a degree five homogeneous polynomial in the homogeneous $\mathbb{P}^4$ coordinates. Its first Chern class vanishes because we can cover $\mathbb{P}^4$ with five patches $U_\alpha = \mathbb{P}^4 \backslash \{X_\alpha = 0\}$, $\alpha = 0, ..., 4$, and in each of these we can choose local coordinates by setting $X_\alpha = 1$ and solving one further coordinate as a function of the three remaining ones. In U_0 we can define a closed everywhere non-vanishing three-form[18]

$$\Omega = \frac{dX_1 \wedge dX_2 \wedge dX_3}{\partial P_5 / \partial X_4} , \tag{3.24}$$

and analogously in the other patches. It is simple to show that these definitions match on the overlaps, and the definition is non-singular provided the embedding is non-singular, i.e. $\{P_5 = 0\} \cap \{dP_5 = 0\} = \emptyset$. Thus the quintic has trivial canonical bundle and is therefore CY.

Note that we have not said what is the polynomial P_5. A parametrization contains 126 complex parameters. Different choices give rise to different *complex structures* unless they are related to each other via linear redefinitions of the $\mathbb{P}^4$ coordinates. This gives us a $101 = 126 - 25$ dimensional *complex structure moduli space*. Moreover, there is a canonical metric on $\mathbb{P}^4$ called the *Fubini-Study* metric,

$$J_{FS} = t \frac{i}{2} \frac{|X|^2 (dX \wedge \overline{dX}) - (\overline{X} dX) \wedge (X \overline{dX})}{|X|^4} , \tag{3.25}$$

for some real parameter t. Its restriction to the quintic gives us the Kähler class of

[16] Strictly speaking the two statements are equivalent only for simply connected CYs. We shall only consider those that are.

[17] There are only two such manifolds up to continuous deformations in the complex structure, $T^2 \times K3$ where $K3$ is the unique CY two-fold and T^6. These preserve twice (four times) as many supercharges in four dimensions.

[18] The definition is asymmetric in the X_i but it is easy to convince oneself that changing the rols of the X_i gives the same form.

the quintic parameterized by a single real parameter t which measures the physical size of the CY. We say, the *Kähler moduli space* is real one-dimensional. In general the Calabi-Yau moduli space factorizes as $\mathcal{M}_{\text{complex structure}} \times \mathcal{M}_{\text{Kähler}}$. Its tangent space is the space of deformations $\delta g_{ij}(x)$ of the metric that leave the metric Ricci flat modulo coordinate reparametrizations. Metric perturbations with mixed indices $\delta g_{a\bar{b}}$ leave the metric hermitian. Therefore they can be thought of as a perturbation of the Kähler form $J \longrightarrow J + \delta J$ which must remain closed. This changes the Kähler class unless δJ is exact, but the new metric will in general not be Ricci-flat anymore. However, the CY theorem guarantees that there exists a unique exact form that can be added to make it Ricci flat again (namely such that δJ is harmonic). So metric perturbations of this type are counted by the dimension of the *Dolbeault* cohomology group $H^{1,1}_{\bar{\partial}}(X)$ where

$$H^{p,q}_{\bar{\partial}}(X) \equiv \frac{\bar{\partial} - \text{closed } (p,q)\text{-forms}}{\bar{\partial} - \text{exact ones}} \,. \tag{3.26}$$

The real dim $H^{1,1} \equiv h^{1,1}$-dimensional moduli space of such deformations is called *Kähler moduli space*. In contrast *complex structure moduli space* corresponds to pure index metric perturbations $\delta g_{ab} = (\delta g_{\bar{a}\bar{b}})^*$. These satisfy the linearized Einstein equations if the $(2,1)$ form

$$\chi_{\delta g} = \delta g_{\bar{a}\bar{d}} g^{\bar{d}e} \Omega_{ebc} d\bar{z}^{\bar{a}} \wedge dz^b \wedge dz^c \,, \tag{3.27}$$

is harmonic. Thus, the number of such deformations is counted by $h^{2,1}$, the dimension of $H^{2,1}$ (in general, $h^{p,q} \equiv \dim H^{p,q}$). To bring back the metric to hermitian form requires a change in complex structure, thus the name. The *Hodge numbers* $(h^{1,1}, h^{2,1})$ are the only independent ones[19] and the quintic has $(h^{1,1}, h^{2,1}) = (1, 101)$.

3.3.1 Type IIB on CY threefolds

Let us now consider compactifying type IIB supergravity on a Calabi-Yau three-fold. First, from the $10d$ metric we obtain the $4d$ graviton plus scalar fields associated with the geometric deformation moduli that parametrize the Kähler class and complex structure of the CY. Second, the massless spectrum contains the axio dilaton $\tau = C_0 + ie^{-\phi}$. But it also contains various *axions* from the reduction of the p form fields B_2, C_2, C_4. It is useful to adopt a basis of dual two and four forms $(\omega^i, \tilde{\omega}_j)$, $i, j = 1, ..., h^{1,1}$, as well as three forms (α^a, β_b), $a, b = 1, ..., h^{2,1} + 1$ that satisfy

$$\int_X \omega^i \wedge \tilde{\omega}_j = \delta^j_i \,, \quad \int_X \alpha^a \wedge \beta_b = \int_{\mathcal{A}^b} \alpha^a = -\int_{\mathcal{B}_a} \beta_b = \delta^a_b \,, \tag{3.28}$$

[19] We have $h^{0,1} = 0$ due to simply-connectedness, $h^{3,0} = 1$ because Ω is the only holomorphic three-form, $h^{p,q} = h^{q,p}$ from complex conjugation and $h^{p,q} = h^{3-p,q}$ from wedging/contracting with Ω and $\bar{\Omega}$.

	bosonic components	multiplicity
gravity multiplet	$g_{\mu\nu}$, A_Ω	1
universal hypermultiplet	$\tau = C_0 + ie^{-\phi}, b_0, c_0$	1
Kähler hypermultiplets	t_i, ρ^i, b_i, c_i	$h^{1,1}$
complex structure vector multiplets	Z_a, A_a	$h^{2,1}$

Table 3.1: The bosonic components of the $\mathcal{N} = 2$ multiplets of type IIB on a CY threefold.

where three-forms α^a, β_b are the Poincaré dual three-forms to the so-called $\mathcal{B}$ respectively $\mathcal{A}$-cycles $(\mathcal{B}_b, \mathcal{A}^a)$ that form a symplectic basis of the third homology group $H_3(X, \mathbb{Z})$. The forms $(\omega^i, \tilde{\omega}_j)$ are Poincaré dual to an integral basis of four and two-cycles $(\tilde{\Sigma}_i, \Sigma^j)$ in $H_4(X, \mathbb{Z})$ respectively $H_2(X, \mathbb{Z})$.

At linear order the p-form fields satisfy equations of motion

$$\left(\partial^2 + \Delta_{\bar{\partial}}\right) \{B_2, C_2, C_4\} = 0\,, \tag{3.29}$$

where ∂^2 is the four-dimensional Laplacian and $\Delta_{\bar{\partial}}$ is the six-dimensional one. Therefore, the massless $4d$ fields appear as coefficients in the expansion of form fields in our basis of harmonic forms,

$$B_2 = \mathcal{B}_2(x) + \sum_{i=1}^{h^{1,1}} b_i(x)\omega^i\,, \quad C_2 = \mathcal{C}_2(x) + \sum_{i=1}^{h^{1,1}} c_i(x)\omega^i \tag{3.30}$$

$$C_4 = \sum_{i=1}^{h^{1,1}} \left(\rho^i(x)\tilde{\omega}_i + \rho'_i(x)\omega^i\right) + \sum_{a=1}^{h^{2,1}+1} \left(A_a(x) \wedge \alpha^a + A'^b(x) \wedge \beta_b\right)\,. \tag{3.31}$$

From C_2 and B_2 we get two $4d$ two-forms $\mathcal{B}_2$ and $\mathcal{C}_2$ that can be dualized to two axions (b_0, c_0). They pair with the axio-dilaton into the *universal hypermultiplet*. Furthermore, there are $2h^{1,1}$ *model-dependent* axions b_i, c_i. Naively, from C_4 we get $2h^{1,1}$ axions corresponding to the ρ^i and the ones dual to the two-forms ρ'_i. But since the five form field strength is required to be self-dual, the two sets of axions must be identified, and there are in total $h^{1,1}$ independent C_4-axions. The $3h^{1,1}$ axions from C_2, B_2 and C_4 combine with the real Kähler moduli t^i into the $h^{1,1}$ Kähler hypermultiplets. Finally, we have listed $2h^{2,1} + 2$ vectors coming from $H^3 = H^{3,0} \oplus H^{2,1} \oplus H^{1,2} \oplus H^{0,3}$, but again, half of them have to be identified with the electric-magnetic duals of the other half to ensure self-duality of F_5. The vectors A_a associated with $H^{2,1}$ combine with the complex structure moduli into $h^{2,1}$ vector multiplets, and the remaining vector A_Ω from $H^{3,0}$ enters the gravitational multiplet. The bosonic field content is summarized in table 3.1. There is a natural set of $h^{2,1} + 1$ complex projective coordinates Z_a complex

structure moduli space defined via the $\mathcal{A}$-cycle respectively $\mathcal{B}$-cycle periods

$$Z_a \equiv \int_{\mathcal{A}^a} \Omega\,, \quad G^a(Z) \equiv \int_{\mathcal{B}_a} \Omega\,. \tag{3.32}$$

The latter are 'functions' of the former.[20] The canonical metric on complex structure moduli space is computed from the Kähler potential

$$K_{cs} = -\log\left(-i\int_X \Omega\wedge\overline{\Omega}\right) = -\log\left(i\overline{Z}_a G^a + c.c.\right)\,, \tag{3.33}$$

and is called the *Weil-Petersson* metric. It is in fact the *physical* field space metric that one computes by dimensionally reducing the $10d$ Einstein Hilbert term. From the form of the Kähler potential one reads off that complex structure moduli space is indeed a *special Kähler* manifold $\mathcal{M}_{sK}$ with holomorphic pre-potential $\mathcal{F}(Z)$ s.t. $G^a = \partial_{Z_a}\mathcal{F}$.

The hypermultiplets parametrize a real $4(h^{1,1}+1)$-dimensional manifold on which there are *three* linearly independent complex structures J^1, J^2, J^3 that satisfy the quaternion algebra (beware, we mean the complex structure of Kähler moduli space, not of the CY). Roughly speaking these are associated with pairing the real Kähler modulus with either one of the three axions into a complex coordinate, while pairing the remaining two into another one. This identifies the moduli space parameterized by the hypermultiplets as a *quaternionic-Kähler* manifold $\mathcal{M}_{qK}$. At leading order in the α' expansion the physical moduli space factorizes as $\mathcal{M}_{CY} = \mathcal{M}_{sK}\times\mathcal{M}_{qK}$ [148, 149].

3.3.2 The conifold

We have repeatedly stated that no analytical CY metrics are known. This is actually true only for *compact* ones. Sometimes, interesting regions in a compact CY are well approximated by regions of non-compact CYs for which the metric is actually known. This is particularly interesting when the CY is singular (or 'almost' singular) at some locus contained in this region. The perhaps most prominent non-compact CY that serves for this purpose is the so-called *conifold* [150]. In a certain sense it arises near the most *generic* singularities of CY compactifications. In what sense this is generic we would like to explain now.

We will focus on algebraic varieties defined as the vanishing locus of a polynomial P in a toric ambient space such as $\mathbb{P}^4$. This is singular if

$$\{P=0\}\cap\{dP=0\}\neq\emptyset\,. \tag{3.34}$$

[20] We say "functions" because they have branch cuts so are not single valued holomorphic functions of the projective coordinates Z_a.

For a generic polynomial this does not occur, but on complex co-dimension one loci in complex structure moduli space this locus is a set of points. In other words, we need to tune one polynomial coefficient to make the polynomial vanish quadratically as opposed to linearly at a point. Locally such a singular embedding can be brought to the form

$$X_{loc} \approx X_{cf} \equiv \left\{ z \in \mathbb{C}^4 | \sum_{i=1}^{4} (z^i)^2 = 0 \right\} , \tag{3.35}$$

which is called the conifold, a node, or an ordinary double point. If we leave the singular locus in moduli space by a small amount $\epsilon \ll 1$ we obtain the *deformed* (non-singular) conifold as the vanishing locus of

$$P = \sum_{i=1}^{4} (z^i)^2 - \epsilon \, . \tag{3.36}$$

By an appropriate redefinition of the phases of the coordinates we can choose ϵ to be real and positive. This space is actually easy to understand. First, let us note that a global $SO(4) = SU(2) \times SU(2)$ symmetry is manifest from the embedding equation. A further $U(1)_R$ factor that rotates the phases of the z^i is restored in the singular limit. Now, let us split the complex coordinates into real and imaginary part, $z^i = x^i + iy^i$, and define a radial coordinate that measures the 'distance' to the singular point,

$$\rho^2 \equiv \sum_{i=1}^{4} |z^i|^2 = \sum_{i=1}^{4} (x^i)^2 + (y^i)^2 \equiv \vec{x}^2 + \vec{y}^2 \, . \tag{3.37}$$

The definition of our radial coordinate and the vanishing of the defining polynomial then take the form

$$\frac{1}{2}(\rho^2 + \epsilon) = \vec{x}^2 \, , \quad \frac{1}{2}(\rho^2 - \epsilon) = \vec{y}^2 \, , \quad \vec{x} \cdot \vec{y} = 0 \, . \tag{3.38}$$

At fixed radial coordinate, the first equation defines a three-sphere S^3, while the second and third describe a fibration of a two-sphere S^2 over the S^3. This fibration is actually trivial, and the *angular* topology is $S^3 \times S^2$ so long as $\rho^2 > \epsilon$. The angular geometry is best thought of as the coset space $\frac{SU(2) \times SU(2)}{U(1)}$ where the $U(1)$ is a diagonal subgroup of the two $SU(2)$ factors. The global symmetry group is $SU(2) \times SU(2) \times \mathbb{Z}_2$ and in the limit $\epsilon \longrightarrow 0$ another $U(1)_R$ symmetry develops.

Clearly the minimal radius is $\rho^2 = \epsilon$ where the S^2 degenerates (smoothly), while the S^3 stays at finite size. The *deformation parameter* ϵ measures the minimal size of this S^3 at the bottom of the conifold. In fact, we may choose this S^3 as one of the $\mathcal{A}$-cycles and it is straightforward to show that the associated complex structure modulus is

proportional to the deformation parameter ϵ. For future reference we record the Ricci flat metric in the singular limit $\epsilon \longrightarrow 0$,

$$ds^2_{cf} = dr^2 + r^2 ds^2_{T^{1,1}} \,. \tag{3.39}$$

It takes the form of a real cone over the Sasaki-Einstein base $T^{1,1}$, with another radial coordinate defined in terms of the one we defined previously as $r = \sqrt{\frac{3}{2}}\rho^{2/3}$. This metric respects the full global symmetry group $SU(2) \times SU(2) \times U(1)_R \times \mathbb{Z}_2$. Note that since $T^{1,1}$ has a two and a three cycle, it possesses harmonic two and three forms (ω_2, ω_3). These are invariant under the the global symmetry group of the singular conifold.

3.4 Conifold transitions and black hole condensation

The conifold is the simplest playground for one of the most fascinating phenomena of CY compactifications in string theory, the possibility of topology changing transitions between CYs that can be fully described by string theory [76, 77, 151]. First, it is important to appreciate that perturbative string theory on certain singular spaces such as orbifolds makes perfect sense. The extended nature of the string essentially means that it does not 'see' the point-like singularity in space. The conifold is *not* of this type, so naively string theory on the singular conifold does not make sense [151]. Fortunately, *non-perturbative* string theory *does* make sense on the conifold. This was first understood by A. Strominger [151] who considered the behavior of the gauge kinetic function of the vector multiplet associated with the conifold $\mathcal{A}$-cycle,

$$\tau = G'(Z) = \log(Z)/2\pi i + \dots \,, \quad \Longrightarrow \quad 8\pi^2/g^2 = \log(|Z|^{-1}) + \dots \,. \tag{3.40}$$

This expression rings a bell: The gauge coupling of a $U(1)$ gauge theory coupled to n_f charged (Weyl-)fermions and n_{cs} complex scalars runs at one-loop according to

$$\frac{8\pi^2}{g^2(\mu)} = \frac{8\pi^2}{g^2(\mu_{\rm UV})} - b \log\left(\mu_{\rm UV}/\mu\right) \,, \quad b = -\left(\frac{1}{3}n_f + \frac{1}{6}n_{cs}\right) \,. \tag{3.41}$$

Below the matter mass-scale m we can integrate out the charged matter so in the IR the gauge coupling is frozen to

$$8\pi^2/g^2_{\rm IR} = -b \log\left(\mu_{\rm UV}/m\right) + \dots \tag{3.42}$$

In $\mathcal{N} = 2$ gauge theories charged matter comes in hypermultiplets, so for a single one $n_f = n_{cs} = 2$, and $b = -1$ (and the one-loop running is exact in perturbation theory). Thus we reproduce the expression (3.40) if we identify $|Z|$ with the mass scale of a

single hypermultiplet (H_1, H_2). Hypermultiplets indeed receive their masses through a superpotential $W(H, Z) = H_1 Z H_2$, so their mass scales linearly with the Coulomb branch coordinate Z. Whatever this hypermultiplet is, its mass should scale linearly with $|Z|$ and it should be electrically charged. As the gauge field comes from the dimensional reduction of C_4 it must be a $D3$ brane, wrapped over the $\mathcal{A}$-cycle S^3 at the bottom of the conifold so that it has electric charge equal to one.[21] As the volume of the S^3 is set by $|Z|$, the mass of this object really does satisfy

$$m_{D3} = T_{D3}\mathrm{Vol}(S^3) \sim |Z|\, M_P\,, \tag{3.43}$$

where T_{D3} is the $D3$ brane tension. So this picture is remarkably consistent. In an effective field theory with cut-off $\Lambda_{\rm UV}$ we must *integrate in* the charged $D3$-brane hypermultiplet once $|Z| \lesssim \Lambda_{\rm UV}/M_P$, and the physics near the conifold singularity is smoothed out. Deforming the conifold is the physical process of going on the Coulomb branch of the $U(1)$ gauge theory.

But there is more: It is crucial that mathematically the conifold can actually be smoothed in *two* radically different ways that both (locally) maintain the CY condition [150]. One is the deformation that we discussed. Another one is called a *resolution*. We recall that the singular conifold embedding was smooth everywhere except at the origin. So let us rewrite the embedding as

$$T \cdot \begin{pmatrix} U \\ V \end{pmatrix} = 0\,, \quad \text{with} \quad T \equiv \sum_{i=1}^{4} z^i \sigma_i = \begin{pmatrix} z_3 + i z_4 & z_1 - i z_2 \\ z_1 + i z_2 & -z_3 + i z_4 \end{pmatrix}\,, \tag{3.44}$$

and $[U : V]$ are the homogeneous coordinates of a $\mathbb{P}^1$. As by definition of $\mathbb{P}^1$ the two coordinates U, V cannot vanish simultaneously, the matrix T must have a zero eigenvalue so its determinant must vanish. But $\det T = -\sum_{i=1}^{4}(z^i)^2$ so we recover the singular conifold embedding equation. Away from the origin $z^i \neq 0$ we can simply solve for the $\mathbb{P}^1$ coordinates as a function of the z^i so there is nothing new. But *at* the origin $T = 0$ and the $\mathbb{P}^1$ coordinate $[U : V]$ is unconstrained. We have replaced the singular point by a two-sphere ($=\mathbb{P}^1$) and one readily checks that the embedding (3.44) is non-singular (this procedure of replacing a singularity by a $\mathbb{P}^1$ is called a *blow up*). The size of the S^2 is controlled by a real parameter t called the *resolution modulus*. We now have two non-singular geometries, the deformed and the resolved conifold that share a common singular locus in moduli space. For both the explicit CY metric is known. Passing from one branch to the other through the singular locus is called a *conifold transition*. We have described this for the non-compact conifold but compact

[21] $D3$ branes wrapped over the $\mathcal{B}$-cycles carry *magnetic* charge.

CY manifolds are frequently connected with each other through such transitions. Let us give the perhaps canonical example of this phenomenon as described in [76]: We start with a complex five-dimensional toric ambient space $\mathbb{P}^4 \times \mathbb{P}^1$, and define our three-fold X_{res} as the vanishing locus of *two* polynomials, of weight $(4,1)$ respectively $(1,1)$. This space is CY[22] and has $(h^{1,1}, h^{2,1}) = (2, 86)$. Thus both polynomials are linear in the $\mathbb{P}^1$ coordinates so we may write

$$\begin{pmatrix} X & Y \\ P & Q \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = 0\,, \tag{3.45}$$

where $[u : v]$ parametrize $\mathbb{P}^1$, while (X, Y) and (P, Q) are degree four respectively linear polynomials in the $\mathbb{P}^4$ coordinates. By the same argument that we used in the non-compact case,

$$\det \begin{pmatrix} X & Y \\ P & Q \end{pmatrix} = XQ - PY \stackrel{!}{=} 0\,, \tag{3.46}$$

which is a degree five polynomial in the coordinates of $\mathbb{P}^4$. Thus it is a quintic three-fold obtained via the shrinking the volume of the blow-up $\mathbb{P}^1$. The locus where the linear polynomials vanish, $P = Q = 0$, is a $\mathbb{P}^2$ sublocus of $\mathbb{P}^4$. On this sub-locus, the two quartic polynomials vanish simultaneously at $16 = 4 \times 4$ points, if the polynomials are chosen generic. These are conifold points. Thus we have described (mathematically) what it means to pass from the quintic three-fold to a different CY manifold that has 15 fewer complex structure moduli but an *additional* Kähler modulus. This transition through the singular locus begs for a physical interpretation. This was given by B. Greene, D. Morrison and A. Strominger [77]:

As the quintic has 101 complex structure moduli, it is apparent that the singular locus occurs at co-dimension $15 = 101 - 86$ in complex structure moduli space. Hence, there is one homology relation among the 16 vanishing three-cycles γ^i which reads

$$\sum_{i=1}^{16} [\gamma^i] = 0\,. \tag{3.47}$$

But it is intuitively obvious that if there are 16 conifold points in the CY there should also be 16 independent hypermultiplets that become massless at the singular locus in moduli space. As there are only 15 independent $\mathcal{A}$-cycles we are in the situation that the $U(1)^{15}$ gauge theory possesses a *Higgs branch* parametrized by the four real

[22] For manifolds defined as vanishing loci of a set of polynomials within a toric ambient space, one computes the Chern class using the so-called *adjunction formula* (see e.g. [152]). The vanishing of the first Chern class constrains the weights of the polynomials under the toric $\mathbb{C}^*$ scalings. The constraints are satisfied for our examples.

scalar components of a single hypermultiplet. Going to the Higgs branch is seen as *condensation* of the $D3$ brane hypermultiplets. This picture involves some intriguing steps: When the deformation S^3 is large, the wrapped $D3$ brane states are solitons with mass above the KK-scale. For sufficiently small $|Z|$ they enter the four-dimensional theory as isolated light particle states, and on the Higgs branch they condense to make up again a geometric modulus, but now of a *different* geometry than the one we started with on the deformation side.

The 'radial' component of this hypermultiplet is of course our resolution modulus while the other three are the axions defined as integrals of B_2, C_2 over the resolution cycle, as well as C_4 over the dual four cycle that reaches into the bulk CY. Two of the newly acquired axions will play a crucial role in chapter 6.

3.5 Orientifolds of the type II string

While CY compactifications of the type II string are a beautiful subject they come with a phenomenological problem: They describe a world very different from ours. There are no non-abelian gauge sectors, there is $\mathcal{N} = 2$ unbroken supersymmetry, and many exactly massless scalar fields. First, we should reduce the amount of supersymmetry, ideally without loosing too much computational control. String theory offers a way to do this which is called an *orientifold projection* (see e.g. chapters 11 and 12 of [125]). In type IIB string theory, given a manifold that has a discrete $\mathbb{Z}_2$ symmetry group $\mathcal{R}$ with even-dimensional fixed point locus, we are allowed to project the degrees of freedom of our theory onto the sector invariant under

$$\Omega\mathcal{R}(-1)^{F_L}\,, \tag{3.48}$$

where F_L denotes left-moving worldsheet fermion number and Ω is worldsheet parity. This means that the $10d$ fields $\{G_{MN}\,, \tau = C_0 + ie^{-\phi}\,, C_4\}$ are required to be even under the geometric action, e.g. $C_4(\Omega x) = C_4(x)$, while B_2 and C_2 are odd. For a CY three-fold the geometric action should be holomorphic in order to preserve some amount of supersymmetry. At the level of the zero mode spectrum this means that within each $\mathcal{N} = 2$ multiplet half of the components are projected out. The orientifold acts on the cohomology groups and is useful to decompose them into their even and odd eigenspaces $H^{p,q} \equiv H^{p,q}_+ \oplus H^{p,q}_-$. For $O7$ orientifolds the fixed point locus of the orientifold is of complex co-dimension one and the massless spectrum is the following: The $\mathcal{N} = 2$ gravity multiplet loses its vector and the universal hypermultiplet loses two of its axions b_0, c_0. The Kähler hypermultiplets are split into $h^{1,1}_+$ chiral multiplets with bosonic components (t_i, ρ^i) and $h^{1,1}_-$ axion chiral multiplets (b_i, c_i). The $\mathcal{N} = 2$

vector multiplets split into $h^{2,1}_+$ $\mathcal{N}=1$ vector multiplets and $h^{2,1}_-$ complex structure chiral multiplets.

As the $O7$ planes carry negative magnetic charge under C_0 (and negative tensions) we must cancel their charges (and tensions) at least globally by including also a number of $D7$ branes. The simplest case is the one where 4 $D7$ branes lie on top of each $O7$ plane, yielding non-abelian $SO(8)$ gauge groups. This brings us much closer to a realistic theory. Generically the seven-brane stacks also carry induced $D3$ brane charge which must be canceled by introducing mobile $D3$ branes.

Ignoring the open string degrees of freedom the effective $\mathcal{N}=1$ supergravity is well known. We focus on the case $h^{1,1}_+=1$ with only a single Kähler modulus T, but arbitrary $h^{1,1}_-$. The Kähler potential reads [153]

$$\begin{aligned} K =& -3\log(F) - \log(-i(\tau-\bar\tau)) - \log\left(-i\int_{CY}\Omega\wedge\overline{\Omega}\right) + const\,, \\ \text{with}\quad F \equiv& \,\mathrm{Vol}(CY)^{2/3} = T+\bar T - \frac{3i}{4(\tau-\bar\tau)}\kappa_{+ij}(\mathcal{G}-\bar{\mathcal{G}})^i(\mathcal{G}-\bar{\mathcal{G}})^j\,, \end{aligned} \tag{3.49}$$

where κ_{+ij} are the triple intersection numbers between the single orientifold even four-cycle and a pair of orientifold-odd ones, $\kappa_{+ij}\equiv\int_{CY}\tilde\omega_+\wedge\tilde\omega_i\wedge\tilde\omega_j$. Here, $\mathcal{G}^i\equiv c^i-\tau b^i$ are the $h^{1,1}_-$ complex axions, and the complex overall volume modulus T is defined as

$$T\equiv \mathrm{Vol}(CY)^{2/3}+i\rho^+\,. \tag{3.50}$$

At this level the superpotential vanishes to all orders in perturbation theory.

3.6 Fluxes and the landscape

We have reduced the amount of unbroken supersymmetry but we have yet to get rid of the remaining massless fields and break supersymmetry by a controlled amount. This is done by adding three-form fluxes on three-cycles. The simplest way to see the effects of three-form fluxes is by remembering that the $D5$ brane and the NS5 brane wrapped over a three-cycle Σ_3 are the domain walls over which the F_3 respectively H_3 flux quanta on the dual cycle $\tilde\Sigma_3$ jump by a unit. The four-dimensional tension of such a $D5$ brane is

$$\frac{T_{DW}}{M_P^3} = \frac{2\pi}{l_s^6}e^{\phi/2}\frac{\mathrm{Vol}(\Sigma_3)}{\mathrm{Vol}(CY)^{3/2}} \sim e^{K/2}\int_{\Sigma_3}\Omega = e^{K/2}\int\omega_{\tilde\Sigma_3}\wedge\Omega\,, \tag{3.51}$$

where $\omega_{\tilde\Sigma_3}$ is Poincaré dual to the cycle Σ_3. It is a standard result that the change in the *superpotential* ΔW across a $\frac{1}{2}$BPS domain wall is related to the tension of the

domain wall by

$$T_{DW} = 2e^{K/2}|\Delta W|\,. \tag{3.52}$$

Comparing with (3.51) one finds that the *flux* superpotential should be given by [154]

$$W \sim \int_{CY} (F_3 - \tau H_3) \wedge \Omega = \int G_3 \wedge \Omega\,, \tag{3.53}$$

which is called the *Gukov-Vafa-Witten* (GVW) flux superpotential. Indeed, across a domain wall of type discussed above F_3 changes by an amount $\omega_{\tilde{\Sigma}_3}$ so the domain wall tension comes out right. It was shown in [40] that the F-terms of the complex structure and the axio-dilaton possess a common solution locus where the three-form fluxes G_3 only have components of Hodge type $(0,3) \oplus (2,1)$, in other words they are *imaginary self-dual* $*G_3 = iG_3$. It is then useful to define the *imaginary-self-dual* (ISD) and *imaginary-anti-self-dual* (IASD) components $G_3^{\pm} \equiv (* \pm i)G_3$. The vanishing of the $(1,2)$ and $(3,0)$ components are $h^{2,1}+1$ complex conditions on the $h^{2,1}+1$ complex structure moduli and the axio-dilaton so for sufficiently generic three-form fluxes we expect the solution set to be a set of points in moduli space. In other words the complex structure moduli and the dilaton are *stabilized* by fluxes.

3.6.1 The GKP solutions

The four-dimensional flux vacua we have introduced in the preceeding section were argued to exist from purely four-dimensional considerations. One may rightfully ask if they actually lift to consistent ten-dimensional ones, and moreover in which regimes they can be trusted. This is slightly subtle because three-form fluxes carry induced $D3$-brane charge due to the Bianchi identity

$$dF_5 = F_3 \wedge H_3 + \rho_{D3}\,, \tag{3.54}$$

where ρ_3 is the $D3$-brane charge density carried by localized objects. Since our internal manifold is compact, the integral over the l.h. side of the Bianchi identity vanishes by Stokes theorem so the integral of the r.h. side must vanish as well. This constrains the allowed choices of three-form quanta in terms of the $D3$ brane charge carried by localized objects,

$$l_s^{-4}\left(\int_{CY} F_3 \wedge H_3\right) + N_{D3} - \frac{1}{4}N_{O3} + N_{\text{induced}} = 0\,. \tag{3.55}$$

N_{D3} denotes the number of D3 branes and we have used that a single O3 plane carries $-1/4$ units of D3 brane charge. Moreover we have added the induced D3-charge N_{induced}

on other localized objects such as seven-branes. E.g. a single D7 brane wrapped on a divisor D with trivial normal bundle carries $\frac{\chi(D)}{24}$ units of induced $D3$ brane charge, where $\chi(D)$ is the *Euler number* of the divisor. This is often times negative which means we have room to add fluxes of positive $D3$ brane charge. In typical CY orientifolds and their F-theory generalizations this gives us enough room to place modest amounts of flux quanta on each three-cycle [42, 152, 155].

Among other things it was shown by S. Giddings, S. Kachru and J. Polchinski (GKP) [40] that the direct dimensional reduction of the $|G_3|^2$ term in the $10d$ action indeed matches with the one derived from the GVW superpotential. For validity of this approximation the ten-dimensional backreaction of the fluxes must be small, which is the case when

$$\frac{|G_3^+|^2}{\mathrm{Im}(\tau)} \sim F_3 \cdot *H_3 \sim l_s^4\,(\text{D3-brane charge density}) \ll \text{typical curvature} \sim 1/L^2\,. \quad (3.56)$$

In other words, fluxes are sufficiently *dilute* to treat their presence as a small perturbation on top of the CY background if the number of $D3$ brane charge units contained in any region of size L is smaller than L^4/l_s^4. When the CY is very homogeneous and isotropic this is just a condition on the overall volume of the CY,

$$R_{CY}^4 \gg \text{total positive D3 brane charge}\,. \quad (3.57)$$

It is however important to appreciate that GKP also showed that even when fluxes are *not* dilute the ISD condition still solves the ten-dimensional equations of motion and all the backreaction on the metric and five form fluxes are controlled by a single function of the CY coordinates called the *warp factor* e^{2A}. Specifically, the metric and five-form ansätze are

$$ds^2 = e^{2A}dx^2 + e^{-2A}ds_{CY}^2\,, \quad F_5 = (1+*)d(e^{4A}) \wedge d^4x\,, \quad (3.58)$$

and all equations of motion are solved given a solution to

$$\nabla_{CY}^2 e^{-4A} = \text{D3-brane charge density}\,. \quad (3.59)$$

Changes in the overall volume of the CY correspond to the freedom to add a constant to any solution of eq. (3.59) [156]. To be precise, given any solution of the metric and axio-dilaton associated with a non-trivial $\mathcal{N} = 1$ preserving seven-brane background at tree-level (i.e. an F-theory solution), we can correct this solution by including ISD three-form fluxes, possibly mobile $D3$ branes and accounting for the induced negative $D3$ brane charge on the seven branes. The only equations that need to be solved are

the ISD condition (which fixes the complex structure moduli and the dilaton), as well as the 'electro-static' warp factor equation (3.59).

3.6.2 Warped throats and exponential hierarchies

A natural question to ask is whether it is possible to find solutions with regions where the warp factor e^{2A} is exponentially smaller than in others. The answer turns out to be yes, as shown by GKP, thereby realizing the Randall-Sundrum (RS) mechanism [157] to generate natural exponential hierarchies. It is easy to convince one-self that the warp factor naturally runs exponentially whenever $D3$ brane charge is not dilute. The famous backreacted solution associated with a stack of N $D3$ branes placed on a flat background $\mathbb{R}^6$ shows this,

$$e^{-4A} = 1 + \frac{4\pi N\alpha'^2}{r^4}\,, \quad ds^2 = e^{2A}dx^2 + e^{-2A}(dr^2 + r^2 d\Omega^2_{S^5})\,. \tag{3.60}$$

Clearly, in the near horizon limit $r^4 \ll N\alpha'^2$, the metric approaches the form

$$ds^2_{nh} = \frac{r^2}{\sqrt{4\pi N}\alpha'}dx^2 + \sqrt{4\pi N}\alpha'\left(\frac{dr^2}{r^2} + d\Omega^2_{S^5}\right) = e^{2ky}dx^2 + dy^2 + k^{-2}d\Omega^2_{S^5}\,, \tag{3.61}$$

which is a patch of $AdS^5 \times S^5$. Here we have introduced a new radial coordinate $y = k^{-1}\log(kr)$ that measures physical distances along the 'throat'$\equiv \mathbb{R} \times S^5$, and $k^{-2} \equiv \sqrt{4\pi N}\alpha'$. It is apparent that the warp factor now runs exponentially as desired. Solutions of this type are called *warped throats* and they offer a remarkable amount of computational control. It is one of the most celebrated results of the past decades, due to J. Maldacena, that such ten-dimensional gravitational backgrounds (in their near horizon limits) are in fact *dual* to four-dimensional QFTs [158]. We have just given the simplest of these. For a stack of N $D3$ branes we know from perturbative string theory that in the deep infrared the brane degrees of freedom decouple from the bulk gravitational ones and realize the $\mathcal{N} = 4$ $SU(N)$ super Yang-Mills gauge theory (which is actually conformal). Its holomorphic gauge coupling is set by the ten-dimensional axio-dilaton at the point probed by the brane stack. The gauge theory is weakly coupled when $g_s \ll 1$ and also the 't Hooft coupling is small $g_sN \ll 1$. Beyond these limits the gauge theory is strongly coupled. But we just saw another way to take the IR limit, namely going into the near horizon limit of the backreacted supergravity solution. The size and inverse curvature scale of the *string frame* metric is $L^4 \sim g_sN\alpha'^2$. So for the gravitational description to be weakly curved we need to precisely go to the opposite regime $g_sN \gg 1$ (but still $g_s \ll 1$). So we see that the large 't Hooft coupling limit of the gauge theory is described by a ten-dimensional string geometry! This is the

simplest example of the famous AdS/CFT correspondence.

To realize the RS idea the infinite $AdS_5 \times S^5$ throat must be reduced to finite length thus breaking the isometry group of AdS_5 down to Poincaré$_4$. From the field theory side this amounts to breaking conformal invariance. But before we get there let us note that we can replace the flat six-dimensional space we started with by the singular conifold geometry. Placing the stack of $D3$ branes at the singular point gives rise to a near horizon limit supergravity solution $AdS_5 \times T^{1,1}$ which is obtained from the $AdS_5 \times S^5$ solution by simply replacing $k^{-4} \longrightarrow \frac{\mathrm{Vol}(T^{1,1})}{\mathrm{Vol}(S^5)} k^{-4} = \frac{27}{16} k^{-4}$. The ten-dimensional $AdS_5 \times T^{1,1}$ background is another example of an infinite length throat but it breaks more supersymmetries than the $AdS_5 \times S^5$ throat. Its field theory dual is an $\mathcal{N} = 1$ superconformal $SU(N) \times SU(N)$ quiver gauge theory, the conifold or Klebanov-Witten (KW) gauge theory, with the bifundamental matter coming in the $(2,1)$ respectively $(1,2)$ of the global conifold symmetry group $SU(2) \times SU(2)$, and with a quartic superpotential [159]. The holomorphic gauge couplings $(\tau_{\mathrm{YM}}, \tilde{\tau}_{\mathrm{YM}})$ of the two gauge group factors are now set by

$$\tau_{\mathrm{YM}} + \tilde{\tau}_{\mathrm{YM}} = \tau\,, \quad \tau_{\mathrm{YM}} - \tilde{\tau}_{\mathrm{YM}} = -\tau + \mathcal{G}/\pi \mod 2(m - n\tau)\,, \tag{3.62}$$

with a torus valued complex axion $\mathcal{G} \equiv \int_{S^2 \subset T^{1,1}} C_2 - \tau B_2$. Here it is understood that τ and $\mathcal{G}$ take values in their appropriate fundamental domain so that both gauge group factors are weakly coupled.

This theory can be perturbed by adding M RR three-form fluxes on the $S^3 \subset T^{1,1}$ [78, 160], $F_3 = M\omega_3$, where ω_3 is the harmonic three-form of $T^{1,1}$. In order to solve the ISD condition the NS three-forms should be given by (setting $C_0 = 0$)

$$H_3 = -g_s * F_3 = \frac{3}{2\pi} g_s M \frac{dr}{r} \wedge \omega_2\,, \quad \text{i.e.} \quad B_2 = \frac{3}{2\pi} g_s M \left(\log(r/l_s) + \text{const.}\right) \omega_2\,, \tag{3.63}$$

where ω_2 is the harmonic two-form of $T^{1,1}$. As the metric is known and the ISD condition solved, all that remains is to compute the warp factor by integrating (3.59). The result is

$$e^{-4A} = \frac{L^4}{r^4} \left(\log(r/r_0) + \frac{1}{4} \right) + const.\,, \quad \text{with} \quad L^4 = \frac{81}{8(2\pi)^4} g_s M^2 l_s^4\,. \tag{3.64}$$

This is called the Klebanov-Tseytlin (KT) solution [78] and it runs into a singularity at $r \lesssim r_0$. The (non-quantized) five form flux runs as $N(r) = \frac{3}{2\pi} g_s M^2 \log(r/r_0)$. The logarithmic running of the warp factor is reminiscent of the RG running of a gauge coupling and indeed, there is a $\mathcal{N} = 1$ gauge theory that is dual to the geometry. It can be understood as a small perturbation of the gauge group ranks of the conifold

gauge theory $SU(N) \times SU(N) \longrightarrow SU(N+M) \times SU(N)$, if $M \ll N$. The two gauge couplings run in opposite directions which precisely matches the radial running of the B_2 field and the identification of eq. (3.62). As a consequence one of the two gauge factors becomes strongly coupled after some RG running. It was realized by I. Klebanov and M. Strassler (KS) that after an appropriate application of Seiberg duality [161] the theory is of the same form as the initial one but with N replaced by $N - M$ [162]. The supergravity dual of this effect is of course the radial running $N(r)$. As a consequence the gauge theory undergoes repeated steps (*cascades*) of Seiberg dualities as it flows to the IR. The KT solution accurately describes the RG flow of the gauge theory only so long as $r \gg r_0$, i.e. $N \gg M$. At last, only a single $SU(M)$ factor remains which confines and undergoes gaugino condensation. This effect is actually captured by the everywhere smooth KS solution which uses the *deformed* conifold as a starting point rather than the singular one [162]. It is intuitive that this is the correct starting point because that KT singularity arises when the S^3 is forced to shrink to zero size while the non-vanishing RR flux really wants to keep it at finite size. We will not need the precise form of the solution but we note a few important properties.

First, we are actually interested in embedding such a *KS throat* into a compact CY. So we should cut-off the throat at some UV value $r_{\rm UV}$ which marks the point where the details of the CY geometry start to depart from the simple conifold. Note that while we do not know the precise form of the solution beyond this point, existence of a smooth interpolation is guaranteed. From the running of the B_2 field we learn that the NS flux on the $\mathcal{B}$-cycle of the throat is set by

$$K \equiv \int_{\mathcal{B}} H_3 = \int_{S^2 \subset T^{1,1}} B_2(r_{\rm UV}) = \frac{3}{2\pi} \log(r_{\rm UV}/r_{\rm IR})\,. \tag{3.65}$$

This implies that the finite hierarchy induced by the fluxes in the throat is of order

$$e^{4A_{\rm IR}-4A_{\rm UV}} \sim \frac{r_{\rm IR}^4}{r_{\rm UV}^4} \sim \exp\left(-\frac{8\pi}{3}\frac{K}{g_s M}\right)\,. \tag{3.66}$$

Even without investigating the detailed from of the KS solution dimensional analysis gives that the conifold complex structure ϵ is fixed at value

$$|\epsilon| \sim \frac{r_{\rm IR}^3}{r_{\rm UV}^3} \sim \exp\left(-2\pi\frac{K}{g_s M}\right)\,. \tag{3.67}$$

It is important to note that we have been a bit sloppy and assumed that the three-form flux quanta K are *all* located within the throat. If the $\mathcal{B}$-cycle reaches only into a single

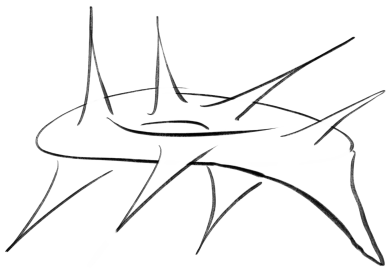

Figure 3.4: In 'typical' flux compactifications there are many warped throats of significant warping that can be thought of as spikes emanating from the CY orientifold.

throat this is not a bad approximation as long as $K \gg g_s M$.[23] In later chapters we will be interested also in cases where the NS three-form fluxes are equally distributed on more than a single throat.

For now we close by noting that the formulas we have extracted from the KS solution nicely match with the F-term solutions of the GVW superpotential [40]. We call the conifold complex structure Z, and the $\mathcal{B}$-cycle period $G(Z)$. From monodromy considerations it is known that $G(Z) = Z\frac{\log(Z)-1}{2\pi i} + \text{holomorphic}$. The GVW superpotential reads

$$W = \int (F_3 - \tau H_3) \wedge \Omega = MG(z) - \tau K Z + W_0 \,. \tag{3.68}$$

where W_0 encodes Z-independent terms from fluxes on other cycles. Using the Kähler potential of eq. (3.33) the F-term condition $D_Z W = 0$ takes the form

$$D_Z W = M\left(\frac{\log Z}{2\pi i} + \mathcal{O}(1)\right) - \tau K + \mathcal{O}(|Z|) \,. \tag{3.69}$$

In the regime $|Z| \ll 1$ the $\mathcal{O}(|Z|)$ corrections can be neglected and the F-term vanishes for

$$Z \sim \exp\left(2\pi i \frac{\tau K}{M}\right) \quad \Longrightarrow \quad |Z| \sim \exp\left(2\pi \frac{K}{g_s M}\right) \sim \epsilon \,. \tag{3.70}$$

The $\mathcal{O}(|Z|)$ can be neglected when $|Z| \ll 1$, i.e. when $K \gg g_s M$. So the $4d$ supergravity F-term reproduces the KS formula. As the conifold singularity is the most generic one it is reasonable to expect that generic flux compactifications come with many warped throats [42, 163].[24] We depict a cartoon of such a compactification in Figure 3.4.

[23] An $\mathcal{O}(g_s M) \ll K$ number of NS flux quanta should sit in the bulk CY to ensure that the ISD condition is satisfied also there. Here we have assumed that the bulk CY takes a rather generic form.

[24] Loosely speaking, only when $K \sim g_s M$ for all pairs of flux quanta, or at exponentially large volumes $\mathcal{V} \ll (N_{D3})^{3/2}|Z|^{-2}$ they do *not* form.

3.7 Moduli stabilization & de Sitter vacua

3.7.1 The Dine-Seiberg problem

So far we have discussed classes of $\mathcal{N} = 1$ 'quasi' vacua in which the fate of a sub-sector of the CY moduli (the Kähler moduli) is generically unknown. They are massless at tree level but supersymmetry is broken by the flux superpotential. The flatness of the scalar potential is due to the *no-scale* structure of the Kähler potential of the Kähler moduli,

$$g^{T^i\bar{T}^{\bar{j}}}\partial_{T^i}K\partial_{\bar{T}^{\bar{j}}}K = 3\,. \tag{3.71}$$

This structure reflects the fact that in string theory the scalar potential always vanishes in the decompactification limit $\mathrm{Re}(T^i) \longrightarrow +\infty$. But there is no symmetry principle that protects the flatness of the scalar potential at subleading order in the inverse volume expansion (the α' expansion). For example, the Kähler potential including the leading α' correction that cannot be absorbed by a field redefinition, the Becker-Becker-Haack-Louis (BBHL) correction [164], takes the form

$$K = -2\log\left((T+\bar{T})^{3/2} + \xi e^{-3\phi/2}\right)\,, \tag{3.72}$$

with $\xi \equiv -\frac{\chi}{2}\zeta(3)$, CY Euler number $\chi = 2(h^{1,1} - h^{2,1})$ and Riemann ζ-function. This correction breaks the no-scale structure and leads to a runaway potential for the Kähler moduli. It is of crucial importance for phenomenological applications of almost any sort to have in hand a mechanism to stabilize the Kähler moduli, i.e. to generate a controlled potential with a discrete set of local minima. This program goes under the name of *moduli stabilization*.

We have already implicitly stated a generic problem with moduli stabilization: In string theory at large volumes the scalar potential of the overall volume tends to zero, while the perturbative α' expansion is precisely an expansion in inverse volume powers. So, if the coefficients in this expansion take generic $\mathcal{O}(1)$ values the scalar potential is dominated by the lowest non-vanishing term that falls off towards infinity as an inverse power law,

$$V(T+\bar{T}) \propto \pm(T+\bar{T})^{-p}\,, \quad p > 0\,. \tag{3.73}$$

If the sign of the leading order correction is positive this leads to a run away behavior to large volume. If it is negative it drives the theory to strong coupling where different orders in the perturbative expansion may compete to give rise to a stable minimum. But in this case a full tower of corrections becomes relevant and generically isn't computable. This is the Dine-Seiberg problem [165] and leads one to conclude that a generic

stabilized vacuum of string theory should lie at strong coupling

$$g_s(T+\bar{T}) = \mathcal{O}(1)\,. \tag{3.74}$$

3.7.2 The KKLT proposal

The Dine-Seiberg problem is a *generic* problem of moduli stabilization but in can be avoided at the cost of *tuning*. This is how Kachru, Kallosh, Linde and Trivedi (KKLT) have proposed to achieve moduli stabilization in a controlled regime [41]. We will describe their construction now.

As the flux superpotential is exact in perturbation theory it is sensible to neglect the difficult to access perturbative corrections all together and consider the easier to determine non-perturbative superpotential corrections. We consider a flux compactification with the dilaton and complex structures integrated out. Moreover we focus again on the case of a single Kähler modulus T. As there is an exact gauged axion shift symmetry $T \longrightarrow T + i\mathbb{Z}$ the non-perturbative superpotential must organize into a series

$$W = W_0 + \sum_{n=1}^{\infty} A_n e^{-2\pi n T}\,, \tag{3.75}$$

at least in the absence of monodromy. The one-loop Pfaffians A_n are in general hard to compute but a natural assumption is that they take $\mathcal{O}(1)$ values as long as the CY is stabilized at a point in complex structure moduli space where it has only a single length scale.

Validity of this assumption is actually not as easy to justify as expected. In general, the instantons that contribute to the superpotential have precisely two fermion zero modes, in particular they are $\frac{1}{2}$BPS. If there are more fermionic zero modes, the superpotential contribution vanishes. The $\frac{1}{2}$BPS instanton associated with the n-th term in the non-perturbative expansion is a euclidean $D3$ brane wrapped n times over a holomorphic representative of the single divisor class D. It was shown by E. Witten [166] that a necessary condition for exactly two zero modes is that the *arithmetic genus* $\chi(D,\mathcal{O}_D) \equiv \sum_{i=0}^{2}(-1)^i h^{i,0}$ of the divisor D is equal to one. A sufficient condition is satisfied when $h^{1,0} = h^{2,0} = 0$, i.e. when the divisor is *rigid*. Strictly speaking the presence of three-form fluxes might lift additional zero modes so that a superpotential might nevertheless be generated when $\chi(D,\mathcal{O}_D) \neq 1$ [167, 168], but we will not consider this option here. Another option is axion monodromy: The simplest way to achieve it is by considering a non-abelian seven brane gauge group that wraps D. In the simplest case, this is a $SU(N)$ gauge theory associated with a stack of N $D7$ branes. If all matter can be made massive, the effective field theory at low energies contains a pure

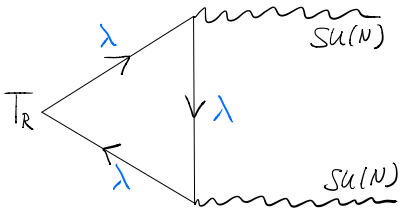

Figure 3.5: The classical $U(1)_R$ symmetry is anomalous since the gaugino λ runs in the above $U(1)_R SU(N)^2$ triangle diagram.

supersymmetric Yang-Mills sector with $4d$ holomorphic gauge coupling set by

$$\tau_{YM} = iT \sim i\text{Vol}(D) - \int_D C_4 \,. \tag{3.76}$$

This is easily seen from a direct dimensional reduction of the classical $8d$ DBI and CS action. Fortunately, $\mathcal{N} = 1$ pure supersymmetric Yang-Mills is a very well-studied field theory and at the level of the renormalizable UV Lagrangian the low energy effective superpotential is known *exactly*.

The classical theory of pure $SU(N)$ SYM enjoys a $U(1)_R$ symmetry that rotates the phase of the gaugino. However, this symmetry is broken at the quantum level by instantons due to the triangle anomaly depicted in Figure 3.5. Assigning the holomorphic scale $\Lambda^{3N} = \mu_{\text{UV}}^{3N} e^{-2\pi T}$ spurious R-charge $2N$ the symmetry is restored. If we view the holomorphic gauge coupling as a dynamical field (as we always do in string theory), we see that the $U(1)_R$ symmetry is broken spontaneously by the expectation value of T. As $T \sim T + i\mathbb{Z}$ a $\mathbb{Z}_{2N} \subset U(1)_R$ remains unbroken. Assuming that the gauge multiplet confines and can be integrated out the only low energy superpotential we can write down that has the correct R-charge is

$$W_{eff} = c\Lambda^3 \,, \tag{3.77}$$

for some coefficient c. By matching this theory with SQCD theories via mass perturbation, allows one to compute the coefficient c via a weakly coupled instanton calculation in SQCD with $N-1$ flavors [169]. The result is $c = N$, and in particular it is *not* zero, indicating that confinement indeed occurs. The non-vanishing superpotential (3.77) signals a further spontaneous breaking of the R-symmetry group down to $\mathbb{Z}_2$, as it has N solution branches for each given value of $\Lambda^{3N} \sim e^{-2\pi T}$. Indeed, it measures the expectation value of the gaugino bilinear [170]

$$\langle \lambda\lambda \rangle \equiv \langle \text{Tr}\left(\bar{\lambda} P_L \lambda\right) \rangle = -16\pi \partial_{F_T} \log Z = -16\pi \partial_T W_{eff}(T) = 32\pi^2 \Lambda^3 \,, \tag{3.78}$$

where Z denotes the partition function. In other words the gauge theory has N vacua distinguished from one another by the phase of the gaugino bilinear. There exist gauge theory domain walls that interpolate between them [171]. These are of course the domain walls in the Kaloper-Sorbo description of axion monodromy. Crucially, the monodromy is *finite* because after passing N domain walls one is back in the gauge theory vacuum one started with.

If higher derivative corrections to the UV Yang-Mills action are present (as they always are in string theory), suppressed by some UV scale $M_{\rm UV}$, one expects the superpotential to also receive higher order corrections [172]

$$W_{F=0} = N\Lambda^3 \left(1 + \sum_{n=1}^{\infty} c_n \left(\frac{\Lambda}{\mu_{\rm UV}}\right)^{3n}\right) . \tag{3.79}$$

So applying this logic to the case of a stack of seven-branes wrapped on a divisor we have an effective superpotential (including the constant flux contribution)

$$W = W_0 + ANe^{-\frac{2\pi}{N}T} + \dots , \tag{3.80}$$

where $A \equiv M_{\rm UV}^3/M_P^3$ is assumed to be of order one. Just keeping the first two terms, and using the tree-level Kähler potential

$$K = -3\log(T+\bar{T}) , \tag{3.81}$$

one computes the F_T-term

$$D_T W = -2\pi A e^{-\frac{2\pi}{N}T} - \frac{3}{T+\bar{T}}\left(W_0 + ANe^{-\frac{2\pi}{N}T}\right) , \tag{3.82}$$

In the regime $(T+\bar{T}) \gg N$ where the gauge theory is weakly coupled and our formula should be valid the F-term equation has a solution with

$$T \approx \frac{N}{2\pi}\log(-W_0) , \tag{3.83}$$

which is self-consistent if $|W_0| \ll 1$. By scanning over the huge set of available flux vacua one should be able to find solutions with $|W_0|$ almost arbitrarily small. So, it is (almost [173]) unanimously accepted that there exist fully stabilized AdS vacua of the KKLT type. The vacuum energy at the SUSY minimum of the scalar potential is given by

$$V_{AdS} = -3e^K|W|^2 \approx -\frac{3}{(T+\bar{T})^3}|W_0|^2 . \tag{3.84}$$

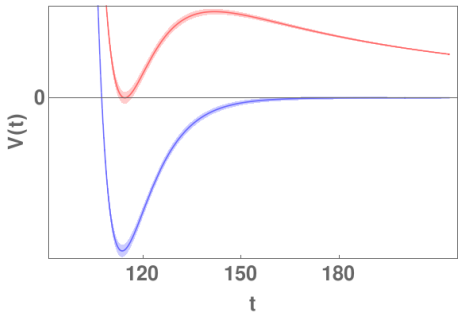

Figure 3.6: In blue: The supersymmetric KKLT potential for parameters $NA = 1$, $\frac{2\pi}{N} = 0.1$ and $W_0 = -10^{-4}$. In red: The uplifted potential with appropriately fine tuned warp factor $a_0^2 \sim |W_0|$. There is a dS minimum at $t \equiv \mathrm{Re}(T) \approx 110$. The maximal possible uplift occurs with vacuum energy $V \sim +|V_{SUSY}|$. At larger values of a_0^2 there is no meta stable minimum anymore.

We depict the scalar potential with its KKLT minimum in Figure 3.6.

KKLT have also proposed how to generate controlled *de Sitter* vacua by incorporating small amounts of meta stable supersymmetry breaking. This idea in general is called an *uplift*. One of the most prominent ideas how to accomplish this is by adding so-called anti-D3 branes into the setup. Locally, these are just D3 branes, but their worldvolume orientation is such that the supersymmetries preserved by the flux background (in the limit $W_0 = 0$) are the ones broken by the brane and vice versa. As a consequence, an anti-brane also carries $D3$-brane charge of the sign opposite to the one carried by fluxes. Thus, if we are to compare two configurations, a compactification with an anti-brane, to one without it, the flux numbers of the two configurations must be slightly different so that one extra unit of $D3$ brane charge carried by fluxes can compensate the negative anti-brane charge. Schematically the scalar potential induced by such a configuration is

$$V_{\text{uplift}} \sim e^{4A} \left(2T_3 + \text{binding energy}\right) , \tag{3.85}$$

where T_3 is the D3 brane tension and the factor of two arises because of the extra unit of flux induced charge that carries the tension of a single D3 brane due to the ISD condition. In a generic flux compactification the anti-brane position moduli are stable if the brane is placed where the warp factor assumes a local minimum. This is precisely the tip of a warped throat, where indeed the gravitational red-shifting is exponentially small. It is usually assumed that this is a KS throat.

A set of natural questions comes to mind that have led to quite some controversy in the past: 1) Is the brane/flux bound state (meta-)stable against mutual annihilation? 2) Is the binding energy sub-leading? The first question has been addressed explicitly

by Kachru, Pearson and Verlinde (KPV) who find that the only allowed decay channel is a brane-flux decay where the anti-brane polarizes into an NS5-brane that winds around the compact three-sphere of the throat thus discharging one unit of NS fluxes on the $\mathcal{B}$-cycle and leaving behind $M-1$ D3 branes [174]. If $M \gtrsim 12$, this is a non-perturbative (i.e. slow) tunneling transition so the anti-brane is meta-stable. This calculation was done in the approximation where the anti-brane is treated as a probe of the background flux geometry. Thus, the second question is related to the question whether or not this 'probe' approximation is actually valid, so it naturally relates with the question of meta-stability. This has been discussed extensively in the literature (see [175–190] for recent literature on this topic) and it is by now widely believed that the probe approximation is valid, at least for a single anti-brane.

It is of course important to know how the anti-brane vacuum energy depends on the volume modulus T in order to decide whether or not the volume modulus is destabilized by the uplift. It is straightforward to compute this in the probe approximation on a classical GKP background. The result is [191]

$$V_{\text{uplift}} \propto \frac{a_0^4}{(T+\bar{T})^2}\,, \tag{3.86}$$

where we have suppressed $\mathcal{O}(1)$ coefficients, and $a_0 \equiv \exp\left(-\frac{8\pi}{3}\frac{K}{g_s M}\right)$ is the hierarchy of the compactly embedded KS throat. Using this one finds de Sitter vacua of tunable positive or negative cosmological constant by making judicious choices of flux quanta (see Figure 3.6), so that $a_0^4 \sim |W_0|^2$.

3.8 The swampland of effective field theories

We would now like to give a brief introduction into some of the ideas that have emerged over the past decade and which go under the name of the *swampland of effective field theories* [29, 30].

In the early days of string theory it was hoped that the theory would produce a more or less *unique* supersymmetry breaking vacuum, that could be compared with real world physics. Although formally speaking this idea is still on the table it is very unlikely that string theory works like this. We have given a brief account of flux compactifications in the type IIB corner of the theory, and it is fairly clear that many different vacua can be generated by using the freedom to dial the flux quantum numbers. At least in very naive terms it is clear that the number of vacua that can be obtained in this way is truly enormous. As a typical CY has $\mathcal{O}(100)$ distinct three-cycles we should be able to choose flux numbers in a reasonably bounded range (say $\mathcal{O}(10)$) for each cycle.

Simple combinatorics then produces estimates for the number of flux vacua such as the 'historical' number 10^{500}. This is actually a huge *under*estimate. Somewhat recently, Taylor and Wang found a CY fourfold[25] that produces $\mathcal{N}_{\text{vacua}} \sim 10^{272,000}$ different vacua [194]. With this unimaginably large number in mind one might even be tempted to believe that almost anything goes in string theory. Certainly, Wilson coefficients of the effective four-dimensional supergravities that describe classes of string vacua should show *some* dependence on the choices flux quantum numbers. This perhaps most clearly seen for the expectation value of the GVW superpotential after integrating out the dilaton and the complex structure moduli, $W_0 \equiv \langle W_{GVW} \rangle$, which sets the physical mass of the gravitino and so the *scale of supersymmetry breaking*. It is widely believed that at least this number can take almost arbitrarily small values down to $|W_0^{\text{minimal}}| \sim 1/\sqrt{\mathcal{N}_{\text{vacua}}}$ [42, 152]. If this were true for all Wilson coefficients it would be impossible to test string theory observationally without accessing energy scales of order the string scale.[26] In many ways the idea of the swampland is that while *some* Wilson coefficients cannot be tightly constrained in string theory, there nevertheless exist clear boundaries in EFT parameter space that divide the (extremely) densely populated *landscape* of string solutions from *completely* empty regions, *the swampland*, that can never be reached by string theory. We may define the landscape as the set of low energy EFTs coupled to gravity that can be realized in string theory, while the swampland consists of those that cannot. We have depicted a cartoon of the total space of EFTs in Figure 1.2. If we can identify clearly cut boundaries that divide the two sets from each other we have extracted a non-trivial prediction of string theory that might be relevant for low energy physics. The *swampland program* is the attempt to do so, and many *swampland conjectures* have been put forward. We will describe some of them shortly. For a more complete introduction to this subject we refer the reader to ref. [195].

3.8.1 Axions and shift symmetries

Axions will play a major role in this thesis so it is worthwhile describing what we mean by an axion. Historically the axion was introduced as a pseudo-scalar field $a(x)$ that couples to QCD via a non-renormalizable interaction,

$$\mathcal{L} \subset -\frac{f_a^2}{2}(\partial a)^2 + \frac{a(x)}{32\pi^2}\text{Tr}\left(G_{\mu\nu}\tilde{G}^{\mu\nu}\right) , \tag{3.87}$$

[25] We have not said what is F-theory [192]: It is the generalization of weakly coupled type IIB that enables one to study general configurations of (p, q) seven-branes. See [193] for a pedagogical introduction.

[26] This scale could lie anywhere between the TeV scale and the Planck scale.

with *axion decay constant* f_a. At the classical level the action is invariant under the continuous shift symmetry

$$a(x) \longrightarrow a(x) + const\,. \tag{3.88}$$

This shift symmetry is broken to a discrete one $a(x) \longrightarrow a(x) + 2\pi$ at the non-perturbative level due to instantons. It was shown by C. Vafa and E. Witten that the effective potential $V_{QCD}(a)$ induced by QCD instantons has a minimum at $a = 0$ [196] thus offering a *dynamical* solution to the strong CP problem of QCD [197].

The axions we are considering are very similar but we do not require them to couple to QCD. They can be thought of as the integrals of the various stringy p-form potentials over the different p-cycles of the CY in question. They are also pseudo-scalars from the $4d$ perspective and they share the defining feature of a *perturbative* shift symmetry. At the $\mathcal{N} = 2$ level these continuous shift symmetries are always exact in perturbation theory, because only the field strengths enter the equations of motion rather than the potentials themselves. There always exists an instanton that breaks the shift symmetry to a discrete one. For the RR p-form axions these are euclidean D$(p-1)$ branes wrapped on the corresponding p-cycle, while for the NS 2-form it is a euclidean string worldsheet that wraps around the associated two-cycle, respectively a euclidean NS5 brane wrapping the whole CY. In the euclidean path integral they contribute with a factor

$$e^{-(S_E + ia(x))}\,, \tag{3.89}$$

with *euclidean instanton action*

$$S_E = 2\pi \begin{cases} g_s^{-1}\mathrm{Vol}(\Sigma_p) & \text{model dependent RR axions \& } C_0\,, \\ \mathrm{Vol}(\Sigma_2) & \text{model dependent } B_2 \text{ axions}\,, \\ g_s^{-1}\mathrm{Vol(CY)} & \text{universal } C_2 \text{ axion}\,, \\ g_s^{-2}\mathrm{Vol(CY)} & \text{universal } B_2 \text{ axion}\,. \end{cases} \tag{3.90}$$

Here, cycle volumes are understood to be measured with the *string frame metric* in units $l_s = 1$. Clearly these non-perturbative effects again break the continuous shift symmetries to discrete ones. We will always define the axion decay constant via a choice of axion normalization such that a shift by 2π is an exact gauge symmetry of the theory. It can however be broken spontaneously: For example, in type IIB flux compactifications the discrete shift symmetry contained in $Sl(2,\mathbb{Z})$ also acts on the three-form fluxes,

$$C_0 \longrightarrow C_0 + 1\,, \quad (H_3, F_3) \longrightarrow (H_3, F_3 + H_3)\,, \tag{3.91}$$

so after applying the shift symmetry the flux quanta have changed and we are in a different vacuum. Thus it is not obvious why the axion potential $V(C_0)$ would have to come back to itself after it passes once around its fundamental domain continuously. Indeed, the GVW superpotential depends explicitly on the C_0-axion.[27] The B_2 and C_2 shift symmetries might generically be broken spontaneously as well if five-form flux quanta were introduced. Since CYs don't have five-cycles this phenomenon does not occur in the type IIB flux landscape. The general idea of spontaneously breaking an axionic shift symmetry is called *axion monodromy* [46, 73, 198].

It is sometimes useful to dualize the axion to a two form b_2. In this language, axion monodromy can be understood as the gauging of a two form shift symmetry with a three-form gauge potential C_3, as advocated by N. Kaloper and L. Sorbo [198, 199]. The relevant action, due to G. Dvali [200], is

$$S = \int -\frac{1}{2\lambda^4} F_4 \wedge *F_4 - \frac{1}{2f^2}(db_2 - C_3) \wedge *(db_2 - C_3)\,, \tag{3.92}$$

with four form field strength $F_4 = dC_3$. The electrically charged objects under C_3 are domain walls. Dualizing back to an axion and integrating out the four form one obtains the usual axion kinetic term and a scalar potential

$$V(a) = \frac{1}{2}\lambda^4 \left(n + \frac{a}{2\pi}\right)^2\,, \tag{3.93}$$

where $n \in \mathbb{Z}$ is the four form flux quantum number. The combined transformation

$$(a, n) \longrightarrow (a, n) + (2\pi, -1)\,, \tag{3.94}$$

is a gauge transformation. It is now clear that when $a \geq \pi$ there is a non-perturbative tunneling instability towards nucleating the domain wall that sends $n \longrightarrow n - 1$ and abruptly lowers the potential.

We would like to emphasize that the defining property of an exact gauged discrete gauge symmetry is physically equivalent to the existence of *cosmic strings* which are the electrically charged objects under b_2 [201]. Passing around the string the axion travels around its fundamental domain once (see Figure 3.7). For the effective four-dimensional axion C_0 this effective string is formed by wrapping a $D7$ brane once around the CY manifold, which indeed gives a spatially one-dimensional object in four dimensions, and a cosmic string analogous to this one exists for every stringy axion. When the gauged axion shift symmetries are broken spontaneously these strings are required to be the

[27] Whether or not the discrete shift symmetry is actually broken in this case is a subtle question (see the end of this section). But it *could* in principle be broken.

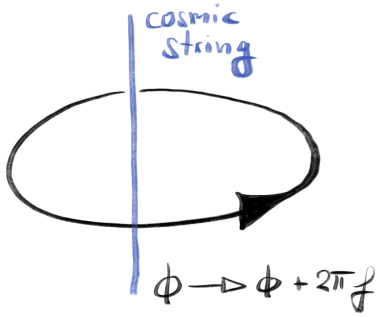

Figure 3.7: Upon encircling a cosmic string of smallest charge (blue) the axion traverses its fundamental domain once.

boundaries of the (thin) domain walls of the Kaloper-Sorbo description.[28] In general we expect that beyond *some* critical field excursion this domain wall becomes 'tensionless'. Near such a point, if it lies in a regime where $4d$ EFT is valid, the domain wall should be resolved in terms of a scalar field that *either* adjusts adiabatically as a function of the axion vev thus bending down the effective axion potential back to zero, *or* becomes tachyonic leading to a fast transition. This can be illustrated by a two field Lagrangian

$$\mathcal{L} = -\frac{f_a^2}{2}(\partial a)^2 - \frac{f_\chi^2}{2}(\partial\chi)^2 - \frac{\lambda^4}{2}\left(\frac{\chi}{2\pi} - \frac{a}{2\pi}\right)^2 - \Lambda^4\left(1-\cos(\chi)\right) . \tag{3.95}$$

If $\Lambda^4/f_\chi^2 \gg \lambda^4/f_a^2$ we may integrate out χ and parametrize the IR dynamics using the single axion $a(x)$. If $\lambda^4 < \Lambda^4/(2\pi)^2$ we are in the broken phase where the effective scalar potential $V_{\text{eff}}(a)$ has multiple branches. At a critical field excursion

$$a_c = \frac{\Lambda^4}{(2\pi)^2\lambda^4} + \frac{\pi}{2} + \mathcal{O}\left(\frac{\lambda^4}{\Lambda^4}\right) , \tag{3.96}$$

the field χ becomes tachyonic and triggers a fast domain wall nucleation process that lowers the energy. For $a \ll a_c$ we have $\chi \approx 2\pi n$ thus reproducing the multi-branched potential of eq. (3.93). This is the broken phase.

In the opposite regime $\lambda^4 > \Lambda^4/(2\pi)^2$ the heavy field χ adjusts adiabatically $\chi = a + \mathcal{O}(\Lambda^4/\lambda^4)$ and the effective potential looks like it is generated by instantons,

$$V(a) = \lambda^4 \sum_{n=1}^{\infty} c_n e^{-nS}(1-\cos(na)) , \quad e^{-S} \equiv \frac{\Lambda^4}{\lambda^4} , \quad c_n = \{1, -1/4, ...\} . \tag{3.97}$$

[28] Strictly speaking a domain wall is attached to the string even when the gauged shift symmetry remains unbroken due to the non-perturbative axion potential. However, in an unbroken phase a low energy observer that can resolve the axion as a dynamical field will also be able to resolve the effective domain wall as an axion profile, so the domain wall is 'thick'. In a broken phase this may not be possible.

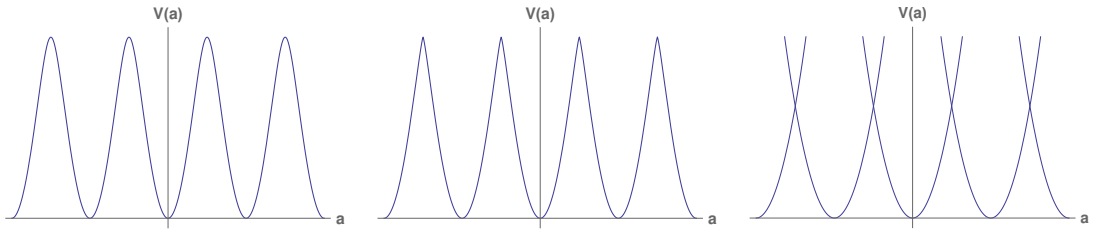

Figure 3.8: The effective axion potential in the unbroken (left), critical (middle) and broken (right) phases. We have plotted the cases $\Lambda^4/(2\pi)^2\lambda^4 = \{0.5, 1, 4\}$. At the critical value $(2\pi)^2\lambda^4 = \Lambda^4$ the scalar potential develops distinct branches with finite (meta-)stable ranges.

This is the unbroken phase. We depict both regimes and the critical intermediate one where branches of the potential 'reconnect' in Figure 3.8.

Finally, let us note that the way we think about axions is fundamentally *perturbative*. We require the continuous shift symmetry of the axion to be exact to all orders in perturbation theory, so as we pass over loci of strong coupling in moduli space it is not guaranteed that the dual weakly coupled description that we encounter on the other side of the locus even possesses anything like an axion. The mere notion of an axion is thus equally vaguely defined as that of a gauge group in QFT.[29] We should think about it as an *emergent* weak coupling phenomenon.

3.8.2 The weak gravity conjecture

We would now like to introduce a conjecture that was put forward more than a decade ago by Arkani-Hamed, Motl, Nicolis and Vafa [31]. It is called the *weak gravity conjecture* and turns out to be applicable to many weak coupling phenomena such as gauge theories as well as axions. As such it is relevant in principle for both particle physics and inflationary cosmology. In its simplest version it is applied to the situation of a $U(1)$ gauge theory. In such a theory coupled to gravity there exist Reissner-Nordstöm black holes characterized by their mass and charge (M, Q) subject to an extremality bound $Q \lesssim M/M_P$, in order to avoid naked singularities. Due to Hawking radiation any initial configuration (Q, M) will evaporate down to extremal ones. But if this is their only decay channel there exist an infinite finely spaced tower of extremal black hole states labeled by their charge Q. As these are macroscopic objects one would expect the charge and thus mass spacing to be $\mathcal{O}(1)$. This is a slightly awkward situation because there exists an infinity of states that are all absolutely stable while no symmetry principle forbids their decay. For all we know this possibility is not obviously inconsistent, but

[29] It happens frequently that a single gauge theory has two complementary weak coupling descriptions that involve two *different* gauge groups or even no gauge group at all on one side of the self-dual locus, see e.g. [202, 203]

slightly discomforting. In any case, the unease about this was sufficient for the authors of [31] to conjecture a further decay process: There should exist a light particle of mass (m, q) with charge to mass ratio bigger than unity, $q \gtrsim m/M_P$. Then, extremal black holes can decay via Schwinger pair production. To date it is not fully understood why really such a conjecture would have to hold (see however [63, 204]), while the strongest evidence for the conjecture stems from the fact that no fully understood examples seem to exist in string theory that would violate it.

This conjecture can be generalized to p-form gauge theories, coupled to $(p-1)$-brane states, schematically summarized by an action

$$S_p = -\int \frac{1}{2g^2} F_{p+1} \wedge *F_{p+1} - T_{p-1} \int_\Sigma d^p x \sqrt{-g^{\text{ind}}} + \mu_{p-1} \int_\Sigma A_p \,, \tag{3.98}$$

with field strength $F_{p+1} = dA_p$, and T_{p-1} is the tension, μ_{p-1} is the charge and Σ is the world volume of the electrically charged object. A special case is the one with $p = 0$: A zero-form gauge potential is an axion, and its electrically charged object is an instanton (the magnetically charged ones are cosmic strings). The 'mass' is the euclidean action and the gauge coupling is the inverse axion decay constant.

The weak gravity conjecture applied to the axion case says that there should exist an instanton with euclidean action S_E that induces shift symmetry breaking effects that preserve a discrete periodicity $2\pi q$ such that

$$S_E \lesssim q M_P / f_a \,. \tag{3.99}$$

By definition, the axion decay constant is set by requiring $q \in \mathbb{Z}$, and with minimal charge $q_0 = \pm 1$. If the theory breaks supersymmetry instantons generate contributions to the scalar potential of the form

$$V_{q,S_E}(a) = M_P^4 e^{-S_E} \left(1 - \cos(qa)\right) + \mathcal{O}(e^{-2S_E}) \,. \tag{3.100}$$

For such a contribution to the potential to be calculable and small (i.e. for the *dilute instanton approximation* to hold), the euclidean instanton action must be larger than unity. But this means that the harmonic induced by the WGC fulfilling instantons oscillates on sub-Planckian distances in field space,

$$\frac{\Delta\phi_a}{M_P} \sim \frac{f_a}{q M_P} \lesssim S_E^{-1} < 1 \,. \tag{3.101}$$

This alone does not mean that axion inflation with non-perturbatively generated axion potentials, i.e. *natural inflation*, is impossible simply because so far we have not said

that the WGC fulfilling instanton gives a dominant contribution to the scalar potential. The *strong form* of the conjecture asserts that it always does. Then, if true, single field natural inflation is not possible. The particle version of this form would be that the *lightest* charged particle fulfills the WGC (as does the electron). This conjecture has been extended from its most basic form. For instance, B. Heidenreich, M. Reece and T. Rudelius have conjectured a *sub-lattice* WGC which states that on a finite index sub lattice of the full charge lattice, each site must be filled with an object satisfying the WGC [64].

3.8.3 The distance conjecture

Another conjecture that is relevant for phenomenology is called the *distance conjecture* [30]. It states that as parametrically large super-Planckian *geodesic* distances d (measured in Planck units) are traversed in field space, there should exist an infinite tower of states indexed by some integer n with masses m_n that becomes light as

$$m_n(d) \leq m_n(0) e^{-c\,d}\,, \quad \text{as } d \longrightarrow \infty\,, \tag{3.102}$$

for some $\mathcal{O}(1)$ coefficient c. A strong form of this conjecture is that the onset of this exponential behavior starts at $d \gtrsim \mathcal{O}(1)$ [205]. Part of this statement is easily understood from simple KK reduction. There always exists a *radion* R (the volume modulus) that measures the physical size of the compactification space. Its kinetic term takes the form

$$\mathcal{L}_{kin} \propto -M_P^2 (\partial R)^2 / R^2\,, \tag{3.103}$$

so the canonically normalized field is $\phi_R \propto M_P \log(R/R_0)$. The masses of the tower of KK modes (there always exists at least a spin 2 tower) scale with an inverse power of R, so in terms of the canonically normalized field distance the formula (3.102) holds toward large R, i.e. towards weak coupling. In string theory, we expect to always be able to continue past strong coupling points into a different weak coupling regime. For simple circle compactification this is just the decompactification limit of the T-dual theory for which there exists a KK tower again. From the point of view of the original theory the tower that comes downs as the small volume locus is approached is seen as a tower of wrapped extended objects, i.e. winding modes associated to the 'fundamental' string or extended solitons such as Dp branes. At least at the level of $\mathcal{N} = 2$ CY moduli spaces, this conjecture seems to always hold [206, 207] (but an interesting potential counter example has recently been proposed where flux backreaction is strong and yet controlled by warped throat solutions [208]).

Chapter 4

dS uplifts: A 4d point of view

In the last section we have introduced the KKLT proposal for generating de Sitter vacua. Of course many different proposals have followed since KKLT made their proposal (see e.g. [209] for a review), but many of them are fairly analogous to KKLT: The logic is to first engineer parametrically controlled AdS vacua, and as a second step perturb them with a SUSY breaking object such as an anti-brane to *lift* the vacuum energy above zero. This general procedure is called a *de Sitter uplift*. In this section we would like to comment on the robustness of such constructions in general although for definiteness we will employ the original KKLT construction.

First, we will devise a short list of options how the KKLT proposal and other ideas for uplifting *could* fail (section 4.1, based on [3, 5]). This will be based purely on four-dimensional EFT logic, and can be divided into two simple possibilities:

(a) The uplift cannot be decoupled from the stabilization sector to a sufficient degree leading to substantial amounts of backreaction on the Kähler modulus as the vacuum energy increases. As the vacuum energy approaches zero from below, backreaction on the volume modulus is sufficiently strong so that the neglect of perturbative corrections to the scalar potential becomes questionable, or the internal space decompactifies altogether. We will call this *uplift-flattening*.

(b) The standard parametrization of uplifts within four-dimensional EFT is correct but the appropriate values of parameters that would realize a successful uplift to dS are not available in string theory.

Second, we will give simple concrete examples where such problems actually *do* arise (section 4.2). In fact the flattening of the uplift seems to be the physical reason why there are no-go theorems against dS vacua in the classical corners of string theory [89–103], so it is tempting to suggest that uplift-flattening is the generic problem that makes

constructions of de Sitter vacua difficult in string theory. Many of these no-go theorems can be derived by considering higher-dimensional *tadpole cancellation conditions*. We will introduce this concept as a computational tool, demonstrate its usefulness in simple six-dimensional flux compactifications, and relate the no-go theorems with uplift-flattening. Finally, we will argue that a class of models in which uplift-flattening could be refuted based purely on four-dimensional EFT arguments are in surprising tension with the weak gravity conjecture for axions 4.3. This is based on [4].

4.1 Uplifts and decoupling

As promised we will give a brief account of simple reasons why a given uplift idea might fail from the four-dimensional EFT point of view. Throughout we will employ KKLT as an example but the general ideas readily carry over also to other types of uplifts.

The $10d$ warped throat supergravity solution that is used for the uplift is dual to the Klebanov-Strassler (KS) gauge theory [162], so one should be able to equivalently describe the anti-brane as a state of the KS gauge theory that breaks supersymmetry spontaneously rather than explicitly [188]. If this is the case, it has been argued that at very low energies the only degrees of freedom are the nilpotent goldstino multiplet S (nilpotency means $S^2 = 0$, for constrained superfields see e.g. [210]) and the volume modulus T [211–219]. The following Kähler- and superpotential have been proposed,

$$K = -3\ln(T + \bar{T} - S\bar{S}) \quad \text{and} \quad W = W_0 + Ae^{-aT} + a_0^2\mu^2 S\,. \tag{4.1}$$

Here, a_0^2 parametrizes the strength of supersymmetry breaking and is again identified with the warp factor at the tip of the throat, while μ is related to the unwarped tension of the anti-$D3$ brane as $|\mu|^4 \sim T_3$.

In deriving the scalar potential one should treat S as a usual chiral multiplet and in the end set $S = 0$. For real parameters W_0 and A, the scalar potential reads

$$V(\rho) = \frac{aAe^{-a\,\mathrm{Re}(T)}}{6\,\mathrm{Re}(T)^2}\left[Ae^{-a\,\mathrm{Re}(T)}(a\,\mathrm{Re}(T)+3) + 3W_0\cos\left(a\,\mathrm{Im}(T)\right)\right] + a_0^4\,\frac{|\mu|^4}{12\,\mathrm{Re}(T)^2}\,. \tag{4.2}$$

The reason why this form is expected to be correct comes from taking different limits: In the limit of vanishing non-perturbative stabilization $A \longrightarrow 0$ one recovers the known runaway potential that is easily read off of the anti-brane DBI+CS action, while in the limit $\mu \longrightarrow 0$ one recovers the supersymmetric KKLT potential. The corresponding potential is simply the sum of the (would-be) runaway $\overline{D3}$ potential and the (would-be) supersymmetric KKLT potential.

Interestingly, within this description of the $\overline{D3}$-induced uplift, the potential energy of the $\overline{D3}$ adds on top of the negative potential at the supersymmetric AdS minimum to good approximation until a maximum uplift of about $\delta V \sim 2 \times |V_{AdS}|$. Beyond that we encounter run-away behavior $\mathrm{Re}(T) \to \infty$.

However, we would like to point out that these limits alone do not uniquely determine the form of the superpotential. The ambiguity arises from the $4d$ point of view by the appearance of a new mass scale due to non-perturbative Kähler moduli stabilization, which was zero in the classical dimensional reduction from which the form of the $\overline{D3}$-induced scalar potential was deduced. For example, a superpotential of the form

$$W = W_0 + b \cdot S + A(1 + c \cdot S)e^{-aT} \,. \tag{4.3}$$

is fully consistent with the classical limit $A \longrightarrow 0$ and the SUSY restoring limit $(b, c) \longrightarrow 0$. The special choice $c = 0$, $b = a_0^2\mu^2$ corresponds to the standard KKLT potential. The other extreme case would be to set $b = 0$. In this case the SUSY KKLT vacuum corresponding to $c = 0$ cannot be uplifted to de Sitter for any value of c. From this one can see one way how a potential dS uplift proposal *might* fail:

> *As the uplift potential (say the IR warp factor)*
> *is dialed up, the volume modulus is pushed to*
> *larger volumes in such a way that it is impossible* (4.4)
> *to reach positive vacuum energy (see Figure 4.1).*

There is a discussion in the literature [3, 5, 220, 221] about the question to what extent a non-suppressed coefficient c would endanger the existence of de Sitter vacua in this way. So it is worthwhile to expand on what we have said so far. First, only by choosing a *large enough* coefficient c (where 'large enough' will be made more precise below) one matches the qualitative behavior described in 4.4.

One may convince oneself that for $b = 0$ there are no de Sitter vacua and the coupling proportional to c mediates a large back-reaction on the Kähler modulus T as the vacuum energy increases. For any given value of c we can turn on the small coefficient b until eventually one reaches positive vacuum energy and the *additional* backreaction on T that comes from turning on $b \neq 0$ is small. However, if c is *large enough* to provide the dominant part of uplifting to zero vacuum energy, such de Sitter 'vacua' cannot be trusted within a truncation to the leading order Kähler potential. This is because contributions to the scalar potential from perturbative corrections to the Kähler potential can no longer be argued to be negligible. The model thus implements (4.4) within the margin of theoretical error if c is large enough in the sense we now

describe.

The T-dependent scalar potential around the supersymmetric AdS KKLT vacuum ($b = c = 0$) looks rather similar to the potential around the non-supersymmetric AdS point with $c \neq 0$. Naively one might believe that both vacua are equally well controlled. However, perturbative corrections to the scalar potential in the form of volume suppressed α' corrections are expected to take the form

$$\delta V \sim e^{K_0}|W_0|^2 \times \frac{1}{(T+\bar{T})^p}\,, \quad p > 0\,, \tag{4.5}$$

where K_0 is the tree level Kähler potential and $p = 1/2, 1, 3/2$ for corrections arising at $\mathcal{O}(\alpha'), \mathcal{O}(\alpha'^2), \mathcal{O}(\alpha'^3)$, respectively.

The vacuum energy of the supersymmetric vacuum is given by

$$V_{\rm SUSY} = -3e^K|W|^2 \approx -3e^{K_0}|W_0|^2\,. \tag{4.6}$$

Therefore, in the large volume regime one may neglect α' corrections to the scalar potential expanded around the SUSY vacuum. In contrast, the vacuum energy of the non-supersymmetric vacuum is parametrized by the value of c. If c is large enough, one has $|V_{\rm SUSY}| \gg |V_{\cancel{\rm SUSY}}| \sim \delta V$ and perturbative corrections start to give important contributions to the scalar potential. We plot both scalar potentials with their margins of theoretical error in Figure 4.1.

Hence, there is a critical value of c,

$$c_{crit} \equiv \gamma A\sqrt{a\log(|A/W_0|)}\,, \tag{4.7}$$

with numerical coefficient γ, such that

- for $c \ll c_{crit}$ using the b-coupling to provide the missing uplift to zero vacuum is fully controlled,

- while for $c \gtrsim c_{crit}$ any dS minima created by adding the b-coupling are in the regime $|V_{\rm SUSY}| \gg |V_{\cancel{\rm SUSY}}| \sim \delta V$ where the scalar potential cannot be reliably predicted.

Thus the model illustrates the effect of unsuppressed exponential couplings but is by far not the unique one to do so. It can easily be generalized to a whole class of models that all exhibit the effect that we are after. One simply starts with the superpotential of eq. (4.3), transforms the classical warp factor b into the Kähler potential by a field

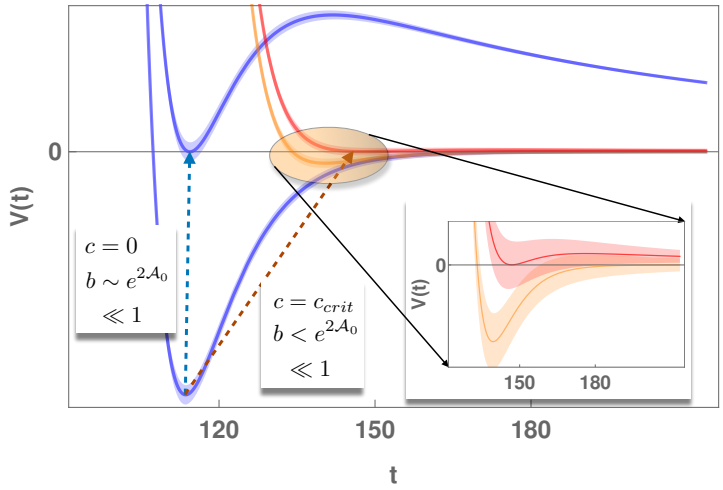

Figure 4.1: In blue: the supersymmetric KKLT potential with an uplift by $c = 0$, $b \sim e^{2\mathcal{A}_0}$ in blue error bars in orange (assuming $p = \frac{1}{2}$). Perturbative corrections to the potential are negligible. In orange: partially uplifted scalar potential in the limit of marginally controlled uplift from the c-coupling at $c = c_{crit}$ (and $\gamma \approx 6$), $b = 0$. Perturbative corrections can no longer be neglected. In red: uplifted scalar potential in the limit of marginally controlled uplift from the c-coupling at $c = c_{crit}$, and additional b-coupling uplift with $0 < b < e^{2\mathcal{A}_0}$. Again, perturbative corrections can not be neglected, and putative dS minima in this regime are not trustworthy. In all cases we have chosen $A = 1$, $a = 0.1$, $W_0 = -10^{-4}$.

redefinition of S as in ref. [222],

$$K = -3 \ln \left(T + \bar{T} - \frac{S\bar{S}}{b^2} \right) \tag{4.8}$$

and then replaces

$$b^2 \to b^2 + b(f(T + \bar{T})e^{-aT} + c.c.) + g(T + \bar{T})e^{-2a\mathrm{Re}T} \,, \tag{4.9}$$

with some power law functions f and g. For the special choice $f = \bar{c} \in \mathbb{C}$, $g = |c|^2$ we obtain the simple parametrization that was originally proposed.

However, for example, we may instead choose $g(T+\bar{T}) = g_1 \cdot (T+\bar{T})$, with $g_1 \in \mathbb{R}_+$ and $f = 0$. One can check that the bound analogous to (4.7) reads

$$g_1 \gtrsim g_{crit} \equiv \gamma' a^2 A^2 \,, \tag{4.10}$$

again for some numerical constant γ'. Such a model implements the behavior of (4.4) in a less contrived way than the one we started with. We hope it has become clear that there are many ways to write down 4d supergravity models that would reproduce known supersymmetric or classical limits of KKLT without guaranteeing the existence

of controlled dS vacua.

Now that we have explained what such unsuppressed exponential couplings *could* in principle do, let us comment on the question whether or not we should expect this correction to be present. Of course it would be rather surprising if the term proportional to c were completely absent for the following reason. It is well known that if mobile $D3$ branes are present, the coefficient of e^{-aT} in the superpotential is a holomorphic function of their position moduli [105, 223]. Their moduli space is therefore lifted by the same non-perturbative effects that lead to volume stabilization. If $D3$-branes modify the coefficient of the exponential term in the superpotential, one would expect an $\overline{D3}$-brane to do so as well.

This modification is relevant if the coefficient c that multiplies the gaugino condensate is *not* further suppressed by whatever mechanism suppresses the scale of the classical uplift, in this case warping. In general this is an extra requirement that isn't obviously satisfied automatically once the classical uplift energy has been tuned small. Without a 4d EFT reason that would forbid such a term we should, in the spirit of Wilsonian effective field theory, consider the option that it is sizable.

The observation that the fate of the uplift depends so heavily on the details of the moduli potential can be embedded into a more systematic 4d approach towards a de Sitter uplift: For simplicity let us consider only a single light field ϕ in the 4d effective field theory. We may Taylor-expand the scalar potential (before any attempt to uplift it) around its SUSY *AdS* (or Minkowski) minimum,

$$V = V_0 \left(1 + \sum_{n=2}^{\infty} c_n \left(\frac{\phi}{M_P} \right)^n \right) , \tag{4.11}$$

assuming canonical normalization. Including an uplift means adding a further term $\delta V(\phi)$ to the potential,

$$V(\phi) \longrightarrow \tilde{V}(\phi) = V(\phi) + \delta V(\phi) \, . \tag{4.12}$$

As the uplift breaks supersymmetry it is usually not easy to engineer a lot of structure into the functional form of the uplift potential, but we really only have good control over the overall scale ϵ in

$$\delta V(\phi) = \epsilon \sum_{n=0}^{\infty} \tilde{c}_n \left(\frac{\phi}{M_P} \right)^n , \tag{4.13}$$

i.e. we should assume that $\tilde{c}_n = \mathcal{O}(1)$. Then, it is easy to see that after the uplift is

included, the scalar field ϕ obtains a non-trivial vev

$$\frac{\phi_{\text{uplift}}}{M_P} \approx -\frac{\epsilon}{V_0}\frac{\tilde{c}_1}{c_2} + ... \sim (c_2)^{-1} \sim \frac{|V_0|}{m_{\phi,0}^2 M_P^2} + ... \,, \tag{4.14}$$

which is a good approximation if $\phi \ll M_P$, i.e. when

$$|V_0| \ll m_{\phi,0}^2 M_P^2 \,. \tag{4.15}$$

Here we have inserted the physical mass $m_{\phi,0}$ of the lightest modulus in the supersymmetric minimum and we have used that for an uplift to dS we need $\epsilon \gtrsim -V_0$. This relation is important to keep in mind: Given a supersymmetric model where all moduli are stabilized in a way that the bound (4.15) is satisfied, success or failure of an uplift depends *only* on the ability to find a meta stable SUSY breaking source of overall scale $\epsilon \sim -V_0$. In other words, the fate of the uplift depends only on the ability to tune a single Wilson coefficient, or the uplift is controllable using 4d EFT.

In general, the value of the scalar potential of the uplift is given by

$$V_{\text{uplift}} = (V_0 + \epsilon) + \epsilon\, \mathcal{O}\left(\frac{|V_0|}{m_{\phi,0}^2 M_P^2}\right) . \tag{4.16}$$

So, if the bound (4.15) is violated the fate of the uplift depends on the precise values of a large (or even infinite) tower of Planck suppressed operators as all the higher order corrections in (4.16) become important. This *UV sensitivity* is completely analogous to the one of large field inflation. This analogy is actually quite sharp: On the one hand, models that satisfy the bound (4.15) are not UV sensitive but are necessarily tuned just as the predictions of models of small field inflation depend only on a handful of Wilson coefficients at the price of tuning. On the other hand, models that violate the bound are far more generic but UV sensitive just as large field inflation is.

In the simplest examples of moduli stabilization such as KKLT [41], but also the *large volume scenario* (LVS) the value of the cosmological constant at the supersymmetric minimum V_{AdS} is tied to the mass-scale m_ϕ of the lightest modulus

$$|V_{AdS}| \sim m_\phi^2 M_{\rm P}^2 \,, \tag{4.17}$$

and the uplift is UV sensitive. This is precisely the reason why it was possible to write down bottom-up consistent modifications of KKLT that would prevent the uplift. But it is important that this does not imply that the uplift really fails! There are in our opinion (at least) two ways to make progress:

(a) Investigate the viability of more elaborate schemes of moduli stabilization that allow to decouple the scales V_0 and $m_{\phi,0}$ from one another at the price of tuning. In section 4.3 we will follow this road, and encounter a generic tension with the strong form of the weak gravity conjecture for axions.

(b) Pin down the form of the uplift in models such as KKLT using UV input. In the following sections we will argue that uplift flattening is a *generic* problem in higher-dimensional theories and that this problem is the reason why classical no-go theorems exist in various classical corners of string theory. We will postpone a discussion of KKLT to chapter 5. We will find that uplift flattening does *not* occur in KKLT, but rather that there is a generic difficulty to engineer the right scale of the uplift potential.

4.2 Higher-dimensional tadpole cancellation

In view of the surprising difficulties that one usually encounters when trying to construct consistent de Sitter vacua in string theory we find it worthwhile to investigate if the seemingly conspirative modification of the 4d effective field theory that would prevent an uplift to de Sitter indeed occurs.

Clearly the cleanest way to do this would be to derive the correct effective field theory of the volume modulus together with all its SUSY breaking states from first principles. Due to the obvious difficulty of this approach we will opt for another one. Instead of deriving the off-shell 4d effective potential we will confront it with 10d tadpole cancellation constraints. Before turning to the 10d setup we will outline general aspects of compactifications that will later be relevant and use them to explain the use of tadpole cancellation constraints and their interpretation. We will use a simple 6d toy model to develop a physical intuition that we believe is applicable in general.

When a $D = 4 + d$-dimensional theory is compactified on some d-dimensional internal manifold the effective 4d potential is easily obtained from the higher-dimensional Einstein equations. One simply starts with the most general metric ansatz

$$ds^2 = e^{2A(y)}\tilde{g}^4_{\mu\nu}(x)dx^\mu dx^{nu} + g_{mn}(y)dy^m dy^n\,, \tag{4.18}$$

with warp factor e^{2A}, 4d coordinates x^μ and internal coordinates y^m. The higher-dimensional Einstein equations read

$$M^{D-2}\left(R_{MN} - \frac{1}{2}g_{MN}R\right) = T_{MN}\,, \tag{4.19}$$

with D-dimensional Planck mass M, and stress energy tensor $T_{MN} = -\frac{2}{\sqrt{-G}}\frac{\delta S^{\text{matter}}}{\delta G^{MN}}$. The effective 4d potential is then determined in terms of the internal curvature R_d and the higher-dimensional stress energy tensor as

$$V \cdot M_{\text{P}}^{-4} = \mathcal{V}_w^{-2} M^d \int d^d y \sqrt{g} e^{4A} \left(-\frac{1}{4M^D} T_\mu^\mu - \frac{1}{2M^2} R_d \right) , \tag{4.20}$$

where M is the D-dimensional Planck mass, $\mathcal{V}_w = M^d \int d^d y \sqrt{g} e^{2A}$ is the warped volume and $M_{\text{P}}^2 = M^2 \mathcal{V}_w$ is the $4D$ Planck mass. Four-dimensional vacua of the higher-dimensional theory correspond to local minima of this potential which encodes the value of the cosmological constant as well as the scalar mass-spectrum.

In many instances it is hard to determine this potential in full explicitness but the possible values it can take are severely constrained due to higher-dimensional tadpole cancellation conditions. For example, we can take the trace over the D-dimensional Einstein equations and insert the solution for R back into them. The result is the trace reversed Einstein equations

$$M^{D-2} R_{MN} = T_{MN} - \frac{1}{D-2} T g_{MN} \, . \tag{4.21}$$

Then, we can trace over the four-dimensional components, and insert the warped ansatz of eq. (4.18) to obtain

$$\tilde{\nabla}^2 A = \frac{1}{4} e^{-2(1+k)A} \tilde{R}_{4d} - \frac{M^{-(D-2)}}{D-2} \left(\frac{D-6}{4} T_\mu^\mu - T_m^m \right) , \tag{4.22}$$

where $\tilde{\nabla}^2$ is the scalar Laplacian associated with the fiducial metric $\tilde{g}_{mn} \equiv e^{2kA} g_{mn}$, and $k \equiv 4/(D-6)$. As the l.h. side is a total derivative, the integral over the compact internal manifold with measure $\sqrt{\tilde{g}_{mn}}$ must vanish. Thus, also the integral over the r.h. side vanishes, giving the tadpole cancellation condition

$$0 \stackrel{!}{=} \int d^d y \sqrt{g} \left(e^{(4k-2)A} \tilde{R}_{4d} - e^{4kA} \frac{4M^{-(D-2)}}{D-2} \left[\frac{d-2}{4} T_\mu^\mu - T_m^m \right] \right) . \tag{4.23}$$

It is important to keep in mind this expression holds only *at the minimum* of the scalar potential (4.20) [91, 156]. This has several consequences: for example, if a higher-dimensional source contributes an energy momentum tensor δT_{MN} with

$$\frac{d-2}{4} \delta T_\mu^\mu - \delta T_m^m < 0 \, , \tag{4.24}$$

it does *not* imply that the $4D$ cosmological constant decreases when the source is

included. This is because a critical point of the scalar potential is in general not a critical point of the combination of the stress energy tensor that enters the tadpole integrand. Therefore, as an extra source with small stress energy δT_{MN} is added to a stabilized setup the background stress energy tensor T_{MN} reacts at *linear order* in the perturbation rather than only at quadratic order. Although this means that tadpole bounds have to be interpreted carefully one can derive powerful statements from it. For example, in the absence of *any* source violating (4.24) the $4D$ vacuum energy can never be positive. Using this, one may show that if one only allows for p-form fluxes with $1 \leq p \leq D-1$ and localized objects of positive tension and co-dimension ≥ 2, de Sitter solutions are ruled out [90]

This means that in many interesting examples the tadpole bound encodes subtle back-reaction effects that correspond to the correction terms in (4.16): whenever a source of positive higher-dimensional energy is turned on that satisfies the condition (4.24) in a compactification that is (fully) stabilized by sources that also satisfy it, the higher order corrections in (4.16) must conspire to keep the overall potential energy negative.

4.2.1 A simple example: Freund-Rubin compactification

We will now demonstrate this behavior using the well known Freund-Rubin compactification. This is a $6D$ theory compactified on a S^2, with the S^2 stabilized by 2-form fluxes [224]. The $6D$ action is

$$S_6 = \frac{M^4}{2} \int \left(*R_6 - \frac{1}{2} F_2 \wedge *F_2 \right) , \tag{4.25}$$

with a 2-form field-strength $F_2 = dA_1$. The equations of motion/Bianchi identity are

$$R_{MN} = \frac{1}{2} F_{MP} F_N{}^P - \frac{1}{8} g_{MN} |F_2|^2 , \quad dF_2 = 0 = d * F_2 . \tag{4.26}$$

These admit a solution where the $6D$ geometry is a product $AdS_4 \times S^2$ and the S^2 is threaded by N units of 2-form flux $F_2 = \frac{N}{2q} \omega_2$. Here, ω_2 is the volume form of the S^2 normalized to $\int_{S^2} \omega_2 = 4\pi$ and q is the $U(1)$ charge of the particle that couples electrically to A_1 with smallest charge. The S^2 radius is fixed at

$$L_0^2 = \frac{3N^2}{32q^2} , \tag{4.27}$$

and in agreement with the tadpole bound (4.23) the $4D$ vacuum energy reads

$$V \cdot M_{\rm P}^{-4} = -(12\pi M^4 L_0^4)^{-1}\,, \tag{4.28}$$

and is manifestly negative.

One may try to uplift the four-dimensional vacuum energy by adding a number N_3 of three-branes of positive tension T_3 smeared over the internal two-sphere. Clearly they are a source of energy density and therefore the expectation is that they give rise to an increase of the four-dimensional vacuum energy. However, their stress energy tensor satisfies $(D-6)T^\mu_\mu - 4T^m_m = 0$ and so there is no new contribution to the integrand of eq. (4.20). Even without knowing the full solution to the 6d equations of motion we can solve the tadpole bound for the 4d vacuum energy as a function of the a priori unknown size of the two-sphere:

$$V \cdot M_{\rm P}^{-4} = \mathcal{V}^{-2} M^2 \int d^2y \sqrt{g^{S^2}} \frac{(-2|F_2|^2)}{16M^2} = -\frac{M^2 L_0^2}{12\pi M^6 L_1^6}\,, \tag{4.29}$$

where L_0 is given in eq. (4.27) and L_1 is the adjusted length-scale of the two-sphere. From eq. (4.29) one follows that no matter how much three-brane tension is added, the vacuum energy cannot increase beyond zero but at most approaches zero from below. In this simple 6d case one can of course do better and from the internal Einstein equations determine L_1 as a function of the three-brane tension:

$$L_1^2 = (1-\mathcal{T}_3)^{-1} L_0^2\,, \quad \text{with} \quad \mathcal{T}_3 \equiv \frac{N_3 T_3}{4\pi M^4}\,. \tag{4.30}$$

Plugging this into eq. (4.29) we see that indeed the vacuum energy approaches zero from below as we increase the three-brane tension $\mathcal{T}_3 \longrightarrow 1$. The higher order terms conspire to prevent an uplift to de Sitter. In the limit $\mathcal{T}_3 = 1$ the S^2 decompactifies. Note that as predicted by (4.14) the expansion parameter that controls back-reaction is given by $\delta V/m_{KK}^2 M_{\rm P}^2$ because the KK-scale is the mass of the lightest degree of freedom.

One might be concerned that three-branes and two-form fluxes share an intrinsic property making them unsuitable uplifting ingredients because they appear with the wrong sign under the integral of (4.23). Of course, if one *only* included these ingredients in the compactification de Sitter solutions would be ruled out [90]. But as we now demonstrate it is enough to include also a positive 6d c.c., or equivalently a five-brane of tension $T_5 \equiv \mathcal{T}_5 M^6$, for an uplift to de Sitter *by three-branes or fluxes* to be possible (see also [225] for related conclusions). In this case the size of the 2-sphere is bounded

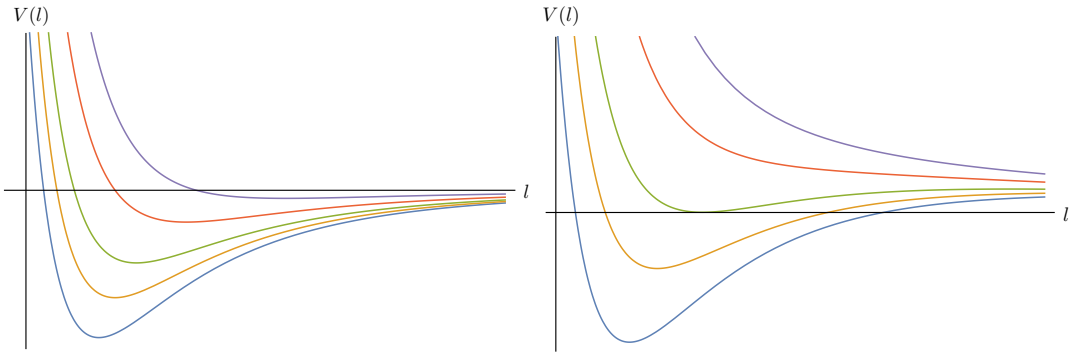

Figure 4.2: Left: Scalar potential $V(l)$ for the S^2 volume modulus in the case without a $6D$ c.c. for $n = 25$ flux units and different values of the dimensionless three-brane tension $\mathcal{T}_3$: $\mathcal{T}_3 = 0$ in blue, $\mathcal{T}_3 = 0.1$ in yellow, $\mathcal{T}_3 = 0.21$ in green, $\mathcal{T}_3 = 0.4$ in red and $\mathcal{T}_3 = 0.6$ in purple. As can be seen, the more energy density, the higher the vacuum energy, but the flattening prevents the minimum to go above zero. Right: Scalar potential $V(l)$ for the S^2 volume modulus in the case with a $6D$ c.c. for $n = 25$ flux units, $\mathcal{T}_5 = 0.004$ and different values of the dimensionless three-brane tension $\mathcal{T}_3$: $\mathcal{T}_3 = 0$ in blue, $\mathcal{T}_3 = 0.1$ in yellow, $\mathcal{T}_3 = 0.21$ in green, $\mathcal{T}_3 = 0.4$ in red and $\mathcal{T}_3 = 0.6$ in purple. This time it is also possible to find de Sitter minima once enough three-brane tension has been added.

from above via

$$l^2 \equiv L^2 M^2 = \frac{1-\mathcal{T}_3}{\mathcal{T}_5}\left(1-\sqrt{1-\frac{3}{16}\frac{\mathcal{T}_5 n^2}{(1-\mathcal{T}_3)^2}}\right) \leq \frac{1-\mathcal{T}_3}{\mathcal{T}_5}\,, \tag{4.31}$$

where $n \equiv N \cdot M/q$ corresponds to the number of two-form flux quanta. When $n^2 > n^2_{max} \equiv \frac{16}{3}\mathcal{T}_5^{-1}(1-\mathcal{T}_3)^2$ the sphere decompactifies. Thus, in order for the curvature of the sphere to be sub-planckian we need both a small positive 6d cosmological constant $\mathcal{T}_5 \ll 1$ as well as a large number of two-form fluxes n. Since the 6d cosmological constant violates the bound (4.24) dS solutions are now possible. Somewhat amusingly, even allowing for a positive 6d cosmological constant, lower-dimensional de Sitter vacua appear only in a very narrow window of parameter space. We give a concrete example in Figure 4.2.

The scalar potential reads (plotted in Figure 4.2)

$$V \cdot M_{\rm P}^{-4}(l) = \frac{1}{16\pi}\left[\frac{4\mathcal{T}_5}{l^2} + \frac{n^2}{4l^6} - \frac{4(1-\mathcal{T}_3)}{l^4}\right]. \tag{4.32}$$

Evidently, the flattening behavior observed for the case with only fluxes and three-branes does not exhibit any intrinsic feature of branes and fluxes but is merely a property of the simple scheme of moduli stabilization. By including a positive 6d

c.c. it was possible to somewhat decouple the lightest modulus mass from the value of the 4d c.c. such that a small perturbation could uplift it to $4D$ de Sitter.

Unfortunately, in string theory we never have to our disposal a positive ten-dimensional cosmological constant which is part of the reason why finding lower-dimensional dS vacua is hard.[1] More so, no-go theorems of the type we have described can be derived in many circumstances [89–103]. We will soon consider an improved version of the statement valid for type IIB string theory.

Whenever no-go theorems of this type can be derived, the following situations occur,

(a) Uplifts from AdS fail due to the flattening of the uplift as depicted in Figure 4.2.

(b) Uplifts from Minkowski fail because there is a massless mode that is sent into a runaway behavior by the uplift.

In both cases, the failure to stabilize moduli with sufficient 'rigidity' is at the heart of the problem. This is why the problem of moduli stabilization is a crucial one in all phenomenological applications of higher-dimensional theories, in particular string theory.

4.3 Racing through the swampland

The KL racetrack vs the weak gravity conjecture

Before we turn to the tadpole cancellation constraints in KKLT we would like to remark on a possible relation between the weak gravity conjecture and the difficulty to uplift to de Sitter [4]. For this purpose we focus on uplift ideas where it can be argued that the classical uplift can be tuned almost arbitrarily small but there is little control over the cross couplings between uplift sector and stabilization sector. In this case substantial uplift flattening would be expected according to what we said in section 4.1. We specifically leave out the anti-brane uplift because we will argue later that there is no substantial uplift flattening in this case. However, all uplifts via bulk fields such as complex structure moduli that directly enter the one-loop Pfaffian in the non-perturbative superpotential would suffer from this problem. In this case, one may simply accept this and look for more elaborate schemes of moduli stabilization in which the mass of the lightest modulus can be parametrically decoupled from the depth of the scalar potential at the supersymmetric starting point. In this case dS uplifts would be generic as we have explained in the preceeding section. In this section we focus on

[1] Note that massive type IIA and the $SO(16)^2$ heterotic string have positive vacuum energy, but due to a dilaton dependence this does not lead to ten-dimensional dS vacua.

the only scheme of moduli stabilization that we are aware of that accomplishes this goal. This is a variant of the old *racetrack* idea and was proposed by Kallosh and Linde (KL) [104].

The reason why one might expect the WGC to constrain de Sitter uplifts is that the volume modulus is always accompanied by an axionic partner which in type IIB string theory can be thought of as the integral of the RR 4-form over the 4-cycle associated to the volume modulus. Given a stabilization mechanism for the volume we can hence inquire about the axion potential and whether or not it is consistent with the WGC. For instance, for the KKLT scenario with a single gaugino condensate $W = W_0 + A\exp(-aT)$, one can easily verify[2] that

$$f/M_p \sim (a\mathrm{Re}(T))^{-1}\,, \tag{4.33}$$

up to order 1 coefficients. If higher order non-perturbative corrections to the superpotential

$$\delta W_{\text{n.p.}}^{\text{higher order}} = \sum_{n=2}^{\infty} A_n e^{-naT} \tag{4.34}$$

can be ignored we necessarily have $a\mathrm{Re}(T) > 1$ and hence f is sub-Planckian.[3] Therefore, an axion potential generated via gaugino condensation remains sub-Planckian for very much the same reason as an instanton generated potential does. We conclude that the WGC should apply to gaugino condensation.

The KL racetrack scheme of moduli stabilization [104] is defined via the tree level Kähler potential as given in (3.81) but with a superpotential

$$W = W_0 + AN_1\exp(-2\pi T/N_1) + BN_2\exp(-2\pi T/N_2)\,, \tag{4.35}$$

Usually one assumes that this arises from gaugino condensation for the product gauge group $SU(N_1)\times SU(N_2)$ with gauge coupling set by the modulus T (and no massless matter is assumed) [226]. W.l.o.g in the following we take $N_1 \geq N_2$.

Splitting the real and imaginary (axionic) parts of $T = t + i\phi$, the scalar potential $V(t,\phi)$ reads [227]

$$\begin{aligned} V(t,\phi) =& V_0(t) + V_1(t)\cos\left(\frac{2\pi}{N_1}\phi - \alpha\right) + V_2(t)\cos\left(\frac{2\pi}{N_2}\phi - \beta\right) \\ &+ V_{1-2}(t)\cos\left(\frac{2\pi(N_2-N_1)}{N_1N_2}\phi - \gamma\right)\,, \end{aligned} \tag{4.36}$$

[2]The Kähler metric reads $g_{T\bar{T}} = \frac{3M_P^2}{(T+\bar{T})^2}$, so at fixed $\mathrm{Re}(T)$ the canonically normalized axion is $\phi_c \sim M_P\mathrm{Im}(T)/\mathrm{Re}(T)$. The factor of a^{-1} appears because the superpotential is invariant under $T \longrightarrow T + 2\pi i a^{-1}$.

[3]Such corrections would likely be generated by higher-derivative corrections of the gauge theory [172].

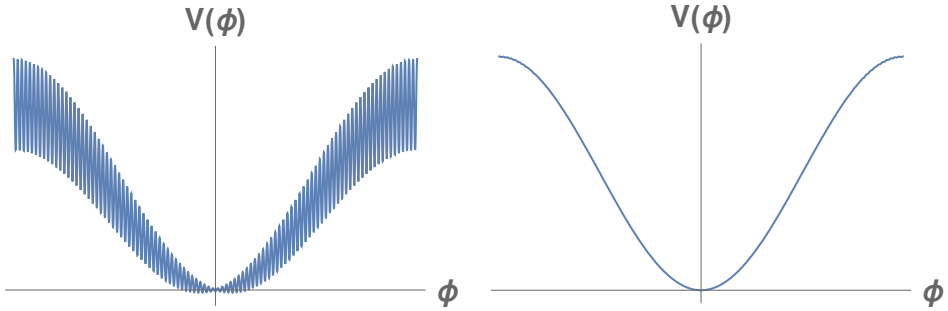

Figure 4.3: We depict the axion potential on a fundamental domain for choices of parameters $(A, B, N_1, N_2) = (1, -1.1, 100, 99)$. On the left, we plot the potential at $t = 1.6t_0$ (where t_0 is defined in eq. 4.43) and one notices that the small wavelength oscillations play a dominant role. On the right we plot the potential for $t = t_0$ where the long wavelength oscillation dominates. For $t = t_0$ the axion decay constant in Planck units is $\mathcal{O}(100)$.

with $\alpha \equiv \arg(A\overline{W_0})$, $\beta \equiv \arg(B\overline{W_0})$, $\gamma \equiv \arg(A\overline{B})$ and coefficient functions

$$V_0(t) = \frac{1}{2t^2}\left(2\pi N_1|A|^2\left(1+\frac{2\pi t}{3N_1}\right)e^{-\frac{4\pi}{N_1}t} + 2\pi N_2|B|^2\left(1+\frac{2\pi t}{3N_2}\right)e^{-\frac{4\pi}{N_2}t}\right), \tag{4.37}$$

$$V_1(t) = 2\pi\frac{|AW_0|}{2t^2}e^{-\frac{2\pi}{N_1}t}, \quad V_2(t) = 2\pi\frac{|BW_0|}{2t^2}e^{-\frac{2\pi}{N_2}t}, \tag{4.38}$$

$$V_{1-2}(t) = \frac{|AB|}{2t^2}\left(2\pi(N_1+N_2)+\frac{8\pi^2 t}{3}\right)e^{-(\frac{2\pi}{N_1}+\frac{2\pi}{N_2})t}. \tag{4.39}$$

There exist three distinct harmonics for the axion ϕ with coefficient functions V_1, V_2 and V_{1-2} (see figure 4.3 for a plot of the scalar potential along the axion direction in field space). The last one sets the axion periodicity to $\frac{N_1N_2}{N_1-N_2}$ so the axion decay constant is

$$f_\phi/M_P \sim \frac{N_1N_2}{N_1-N_2}\cdot\frac{1}{t}. \tag{4.40}$$

This is super-Planckian when $N \equiv N_1 \approx N_2$ and $N < t < N^2$, a regime where the fractional instanton expansion would seem to be under control. The strong form of the WGC would require the short wavelength wiggles with amplitudes V_1 and V_2 to dominate over the long wavelength harmonic with amplitude V_{1-2}. This has to be true all the way down to the breakdown of the fractional instanton expansion at $t \sim N$. In order for the KL racetrack to be compatible with this (i.e. $V_{1-2} \overset{!}{\leq} \max(V_1, V_2)$) we would have to demand that

$$|W_0| \gtrsim \min(|A|, |B|). \tag{4.41}$$

However, we would find such a strict bound on the flux number W_0 very surprising, in particular because it seems that no such bound can be derived for the single gauge group

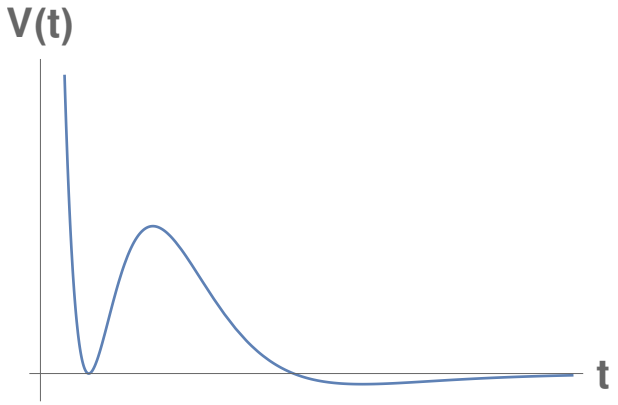

Figure 4.4: The KL racetrack potential along the radial direction in the limit of (4.42), with the same choices of parameters as in figure 4.3. One notices a Minkowski minimum at $t = t_0 \approx 150$ and a standard KKLT minimum at large values of t.

KKLT model. Moreover, explicit studies of the classical flux superpotential indicate that the genericity arguments for a small W_0 are valid [43].

The phenomenological virtue of this model is that when the parameters A, B, W_0 of the model are tuned to satisfy

$$-W_0 = AN_1\left(-\frac{A}{B}\right)^{N_2/(N_1-N_2)} + BN_2\left(-\frac{A}{B}\right)^{N_1/(N_1-N_2)}, \tag{4.42}$$

there exists a SUSY Minkowski minimum at

$$2\pi T_0 \equiv 2\pi(t_0 + i\phi_0) = \frac{N_1N_2}{(N_1-N_2)}\log\left(-\frac{B}{A}\right), \tag{4.43}$$

and the mass of the volume modulus is finite (a corresponding minimum exists also when the relation (4.42) is detuned, but the vacuum will be of Anti-de-Sitter type.). See figure 4.4 for a plot of the scalar potential in this limit. In order for this minimum to lie at positive volume it is required that $|B| > |A|$.

If the tuning of eq. (4.42) holds, one has that

$$|W_0| \leq |Ae^{-2\pi t_0/N_1}| + |Be^{-2\pi t_0/N_2}|. \tag{4.44}$$

It then follows that if we take $|A/B| = \mathcal{O}(1)$ the WGC-type bound (4.41) cannot possibly be satisfied unless $e^{-2\pi t_0/N} \gtrsim \mathcal{O}(1)$. In other words, the Minkowski racetrack minimum would lie outside the (naive) validity of the controlled fractional instanton expansion. As a consequence, *if* the WGC holds, the racetrack minimum cannot be used as a controlled starting point for uplifting to de Sitter space, i.e. there is no

parametricallly controlled de Sitter uplift within the racetrack scheme.

Of course this conclusion can be evaded if hierarchically different values for the one-loop Pfaffians A, B are chosen [4, 221, 228]. Given that the two seven-brane stacks have to wrap the same divisor class we do not find it reasonable to expect that this can be done.

There are of course many ways how this tension could get resolved. The simplest options would be that

(a) the required gauge theory configurations cannot be engineered in the type IIB corner of string theory, or

(b) the bound of (4.41) holds, or

(c) the strong form of the WGC does not hold in the axion context.

Interestingly, we will find explicit racetrack type superpotentials in chapter 6 where a condition on the ranks of the gauge groups involved prevents long wavelength dominant contributions to the scalar potential. This can be taken as evidence for option (a).

Whichever option holds, we observe an unexpected relation between the ability to realize parametrically large axion decay constants and parametrically controlled de Sitter vacua. Success or failure to achieve the former, possibly determines the viability of the latter.

4.4 Discussion

We would like to give a brief summary of the conclusions we have drawn in this section. There are, in our opinion, two conceivable mechanisms by which potential uplifts do de Sitter vacua could fail in principle. The first, we have dubbed uplift flattening: In the simplest schemes of moduli stabilization it is not possible to engineer a hierarchy between the depth of the scalar potential in its AdS minimum V_{AdS}, and the mass scale of the lightest modulus m_T, concretely

$$m_T^2 M_P^2 \sim |V_{AdS}| \,. \tag{4.45}$$

This is true for simple Freund Rubin compactifications as illustrated in section 4.2.1, but also for the KKLT mechanism.[4] In this case, given only the existence and metastability of (4d-)spacetime filling objects that can perturb the AdS vacuum and raise the vacuum energy (an uplift), the existence of de Sitter vacua is not guaranteed due to the possibly significant backreaction of the uplift on the lightest modulus. Significant

[4] It also holds for the *Large volume scenario* (LVS) [75].

uplift flattening means that backreaction is sufficiently strong to preclude the existence of positive energy solutions in regimes of parametric control over the given perturbative expansion scheme.

Whether or not this occurs depends on the detailed *form* of the uplift potential as a function of the lightest modulus. We have argued in section 4.1 that the form of the uplift potential is generically hard to compute reliably or even estimate from an EFT point of view due to its sensitivity to a large number of Planck suppressed operators. Together with the observation that uplift flattening occurs rather generically in higher-dimensional setups, we are motivated to investigate whether or not this happens also in KKLT. Investigating this from a ten-dimensional perspective will be the focus of the next section.

The obvious way to return the question of the existence of de Sitter uplifts back into the realm of four-dimensional EFT would be to engineer alternative schemes of moduli stabilization where the relation (4.45) can be violated parametrically. In section 4.3 we have considered the KL racetrack scheme which is designed to do precisely this, and have shown that such models are in tension with the weak gravity conjecture for axions. So, rather surprisingly, we have given circumstantial evidence that realizing parametrically controlled de Sitter vacua from this line of thought may be difficult for the same reasons that engineering models of large field inflation is difficult (or even impossible) in string theory.

Chapter 5

KKLT in ten dimensions

In this section we would like to put the KKLT proposal under scrutiny. We focus on this model because 1) it is one of the most studied models [105–107], 2) it is consistent with the general expectation that de Sitter vacua are non-generic and at best metastable [41, 165, 229, 230], and 3) there is evidence that the tuning requirements that make it non-generic can actually be met [152]. So in many ways the KKLT proposal offers concrete evidence for the existence of a large landscape of dS solutions in string theory. This evidence is hard to simply dismiss. Nevertheless, the question whether or not de Sitter vacua exist in string theory has received renewed attention, in part due to the recently proposed conjecture that such solutions cannot exist as a matter of principle [79–81, 231]. Put in the jargon of the field, it has been claimed that all EFTs coupled to gravity with de Sitter (dS) vacua reside in the swampland.[1] This criterium is referred to as the *no-dS conjecture*.

The *no-dS conjecture* is in stark contrast to what an effective field theorist would conclude from the observational fact that our universe is undergoing accelerated expansion. Arguably, from her point of view a tiny but positive cosmological constant would be the simplest and most natural fit to the data, see section 2.3. In particular, she would conclude that in the far future the geometry of our universe will be well described by a patch of de Sitter (dS) space.

Some of the evidence for the conjecture comes from the fact that in various classical corners of string theory there exist no-go theorems against the existence of de Sitter solutions [89–96, 98, 99, 101–103, 232]. In section 4.2.1 we have related these to the problem of uplift flattening. Due to the existence of these theorems (and also due to the Dine Seiberg problem [165]) it seems natural to expect that if any dS solutions exist at all, they will require a competition between classical and quantum effects that

[1] For a discussion regarding the form and viability of the conjecture we refer the reader to references [82–88].

cannot be mapped into a purely classical effect by any duality transformation.[2] As we have explained, the KKLT proposal is one of the most convincing proposals of such a kind.[3]

There are two interesting mutually exclusive but as far as we can judge today equally likely options that we will entertain in the following:

(a) The KKLT proposal withstands sufficiently many lines of attack so that it can be established beyond reasonable doubt. In this case the no-dS conjecture would clearly be wrong and the dS landscape of string theory should be continued to be explored in as many ways as possible.

(b) The KKLT proposal turns out to be inconsistent, and we should focus on devising new ideas for realizing dS vacua in string theory or alternative ideas of dark energy.

We will do so by considering a ten-dimensional consistency requirement in the form of a tadpole cancellation very similar to the one of eq. (4.23). This is obtained as follows. Starting from the 10d Einstein frame action of Type IIB supergravity, and the usual warped ansatz (4.18) for the ten-dimensional metric and five form one may combine the trace reversed Einstein equations with the five-form Bianchi identity to obtain [40, 91]

$$\tilde{\nabla}^2\Phi^- = \tilde{R}_{4d} + \frac{e^{2A}}{4\mathrm{Im}(\tau)}|G_3^-|^2 + e^{-6A}|\partial\Phi^-|^2 + e^{2A}\frac{\Delta^{loc}}{2\pi}\,, \tag{5.1}$$

where

$$G_3^{\pm} \equiv (*_6 \pm i)G_3\,, \quad \Phi^{\pm} \equiv e^{4A} \pm \alpha\,, \quad \text{and} \quad \Delta^{loc} \equiv \frac{1}{4}\left(T_m^m - T_\mu^\mu\right)^{\mathrm{loc}} - T_3\rho_3^{\mathrm{loc}}\,. \tag{5.2}$$

Here G_3 is the complexified three-form $F_3 - \tau H_3$. Moreover T^{loc}_{MN} and $T_3\rho_3^{loc}$ are the stress energy tensor and $D3$-brane charge density of localized objects. We may integrate this equation over the internal manifold to obtain the tadpole cancellation condition [40, 91]

$$0 = \int d^6y\sqrt{g^6}\left[e^{6A}\tilde{R}_{4d} + e^{8A}\frac{\Delta}{2\pi} + |\partial\Phi^-|^2\right]\,, \tag{5.3}$$

[2] Whether this intuition is correct remains an open question as there are many proposals for purely classical meta-stable dS solutions, e.g. supercritical strings [233–235], type IIB string theory compactified on orientifolded products of Riemann surfaces [225], proposals involving combinations of 05 and 07 planes [236], and more recent work in the context of F-theory [237–239]. On the type IIA side there were studies of dS on (generalizations of) twisted tori [83, 93, 99, 240]. These proposals should be further scrutinized in the future as they form possible counter examples against the no-dS conjecture.

[3] For further dS proposals involving balancing classical and quantum effects, see e.g. [73, 75, 83, 225, 241–251], and the recent review [209]. For a new perspective on dark energy from F-theory, see [252, 253].

where

$$\Delta \equiv 2\pi \frac{|G_3^-|^2}{4\mathrm{Im}(\tau)} + \Delta^{loc} \,. \tag{5.4}$$

From this expression it follows immediately that as long as all localized sources satisfy $\Delta^{loc} \geq 0$ the unique classical Minkowski solutions of type IIB string theory are the ISD solutions,

$$G_3^- = \Delta^{loc} = \tilde{R}_{4D} = \Phi^- = 0 \,. \tag{5.5}$$

Under the same assumption de Sitter solutions are ruled out as well. Therefore, a necessary condition for realizing $4D$ de Sitter solutions is that there exists at least one localized object that satisfies $\Delta^{loc} < 0$.[4] This is a remarkably strong condition because it is the opposite of a BPS bound for D3 branes. In particular sources like D7 branes, O7 planes (which carry induced D3 brane charge), D3 branes, O3 planes and even anti-D3 branes are not enough to violate the bound [40].

Another problem is the appearance of eight powers of the warp factor in front of the contribution from fluxes and localized sources. This means that the stress energy of all warped uplifts essentially does not contribute to the tadpole. Whatever object sources the stress energy that would allow the four-dimensional vacuum energy to lift beyond zero, it must be a *bulk* effect. This is slightly discouraging because the whole idea of warped de Sitter uplifts is that some isolated strongly warped supersymmetry breaking source contributes to the four-dimensional vacuum energy while its effects on the bulk geometry are under parametric control. In KKLT, the only ISD breaking source aside from the uplift itself is the physics of gaugino condensation on the stack of seven-branes. Thus the stress energy induced by gaugino condensation must not only change significantly, it must even change in sign. This is the tadpole cancellation problem as formulated in [5]. Whether or not this problem can be solved is still a subject of discussion. We will argue shortly, based on [1] and in agreement with ref. [110], that this problem is solved dynamically in KKLT. However, the authors of [111] disagree with us, and we will explain where the disagreement lies.

Before we get there we must introduce what is known about the ten-dimensional view on *four-dimensional* gaugino condensation. A conjectured $10d$ lift of the KKLT vacua goes via the by-hand insertion of a non-vanishing $\mathbb{C}$-valued expectation value for the seven-brane gaugino bilinear $\langle\lambda\lambda\rangle \neq 0$ [106]. This conjecture is supported for instance by the fact that the non-perturbative superpotential for $D3$-brane position moduli can be accurately computed from ten-dimensional supergravity via the insertion of the gaugino bilinear as a classical source term [106], and the ability to find supersymmetric

[4] Alternatively one might invoke quantum corrections that *cannot* be described in 10d.

backreacted solutions [107]. We will call this the $10d$ *gaugino condensation conjecture*. If this conjecture were true in general, roughly speaking, it would allow to constrain de Sitter vacua of the KKLT type much via the same tools that are used to exclude purely classical solutions in [90].

We will explain that while the $10d$ gaugino condensation conjecture can be argued to be valid for the supersymmetric KKLT AdS vacua, it will fail to hold once supersymmetry is broken: there are additional contributions to the $10d$ tadpole equation that can be shown to arise from demanding *only* the consistency of the supersymmetric KKLT construction and that become relevant *only* once SUSY is broken. We find it unlikely that these contributions can be captured by a *local* $10d$ action in the above sense. Moreover, under the assumption that arbitrarily strongly warped regions exist in the flux compactification, these new contributions can be shown to precisely cancel the tadpole once SUSY is broken by a warped uplift. In total, we see no reason to expect the failure of KKLT uplifts from considerations of $10d$ tadpole cancellation alone. This can be interpreted as evidence that the problem of uplift-flattening does *not* occur in KKLT. In other words, the coefficient c in eq. (4.3) is sufficiently suppressed to play no role.

We will later contrast this result by arguing that a successful uplift to a dS vacuum via warped uplifts, and more so to a SUSY breaking AdS vacuum, is highly constrained by the geometrical consistency requirement that the warped throat used for the uplift must fit into the bulk CY.[5] We will show that the simplest examples with a single Kähler modulus can hardly satisfy this basic requirement. For the case of many Kähler moduli we speculate that this problem becomes even more severe unless the compactification manifold satisfies additional geometrical properties that we believe are highly non-generic. It would be very interesting to investigate whether such geometries can be realized in a controlled manner. This is, in our opinion, a good physical motivation to try to understand in detail the *geometriy* of CY compactifications beyond topological data. We will however not pursue this goal in this thesis.

5.1 Non-perturbative $D3$ brane potentials: Three perspectives

As explained in the introduction of this section we would like to study moduli stabilization and the uplift to de Sitter space from a ten-dimensional point of view. The classical part is extremely well understood: the Gukov-Vafa-Witten superpotential can be lifted to the ten-dimensional three-form potential of type IIB supergravity [154] and the 4d SUSY conditions that determine the three-form fluxes to be of Hodge-type $(2,1)$

[5] Note that the same requirement has been used to constrain inflationary models in [46] and in [230] to argue that the flux superpotential must be tuned extremely small for the KKLT construction to be consistent.

lift to the 10d SUSY conditions of *B-type* [254]. Furthermore the 4d scalar potential is minimized precisely when the 10d equations of motion are solved by the imaginary self-dual (ISD) solutions of [40].

An analogous $10d \longleftrightarrow 4d$ correspondence of Kähler moduli stabilization is somewhat harder to establish, both conceptually as well as technically:[6] the dynamical origin of the exponential superpotential is the condensation of gaugino bilinears in the 4d Yang Mills gauge theory (or euclidean $D3$ brane instantons). The scale below which the condensation occurs is the dynamical scale of the gauge-theory which typically lies far below the Kaluza-Klein scale. So, how can it be possible even in principle to include the non-perturbative effects in a higher-dimensional setup? First, there certainly exist geometrical setups compatible with the correct order of scales: an example is that of an 'anisotropic' Calabi-Yau space in which the four-cycle that the 7-branes wrap is much smaller than the typical length-scale of the transverse space [106]. In this case the non-perturbative scale of gaugino condensation can lie far below the Kaluza-Klein scale of the four-cycle and at the same scale as the transverse Kaluza-Klein scale. Another situation of this type corresponds to a compactification space that is equipped with warped throats of significant warping. In this case the warped Kaluza-Klein scale lies exponentially below the bulk KK-scale.

There has however been crucial progress in recent years in establishing a far more general ten-dimensional picture of gaugino condensation [105, 107, 255, 256]. First, note that if a mobile $D3$-brane is present, the classical moduli space of the world-volume scalars is identified with the compactification geometry. In the absence of non-perturbative effects there is no potential for the world-volume scalars and the internal geometry can thus be probed at arbitrarily small energies. Thus, even if non-perturbative effects generate a potential for the world volume scalars one may probe the (quantum-deformed) internal flux geometry at scales that lie far below the KK-scale. With this in mind one should be able to effectively describe the SUSY vacua with non-perturbative Kähler stabilization by the $10D$ equations of motion, corrected at order of the value of the gaugino condensate $\langle\lambda\lambda\rangle$.

As there is a controversy surrounding the question how the ten-dimensional lift of KKLT vacua should be implemented [1, 110, 111] we will now explain in some detail what kinds of technical problems one encounters and how these are resolved eventually.

Remarkably, as a first step, the following simple prescription advocated by the authors of [106, 107, 257] captures an important set of physical effects,

(a) Start with the classical type IIB supergravity together with the DBI and CS actions for localized objects to quadratic order in the worldvolume fermions.

[6] We thank Arthur Hebecker for discussions concerning this point.

(b) Solve the 10d equations of motion, assuming a non-vanishing $\mathbb{C}$-valued expectation value of the fermion bilinear associated with the 7-brane gaugino.

According to this the relevant term in the action is [107]

$$S_{D7} \supseteq \int_{M_{10}} \pi \delta_D^{(0)} e^{\phi/2} e^{-4A} \frac{\langle \bar{\lambda}\bar{\lambda} \rangle}{16\pi^2} G_3 \wedge *\Omega + c.c. \,, \tag{5.6}$$

and acts as a source for the three-form fluxes. Here, $\lambda\lambda \equiv \mathrm{Tr}\left(\bar{\lambda} P_L \lambda\right)$.

Clearly this approach needs to be justified. For this it is useful to consider the non-perturbative lifting of the $D3$-brane position moduli space. Recall that in classical ISD solutions D3 branes can be moved without energy cost. The 4d fields that parameterize their positions in the internal CY are massless moduli. However, at the non-perturbative level, this moduli space is generically *lifted*. This effect can be studied from three different angles. First, there is the standard 4d perspective. In compactifications with both $D7$-branes and mobile $D3$ branes the gauge-kinetic function f of the $D7$ brane gauge theory depends on the open-string $D3$-brane position moduli z^i via one-loop open string threshold corrections which were calculated explicitly e.g. for a $T^4/\mathbb{Z}_2 \times T^2$ orientifold of type IIB string theory [223]. Then, at low energies the non-perturbative superpotential

$$W \propto e^{\frac{2\pi i}{N} f(z^i, T)} \tag{5.7}$$

is a function of the position moduli z^i which obtain a non-trivial scalar potential.

Interestingly, the open string calculation of [223] was perfectly matched with a dual *closed string* calculation in [105, 255] as follows: the position moduli y^i_{D3} of a mobile $D3$ brane treated as a classical localized source in the 10d supergravity enter the electro static equation for the warp factor

$$\nabla^2_y e^{-4A}(y; y_{D3}) \propto \frac{\delta^6(y - y_{D3})}{\sqrt{g_{CY}}} - \rho_{\text{background}} \,, \tag{5.8}$$

where the second term is a background charge that integrates to one. One can then show that

$$\nabla^2_{y_{D3}} e^{-4A}(y; y_{D3}) \propto \frac{\delta^6(y - y_{D3})}{\sqrt{g_{CY}}} - \frac{1}{\mathrm{Vol}(CY)} \,, \tag{5.9}$$

in other words the perturbation of the warp factor at some position y induced by a moving D3 brane does not depend on the form of background charge distribution.

Then, the divisor volume which is identified with the imaginary part of the seven-

brane gauge kinetic function is

$$\mathrm{Im}(f(y_{D3})) \sim \mathrm{Vol}(D) = \int_D d^4y\sqrt{g}\, e^{-4A}(y_{D3})\,. \tag{5.10}$$

Therefore, it acquires a dependence on the position moduli of the D3 branes. This can be solved explicitly in toroidal setups and reproduces the open string calculation of [223]. Again, the $D3$ brane position moduli enter the non-perturbative superpotential in the 4d EFT[7]. The closed string computation is particularly useful as it readily generalizes beyond simple toroidal orientifolds. In particular, for a stack of N $D7$-branes with holomorphic embedding equation $h(z) = 0$, the gauge-kinetic function $f(T, z^i)$ of the $D7$ gauge theory depends on the volume modulus T as well as the $D3$ position moduli z^i [105]

$$f(T, z) = iT + \frac{\ln h(z)}{2\pi i}\,. \tag{5.11}$$

Using this dependence of the gauge-kinetic function f on the $D3$-brane position moduli one may determine the $4D$ non-perturbative superpotential to be

$$W \propto e^{\frac{2\pi i}{N}f} = h(z)^{1/N} e^{-\frac{2\pi}{N}T}\,. \tag{5.12}$$

So far, classical 10d physics has been used only to obtain the gauge kinetic function (5.11) while the generation of a non-trivial potential for the $D3$-brane moduli is dealt with entirely within 4d effective field theory. Crucially these two steps could be separated because the classical back-reaction of a $D3$-brane on the classical $10D$ supergravity solution is finite. This is clearly not the case for an $\overline{D3}$-brane due to the run-away instability. Thus it is desirable to have in hand a quantum corrected 10d action. The key points were derived in [106], where the authors analyzed the generation of a non-trivial *classical* potential for the position moduli of $D3$ branes in ISD backgrounds subject to harmonic non-ISD perturbations. Crucially, it was shown that in conifold backgrounds every superpotential that can be written down for the position moduli in the 4d effective field theory can be matched to a non-compact *classical* 10d supergravity solution such that the scalar potentials coincide. Hence, the quantum corrected 10d supergravity that reproduces the correct $D3$ brane potential is only corrected by terms that are localized away from the warped throat. Such localized terms are necessary because the entirely uncorrected type IIB supergravity equations do not admit static non-ISD perturbations in the compact case due to the *global* constraints of [40].

It is tempting to identify these localized terms with the terms in the 7-brane action that are proportional to the gaugino bilinear $\langle\lambda\lambda\rangle$. Indeed, in non-compact examples,

[7] See also [258, 259] for a derivation using the language of *generalized complex geometry*.

the superpotential (5.12) can be encoded in so-called *series I* three-form flux

$$(G_3)_{i\bar{j}\bar{k}} \propto \langle\lambda\lambda\rangle \nabla_i \nabla_l \mathrm{Re}(\ln h(z)) g^{l\bar{m}} \bar{\Omega}_{\bar{m}\bar{j}\bar{k}}\,, \tag{5.13}$$

where Ω is the holomorphic three-form of the Calabi-Yau [106]. This is precisely the perturbation of three-form fluxes that is sourced by the fermionic bilinear term in the action (5.6).

5.2 UV ambiguities and their resolution

Guided by the non-trivial consistency check that we just described one would conclude that the relevant details of non-perturbative volume stabilization are indeed captured by the classical 10d supergravity action assuming a non-vanishing expectation value of the gaugino bilinear. However, this prescription is not fully complete which can be seen from the gravitational backreaction sourced by the action (5.6):

In a compact setup the action (5.6) sources a three-form profile [107]

$$G_3 = G_3^{\mathrm{h}} + e^{-\phi/2} \frac{\langle\lambda\lambda\rangle}{16\pi^2} \frac{dX}{2\pi}\,, \quad X \equiv \partial_i \Psi g^{i\bar{j}} \bar{\Omega}_{\bar{j}\bar{k}\bar{l}} \frac{1}{2} d\bar{z}^{\bar{k}} \wedge d\bar{z}^{\bar{l}}\,, \tag{5.14}$$

where Ψ is a scalar Green's function,

$$\nabla^2 \Psi = 2\pi \left(\delta(\Sigma) - \frac{\mathrm{Vol}(\Sigma)}{\mathrm{Vol}(CY)} \right)\,, \quad \text{and} \quad \bar{\partial} X = \frac{1}{2} (\nabla^2 \Psi) \overline{\Omega}\,. \tag{5.15}$$

Here, $\delta(\Sigma)$ is the scalar delta function that localizes on the four-cycle that the 7-branes wrap, and G_3^{h} are the harmonic ISD background fluxes. Note that in a non-compact setup Ψ is identified with $\mathrm{Re}\log h(z^i)$, where $h(z^i) = 0$ is the holomorphic embedding equation of the 7-brane divisor [106].

But this implies that the ISD component of G_3

$$G_3^{\mathrm{ISD}} \propto \bar{\partial} X \propto (\nabla^2 \Psi) \overline{\Omega} \supset \pi \delta(\Sigma) \overline{\Omega}\,, \tag{5.16}$$

contains a term proportional to $\delta(\Sigma)\overline{\Omega}$, and plugging this back into the action (5.6) produces an ill-defined term proportional to $\delta(0)$. Clearly this is a short-distance singularity. With a hard UV cutoff Λ_{UV}, it scales as [5, 260]

$$S_{on-shell} \sim |\langle\lambda\lambda\rangle|^2 \delta_\Sigma(0) \sim |\langle\lambda\lambda\rangle|^2 \Lambda_{\mathrm{UV}}^2 + \text{finite}\,. \tag{5.17}$$

Similarly, the contribution of gaugino condensation to the stress energy tensor diverges in this way. Early attempts to quantify the contribution of gaugino condensation to

the tadpole cancellation conditions of type IIB supergravity were based on cutting off divergent integrals of this sort at the string scale. However, this procedure could not be reconciled with the existence of four-dimensional de Sitter vacua of the KKLT type [5, 260].

It was then understood in ref. [108, 109] that the action of eq. (5.6) must in fact be completed to a *perfect square*. We quote from ref. [110], slightly modified to apply to the CY threefold case, and adapted to our conventions,

$$S_{IIB} \supset -\pi \int_{M_{10}} d^{10}x\sqrt{-G}e^{\phi} \left| G_3 - P\left(e^{-\frac{\phi}{2}}\delta(\Sigma)\frac{\langle\lambda\lambda\rangle}{16\pi^2}\overline{\Omega}\right)\right|^2 , \tag{5.18}$$

where $P(\cdot)$ projects onto the space of closed forms.[8] This result is valid for constant dilaton. Expanding this to linear order in $\langle\lambda\lambda\rangle$ one recovers the kinetic term of G_3 and the action (5.6) so the equations of motion for G_3 are left unaltered. This modification of the action was motivated by analogies to the heterotic string [108] and shown to be required by supersymmetry [109]. Since[9]

$$P(\delta(\Sigma)\overline{\Omega}) = P\left(\left(\frac{1}{2\pi}\nabla^2\Psi + \frac{\mathrm{Vol}(\Sigma)}{\mathrm{Vol}(CY)}\right)\overline{\Omega}\right) = \frac{dX}{2\pi} + \frac{\mathrm{Vol}(\Sigma)}{\mathrm{Vol}(CY)}\overline{\Omega}\,, \tag{5.19}$$

it follows that the on-shell action becomes

$$S_{\text{on-shell}} = -\pi \int_{M_{10}} d^{10}x\sqrt{-G}e^{\phi}\left(\left|G_3^{\mathrm{h}} - e^{-\frac{\phi}{2}}\frac{\langle\lambda\lambda\rangle}{16\pi^2}\frac{\mathrm{Vol}(\Sigma)}{\mathrm{Vol}(CY)}\overline{\Omega}\right|^2 - G_3^{\mathrm{h}}\cdot(*_6\overline{G_3^{\mathrm{h}}})\right)\,, \tag{5.20}$$

which is manifestly finite and vanishes in the limit of infinite transverse volume and ISD harmonic fluxes.[10] Upon dimensional reduction to four dimensions, this action can be compared with the classical four-dimensional $\mathcal{N}=1$ supergravity action and there is perfect agreement [108].

5.3 10d vs 4d supersymmetry conditions

Now we would like to perform a further consistency check of the proposed lift of KKLT to ten dimensions. In ref. [107] it was shown at linear order in the expectation value of the gaugino condensate that the background sourced by the quantum corrected action (5.6) maintains supersymmetry in a non-compact setup. Here, following [5], we wish to comment on a generic obstruction against unbroken supersymmetry in a compactly

[8] The *Hodge decomposition theorem* states that every p-form ω_p can be uniquely decomposed into a harmonic, exact and co-exact component, $\omega_p = \omega_p^h + d\alpha_{p-1} + d^\dagger\beta_{p+1}$. $P(\omega_p) \equiv \omega_p^h + d\alpha_{p-1}$.

[9] We have that $(\nabla^2\Psi)\overline{\Omega} = 2\bar{\partial}X = dX - (\partial - \bar{\partial})X = dX + \frac{1}{8}d^\dagger(\partial\Psi\wedge\overline{\Omega})$.

[10] The second term comes from the CS term of the 10d action.

embedded scenario: Away from the location of the divisor Σ, the Hodge type $(0,3)$ component of the three-form fluxes as perturbed by the gaugino bilinear read

$$G_3|_{(0,3)} = \left(g^{0,3} + e^{-\phi/2} \frac{\langle \lambda\lambda \rangle}{16\pi^2} \frac{\nabla^2 \Psi}{4\pi} \right) \overline{\Omega} \overset{y \notin \Sigma}{=} \left(g^{0,3} - e^{-\phi/2} \frac{\langle \lambda\lambda \rangle}{32\pi^2} \frac{\mathrm{Vol}(\Sigma)}{\mathrm{Vol}(CY)} \right) \overline{\Omega} \,, \tag{5.21}$$

where $g^{0,3} = const.$ is the $(0,3)$ piece of the harmonic background fluxes. This means that for generic values of $g^{0,3}$ and the gaugino bilinear, the $(0,3)$ component of the three-form fluxes are non-vanishing. This however signals supersymmetry breaking as gauginos on a probe brane (say a D3 brane) are known to obtain a soft mass term from such fluxes [261–263]. Thus, a necessary condition for unbroken supersymmetry is that the classical and the quantum term cancel against each other,

$$g^{0,3} \overset{!}{=} e^{-\phi/2} \frac{\langle \lambda\lambda \rangle}{32\pi^2} \frac{\mathrm{Vol}(\Sigma)}{\mathrm{Vol}(CY)} \,. \tag{5.22}$$

This condition should be understood as a constraint on the value that the gaugino condensate takes, and can be compared with the four-dimensional F-term equation of the volume modulus. To facilitate this, first it is necessary to rescale the gaugino according to $\lambda \longrightarrow \mathrm{Vol}(CY)^{3/4}\lambda$ in order to obtain canonical normalization in four-dimensional Einstein frame. Second, the flux number W_0 that appears in the 4d KKLT description is given by

$$W_0 = e^{\phi/2} \frac{\int G_3^{\mathrm{harmonic}} \wedge \Omega}{||\Omega||} = g^{0,3} ||\Omega|| e^{\phi/2} = g^{0,3} e^{\phi/2} \sqrt{\mathrm{Vol}(CY)} \,, \tag{5.23}$$

so the ten-dimensional SUSY condition can be written as

$$W_0 \overset{!}{\sim} \mathrm{Vol}(CY)\mathrm{Vol}(\Sigma)\langle \lambda\lambda \rangle \,. \tag{5.24}$$

The four-dimensional F-term equation reads

$$D_T W \sim -2\pi A e^{-2\pi T/N} - \frac{3}{T+\bar{T}} W_0 \overset{!}{=} 0 \,, \tag{5.25}$$

which upon identifying $T + \bar{T} \sim \mathrm{Vol}(\Sigma)$, and

$$\langle \lambda\lambda \rangle = 32\pi^2 e^{K/2} A e^{-2\pi T/N} \sim \mathrm{Vol}(CY)^{-1} A e^{-2\pi T/N} \,, \tag{5.26}$$

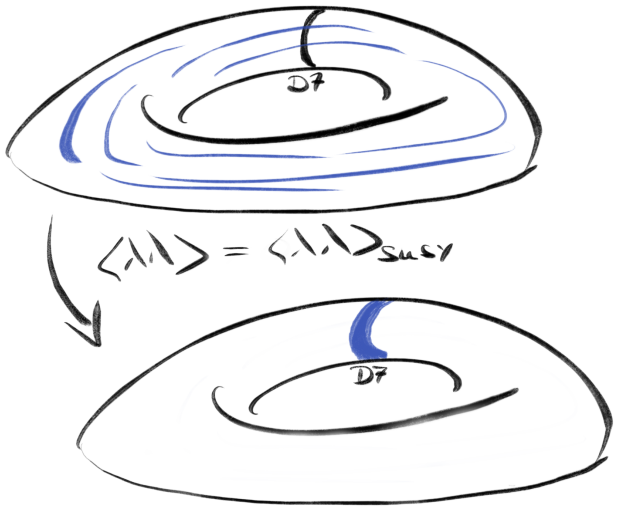

Figure 5.1: Cartoon of backreaction of supersymmetric gaugino condensation on the ten-dimensional flux background. The Hodge type $(0,3)$ background flux (depicted in blue) is fully localized onto the seven-divisor.

indeed reproduces the ten-dimensional SUSY condition (5.24).[11] This means that we now understand from a ten-dimensional point of view the physical mechanism by which gaugino condensation can restore the classically broken supersymmetry in the bulk spacetime: It localizes the $(0,3)$ component of the three-form fluxes onto the divisor Σ so that branes probing the fluxed geometry away from the divisor see only fluxes of Hodge type $(2,1)$, compatible with unbroken SUSY on their worldvolume (see Figure 5.1), despite the fact that $[G_3] \notin H^{2,1}(X,\mathbb{Z})$.

Once a SUSY breaking source such as an anti-brane in a warped throat is introduced which pulls on the volume modulus, the relation between W_0 and $\langle\lambda\lambda\rangle$ will of course be detuned. This effect reintroduces a *bulk* $(0,3)$ component with strength set by the amount of SUSY breaking from the uplift that is seen by probe D-brane gauge theories.

5.4 Gaugino bilinears as classical sources?

In the previous sections we have seen that for many purposes it is remarkably consistent to treat quantum gaugino bilinears as effective classical source terms. We would now like to estimate the limitations of such a procedure. We consider the microscopic Lagrangian of 4d $\mathcal{N}=1$ pure $SU(N)$ Yang-Mills coupled to the Kähler modulus T,

[11] In comparing with eq. (3.78) the reader will notice a further factor $e^{K/2}$. As the superpotential is a section of a line bundle in supergravity, this factor is introduced to relate the holomorphic superpotential to a physical scale.

and the gravity multiplet. Our conventions are as in [145] such that the gauge kinetic term reads

$$e^{-1}\mathcal{L}_{gauge} = -\frac{1}{4}\mathrm{Re}(f_{AB}(T))F^A_{\mu\nu}F^{\mu\nu\,B} + \dots\,, \tag{5.27}$$

with gauge kinetic function $f_{AB} = \frac{T}{4\pi}\delta_{AB}$. We consider the tree-level Kähler as in eq. (3.81) and classical constant flux superpotential $W = W_0$. It is useful to define the composite glueball field S as

$$S \equiv \frac{\delta_{AB}\langle\bar{\lambda}^A P_L \lambda^B\rangle}{16\pi} \equiv \frac{\langle\lambda\lambda\rangle}{16\pi}\,. \tag{5.28}$$

We focus on the gaugino mass term and quartic gaugino interactions in the four-dimensional Lagrangian,

$$\begin{aligned}\mathcal{L}_\lambda =& -\frac{1}{16\pi}(T+\bar{T})^{-1/2}\overline{W_0}\mathrm{Tr}\left(\bar{\lambda}P_L\lambda\right) + h.c. - \frac{1}{48}\left(\frac{T+\bar{T}}{4\pi}\right)^2 |\mathrm{Tr}\left(\bar{\lambda}P_L\lambda\right)|^2 \\ &+ \frac{3}{64}\left(\frac{T+\bar{T}}{8\pi}\right)^2 \mathrm{Tr}\left(\bar{\lambda}\gamma_\mu\gamma_*\lambda\right)\mathrm{Tr}\left(\bar{\lambda}\gamma^\mu\gamma_*\lambda\right)\,.\end{aligned} \tag{5.29}$$

It is straightforward to check that the first line matches with the reduction of the 10d on-shell action. The second line is not yet reproduced by the 10d action as we have not included non-trivial expectation values for the (pseudo-)vector bilinear, but this is of no relevance here since we are interested in Lorentz invariant four-dimensional vacua.

Given the manifestly finite, and appropriately 'quantum corrected' ten-dimensional action described in section 5.2, the existence of a well-defined ten-dimensional lift of KKLT can be considered an established fact. One might hope that these vacua are described perfectly well by choosing an appropriate $\mathbb{C}$-valued expectation value for the gaugino bilinear and compute the backreaction. While this can be done without encountering singular behavior, we will now argue that this approach is not the correct one in general.

First, let us ask (within the 4d supergravity) what is the effective scalar potential $V(T,\bar{T},S,\bar{S})$ once we insert by hand a non-vanishing expectation value $S \neq 0$. Due to Lorentz invariance $\langle\mathrm{Tr}\left(\bar{\lambda}\gamma^\mu\gamma_*\lambda\right)\rangle = 0$. Clearly, we obtain

$$V_{\text{microscopic}}(T,\bar{T},S,\bar{S}) = \frac{(T+\bar{T})^2}{3}|S|^2 + \frac{S\,\overline{W_0} + h.c.}{(T+\bar{T})^{1/2}}\,. \tag{5.30}$$

Let us call this the *microscopic potential*. It is not obvious that this expression is physically meaningful at all. Really, we should have used the fact that the gauge theory is gapped, integrated out all of its degrees of freedom and determined the effective scalar

potential for the Kähler modulus. The effective superpotential below the mass scale of the gauge multiplet is given in eq. (3.80), so the scalar potential is given by the usual F-term scalar potential

$$\begin{aligned} V_{\text{effective}}(T,\bar{T}) &= e^K \left(g^{T\bar{T}} |D_T W|^2 - 3|W|^2 \right) \\ &= \frac{1}{3(T+\bar{T})} \left(1 + \frac{3}{2\pi \frac{\text{Re}(T)}{N}} \right) \left| 2\pi A e^{-2\pi T/N} \right|^2 + \frac{2\pi A e^{-2\pi T/N} \overline{W_0} + h.c.}{(T+\bar{T})^2} \,. \end{aligned} \tag{5.31}$$

This is the physically meaningful low energy effective potential. However, one notices immediately that in the limit of small 't Hooft coupling $\text{Re}(T) \gg N$, the low energy effective potential is actually the same as the microscopic potential upon plugging in the well known value for the gaugino condensate of eq. (3.78) (weighed by a supergravity normalization factor $e^{K/2}$)

$$S = 2\pi e^{K/2} A e^{-2\pi T/N} \,. \tag{5.32}$$

So we see that to good approximation the scalar potential is given by (minus) the classical action evaluated at the correct value of the gaugino bilinear. The strong gauge dynamics essentially only freezes the gauge degrees of freedom and sets the expectation value of the gaugino condensate. Up to these effects, the classical action seems to approximate the low energy effective potential very well. Therefore we expect that the approximate equality between effective and microscopic potential holds also when the $4d$ theory is lifted to the full $8d$ gauge theory on the seven-brane stack embedded into the $10d$ bulk. However, at this point we would like to emphasize the fact that

$$\partial_T V_{\text{effective}} \approx \partial_T V_{\text{microscopic}} + \frac{\partial S(T)}{\partial T} \partial_S V_{\text{microscopic}} \neq \partial_T V_{\text{microscopic}} \,, \tag{5.33}$$

so whenever derivatives of the scalar potential with respect to the Kähler modulus become relevant, special care is needed. In particular, this implies that all physical observables that are sensitive to derivatives of the scalar potential cannot be computed by treating gaugino condensates as local classical source terms in the $10d$ action. This is because the definition of T is non-local from the 10d perspective, and S varies exponentially with T. In section 5.5 we will show that $V'(T)$ indeed plays a crucial part in ten-dimensional tadpole cancellation requirements.

5.5 Gravitational backreaction: How to cancel a tadpole

We now wish to determine the contribution of gaugino condensation to the integrand of the 10d tadpole constraint. In other words we need to compute the stress energy

tensor of gaugino condensation. Let us first be open minded to what is the precise form of the $D7$-brane action that perturbs the GKP background. We only write

$$S^{D7}[G] = \int d^4x \int d^6y \sqrt{-G}\tilde{\mathcal{S}}[g_6](x,y)\,, \tag{5.34}$$

where $\tilde{\mathcal{S}}$ is some yet unspecified space-time dependent functional of the internal metric. We now restrict ourselves to trivial warping. The internal and external components of the stress energy tensor of $S^{D7}[G]$ read

$$T^{D7}_{\mu\nu} = g^4_{\mu\nu}\tilde{\mathcal{S}}[g_6](y)\,, \tag{5.35}$$

$$T^{D7}_{mn} = -\frac{2}{\sqrt{g_6}}\frac{\delta\mathcal{S}[g_6]}{\delta g_6^{mn}}\,, \quad \text{with} \quad \mathcal{S}[g_6] \equiv \int d^6y\sqrt{g_6}\tilde{\mathcal{S}}[g_6]\,. \tag{5.36}$$

We are now ready to evaluate the contribution to the tadpole coming from $S^{D7}[g]$.[12] We may expand the (inverse) internal metric g^{mn} in a complete set of symmetric two-tensors $\{S_i^{mn}\}$,

$$g^{mn} = \sum_i a_i S_i^{mn}\,, \tag{5.37}$$

with Fourier coefficients a_i. These can for instance be taken as eigen-functions of the linearized Einstein equations which corresponds to the usual KK-mode expansion. We take the completeness relation to be

$$\int_{CY} d^6y\sqrt{g_0}S_i^{mn}S_{j\,mn} = \delta_{ij}\,, \tag{5.38}$$

where g_0 is a solution to the $10d$ equations of motion (before the inclusion of S_{D7}). Hence, the inverse relation is

$$a_i = \int d^6y\sqrt{g_0}\,S_{i\,mn}g^{mn}\,. \tag{5.39}$$

We can now vary the functional $\mathcal{S}$ with respect to the Fourier coefficient a_i,

$$\frac{\partial\mathcal{S}[g]}{\partial a_i} = \int d^6y \frac{\partial g^{mn}(y)}{\partial a_i}\frac{\delta\mathcal{S}[g]}{\delta g^{mn}(y)} \tag{5.40}$$

$$= \int d^6y\sqrt{g}\,S_i^{mn}\frac{1}{\sqrt{g}}\frac{\delta\mathcal{S}[g]}{\delta g^{mn}} = -\frac{1}{2}\int d^6y\sqrt{g}S_i^{mn}T^{D7}_{mn}\,, \tag{5.41}$$

[12] For simplicity we will neglect its dependence on C_4, and thus any contributions to ρ_3^{loc}. The background induced $D3$ brane charge will cancel against the 7-brane tension at order α'^2 [40], and contributions from the non-perturbative stabilization of the C_4 axion will vanish as long as the axion is not displaced from its minimum. We will not consider sources that displace the C_4 axion from its SUSY minimum.

where in the last equality we have implemented the definition of the stress-energy tensor. We may now choose our complete set of tensors S_i^{mn} such that $S_0^{mn} = g_0^{mn}$, so that $a_0 \equiv \lambda$ is the Fourier coefficient corresponding to the overall volume modulus. Moreover, we may consider a one-parameter family of solutions $g^{mn} = \lambda g_0^{mn}$. Then,

$$-\frac{1}{2}\lambda\frac{\partial\mathcal{S}[g]}{\partial\lambda} = \int d^6y\sqrt{g}\frac{1}{4}T_m^m\,. \tag{5.42}$$

Furthermore, it is easy to see that

$$\mathcal{S}[g] = \int d^6y\sqrt{g}\frac{1}{4}T_\mu^\mu\,, \tag{5.43}$$

so there is a contribution to the tadpole,

$$\int d^6y\sqrt{g}\Delta^{loc,D7} = -\left(\frac{1}{2}\lambda\frac{\partial}{\partial\lambda} - 1\right)\mathcal{S}[g(\lambda)]\,. \tag{5.44}$$

Since we consider static solutions, we should interpret $\mathcal{S}[g]$ as a contribution to the $4d$ scalar potential from the seven-brane stack. Concretely, let us define the overall volume of the CY as

$$t^{3/2} \equiv \int d^6y\sqrt{g_6}\,, \tag{5.45}$$

and let t_0 be its value for $g = g_0$ (so $\lambda = (t_0/t)^{1/2}$). Then, the four-dimensional Einstein frame metric is

$$g_{\mu\nu}^E = (t/t_0)^{3/2}g_{\mu\nu}^4\,, \tag{5.46}$$

and the action S^{D7} reads

$$S^{D7}[G] = \int d^4x\sqrt{-g^E}\left(\frac{t_0}{t}\right)^3\mathcal{S}[g]\,. \tag{5.47}$$

Therefore, the four-dimensional scalar potential is

$$V_{D7}(t) \equiv -\left(\frac{t_0}{t}\right)^3\mathcal{S}[g(t)]\,. \tag{5.48}$$

We may then write the tadpole bound in the simple form

$$\frac{1}{4}R^E M_P^2 - V_{D7}(t) - \frac{1}{2}t\partial_t V_{D7}(t) = 0\,. \tag{5.49}$$

R^E denotes the $4d$ Ricci scalar of the Einstein frame metric g^E. We have used that $M_P^2 = 4\pi t_0^{3/2}$, and that $R^E = (t_0/t)^{3/2}R_{4D}$, and finally that there are no further contri-

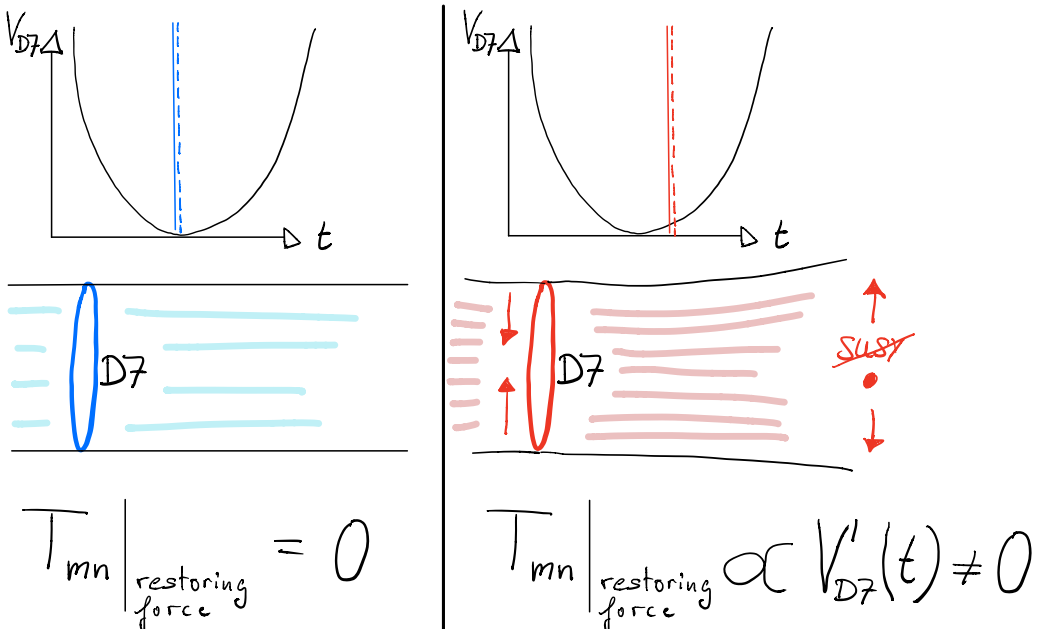

Figure 5.2: A cartoon of the supersymmetric configuration and the 'uplifted' one. A small pull on the volume modulus exerted by the uplift triggers a restoring force on the brane stack of equal strength. This in turn has a significant impact on the gravitational backreaction of gaugino condensation in the internal directions.

butions to the tadpole. In a general setup, one should replace $V_{D7}(t)$ by the contribution to the scalar potential that comes from the *unwarped* part of the compactification.

As a consequence the tadpole is canceled whenever only the seven-branes contribute to the scalar potential, and if $V_{D7}(t)$ has a minimum. This is because (on a maximally symmetric background) the $4d$ Einstein equations reduce to $R^E M_P^2 = 4V_{D7}(t)$, while the equation of motion for the volume modulus are solved when $V'_{D7}(t) = 0$.

If the approximate equivalence between the microscopic (5.30) and the effective scalar potential (5.31) lifts also to ten dimensions there is good reason to believe that the supersymmetric four-dimensional KKLT vacua can be lifted to solutions of the ten-dimensional equations of motion, treating the gaugino bilinear as a classical source term because for such solutions $\partial_t V_{D7} = 0$. This is the approach followed in [5, 106, 107, 260]. However, once the SUSY minimum is left, there is a qualitatively new contribution to the tadpole which is proportional to $V'_{D7}(t)$. This term is interpreted as the *restoring force* that non-perturbative seven-brane effects exert on the volume modulus (see Figure 5.2). As we saw in section 5.1, such terms receive dominant contributions from derivatives of $e^{-2\pi T/N}$ with respect to T. We find it very unlikely that such contributions to the tadpole can be encoded in a *local* 10d action. Rather, we believe that the ability to describe KKLT vacua by inserting a local 10d action is

limited to cases without stress from non-perturbative restoring forces.

We would now like to apply this result to warped uplifts. Strictly speaking we have not considered warping in the above discussion. But it seems very reasonable to us that the existence of strongly warped regions does not alter the stress energy tensor of the seven-brane stack. This is because we assume that it wraps a four-cycle in the essentially unwarped bulk. A warped uplift is characterized by sub-leading contributions to the integrand of the tadpole via its stress energy tensor [5]. This is because such contributions are suppressed by eight powers of the warp factor in the tadpole constraint (5.3). However, an uplift (if sufficiently small) will affect the solution in two ways:

(a) It will pull the volume modulus toward larger values, $t - t_{SUSY} > 0$.

(b) It will raise the vacuum energy.

This must happen in such a way that the $4d$ Einstein equations as well as the equations of motion of the t modulus are solved, i.e.

$$\frac{1}{4}R^E M_P^2 = V_{D7}(t) + V_{uplift}(t)\,, \quad 0 = V'_{uplift}(t) + V'_{D7}(t)\,. \tag{5.50}$$

Plugging this into the tadpole constraint of eq. (5.49) we find that it is canceled provided that also

$$V'_{uplift}(t) = -\frac{2}{t}V_{uplift}(t)\,. \tag{5.51}$$

This equation is actually satisfied if the warped uplift potential is well approximated by the classical expression of (3.86).

Thus, we have shown that any seven-brane action that reproduces the KKLT scalar potential upon dimensional reduction to four dimensions will automatically ensure tadpole cancellation upon uplifting by warped SUSY breaking sources. The 10d action proposed in ref. [108, 109] of course does this. The $4d$ microscopic potential is approximately equivalent to the true effective potential (as we saw in section 5.4). Thus, by inserting,

$$\langle\lambda\lambda\rangle \propto e^{-2\pi T[g,C_4]/N}\,, \quad \text{with} \quad T[g,C_4] \equiv \int_\Sigma \left(d^4y\sqrt{g_\Sigma} + iC_4\right)\,, \tag{5.52}$$

in the $10d$ action [108, 109], one obtains a *non-local* action $S_{D7}[g, C_4]$ that generates the $4d$ KKLT scalar potential, the correct potential for $D3$ brane position moduli as in [106], *and* is consistent with tadpole cancellation upon uplifting by warped sources.

The above reasoning is orthogonal to the question whether or not it is actually possible to generate sufficiently long warped throats and decouple the stabilization

sector from the uplift sector efficiently. We have assumed that this can be done. Rather it shows that *if* these requirements are assumed to be met in a controlled manner, $10d$ tadpole cancellation does not indicate an inconsistency of the assumptions that were made. We will comment on difficulties to actually realize this in section 5.6.

As a side remark we note that the above gives further justification to recent approaches to tadpole cancellation in $10d$ calculations where the tadpole is canceled via the by-hand addition of extra source terms [264]. These terms should really be attributed to the dynamically adjusting restoring force against decompactification that is generated by the non-local interactions on the D7-branes world volume.

Finally, we comment on the discrepancy between the results of [1, 110] and the ones of [111]: In [1, 110] the 10d action is varied with respect to the metric *after* inserting the exponential relation between the gaugino condensate and the volume modulus, while in ref. [111] the expectation value of the gaugino condensate is treated as a purely classical source term. This amounts to neglecting the in our opinion crucial contribution to the stress energy tensor from the restoring force of the stabilization mechanism which we have argued to receive dominant contributions from terms proportional to $\partial_T\langle\lambda\lambda\rangle$. In principle it might be consistent to treat the gaugino condensate as an independent field. But, one would have to start with an off-shell action of $\langle\lambda\lambda\rangle$ and T which involves the Veneziano-Yankielowicz scalar potential [265] that stabilizes $\langle\lambda\lambda\rangle$ in terms of T. We think that including this potential in the seven-brane Lagrangian would resolve the tension between our results and those of ref. [111].

5.6 de Sitter vacua at weak coupling?

In this section we will comment on the question if sufficiently long warped throats exist so that the KKLT effective field theory can be realized with appropriate values of parameters. By appropriate we mean that Kähler moduli stabilization occurs at sufficiently large volume, and the uplift potential is sufficiently small to prevent destabilization. In short, *are parametrically small warped uplifts part of the landscape, or are they in the swampland*? Before we start we define small/large uplifts as follows.

$$\begin{aligned} &\textit{A small uplift is one that does not destabilize any of the moduli.} \\ &\textit{A large uplift is one that is not small.} \end{aligned} \tag{5.53}$$

We will refer to a parametrically small uplift as one with parametrically negligible backreaction on all moduli. The moduli that will be relevant (i.e. the lightest ones) are the Kähler moduli which we assume to be stabilized non-perturbatively as in KKLT.

Throughout this section we assume that both the flux number W_0 in the KKLT

superpotential (4.1) as well as the infra-red warp factor a_0^2 of the KS throat can be tuned arbitrarily well for all practical purposes. Thus, we will ignore the fact that the possibly finite number of flux vacua and $D3$-brane charge cancellation limit the extend to which this can actually be done [41, 152]. Rather we will ask only for geometrical consistency of the setup once the volume modulus is stabilized via KKLT and the warp factor is small enough to prevent decompactification. We will see that this is surprisingly hard to achieve.[13]

First, let us briefly recall what are the qualitatively different regimes of values that the overall volume modulus T can take [40, 91, 156, 266], and argue for a slightly non-trivial minimum value of the Kähler modulus near which the 10d supergravity approximation starts to break down. The Einstein frame $10d$ metric of a one-parameter family of solutions to the equations of motion takes the form

$$ds^2 = t^{-1}e^{2A}dx^2 + \alpha' e^{-2A}ds^2_{CY}\,, \quad e^{-4A} \equiv e^{-4A_0} + (t - t_0)\,, \tag{5.54}$$

where $t_0 \equiv \int_{M_6} \sqrt{g_{CY}} e^{-4A_0}$, and e^{2A_0} is some reference solution for the warp factor. The metric ds^2_{CY} is a CY metric normalized to unit volume (or more generally a solution coming from F-theory). The variable t is the real modulus of the solution, and at sufficiently large values (we will make this more precise below) the metric approaches

$$ds^2 \longrightarrow t^{-3/2}dx^2 + \alpha' t^{1/2} ds^2_{CY}\,, \tag{5.55}$$

so in this regime we may identify $t^{3/2}$ with the compactification volume in units of α'.

In general, recall from section 3.6.1 that for a GKP type solution, the warp factor is determined by solving a $6d$ Laplace equation

$$\nabla^2_{CY} e^{-4A} \propto \rho_{D3}\,, \tag{5.56}$$

where ρ_{D3} is the $D3$ brane charge density as measured using the CY metric g^{CY}_{mn} [156]. Hence, e.g. near a point like (or smeared along a real co-dimension two locus) source of N_{D3} units of $D3$ brane charge the solution takes the form

$$e^{-4A} \sim \begin{cases} N_{D3} r^{-4} + const. & \text{near a point-like source} \\ N_{D3} \log(1/r) + const. & \text{near a co-dimension two source} \end{cases}\,, \tag{5.57}$$

[13] Note that the results of [106] show that in KKLT setups (at fixed value of $|W_0|$) indefinitely small uplifts are beyond the regime where the throat is well approximated by the KS solution. This is due to relevant perturbations of the KS gauge theory that are activated by gaugino condensation in the bulk CY and grow toward the infrared. However, parametrically small uplifts in the sense of (5.53) are not straightforwardly excluded by this.

where r measures the transversal distance to the source (using the dimensionless CY metric). We are interested in cases where the negative $D3$ brane charge is effectively smeared over $D7/O7$ stacks while the positive charge is stored in the fluxes of a KS throat (in such a way that the overall $D3$ brane charge vanishes).[14] W.l.o.g. we may assume that the particular solution $e^{-4A} = e^{-4A_0}$ corresponds to the case where the overall volume is sufficiently small (and not much smaller) that backreaction from $D3$ brane charge cannot be neglected anywhere, but the vanishing locus of the inverse warp factor is still (marginally) aligned with the loci that carry the negative $D3$ brane charge (see figure 5.3). If we now use the one-parameter freedom of the GKP solutions to set

$$e^{-4A} = e^{-4A_0} + (t - t_0)\,, \tag{5.58}$$

we see that the vanishing locus of e^{-4A} merges into the location of negative $D3$ brane charge as we take $t - t_0 \gg N_{D3}$, while for $t - t_0 < 0$ the vanishing locus quickly moves into the bulk and the $10d$ solution becomes pathological.

Moreover by inspecting e.g. the solution near a stack of $D3$ branes it is easy to see that $t_0 \gtrsim N_{D3}$. Therefore, in order for a controlled $10d$ solution to exist where all the (would-be) pathological behavior is concentrated close to the seven-brane stacks, we need $t > N_{D3}$. In this case the physical volume of the CY is also bigger than $N_{D3}^{3/2}$ and well approximated by the value of $\alpha'^3 t^{3/2}$. This is also the regime where we may think of warped throats as isolated regions of strong warping embedded into an essentially trivially warped bulk CY. Note that for an approximately isotropic CY this bound is identical with the dilute flux limit described in section 3.6.1.

But whenever at least one complex structure modulus z is stabilized near a conifold singularity in complex structure moduli space, $|z| \ll 1$, the dilute flux regime corresponds to far bigger volumes of order [156]

$$\mathrm{Re}(T) \gg N_{D3}|z|^{-4/3}\,, \tag{5.59}$$

where N_{D3} is the total $D3$ brane charge stored in fluxes that thread the $\mathcal{A}$ and $\mathcal{B}$ cycle of the conifold (and possibly in mobile $D3$ branes).

The warped throat regime occurs for a large range of intermediate values

$$N_{D3}|z|^{-4/3} \gg \mathrm{Re}(T) \gg N_{D3}\,. \tag{5.60}$$

For values in this range, at least one warped throat forms with localized significant backreaction via fluxes that drive the non-trivial warping. The warped throat can be thought of as an object of size $R^4_{\text{throat}} \sim N_{D3}$ glued into a much larger bulk CY

[14] Here, the negatively charged objects are the $\frac{1}{2}$BPS objects with *negative* (induced) tension.

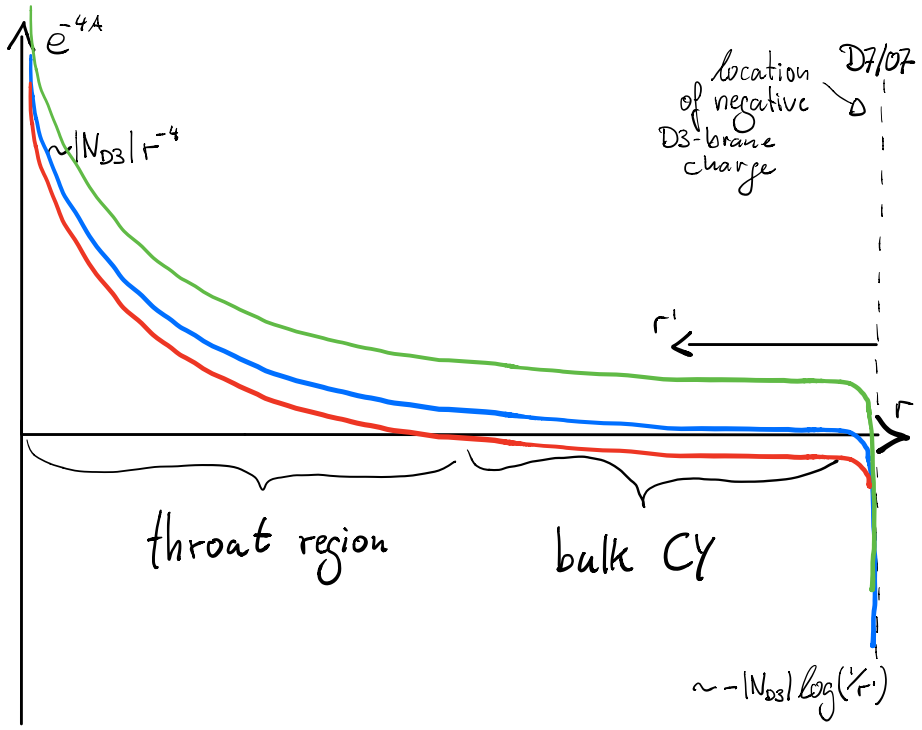

Figure 5.3: We plot the schematic form of the inverse warp factor in a flux compactification for different values of the volume modulus. It diverges as $|N_{D3}|r^{-4}$ near the position of localized (or approximately localized) positive $D3$ brane charge, and as $-|N_{D3}|\log(1/r')$ near the locus of negative induced $D3$ brane charge on $D7/O7$ stacks. r is the transversal distance to the positive charge while r' is the transversal distance to the negative charge. The blue curve corresponds to taking $t \gtrsim N_{D3}$ where the vanishing locus of e^{-4A} is marginally aligned with the $D7/O7$ loci. The green curve corresponds to the case $t \gg N_{D3}$ where the inverse warp factor vanishes only very close to the seven-brane stacks. The $10d$ geometric picture is under parametric control. In the opposite regime $t < N_{D3}$, shown in red, the singular locus reaches far into the bulk.

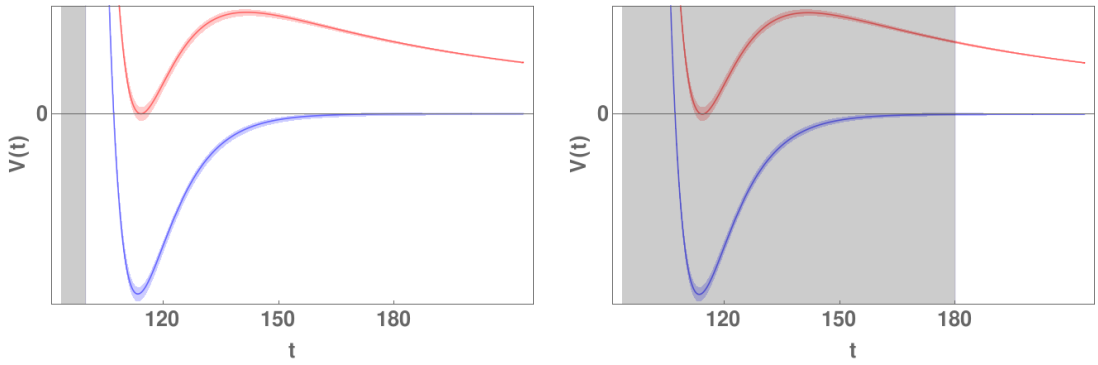

Figure 5.4: We plot the KKLT uplift as in figure 3.6, but with a shaded area that marks the region $\mathrm{Re}(T) < N_{D3}$ where the $10d$ geometrical setup is beyond control. Left: If (say) $N_{D3} = 100$ can be realized simultaneously with the parameter choices of figure 3.6 the uplifted vacuum lies in a controlled region and the uplift can be trusted. Right: If $N_{D3} \gg 100$ is required, the vacuum region cannot be trusted. We find that the scenario on the left is hard or impossible to realize, while generically we are forced into the scenario displayed on the right.

that remains unaffected by flux backreaction. Changing the value of $\mathrm{Re}(T)$ essentially only rescales the bulk CY but leaves the throat unchanged. While flux backreaction is significant, it is controlled by the KS solution [162] which is smoothly glued into the bulk CY (see figure 5.5).

This regime ends once we set $\mathrm{Re}(T) \sim N_{D3}$ as explained above. We will constrain KKLT de Sitter uplifts by demanding that the point in field space where (supersymmetric) Kähler moduli stabilization occurs must not lie in the uncontrolled regime $\mathrm{Re}(T) \lesssim N_{D3}$ (see figure 5.4). We will find that this constraint can generically not be met simultaneously with the constraint that the warped uplift does not trigger decompactification.

We would like to emphasize that the arguments that follow are very similar to (and were inspired by) the ones used in ref. [46, 230]. In particular, in [230] it was argued that W_0 has to be tuned extremely small in order for the KKLT construction to be able to work. However, we will argue that small uplifts are severly constrained even if W_0 can be tuned arbitrarily small.

5.6.1 Single modulus KKLT

We consider the setup of section 3.7.2, i.e. a type IIB Calabi-Yau orientifold with only a single Kähler modulus T and with all complex structure moduli integrated out consistently.

Let us now see if we can make the warped uplift small in the sense defined in (5.53).

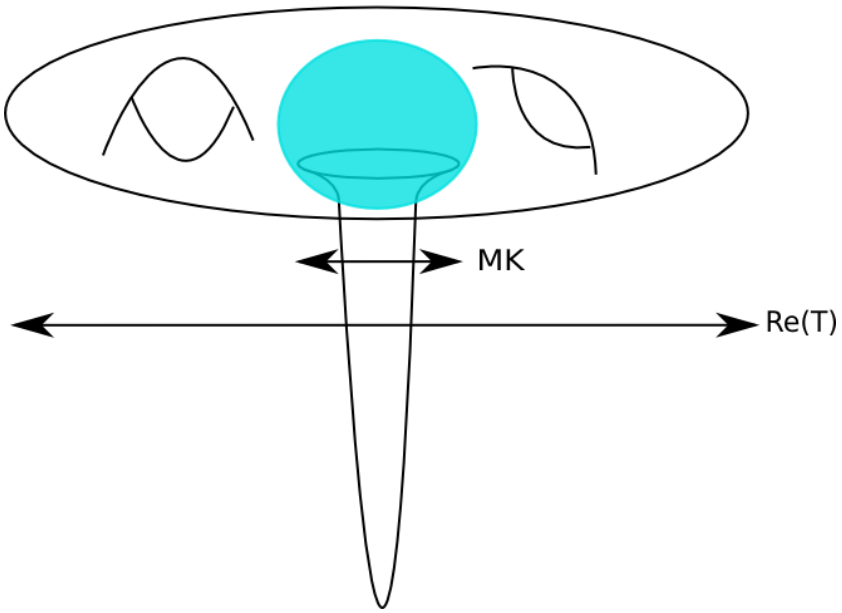

Figure 5.5: We depict a cartoon of a CY that contains a warped throat region. For geometrical consistency the throat region must be smaller than the overall size of the CY.

In order to suppress backreaction on T we need that $V_{uplift} \lesssim |V_{SUSY}|$ [41]. Comparing with eq. (3.84) and eq. (3.86) we see that we need $\alpha \equiv \frac{a_0^4}{|W_0|^2} \lesssim 1$, and as always $|W_0|^2 \ll 1$. It is useful to define an uplift parameter U as follows,

$$U \equiv \frac{\log(|V_{SUSY}|^{-1})}{\log\left(|V_{uplift}|^{-1}\right)} \approx \frac{\log|W_0|^{-2}}{\log a_0^{-4}} = 1 + \frac{\log(\alpha)}{\log(a_0^{-4})}\,, \tag{5.61}$$

in units $M_P = 1$. The uplift is small when $U \lesssim 1$. The interesting regime where an uplift to de Sitter space might take place corresponds to taking $\alpha = \mathcal{O}(1)$, so $|U-1| \ll 1$. Once $U - 1 = \mathcal{O}(1)$ the uplift becomes *exponentially* uncontrollable, i.e.

$$\alpha = \left(|W_0|^{-2}\right)^{\frac{U-1}{U}} \gg 1\,. \tag{5.62}$$

The results of ref. [106] imply that for control of relevant throat perturbations one needs $U \geq 3/4$, but only slightly bigger values allow to keep those perturbations under parametric control. We will now explain why only parametrically large values of U might be possible in a geometrically consistent setup.

This constraint comes from the following requirements. The throat that carries the SUSY breaking source has to be sufficiently small in circumference that it can fit into the bulk Calabi-Yau so that it can be well separated from the seven-brane stack that is responsible for Kähler moduli stabilization (see Figure 5.5). As recalled in the

introduction of this section it is well-known that the (Einstein frame) size of the throat is set by its $D3$-brane charge N_{D3} as

$$R^4_{throat} \sim N_{D3} = MK\,, \tag{5.63}$$

where M and K are the RR and NS-NS flux quanta that stabilize the throat. Thus we should require that

$$\mathrm{Re}(T) \gg MK\,, \tag{5.64}$$

for validity of the $10d$ geometric picture.

However, at the KKLT minimum the value of $\mathrm{Re}(T)$ is set by W_0 according to eq. (3.83) while the IR warp factor is set by eq. (3.66). Plugging these formula into the requirement of eq. (5.64) we obtain

$$N \overset{!}{\gg} \frac{MK}{\log|W_0|^{-1}} \sim \frac{MK}{U\log a_0^{-4}} \sim \frac{g_s M^2}{U}\,. \tag{5.65}$$

This can be turned into a *lower* bound on the uplift parameter U,

$$U \gg (g_s M)^2 g_s^{-1}\frac{1}{N}\,. \tag{5.66}$$

Clearly we cannot make U arbitrarily small. More so, it is not clear to us if it can even be smaller or equal to one as would be required for a controlled uplift. The size of the IR end of the throat as measured in string units is $g_s M$, so we need $g_s M \gg 1$ for control of the α' expansion, as well as $g_s \ll 1$ for control of the string loop expansion. So in order to get $U \approx 1$ or smaller, the rank of the seven-brane gauge group really has to be parametrically large,

$$N \gg (g_s M)^2 g_s^{-1} \gg 1\,. \tag{5.67}$$

There is at least some indication that it is hard to make N arbitrarily large while maintaining $h^{1,1} = 1$. Indeed one of the swampland criteria states that in an EFT coupled to gravity the number of massless fields cannot be arbitrarily large [267], and in this case an arbitrarily large N would imply an arbitrarily large number of gluons. This generic swampland criterium can be checked explicitly in some setups. For example, in [245] the relation between $h^{1,1}$ and the largest possible N was investigated numerically using a subset of the Kreuzer-Skarke database. For $h^{1,1} = 1$, the largest possible N was found to be $\mathcal{O}(10)$, while more generally a linear bound of the form

$$N \leq \mathcal{O}(10)h^{1,1}\,, \tag{5.68}$$

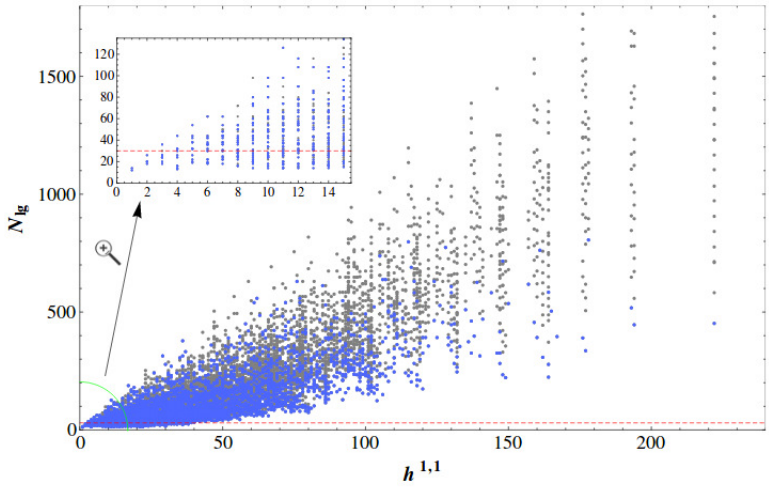

Figure 5.6: For a subset of the Kreuzer-Skarke database [268], the maximal possible gauge group rank in perturbative type IIB string theory grows approximately linearly with $h^{1,1}$. This argument and the above figure are taken from [245].

was found to be obeyed within the limited example set, see figure 5.6. Of course, it could be true that much larger values of N exist for $h^{1,1} = 1$ that are not contained in the example set. This possibility forms a potential loophole. We conclude the following.

In single modulus KKLT with $N = \mathcal{O}(1)$ and $A = \mathcal{O}(1)$, whenever the supersymmetric starting point lies in a regime of parametric control of the supergravity theory, $Re(T) \gg MK$, all warped uplifts will destabilize the Kähler modulus.

Assuming that the empirical relation of eq. (5.68) holds, the loophole $N \gg 1$ is closed. The obvious remaining loophole is to take $h^{1,1} \gg 1$ which is anyway satisfied by generic CYs, and/or assuming $|A| \gg 1$. We will comment on these options momentarily.

5.6.2 Multi modulus KKLT

The arguments of the previous subsection indicate that it is impossible to realize parametrically small uplifts in the case in which the CY orientifold has just one Kähler modulus, i.e. whenever $h^{1,1}_{+} = 1$.[15] One could then try to evade this reasoning, by considering a case with many Kähler moduli. So, let us now consider the potential loophole $h^{1,1}_{+} \gg 1$.

We would like to emphasize immediately that most of the conclusions we draw in this section are based on assumptions about the geometry of CY manifolds which we

[15]The argument is not significantly altered for more general but still $\mathcal{O}(1)$ values of $h^{1,1}_{+}$.

believe to hold *generically*. As such we cannot exclude that non-generic CYs exist for which our discussion does not apply. We will comment on this possibility at the end of this section.

Let us consider a number of Kähler moduli $\{T_1, ..., T_{h^{1,1}_+}\}$, and KKLT type superpotential

$$W = W_0 + \sum_{i=1}^{n} N_i A_i \exp\left(-\frac{2\pi}{N_i}\sum_{\alpha=1}^{h^{1,1}_+} k_i^\alpha T_\alpha\right) , \tag{5.69}$$

with some integer-valued $n \times h^{1,1}_+$ charge matrix k. For the Kähler potential in the multi-modulus case we refer the reader to ref. [153]. Here, we have assumed that there are n superpotential terms that are generated via various confining gauge theories ($N_i > 1$) or euclidean D3 brane instantons ($N_i = 1$).

The F-term equations read

$$D_{T_\alpha} W = -\sum_{i=1}^{n} 2\pi k_i^\alpha A_i \exp\left(-\frac{2\pi}{N_i}\sum_{\beta=1}^{h^{1,1}_+} k_i^\beta T_\beta\right) - \frac{2v^\alpha}{\mathcal{V}} W \,, \tag{5.70}$$

where $\mathcal{V}$ is the overall volume, and v^α is the volume of the two-cycle dual to the four-cycle Σ^α. We expect the generic KKLT type solutions to satisfy,

$$\sum_{\alpha=1}^{h^{1,1}_+} k_i^\alpha \mathrm{Re}(T_\alpha) \sim \frac{N_i}{2\pi}\log(|W_0|^{-1}) \quad \forall\, i, \tag{5.71}$$

so that generically the four-cycle volumes are again bounded as,

$$\mathrm{Re}(T_\alpha) \lesssim \frac{N_{max}}{2\pi}\log(|W_0|^{-1}) \,, \tag{5.72}$$

where N_{max} is the maximal available dual Coxeter number. So far, the story is very similar to the case $h^{1,1}_+ = 1$, except that we expect to have the freedom to take $N_{max} \gg 1$, due to the distribution shown in figure 5.6. Again we need to require that the throat fits into the bulk Calabi-Yau,

$$MK < \mathcal{V}^{2/3} \,. \tag{5.73}$$

Now, we find it reasonable to demand something stronger: The throat must fit into a region in the bulk Calabi-Yau that is well approximated by a conifold region (or more generally some cone over a Sasaki-Einstein base). As a consequence we should really require that zooming into such a region, all the topological structure that the Calabi-Yau possesses should become invisible. This is because we would like to isolate the non-perturbative stabilization sector associated with *each* 4-cycle from the uplift sector

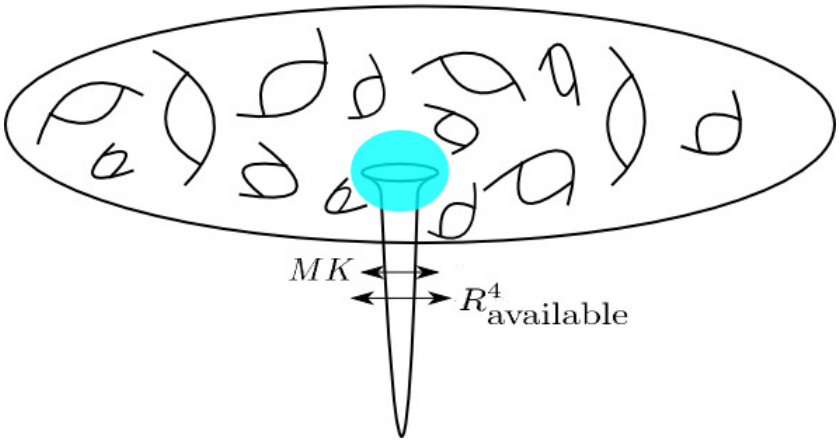

Figure 5.7: We draw a cartoon of a more complicated CY with $h^{1,1} \gg 1$. We expect that the rich topological structure of the manifold leaves us less room to place warped throats in comparison to a simpler CY with $h^{1,1} = 1$ with the same overall volume (compare with figure 5.5).

associated to the throat (see Figure 5.7). At large $h^{1,1}_+$ it is natural to expect that in generic CY manifolds the amount of such freely available volume where warped throats can fit scales with the overall volume of the Calabi-Yau, but also that it decreases monotonically with $h^{1,1}_+$. For definiteness let us parameterize this expectation as

$$R^4_{\text{available}} \propto \frac{\mathcal{V}^{2/3}}{(h^{1,1}_+)^p}\,, \tag{5.74}$$

for some undetermined positive coefficient p. It is easy to see that if we implement the bound on the maximal available dual Coxeter number of eq. (5.68), one obtains a bound

$$U \gg (g_s M)^2 g_s^{-1}\ (h^{1,1}_+)^{p-1}\,. \tag{5.75}$$

It is apparent that choosing large values of $h^{1,1}_+$ will relax the bound only if the freely available volume scales very weakly with the number of Kähler moduli i.e. $p < 1$. So we should ask ourselves how large we expect p to be. Instead of asking what is the maximal region around a conifold singularity that is well described by the non-compact conifold solution, it is simpler and arguably less restrictive to ask the following instead. *What is the spherical region of maximal size around a generic point?* We may build a chart U_p around a generic point p so that every point in ∂U_p is geodesically equidistant from the center p by a distance R_p. Then, what is the largest possible radius R_p? Roughly speaking this gives the largest 5-sphere that can be expanded around a generic point. We do not know how to answer this question for CYs but we expect that generically

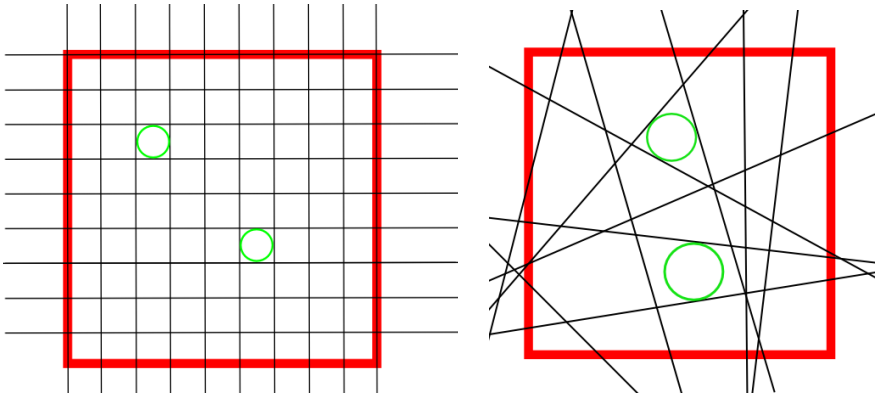

Figure 5.8: On the l.h. side we plot a square intersected by a grid formed of n lines. The freely available space is of order $1/n^2$. On the r.h. side we show a more generic intersection pattern. We expect the freely available size surrounding a generic point to be of the same order as for the case of a regular grid.

the size $R_{available}$ available for fitting a warped throat would be bounded by this.

Let us now consider a somewhat different but related question. For full moduli stabilization to occur we expect that each divisor class has a representative that is wrapped by a seven brane or euclidean $D3$ brane instanton. Our conifold region should not be intersected by any of these divisors. In particular none of the triple intersection points should be contained in the conifold region. So in addition one may ask what is the largest available 5-sphere that does not contain any of the triple intersection points. We expect that both types of largest possible spheres will be bounded for similar reasons.

So let us consider a very simple toy setup where this last question can be easily addressed: Given a six-dimensional cube of unit volume, let us randomly intersect it with n real co-dimension 2 planes. Let us call R_x the radius of the largest 5-sphere centered at a point x that can be drawn without meeting any of the intersecting planes (see Figure 5.8). Since a generic triplet of planes intersects at a point we expect $\mathcal{O}(n^3)$ triple intersection points in the cube, distributed randomly. The largest sphere that can be fit must in particular not contain any of these points in its interior so one finds $R_x^6 < \mathcal{O}(n^{-3})$ for a generic point x. For a cube of overall volume $\mathcal{V}$, this is replaced by

$$R_x^6 < \mathcal{O}(n^{-3}) \cdot \mathcal{V} \,. \tag{5.76}$$

Obviously cubes intersected by co-dimension 2 planes are not a good approximation of CYs with wrapped divisors but it may give us some intuition how intersecting co-dimension 2 branes limit the available un-intersected volume.

In ref. [269] another likewise related question was posed for CY three-folds and

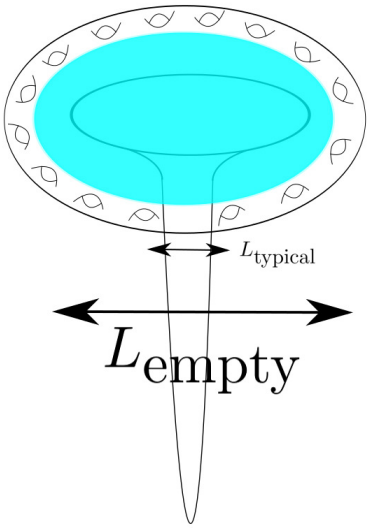

Figure 5.9: We depict a cartoon of a CY manifold that would evade our conclusions. There exists a large topologically trivial area in the interior where a large KS throat can fit. All of the topological structure is densely aligned around it.

numerical answers were given using the Kreuzer-Skarke database: *How large does the overall volume of the compactification have to be in order to ensure that the α' expansion is under marginal control?* Numerically, the answer appears to be

$$\text{Vol}(CY) \gtrsim (h^{1,1})^7 \,. \tag{5.77}$$

Of course it is tempting to interpret this result as the statement that $p = \frac{2}{3}\cdot 7$. However, the above only means that if all the non-trivial curves are required to be bigger than α' the overall volume must be large. In principle the CY might nevertheless contain large *empty* regions. It would be very interesting to explore how precisely the freely available volume within a CY scales with the number of Kähler moduli in order to place the above considerations on a firm footing.

Based on these simplified preliminary observations we find it reasonable to expect that generically it will be hard to engineer $p < 1$, while actually proving this to be impossible is beyond the scope of this thesis. In this case we expect that *generic* CYs do not admit sufficiently small de Sitter uplifts even if W_0 and the IR warp factor a_0^2 can be tuned at will. Again, we want to stress that non-generic CYs (or CYs at non-generic points in moduli space) might evade this argument. For a cartoon of how such a CY could look like, see figure 5.9. It would be very interesting to see if such non-generic CYs can be engineered for dS uplifts.

As promised, let us consider the potential loophole of exponentially small/large coefficient A. It has been explained in ref. [155] that this is indeed possible due to induced D_{-1}-brane charge or induced $D3$ brane charge for the case of euclidean $D3$ brane instantons respectively gaugino condensation on seven branes,

$$|A| \sim e^{\frac{2\pi}{g_s N}\frac{\chi(\Sigma)}{24}} \,. \tag{5.78}$$

This would change our discussion only if one can find solutions where $\log(|A/W_0|)$ is dominated by $\log(|A|)$, so that

$$\mathrm{Re}(T) \sim \frac{N}{2\pi}\log(|A|) \sim \frac{\chi(\Sigma)}{24 g_s} \tag{5.79}$$

This does not seem to be a controlled regime as can for instance be seen by inspecting the euclidean $D3$ brane action. To low orders in the α'-expansion it takes the form

$$S_{ED3} = \frac{2\pi}{g_s}\Big(\underbrace{g_s \mathrm{Re}\, T}_{\text{tree level}} + \underbrace{(-1)\frac{\chi(\Sigma)}{24}}_{\mathcal{O}(\alpha'^2)\text{ correction}} + \mathcal{O}(\alpha'^3)\Big)\,. \tag{5.80}$$

In the regime of (5.79) we see that the tree-level contribution is of the same order as the $(\alpha')^2$ correction, indicating loss of control over the α' expansion.[16]

As a final and perhaps most interesting loophole, let us note that the KS throat also has a dual description in terms of a confining gauge theory [78, 162]. The parameter $g_s M$ sets the 't Hooft coupling of the last steps of the cascade of Seiberg dualities that the gauge theory is undergoing. So the regime $g_s M \ll 1$ is controlled by the gauge theory side of the correspondence. If the SUSY breaking anti-brane state also exists in this regime (and remains meta-stable), the bound on the smallness of the uplift becomes considerably weaker. Even if this holds, it is not obvious whether the uplift could then be made sufficiently small. We can interpret the r.h. side of the bound in eq. (5.67) as the ratio between the size of the IR end of the throat $R^4_{\mathrm{IR}} \sim (g_s M)^2$ and the characteristic 'size' of the anti-brane $R^4_{\overline{D3}} \sim g_s$. Whenever the latter exceeds the former we would expect the brane not to be able to sink down all the way to the bottom of the throat, thus again preventing the uplift potential from becoming small. Whether or not this (after all geometric) intuition carries over to the small $g_s M$ regime remains to be seen. If these considerations are valid, it seems possible that the r.h. side of the bound of eq. (5.67) could take $\mathcal{O}(1)$ values. In this case, moderately large rank

[16] This is qualitatively very different to e.g. the KKLT expansion of the superpotential where also the first two terms in the expansion are of the same order. This is achieved via a fine tuning of the first term so one does not expect a breakdown of the non-perturbative expansion scheme.

gauge groups $N \sim 10$ may be enough to marginally fulfill all constraints, though never parametrically.[17]

5.7 Conclusions

In this chapter we have summarized recent progress made in understanding the ten-dimensional lift of the four-dimensional KKLT vacua [1, 5, 105–110], with the goal to confront the de Sitter uplift proposal of KKLT with ten-dimensional tadpole cancellation conditions. First, building on [107] we find that the ten-dimensional and four-dimensional conditions for unbroken supersymmetry are remarkably consistent with each other [5]. Second, we find that the de Sitter uplift proposal as made by KKLT is fully consistent with ten-dimensional tadpole cancellations [1, 110]. We interpret this as evidence that the coefficient c in eq. (4.3) is suppressed to a sufficient degree that it can be ignored. Thus, significant uplift flattening does not occur in KKLT.

In contrast to the above partial *confirmation* of the KKLT proposal, we have presented a geometric consistency requirement that prevents one from bringing de Sitter vacua of the KKLT type into regimes of parametric control (from a ten-dimensional perspective). In the best case, this means that such vacua live in marginally controlled regimes of the perturbative expansion schemes of string theory, while in the worst case the construction might not work. Deciding which of the options holds depends in a subtle way on the detailed geometry of CY compactifications. Much more work in the spirit of [269] is required to answer this question conclusively.

To conclude, the question whether or not there exist de Sitter vacua in string theory remains a fascinating fundamental issue. While with currently available tools it is hard to prove or disprove the existence of such vacua, we are confronted with non-trivial evidence that such vacua cannot live 'far out' in moduli space, consistent with but independent of results such as the ones obtained in ref. [270]. It is thus tempting to speculate that de Sitter vacua are phenomena that appear in the strongly coupled (or marginally weakly coupled) *interior* of stringy moduli spaces, see Figure 5.10. If this turns out to be correct the quest of understanding stringy de Sitter vacua would require radical progress in the understanding of non-perturbative string theory (the 'missing corner' of [267]).

Another speculative line of thought is that de Sitter vacua could exist in string theory even in weakly coupled (but tuned) corners, but at least the ones with small positive cosmological constant cannot be obtained by small perturbations of anti-de Sitter vacua with small *negative* vacuum energy, see figure 5.11. This would be a far

[17] We thank M. Reece for discussions on this point.

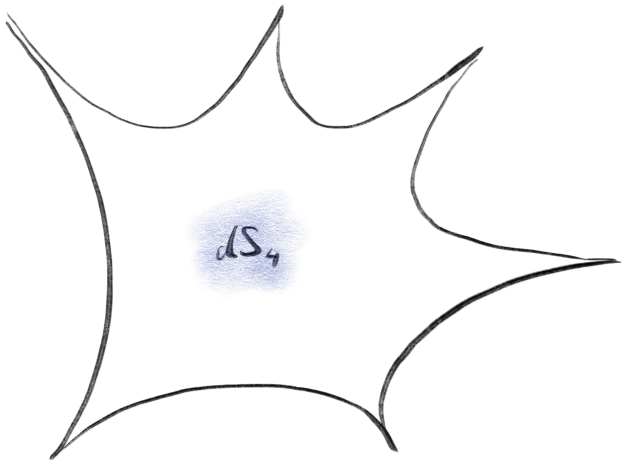

Figure 5.10: A cartoon of the idea that de Sitter vacua all reside near the 'interior' of stringy moduli spaces.

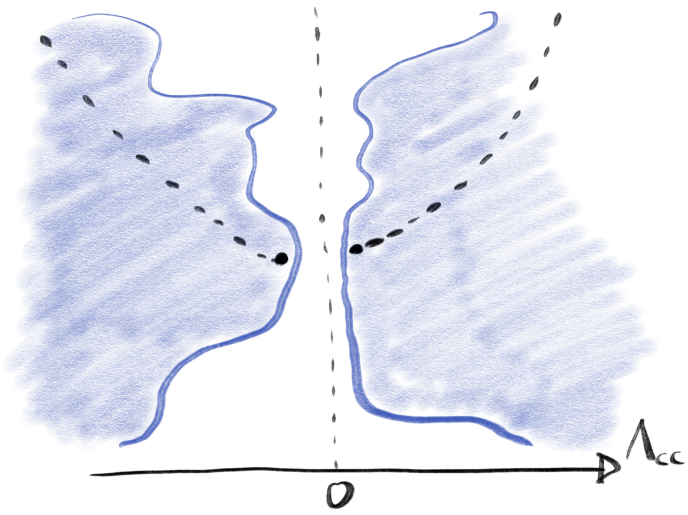

Figure 5.11: Speculative cartoon of stringy moduli space (blue) and their cosmological constants. Two vacua with similarly small vacuum energy but with opposite sign are far away from each other in stringy moduli space, so in particular they are not mapped to each other by an 'uplift' or non-perturbative decay process.

weaker statement than the ones of the no-dS conjecture [79–81], as it would doubt only the idea of a de Sitter 'uplift' rather than the existence of de Sitter vacua all-together.

We are hopeful that *either* it can be shown in the near future that obstacles such as the one presented in section 5.6 are insurmountable *or* the existence of de Sitter vacua of KKLT type will be established beyond reasonable doubt, by making use of the small but perhaps sufficiently controlled window in parameter space.

Chapter 6

Thraxions

New ultralight throat axions
-or-
axion inflation near the conifold transition?

We now wish to switch gears to some extent. From the late time expansion of our universe we now switch to the inflationary era. More precisely we consider the problem of large field inflation in string theory. Recall from the introductory section 2.2 that inflationary models of large field type are the ones with the most exciting observational signatures: Measurable inflationary tensor modes. Moreover they are the least tuned ones. We have explained that EFT approaches to large field inflation are not predictive in the usual sense in that the full inflationary dynamics is not determined by a handful of Wilson coefficients. After all it would be a bit disappointing if a mechanism would allow us to detect the *quantized* graviton, without any sensitivity to the UV completion. Nevertheless, *if* we are given models of large field inflation that are well motivated from string theory they could probably be parameterized by some scalar potential $V(\phi)$. Therefore, even if such a model predicts tensor modes that are then observed, it would be difficult to convince a skeptic that this is a distinctive prediction of string theory. After all, large field potentials are the simple ones, so what do they need string theory for? So perhaps it is in fact much more exciting to be able to *rule out* the possibility of large field inflation in string theory. After all, the upper bounds on the tensor to scalar ratio are already surprisingly low. If there was an inflationary era, as observation strongly suggests, why would it not be governed by the simple large field potentials, if not for a deep string theory constraint?

If one could show that large field inflation is not possible in string theory, it would be natural to expect that the traversed inflationary field distance is as large as it can be found in the theory because otherwise the scalar potential is unnecessarily tuned.

In our opinion, one of the most intriguing hints that string theory is the right theory of quantum gravity would be a detection of the tensor to scalar ratio at the level of

$$r_{\text{string}} \sim 10^{-3}\,, \tag{6.1}$$

which incidentally is just at the border of what we can hope to detect in the future with CMB stage 4 [271]. In fact, some authors have conjectured that string theory predicts this value [51, 272].

However, it is clear that we are nowhere near a consensus that large field inflation is impossible in string theory. In order to narrow down this complicated question it is useful (in my opinion) to focus on the most promising inflaton candidates. As the inflationary scalar potential must remain flat over a super-Planckian range in field space, it is natural to consider axions as inflatons due to their perturbative shift symmetry. Realizing a model of axion inflation is all about generating a small monotonic scalar potential in a controlled way. The simplest idea is the one of *natural inflation* where the inflaton potential is of instanton generated type

$$V(\phi) \sim e^{-S_E}(1-\cos(\phi/f))\,. \tag{6.2}$$

For such a model to realize 60 e-folds of slow roll inflation, super-Planckian decay constants are required $f \gg M_P$, and also such models are basically ruled out by observation. Nevertheless, it is important to study these kinds of potentials because they might reveal that large field inflation is possible in string theory as a matter of principle. This possibility is of course challenged by the WGC for axions [31, 54], which implies that $f < M_P$ in the simplest cases. Conversely, by studying these kinds of models we learn to what extent the WGC holds in string theory.

The most prominent ideas to achieve sufficiently long axion field ranges in string theory are the following,

- N-flation: A large number N of axions traverses a $\mathcal{O}(\sqrt{N})$ enhanced diagonal in field space [45]. Related with this is the idea of *kinetic alignment* [273].
- KNP alignment: A potential can be generated that forces the inflationary trajectory to wind around the at least two-dimensional axion fundamental domain many times [44].
- Axion monodromy: The gauged discrete shift symmetry is broken spontaneously [46, 198, 274]. We introduced this concept already in section 3.8.1, but its most important application could be the one of large field inflation.

The major challenge for the first two approaches is to find an axion potential that

1) forces the inflationary trajectory onto the special long path, and 2) to ensure that the effective potential along the valley does not oscillate on sub-Planckian scales. If only the first condition can be satisfied large field inflation cannot be realized but we nevertheless learn a lot about the structure of stringy EFTs: The (strong form of the) distance conjecture requires a tower of states to become exponentially light as a super Planckian geodesic distance is traversed within a low energy EFT [205]. Certainly this does not occur for axions with non-perturbative harmonic potentials. As a consequence we would learn that strong forms of the distance conjecture are false. We will report evidence in this direction shortly.

The most difficult challenges for axion monodromy lie in realizing a setup were the discrete gauge symmetry is broken to a sufficiently sparse subgroup, and also to engineer a sufficiently monotonic scalar potential. As we have explained in section 3.8.1 we expect axion monodromy to always be finite, so the three different ideas are not completely distinct. A general axion model may display aspects of all three ideas.

In this section, we present a new class of ultralight axions which, we believe to be a generic feature of the type IIB part of the string theory landscape. This idea is an extension of a proposal made in [74]. Somewhat surprisingly, the mass of this axion is suppressed to a much further extent than would be predicted by the WGC for axions. In other words, its scalar potential is far smaller than the scale $M_{\rm P}^4 \exp(-M_{\rm P}/f)$ of generic axion potentials. It is therefore justified to identify them as a new class of axions that has so far not been accounted for in the study of CY flux compactifications.

We consider again type IIB Calabi-Yau orientifolds or F-theory models stabilized by fluxes and non-perturbative effects such as KKLT. We have explained in section 3.6.2 that Klebanov-Strassler (KS) throats [162] with warp factor $a_0 \ll 1$ are expected to be present in an order-one fraction of such models [42, 163, 275]. We recall that this warp factor is naturally *exponentially* small,

$$a_0^3 \sim \exp\left(-2\pi \frac{K}{g_s M}\right) . \tag{6.3}$$

Before we get to the details we wish to summarize the results in terms of their parametric dependencies on the warp factor a_0. Naively, the lightest states are then the glueballs (or warped-throat KK modes) with mass $\sim a_0$ (in Planck units). However, our axion is exponentially lighter than this scale with mass of order $\sim a_0^3$. To be more precise, this happens at least in all cases where the fluxes stabilize the complex structure moduli near a conifold transition locus in moduli space, and if orientifold projections do not interfere with the setup.

Our axion has a decay constant $f \sim \mathcal{O}(1)$ in the simplest models[1], which can be enhanced by products of flux numbers even to parametrically super-Planckian values in more general settings. The effective potential is much smaller than the naive expectation $V \sim \exp(-1/f)\cos(\phi/f)$ (again in Planck units). We believe that this has potentially many interesting applications, from the WGC for axions to inflation and uplifting.

We start with the background solution in section 6.1.1. We consider a Calabi-Yau with a conifold locus in complex structure moduli space at which multiple three-cycles degenerate simultaneously. We explain why this is a generic feature of Calabi-Yaus. Concentrating on the case of two degenerate three-cycles, we introduce separate deformation parameters z_i with phases $\varphi_i = \arg z_i$, $i = 1, 2$, for the two deformed conifold regions. Crucially, the two conifolds, or more precisely the three-spheres located at their apices, are related in homology. As a result, the Calabi-Yau condition ensures that only one complex structure modulus, $z = z_1 = z_2$, is present. Perturbations with $z_1 \neq z_2$ are massive. We then introduce fluxes stabilizing the complex structure modulus z near the conifold point $|z| \ll 1$. The resulting geometry is illustrated in figure 6.1. One can see that the so-called $\mathcal{B}$-cycle is an S^3 which can be thought of as a family of S^2's. This S^2 family reaches into both throats such that the S^2's collapse at the apices. The corresponding dual $\mathcal{A}$-cycle is an S^3 over every point of the double throat in figure 6.1.

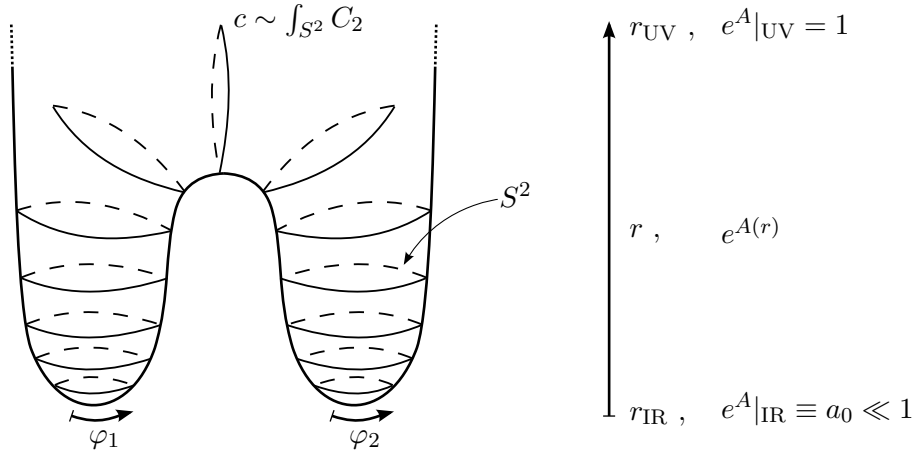

Figure 6.1: An illustration of the setup of the double throat including the phases φ_i and the axion c. The phases φ_i describe physical rotations of each throat. We have not drawn the S^3 over every point of the double throat.

[1]Note that despite the fact that strongly warped throats are needed to generate a small scalar potential for the axion, the decay constant is not suppressed by warping effects. This is because its internal field-profile is *not* localized at the bottom of the throats, in contrast to some examples that have appeared in the literature [276, 277].

In section 6.2, we introduce the axion $c \sim \int_{S^2} C_2$, called *thraxion* from now on.[2] An excursion of the thraxion generates non-zero opposite values of the RR-field strength F_3 at the ends of the two throats. Local backreaction of the resulting energy density then deforms the two throats independently: While the phase φ_1 of the local deformation parameter of one throat is displaced by fluxes, the phase φ_2 of the other throat is displaced in the opposite direction by anti-fluxes. This breaks the constraint $\varphi_1 = \varphi_2$ coming from the CY condition and the homology relation between the two throats. In section 6.3, we calculate the potential induced by non-vanishing 10d Ricci curvature that stabilizes the two deformation parameters against each other. After integrating out heavy degrees of freedom, the result is an effective potential for the thraxion with the properties described above and discussed in section 6.4.

Section 6.6 is devoted to the construction of a 4d supergravity model based on a proposed extension of the Gukov-Vafa-Witten (GVW) superpotential [154] that includes the axion. Besides reproducing the results in 4d supergravity language, we also identify the *saxion* partner of the axion. In section 6.6.3 we generalize our results to general multi throat systems where one or more ultra-light thraxions can appear.

In section 6.7 we explain that our results are very much consistent with the holographic dictionary applied to the Klebanov-Strassler theory [78, 162]. This is done by matching the enhancement of the decay constant of our axion $\int C_2$ with gaugino condensation on the gauge theory side. Applications and implications of these results are the content of section 6.9. This allows us to make the connection to the Kaloper-Sorbo description of axion monodromy quite explicit, and serves as a consistency check. Moreover, it indicates that the model can be trusted well into the gauge theory regime where the throat curvatures are large in string units.

We consider as a semi-explicit example the quintic three-fold stabilized near a conifold transition point. We study the scalar potential for judicious choices of flux quanta. Interestingly, the overall monodromy enhancement is given by the least common multiple of *all* the flux quanta which can easily become parametrically super-Planckian. However, the presence of sub-Planckian modulations generically prevents successful slow-roll inflation. The underlying idea is that a large monodromy is generated by unsynchronized phases (of monodromies of individual throats) drifting away from one another. We call this mechanism *drifting monodromies*. It is completely analogous to the beat phenomenon in accoustics: The interference of harmonics with slightly different small wavelengths leads to large wavelength oscillations, modulated by many

[2] At this stage it may not be obvious why the name 'axion' is appropriate. We will justify this in more detail in later sections. For now let us note that a bulk observer without access to the IR regions of the throats will not notice the fact that the sphere is trivial in homology. In particular an induced scalar potential from the IR regions will be suppressed exponentially, so an approximate shift symmetry is manifest.

smaller ones. For related alternative possibilities of generating large decay constants see [44, 50, 71, 208, 278–282][3].

Finally we describe a clash with the WGC: The effective Euclidean instanton action determined from the scale of the effective potential violates the axionic WGC ($S \lesssim qM_{\rm P}/f_{\rm eff}$) parametrically, but instead a weaker inequality holds,

$$S \lesssim (qM_{\rm P}/f_{\rm eff})^2 \,. \tag{6.4}$$

This is a weaker condition because at fixed control parameter $1/S_E < 1$ the axion decay constant is less constrained as in the original WGC. Conversely, at fixed value of the axion decay constant, the scalar potential can be much smaller than predicted by the WGC.

We finally consider interesting possibilities for uplifting to de Sitter vacua and draw our conclusions in section 6.10.

6.1 Thraxion potential from 10d

6.1.1 Geometry and Flux-Background

First we will explain the basic geometric requirements for our discussion to apply. We will explain why we expect them to be *generically* met.

First we consider again compactifications of type IIB string theory on a Calabi-Yau (CY) threefold, which leads to an effective $\mathcal{N} = 2$ supergravity theory in four dimensions. Now we go to a conifold *transition* locus in complex structure moduli, where n three-cycles degenerate to zero volume [150, 283] that satisfy m relations in homology. We have encountered precisely this setup already in section 3.4: The ($\mathcal{N} = 2$) $U(1)^{n-m}$ gauge theory associated with the $n-m$ deformation complex structure moduli has a Higgs branch parametrized by the scalar components of m hypermultiplets that is properly thought of as the geometric resolution branch. It might come as a surprise that such an intricate configuration should occur *generically*, but it is widely believed that a *generic* CY threefold is in fact related to other CY threefolds via such conifold transitions [76, 284]. This subject isn't closed, but there are large classes of CYs for which this has been shown [285–290][4]. Therefore, a generic CY is believed to have loci

[3]Note in particular the following two papers: The work of [282] is closely related to ours in making use of the conifold complex structure modulus z to create super-Planckian decay constants, while on the technical level the approach is very different. Ref. [208] defines the 5d axion $\int B_2$ on the KT background, analogously to our thraxion. There, the geometric backreaction via the 5d breathing mode allows for monodromy-induced super-Planckian field ranges to be explored in an anisotropic and inhomogeneous 5d spacetime.

[4]But note that this does not mean that every conifold singularity (or even a generic one) is also such a transition point. For example, the mirror quintic threefold at vanishing complex structure has a single shrunken three-cycle. Hence there is no resolved CY geometry [291].

in complex structure moduli space where multiple three-cycles $\mathcal{A}_i$ degenerate together. We expand on this in section 6.6. Being related in homology, the number of homology classes is smaller than the number of collapsing three-cycles. For now we focus on the case of precisely two cycles $\mathcal{A}_{1,2}$ that degenerate. From the above it immediately follows that they are related in homology, i.e. $[\mathcal{A}] \equiv [\mathcal{A}_1] = [\mathcal{A}_2]$. There is a single symplectic dual three-cycle $\mathcal{B}$ connecting the two singular points. We will call this system a *double conifold*. Its complex structure will be denoted by z and the double conifold singularity develops in the limit $|z| \to 0$.

We introduce the fields z_1 and z_2 as illustrated in figure 6.1. These fields may be thought of as 'local complex structure deformations' $z_i = \int_{\mathcal{A}_i} \Omega$, with the holomorphic three-form Ω of the CY, and describe independent *local* deformations of the manifold near one of the two apices. Thus, in the vicinity of either conifold region we want to describe the manifold by embedding it into $\mathbb{C}^4$ via

$$w_1^2 + w_2^2 + w_3^2 + w_4^2 = z_i\,, \quad w \in \mathbb{C}^4\,. \tag{6.5}$$

It is easy to see that the homology relation $[\mathcal{A}_1] = [\mathcal{A}_2]$ enforces the condition $z = z_1 = z_2$ on complex structure moduli space. This is because the difference $\mathcal{A}_1 - \mathcal{A}_2$ is the boundary of a 4-chain $\mathcal{C}$. Therefore, one has

$$z_1 - z_2 = \int_{\mathcal{A}_1} \Omega - \int_{\mathcal{A}_2} \Omega = \int_{\partial \mathcal{C}} \Omega = \int_{\mathcal{C}} \mathrm{d}\Omega\,. \tag{6.6}$$

But on complex structure moduli space one has $\mathrm{d}\Omega = 0$, and hence $z_1 = z_2$. Nevertheless, we will consider the massive deformations of the manifold such that $z_1 \neq z_2$, i.e. deformations away from complex structure moduli space[5] so really,

$$\mathrm{d}\Omega \neq 0\,. \tag{6.7}$$

It is important to note that the local phases φ_i of z_i are the Goldstone bosons of the spontaneously broken $U(1)_R$ symmetry of the conifold [159].[6] This symmetry is approximately restored far away from the tip of either conifold, see figure 6.2. In the limit of large radial coordinates, $r^3/|z_i| \to \infty$, the deformed conifold becomes indistinguishable from the singular conifold.

Finally, we note that we focus on the simple double throat case only for ease of exposition. More general multi conifold situations are analyzed in section 6.6. Furthermore, for reasons of tadpole cancellation we are interested in CY threefolds which

[5] For similar considerations with deformations away from Kähler moduli space, i.e. $\mathrm{d}J \neq 0$, see ref. [292].
[6] This is the R-symmetry group of the dual field theory.

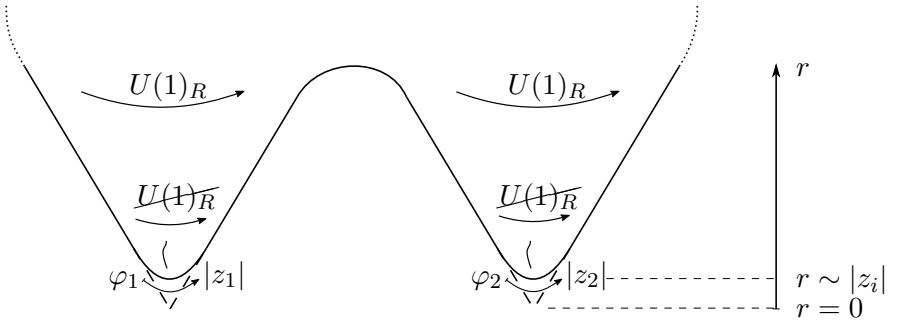

Figure 6.2: The double conifold with asymptotic $U(1)_R$ symmetric regions.

are orientifolded such that O3/O7-planes arise. This projection should leave the conifold transition intact and preserve the key ingredient of a $\mathcal{B}$-cycle reaching down into several conifold regions. In the double throat case, this is realized if two originally present pairs of throats are mapped to each other by the orientifold projection, see figure 6.3. This is completely analogous to the widely-discussed double-throat system of the oldest axion-monodromy models, see e.g. [46, 293] (just simpler, since we need no 2-cycle for the NS5 brane and can hence use standard KS throats). More generally, F-theory solutions with the analogous geometric properties can be considered. Here the tadpole cancellation relies on the fourfold Euler number and no orientifolding is required. Either way, we do not expect that the orientifolding condition or the fourfold embedding endangers the generality of this setting.

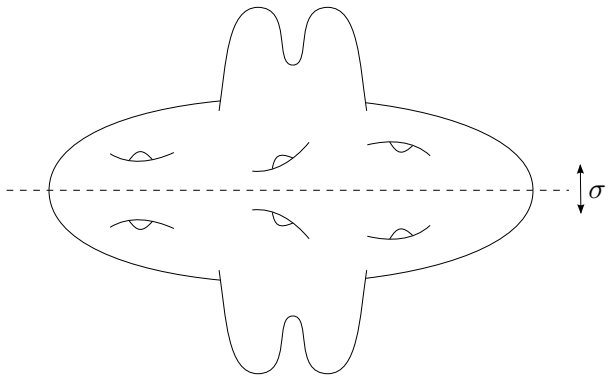

Figure 6.3: A sketch of the the orientifold projection σ. It maps the two originally independent double throat cycles $\mathcal{B}$ and $\mathcal{B}'$ onto one another.

6.1.2 Fluxes on the double throat and a 'Wilson line' axion

We now proceed with our geometrical setup by including three-form fluxes on the $\mathcal{A}$ and $\mathcal{B}$ cycle of the double throat,

$$M \equiv \frac{1}{(2\pi)^2\alpha'}\int_{\mathcal{A}_1} F_3 = \frac{1}{(2\pi)^2\alpha'}\int_{\mathcal{A}_2} F_3\,, \quad K \equiv -\frac{1}{(2\pi)^2\alpha'}\int_{\mathcal{B}} H_3\,. \tag{6.8}$$

Note that while there are two distinct three-cycles $\mathcal{A}_1$ and $\mathcal{A}_2$, their associated flux quanta are identified with each other as they are equivalent in homology. Using that the H_3-flux splits evenly between the two throats, the complex structure modulus is now stabilized at a value

$$|z| \propto \exp\left(-2\pi\frac{K/2}{g_s M}\right) \ll 1\,. \tag{6.9}$$

This even splitting of $\mathcal{B}$-cycle flux will later be shown to arise dynamically, but for now it is enough to recognize that the CY condition $z_1 = z_2$ enforces this in the vacuum.

It is straightforward to show that the phase of the complex structure modulus is set by the RR-3-form flux $Q = \frac{1}{(2\pi)^2\alpha'}\int_{\mathcal{B}} F_3$

$$\varphi = 2\pi\frac{Q}{M} + const\,. \tag{6.10}$$

Locally, we can always set φ to 0 by an appropriate redefinition of the angle. Conversely, without loss of generality, we will choose $Q = 0$.

As explained in section 3.6.2 backreaction of fluxes leads to the formation of warped throats (or Klebanov-Strassler throats). Within these, the metric is well approximated by the Klebanov-Tseytlin solution[7] [78] which we recall for convenience,

$$\mathrm{d}s^2 = e^{2A}\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + e^{-2A}(\mathrm{d}r^2 + r^2\mathrm{d}s^2_{T^{1,1}})\,, \quad e^{2A} \sim \frac{r^2}{g_s M\alpha'}\log(r/r_{\rm IR})^{-\frac{1}{2}}\,, \tag{6.11}$$

with radial coordinate r and warp factor e^{2A}. The radial coordinate is cut off in the IR by the Klebanov-Strassler region and in the UV by the gluing into the bulk CY. One has $a_0 \equiv e^{A(r_{\rm IR})} \sim |z|^{1/3}$. As we have explained, in the vicinity of a conifold transition point a *double throat* (or even *multi throat*) forms[8], see figure 6.1.

The three-cycle $\mathcal{B}$ can be thought of as an S^2 fibered over the radial direction of

[7] Near the bottom of the throat, it has to be replaced by the full Klebanov-Strassler solution [162].

[8] It may not seem obvious that the units of NS-flux on the $\mathcal{B}$-cycle are split democratically so that *each* conifold region is replaced by a warped throat. In fact we will see that there is a light dynamical field that controls this relative distribution (see Section 6.6). In the vacuum however this field is stabilized such that fluxes are indeed distributed democratically.

the conifold [150]. The S^2 collapses at the two *tips* of the deformed conifolds. As introduced in [74], there exists a 4d mode $c(x)$ on the double throat background that can be thought of as the integral of the Ramond-Ramond (RR) two form C_2 over the S^2 as measured far away from the tips of the double throat. This is simple to understand. Simply choose a representative sphere in between the throats that is homologous to the shrinking S^2s. Let us call it $S^2_{\rm UV}$. From the local throat geometries one sees that this is the boundary of three-chains $\mathcal{B}_1$ and $\mathcal{B}_2$,

$$\partial\mathcal{B}_1 = -\partial\mathcal{B}_2 = S^2_{\rm UV}\,, \quad \text{and} \quad \mathcal{B} \equiv \mathcal{B}_1 + \mathcal{B}_2\,. \tag{6.12}$$

These three-chains can be thought of as the local portions of the global three-cycle $\mathcal{B}$ that reach into the respective throats. By Stokes' theorem, a non-trivial 'Wilson' line on the UV sphere satisfies

$$c \equiv \frac{1}{2\pi\alpha'}\int_{S^2} C_2 = \frac{1}{2\pi\alpha'}\int_{\partial\mathcal{B}_1} F_3 = -\frac{1}{2\pi\alpha'}\int_{\partial\mathcal{B}_2} F_3\,, \tag{6.13}$$

so it generates non-quantized pairs of flux and anti-flux on the two respective ends of the cycle $\mathcal{B}$. If we do not allow the throats to backreact geometrically, the potential energy at fixed axion field excursion c is minimized if the flux/anti-flux resides at the bottoms of the throats. By dimensional analysis there is a red-shifted potential,

$$V(c) = \frac{1}{2}m^2c^2 + \ldots\,, \quad \text{with} \quad m^2 \sim a_0^4\,. \tag{6.14}$$

We will consider the geometrical backreaction that was neglected in [74], and turns out to be crucial. In the remainder, we will establish the following points:

- The fields z_1 and z_2 of the two respective throats adjust to the flux/anti-flux pair in such a way that within the two throats supersymmetry is restored locally.
- This adjustment of z_1 and z_2 takes us away from the complex structure moduli space, which is characterized by $z_1 = z_2$ (cf. figure 6.4). For $z_1 \neq z_2$, the CY condition is broken and a scalar potential is generated. This potential is of the order $|z|^2 \sim a_0^6$ and receives its dominant contributions from the bulk CY.
- The backreacted scalar potential is periodic in c with periodicity $2\pi M$. Hence, the naive 2π periodicity of the c-axion is enhanced by a *finite* factor M. While this does *not* allow for a super-Planckian effective axion decay constant $f \gg M_{\rm P}$, approximately Planckian values are possible (see however section 6.9 for a way to also generate large axion periodicities).

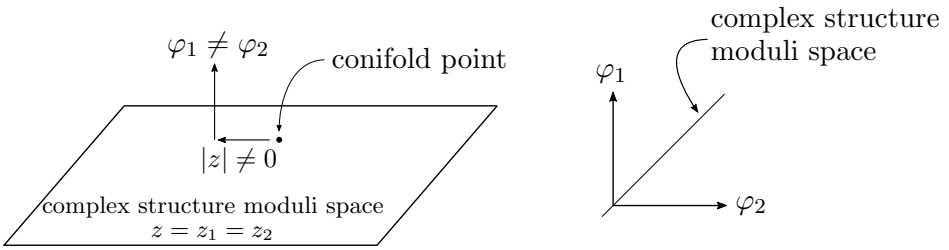

Figure 6.4: Illustration of the z_1-z_2 deformation space. The complex structure moduli space is the subspace $z_1 = z_2$. We only consider deformations away from $z_1 = z_2$ outside the conifold point.

6.2 Local Backreaction in the Throat

We start by discussing how a single throat reacts locally to a finite field excursion c. If the outcome would be that supersymmetry is broken badly within the throats, a description in terms of the GVW superpotential would be questionable. However, we will show that the throat almost perfectly adjusts to produce a locally supersymmetric configuration, so we may use the GVW superpotential self-consistently [40, 154] for the two KS throats. As far as (say) the first local throat is concerned, a non-vanishing field excursion c cannot be distinguished from additional flux $P \equiv c/2\pi$ on the local portion of the $\mathcal{B}$-cycle, say $\mathcal{B}_1$[9], see figure 6.5,

$$c = \frac{1}{2\pi\alpha'}\int_{S^2} C_2 = \frac{1}{2\pi\alpha'}\int_{\mathcal{B}_1} \mathrm{d}C_2 = 2\pi P\,. \tag{6.15}$$

Considering the first throat with complex structure modulus $z_1 \equiv |z_1|e^{i\varphi_1}$, the argu-

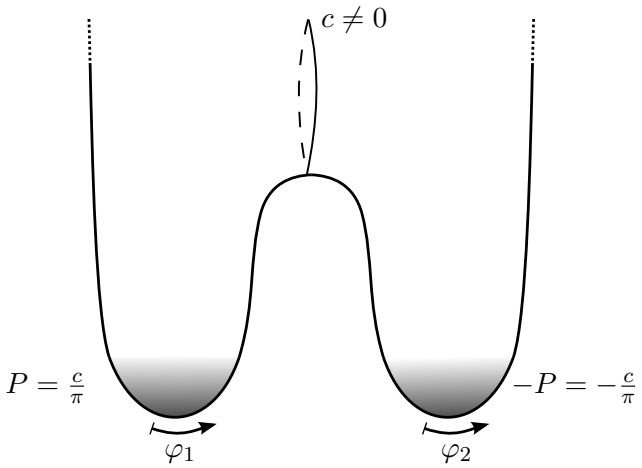

Figure 6.5: Fluxes induced by non-vanishing c are localized at the tips of the throats.

[9]We will use the term flux also for the non-quantized integral $\int F_3$ over some region.

ments of GKP [40] show that there are SUSY configurations for

$$\varphi_1 = 2\pi \frac{P}{M} = c/M \,, \tag{6.16}$$

as in eq. (6.10). Hence, the throat can locally relax the SUSY breaking induced by the extra RR-flux by adjusting the phase of the deformation parameter z_1. However, the second throat sees the field excursion c as the flux $-P = -c/2\pi$ on the $\mathcal{B}$-cycle for which there exists a locally supersymmetric configuration with $\varphi_2 = -c/M$.

Since there is no additional flux that would lead to $|z_1| \neq |z_2|$, we now freeze $|z| = |z_1| = |z_2|$ at the stabilized value (6.10) for what follows. These two modes decouple from the discussion at hand, but will become important later when we introduce the *saxion* partner of the c-axion.

We can encode the discussion above in a 4d EFT potential. To quadratic order, the discrepancy between the local fluxes and local deformations induces a potential

$$V_{\text{flux}}(c, \varphi_1, \varphi_2) = \frac{1}{2}\mu^4 (M\varphi_1 - c)^2 + \frac{1}{2}\mu^4 (M\varphi_2 + c)^2 \,, \tag{6.17}$$

We have $\mu \sim a_0$ since the potential is generated locally near the tip of the throats.

The fact that only the combinations $M\varphi_i \pm c$ appear in the scalar potential can be derived also via ten-dimensional considerations, see appendix A. The key point is that, in the local throats, the combined transformation $\varphi_{1,2} \longrightarrow \varphi_{1,2} \pm \delta$, $c \longrightarrow c + M\delta$ is a diffeomorphism acting on the KS solution. Hence, only the invariant combinations $M\varphi_{1,2} \mp c$ can appear in the scalar potentials that are generated *locally* at the bottom of the throats. Globally, this is not true of course and we will correct for this momentarily.

The potential derived thus far possesses a flat direction which we parametrize by c. This flat direction is given by

$$\varphi_1 = -\varphi_2 = c/M \,. \tag{6.18}$$

This flat direction must of course be lifted due to the fact that we break the CY condition once we set $\phi_1 \neq \phi_2$. We will estimate this effect momentarily.

6.3 The CY Breaking Potential

In the preceding section we have argued that the individual throats react to the field excursion c by adjusting their local deformation parameters z_1 and z_2, more specifically their phases φ_1 and φ_2 respectively. Since the corresponding CY has only one complex structure modulus $z \equiv z_1 \equiv z_2$, the mode z_1/z_2 or rather $\varphi_1 - \varphi_2$ must be massive

already before fluxes are turned on. This eliminates the remaining flat direction in the potential.

We now choose to parameterize the part of the scalar potential that is due to the breaking of the CY condition as

$$V_{\text{CY-breaking}} = \Lambda^4(1 - \cos(\varphi_1 - \varphi_2)) \,, \tag{6.19}$$

with a yet undetermined scale Λ. In writing this we have assumed that the potential is

(a) a function of the difference $\varphi_1 - \varphi_2$ only.

(b) It satisfies $V_{\text{CY-breaking}}(\varphi_1 - \varphi_2) = V_{\text{CY-breaking}}(\varphi_1 - \varphi_2 + 2\pi)$.

(c) The lowest harmonic dominates.

Condition a) must hold because only the local fluxes of the throats stabilize $\varphi_{1,2}$ individually and, without the flux potential, the complex structure modulus $\varphi = (\varphi_1 + \varphi_2)/2$ should be a flat direction. We expect condition b) to hold because we see no reason for a monodromy. Condition c) is a rather unimportant assumption that we make for ease of exposition, but it will be justified in the following.

We now combine the distinct contributions to the scalar potential from fluxes (6.17) and breaking of the CY condition (6.19). We observe immediately that the scalar potential looks very much like the simple two-field potential we wrote down in section 3.8.1 to resolve the Kaloper-Sorbo domain wall in terms of a scalar field. Note in particular the manifest discrete shift symmetry

$$c \longrightarrow c + 2\pi M \,, \quad \phi_{1,2} \longrightarrow \phi_{1,2} \pm 2\pi \,. \tag{6.20}$$

We find it interesting to note that here we encounter two different layers of possible axion monodromy. The first layer is not resolved in terms of a scalar field and corresponds to an M-fold extension of the axion field space. We will postpone a discussion of the domain walls associated with this *finite* layer of monodromy to section 6.7. The 'domain walls' associated with the second layer of monodromy are resolved in terms field profiles of $\phi_{1,2}$. Whether or not this actually leads to a further spontaneous breaking of the axionic shift symmetry depends on the hierarchy between the coefficients Λ^4 and μ^4 as explained in section 3.8.1.

Now, we *assume* that the we are in the unbroken phase of the second layer of monodromy, i.e. we integrate out $\varphi_{1,2}$, assuming $\Lambda^4 \ll \mu^4$ (to be justified below). This corresponds to imposing (6.18). As in section 3.8.1 the effective potential takes the form

$$V_{\text{eff}}(c) = \Lambda^4 \left(1 - \cos(2c/M) + \mathcal{O}(\Lambda^4/\mu^4)\right) \,. \tag{6.21}$$

The height of this potential can be estimated using the 10d solution. To do so, we need to develop a clear picture of how field profiles and 10d geometry change if we displace c. Recall that c is originally defined by a particular 'Wilson line' VEV of C_2 in the UV of the two throats (as well as in the piece of the CY connecting them). Turning on this VEV and focusing on one throat only, we observe a backreaction of the throat geometry which maintains SUSY and corresponds to the motion along a flat direction in 4d field space. This is independently true for the second throat, which backreacts in the opposite way: $\varphi_1 = -\varphi_2 = c/M$.

Now, the crucial point is that the two IR parameters $\varphi_{1,2}$ must by continuity be the boundary values of a smooth higher-dimensional field profile that interpolates between them. We encode this in an effective five-dimensional complex structure field

$$z(x^\mu, r) \equiv |z(x^\mu, r)| e^{i\phi(x^\mu, r)}\,, \tag{6.22}$$

that interpolates between $z_1 = z_0 e^{ic/M}$ and $z_2 = z_0 e^{-ic/M}$ at the respective ends of the throats, so we 'integrate out' the unimportant angular directions of the space $T^{1,1}$. This is illustrated in figure 6.6, which also displays the expected symmetry: The phase of the solution should be antisymmetric under the exchange of the two throats.[10]

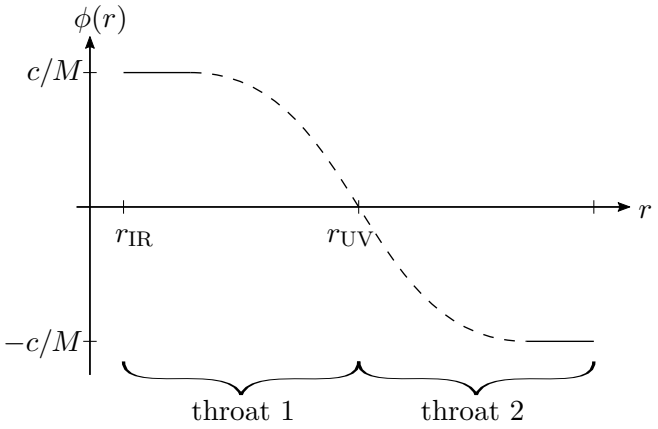

Figure 6.6: The expected profile of the 10d/5d mode along the radial direction.

For computational simplicity, we model the transition region between the throats by a single point, $r = r_{\rm UV}$.[11] In doing so, of course we ignore effects of the unwarped CY region (accepting an $\mathcal{O}(1)$ error). The phase of $z(r)/z_0$ is anti-symmetric under an exchange of the two throats, while the magnitude is symmetric. We may thus limit our attention to one of the two throats when computing the energy density associated with

[10] By a slight abuse of notation we stick with the familiar variable r, although according to our figure this variable must now be growing as one goes down the second throat.

[11] In fact, the exact UV geometry and UV fluxes are irrelevant as long as we do not consider perturbative and non-perturbative corrections [106].

an excursion of c.

The key point is that, after these preliminaries, we are actually able to estimate this energy. It is given by the gradient energy of $z(r)$, which accounts precisely for the clash between the opposite rotations of $\varphi_1 = c/M$ and $\varphi_2 = -c/M$. The relevant action for $z = z(x^\mu, r)$ is obtained by dimensionally reducing the 10d Ricci scalar to quadratic order on the warped conifold background:

$$S[z] = \frac{M_{10\mathrm{d}}^8}{2} \int d^4x \int_{r_{\mathrm{IR}}}^{r_{\mathrm{UV}}} \frac{\mathrm{d}r}{r} \left(-|\partial_r z|^2 - e^{-4A} |\partial_\mu z|^2 \right) . \tag{6.23}$$

Considering the metric (6.11), this form of the 5d action is easily understood. The metric naturally splits into a 5d part $g_{5\mathrm{d}}$ in the external and radial direction and an angular part $\propto g_{T^{1,1}}$. The latter contributes the r-dependent terms $\sqrt{g_{T^{1,1}}} \propto r^5 e^{-5A}$ to the metric determinant. The expectation value of $|z(r)|/r^3$ encodes the degree of $U(1)_R$ symmetry breaking at radius r, compare figure 6.2. As field excursions of the phase of $z(r)$ are obtained by acting with a $U(1)_R$ transformation, any terms in the action that contain the field are multiplied by the factor $|z|^2/r^6$. For $|z| = const.$ this symmetry breaking is of course due to the deformation at the tip of the conifold.

We now apply the static approximation (i.e. disregard the $|\partial_\mu z|^2$ in (6.23)), derive the equation of motion and solve it for the appropriate boundary conditions $z(r_{\mathrm{IR}}) = z_0 e^{\pm ic/M}$. This gives

$$z(r) = z_0 \left(e^{ic/M} - i \frac{r^2 - r_{\mathrm{IR}}^2}{r_{\mathrm{UV}}^2 - r_{\mathrm{IR}}^2} \sin(c/M) \right) . \tag{6.24}$$

Inserting this back into the action (6.23), leads to a 4d potential

$$V \sim |z_0|^2 \left(1 - \cos(2c/M)\right) . \tag{6.25}$$

More generally, for boundary boundary values (z_1, z_2) at the respective throat ends, the field profile is

$$z(r) = z_{0,1} + \frac{1}{2} \frac{r^2 - r_{\mathrm{IR}}^2}{r_{\mathrm{UV}}^2 - r_{\mathrm{IR}}^2} (z_2 - z_1) , \quad \Longrightarrow \quad V(z_1, z_2) \propto |z_1 - z_2|^2 . \tag{6.26}$$

This beautifully matches the form of our proposed scalar potential (6.21), and in particular we infer that $\Lambda^4 \sim |z_0|^2$. Finally inserting the stabilized value $|z_0| \propto a_0^3$ [40], we arrive at $\Lambda^4 \sim a_0^6$. Our assumption $\mu^4 \gg \Lambda^4$ is now justified *a posteriori*. It is also apparent that the effective mass of our ultra-light field is

$$m_c \sim a_0^3 . \tag{6.27}$$

Let us justify the use of the static approximation scheme we have employed. Really, we should have supplemented (6.23) by the kinetic term $S_{\rm kin}[c] \sim \int {\rm d}^4x\,(\partial_\mu c)^2$, imposed the constraint $\phi(x^\mu, r_{\rm IR}) = c(x^\mu)/M$, and determined the mass of the lowest-lying KK mode of the resulting 5d action. However, it is intuitively clear that the UV-dominated kinetic term of c is much more important than the warped-down 4d-gradient term $(\partial_\mu\phi)^2$ in (6.23). Thus, c is the most inert part of the system and it is an excellent approximation to assume that the ϕ profile extremizes just the 5d-gradient-part of the action. To make this quantiative, one may substitute c on the r.h. side of (6.24) with the plane wave $c = \exp(ikx)$ (with $k^2 = -m_c^2$) and check that the resulting $(\partial_\mu\phi)^2$ contribution from (6.23) is negligible compared to $S_{\rm kin}[c]$.

6.4 Discussion of Results

The information we have gathered can be summarized in an effective Lagrangian[12],

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}f_\varphi^2(\partial\varphi_1)^2 - \frac{1}{2}f_\varphi^2(\partial\varphi_2)^2 - \frac{1}{2}f_c^2(\partial c)^2 \\ &-\frac{1}{2}\mu^4(M\varphi_1 - c)^2 - \frac{1}{2}\mu^4(M\varphi_2 + c)^2 - \Lambda^4(1-\cos(\varphi_1-\varphi_2))\,,\end{aligned} \tag{6.28}$$

with coefficients

$$\begin{aligned} f_\varphi^2 &\sim \log(a_0^{-1})^{-3/2}a_0^2\ M_{\rm P}^2\,, & f_c^2 &\approx \frac{2}{9M^2}\log(a_0^{-1})^{-1}\ M_{\rm P}^2\,, \\ \Lambda^4 &\sim \frac{g_s^2}{(g_sM)^4}\log(a_0^{-1})^{-7/2}a_0^6M_{\rm P}^4\,, & \mu^4 &\sim \frac{g_s^4}{(g_sM)^6}\log(a_0^{-1})^{-3}a_0^4\ M_{\rm P}^4\,. \end{aligned} \tag{6.29}$$

The above expressions are valid for the special case of the bulk CY having a single characteristic length-scale, $R_{\rm CY}^6 \sim {\rm Vol(CY)}$, and where the throats marginally fit into the bulk, i.e. $R_{\rm CY}^4 \sim R_{\rm throat}^4 \sim g_sMK\alpha'^2$. For the general case and a derivation of the parametric dependencies on flux quanta and g_s see appendix D of [2].

When we say that the throats should fit into the throat marginally, we mean that the geometrical consistency requirements of section 5.6 that were used to constrain the KKLT construction are approximately saturated. As we are using throats in both setups this is completely analogous: The bulk CY has an overall size $R_{\rm CY}$ that is set by a combination of the Kähler moduli, and the throats can be thought of as objects of a characteristic physical size $R_{\rm throat}$ embedded into the bulk CY. This size $R_{\rm throat}^4$ is set by the local D3 brane charge stored in the fluxes of the throat [40, 162], independently of the size of the bulk CY. For this to be a geometrically consistent configuration, we

[12]For ease of exposition we have written down a diagonal kinetic matrix. This is not quite the case but is not relevant for our discussion. See App. A for details.

must require $R_{\rm CY} > R_{\rm throat}$. Taking $R_{\rm CY} \sim R_{\rm throat}$ is the case where the throats fit into the bulk CY only *marginally*.

Far below the scale $a_0 M_{\rm P}$ we may integrate out $\varphi_{1,2}$, to obtain the effective Lagrangian

$$\mathcal{L}' = -\frac{1}{2} f_c^2 (\partial c)^2 - \Lambda^4 (1 - \cos(2c/M)) \,. \tag{6.30}$$

We would like to highlight the following points,

- Our simplification $R_{\rm throat} = R_{S^2} = R_{\rm CY}$ gives the largest possible value for the decay constant $f_{\rm eff} = M f_c$; any hierarchy $R_{\rm throat} < R_{S^2} < R_{\rm CY}$ suppresses its value. Taking into account logarithmic corrections, the maximal periodicity one can achieve is $\mathcal{O}(M_{\rm P}/\sqrt{\log a_0^{-1}})$ (see Appendix D of [2]). A large hierarchy $a_0 \ll 1$ suppresses the periodicity only very mildly. By taking $g_s M$ and g_s^{-1} to be large, the 10d perturbative expansion becomes better controlled without affecting the axion periodicity. In this sense, our axion can be made approximately Planckian.

- The mass of the axion is $\mathcal{O}(a_0^3)$ which is parametrically smaller than both the warped Kaluza-Klein scale ($\mathcal{O}(a_0)$), and the estimate of [74], where backreaction of the local geometry was not taken into account ($\mathcal{O}(a_0^2)$). The mass-spectrum is essentially *gapped*.

- As pointed out before, the scale of the effective potential is set by the $U(1)_R$ breaking induced by the deformation of the conifold as measured in the UV $\sim |z|^2$. Strictly speaking this is *not* a warp factor suppression, although for moderate CY volumes $|z|$ and a_0^3 are of the same order[13].

The following caveats should be noted: The effective Lagrangians (6.28) and (6.30) are incomplete: We have worked in the regime of classical type IIB solutions so at least the universal Kähler modulus T is not yet stabilized. Moreover, we have not included the b-axion that complexifies c. Finally, there is no parametric separation between the mass scale of the complex structures and the warped Kaluza-Klein scale. Hence, the Lagrangian (6.28) does not define a useful effective field theory in the Wilsonian sense. Equation (6.30) however *does* give rise to a Wilsonian effective Lagrangian once it is completed by the b-axion and the Kähler modulus T.

6.5 The B_2-axion

In the preceding sections we have focused on the ultralight c-axion that can be thought of as the integral of the RR two-form C_2 over a sphere between the two throats. Simi-

[13] One might for instance be tempted to consider the large volume limit where warping becomes negligible. In this case the scale of the potential would still be given by $|z|^2 \ll 1$.

larly, we can define a b-axion by integrating the NS two-form B_2 instead,

$$b \equiv \frac{1}{2\pi\alpha'} \int_{S^2} B_2 \,. \tag{6.31}$$

By the same arguments as before (see (6.15)) a non-vanishing field excursion induces a pair of H_3 flux/anti-flux on the portions of the $\mathcal{B}$-cycle that reach down into the two throats. Now, in the vacuum the $\mathcal{B}$-cycle is already filled with quantized H_3-flux,

$$K \equiv K_1 + K_2 \equiv \frac{1}{(2\pi)^2\alpha'} \left(\int_{\mathcal{B}_1} H_3 + \int_{\mathcal{B}_2} H_3 \right) . \tag{6.32}$$

Here, as in eq. (6.12), $\mathcal{B}_1$ and $\mathcal{B}_2$ are the three-chains that reach into the respective throats and are bounded by the sphere between the throats, so that $\mathcal{B} = \mathcal{B}_1 + \mathcal{B}_2$. Clearly the continuous field excursion of the b-axion does not change the quantized flux integer K. However, by Stokes' theorem it *does* change the relative flux distribution,

$$K_1 \longrightarrow K_1 + \frac{b}{2\pi} \,, \quad K_2 \longrightarrow K_2 - \frac{b}{2\pi} \,. \tag{6.33}$$

By definition, K_1 and K_2 are the (non-quantized) H_3 fluxes that reside in the respective throats. Again, treating the local throat deformation parameters $z_{1,2}$ as independent it is clear that the throats can restore supersymmetry by an appropriate adjustment [40]:

$$|z_{1,2}| \sim \exp\left(-2\pi \frac{K_{1,2}}{g_s M} \right) \;\longrightarrow\; \exp\left(-2\pi \frac{K_{1,2} \pm b/2\pi}{g_s M} \right) . \tag{6.34}$$

Thus, the discussion of the previous section applies also to the b-axion if one replaces the phases of the local deformation parameters by $\log|z_i|$, i.e. the boundary values of the five-dimensional field $z(r)$ depend on the two real axions as

$$z_1 = z_0 e^{ic/M - b/g_s M} \,, \quad z_2 = z_0 e^{-ic/M + b/g_s M} \,. \tag{6.35}$$

In other words, while the c-axion rotates the throats against each other, the b-axion makes one throat longer and the other shorter (see figure 6.7). As a consequence the scalar potential is of order

$$V(c,b) \sim |z_1 - z_2|^2 \sim |z_0|^2 \left| \sin\left(\frac{c + ib/g_s}{M} \right) \right|^2 , \tag{6.36}$$

which we will confirm in the next section from a 4d supergravity point of view.

For now let us emphasize that both the c and b axion are bulk fields. In particular a change in their field excursion is not straightforwardly measured by an observer

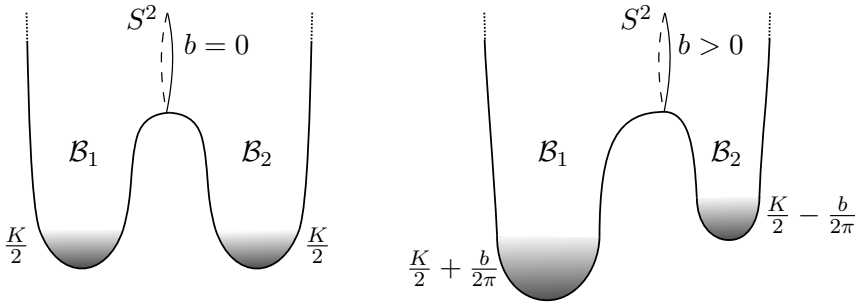

Figure 6.7: The physical effect of a field excursion ofthe b-axion in the double throat system. One throat becomes shorter, whereas the other becomes longer.

located somewhere (say) in the middle of either throat. More precisely, in the limit of an infinitely long KS throat the axion field excursions we consider have no gauge invariant meaning at all (they decouple). Rather, the axion field excursions can be seen only from their global embedding into the bulk CY: The c-axion rotates the throats against each other, whereas the b-axion makes one throat longer and the other shorter (see Figure 6.7).

In contrast, the *local* throat theories remain in their supersymmetric ground states. This is important to keep in mind because there *do* exist physical axionic field excursions also in the infinitely long KS throat, superficially similar to the ones we are considering. By integrating over the angular space $T^{1,1}$ the throat theory is reduced to an effective five-dimensional theory also containing a C_2 and a B_2 axion. But, these are *not* light degrees of freedom. The C_2 axion is eaten by a vector via the Stückelberg mechanism[14] [294] while the B_2 axion receives a mass term from fluxes in the local throats (compare e.g. [208], and for further details see Appendix A). So, the fact that there are no light axions in the effective $5d$ theory is *not* in conflict with the existence of light $4d$ axions in a coupled multi-throat system embedded in a compact bulk CY.

Finally, we find it worthwhile noting that the $D3$ brane charge stored in throat fluxes is redistributed from one throat to the other by a non-trivial field excursion of the b-axion. This is because the local $D3$ brane charge in the throats is set by the local fluxes as

$$N_{D3}|_{\text{throat } 1,2} \equiv MK_{1,2}\,, \tag{6.37}$$

and explains the change in size of the throats which is set by the local $D3$-brane charge as depicted in Figure 6.7.

[14]The effective $5d$ theory contains a $U(1)$ vector field that gauges coordinate reparametrizations along the orbits of the angular $U(1)_R$ isometry of $T^{1,1}$. This vector field becomes massive by eating the $5d$ c-axion.

6.6 Four-dimensional supergravity completion

So far we have discussed how the c-axion and the b axion backreact on the phases and magnitudes of the local deformation parameters of the throats. In this section we propose a completion of the model in the language of 4d supergravity. The C_2-axion pairs with the analogous B_2-axion into a complex field $\mathcal{G} = c - \tau b$.

6.6.1 Counting Moduli Through the Conifold Transition

Throughout section 6.1 we have focused on the case of two S^3-cycles related in homology, i.e. $[\mathcal{A}^1] = [\mathcal{A}^2]$. In general we denote by n the number of collapsing three-spheres $\mathcal{A}^i$, $i = 1, ..., n$ and by m the number of homology relations between them $\sum_{i=1}^{n} p_i^I [\mathcal{A}^i] = 0$, $I = 1, ..., m$.

As we have explained in section 3.4, before the fluxes are turned on and orientifold projections are imposed the physics near conifold transition loci is governed by the Greene, Morrison, Strominger gauge theory: There are $n-m$ complex structure moduli z^i that are the scalar components of $n - m$ vector multiplets. The z^i parametrize the Coulomb branch of the gauge theory. Whenever some of the three-cycles shrink to zero size, charged D3 brane hypermultiplets (*Strominger black holes*) become massless and have to be 'integrated in' [151]. At the origin of the Coulomb branch there are hence n massless charged hypermultiplets and n singular nodes have developed in the CY threefold. There exists an m-dimensional Higgs-branch where the singular nodes are resolved into m (homologically independent) $\mathbb{P}^1$'s [77]. On this branch, the $n-m$ vector multiplets eat $n - m$ D3-brane hypermultiplets and become massive. Geometrically speaking this is the resolution of the conifold [76, 150].

In the $\mathcal{N} = 1$ flux compactification that we are considering the tips of the conifolds become strongly red-shifted. Moreover backreaction of fluxes ensures that even at tiny complex structure the S^3's stay at finite size so that the Strominger black holes cannot ever play an important role. However, since the deformed and the resolved conifold differ only by their strongly red-shifted tip geometries it is natural to expect at least some remnant of the resolved phase of the conifold theory in the light spectrum. As outlined in section 6.1 we expect the 'local complex structures' to decouple from one another so that *all* the n local deformation parameters $z_1, ..., z_n$ become equally light. In other words there are m additional light complex geometric modes. Moreover, on the resolved side of the transition there would be m massless complex axion modes. Since the obstruction for them to be massless is also localized at the tips of the conifolds where the would-be two-cycles collapse we also expect m complex light axionic modes

$\mathcal{G}^I$.[15] As we will argue in the next section, these modes indeed appear quite naturally in the discussion of the flux superpotential.

6.6.2 The Thraxion Superpotential

In this section we make a proposal for the 4d supergravity completion of the Lagrangians (6.28) and (6.30) for a general number of throats n with m homology relations among the shrinking cycles. Throughout this section we work in units $M_{\rm P} = 1$.

As a starting point we consider the GVW flux superpotential for a multi conifold system. All the necessary ingredients are derived in ref. [295] and summarized in Appendix B. We choose to treat the redundant set of the n complex structure parameters z_i associated with the n vanishing cycles $\mathcal{A}^i$ democratically, and impose the m *CY conditions* via a set of Lagrange multipliers λ_I, $I = 1, .., m$. The superpotential reads

$$W(z) = \sum_{i=1}^{n} \left(M_i \frac{z_i}{2\pi i} \log(z_i) + M_i g^i(z) - \tau K^i z_i \right) + \sum_{I=1}^{m} \lambda_I P^I + \hat{\hat{W}}_0(z) \,. \tag{6.38}$$

The m homology relations among the vanishing cycles $\sum_{i=1}^{n} p_i^I \mathcal{A}^i = \partial \mathcal{C}^I$, $I = 1, ..., m$ lead to the following m CY conditions for the $z_i \equiv \int_{\mathcal{A}^i} \Omega$,

$$0 \overset{\mathrm{d}\Omega=0}{=} \int_{\mathcal{C}^I} \mathrm{d}\Omega = \int_{\partial \mathcal{C}^I} \Omega = \sum_{i=1}^{n} p_i^I \int_{\mathcal{A}^i} \Omega = \sum_{i=1}^{n} p_i^I z_i \equiv P^I \,, \quad I = 1, ..., m \,. \tag{6.39}$$

In this language, the m CY conditions $P^I = 0$ are equivalent to the F-term equations of the Lagrange multipliers λ_I, $\partial_{\lambda_I} W \overset{!}{=} 0$. For details we refer the reader to appendix B.[16]

Here, the M_i and K^i are the flux numbers associated to the $\mathcal{A}$- and $\mathcal{B}$-cycle of the i-th throat, and the holomorphic function $\hat{\hat{W}}_0(z)$ denotes contributions to the flux superpotential from other cycles. The $M_i \in \mathbb{Z}$ cannot all be chosen independently but must comply with the m homology conditions

$$\sum_{i=1}^{n} p_i^I M_i = 0 \,, \quad I = 1, ..., m \,. \tag{6.40}$$

[15] It is natural to conjecture that in the absence of fluxes and orientifolds the additional deformation parameters pair with the additional axions into $\mathcal{N} = 2$ BPS hypermultiplets, though we will not follow this line of thought here.

[16] Note that we restrict ourselves to regions in complex structure moduli space close to the conifold transition point, where all throats degenerate simultaneously. This might be more restrictive than is needed for our analysis: If the matrix p_i^I is block-diagonal, we can separate the multi throat system into smaller multi throats whose deformations are independent of one another. In this case we can go through a conifold transition by local degeneration of the throats of a smaller system. Even away from the trivial case of multi throats factorizing, one might be able to achieve small thraxion masses by 'freezing' individual throats with larger deformation z. Given a multi throat with some large z's one has to check the thraxion potential as proposed in this section for flat directions. We leave a more thorough analysis of this possibility for future work.

The K^i can be chosen independently but there is an m-fold redundancy in their definition because we may transform $K^i \longrightarrow K^i + \sum_I \alpha_I p_i^I$ for any $\alpha \in \mathbb{C}^m$ leaving the superpotential invariant upon imposing the constraint equations.[17] Furthermore, there are n unknown functions $g^i(z)$ defined on complex structure moduli space that are holomorphic near the origin.

The complex structure Kähler potential (3.33) is expanded as

$$K_{\rm cs}(z_i,\bar{z}_i) = -\log\left(-i\int \Omega\wedge\overline{\Omega}\right) = -\log\left(ig_K(z) - i\overline{g_K(z)} + \sum_{a=1}^{n-m} i\bar{z}_a G^a + c.c.\right)$$
$$= -\log\left(ig_K(z) - i\overline{g_K(z)} + \sum_{i=1}^{n}\left[\frac{|z_i|^2}{2\pi}\log(|z_i|^2) + i\bar{z}_i g^i(z) - iz_i\overline{g^i(z)}\right]\right)\,, \qquad (6.41)$$

where the holomorphic function $g_K(z)$ encodes contributions from other cycles. We would like to stress that despite the fact that we have written the unknown functions g^i, g_K and $\hat{\hat{W}}_0$ as functions of all the z_i, $i = 1, ..., n$, knowledge of the periods of the various cycles (and the flux quanta) only determines their behavior *along* complex structure moduli space and not beyond.

We are now ready to formulate a proposal for the thraxion superpotential. First, we note the following. By expanding the Lagrange multiplier terms, one may rewrite the superpotential (6.38) as

$$W(z) = \sum_{i=1}^{n}\left(M_i\frac{z_i}{2\pi i}\log(z_i) + M_i g^i(z) + \left[-\tau K^i + \sum_{I=1}^{m}\lambda_I p_i^I\right]z_i\right) + \hat{\hat{W}}_0(z)\,. \qquad (6.42)$$

One observes immediately that the combinations $\sum_{I=1}^m \lambda_I p_i^I$ can be interpreted as an *additional*, unquantized contribution to the complex three-form flux $G_3 = F_3 - \tau H_3$ on the (local portion $\tilde{\mathcal{B}}^i$ of the) $\mathcal{B}$-cycle of the i-th conifold. But we know that such a flux is detected by a boundary integral

$$\hat{\mathcal{G}}_i \equiv c_i - \tau b_i \equiv \frac{1}{2\pi\alpha'}\int_{S^2|_{i\text{-th throat}}}(C_2 - \tau B_2) = \frac{1}{2\pi\alpha'}\int_{\tilde{\mathcal{B}}^i}(F_3 - \tau H_3) \qquad (6.43)$$

over the S^2 at the top of the i-th throat. Crucially, the variables $\hat{\mathcal{G}}_i$ define axionic field

[17] The $n-m$ physical H_3 flux quantization conditions can be stated as $K^a - \sum_{I=1}^m p_a^I K^{n-m+I} \in \mathbb{Z}$, $a = 1, \ldots, n-m$. This is because we can always choose the first $n-m$ of the shrinking cycles to correspond to integral basis elements $[\mathcal{A}^1], ..., [\mathcal{A}^{n-m}]$ in homology. The Lagrange constraints can be stated as $0 = P^I = \sum_{a=1}^{n-m} p_a^I z_a + z_{n-m+I}$, i.e. $z_{n-m+I} = -\sum_{a=1}^{n-m} p_a^I z_a$. In the superpotential the terms that multiply $z_1, ..., z_{n-m}$ are given by the above combination of K^i and correspond to the integer flux numbers on the cycles $\mathcal{B}_1, ..., \mathcal{B}_{n-m}$. Alternatively, one may demand the sufficient but not necessary conditions that $K^i \in \mathbb{Z}$ for $i = 1, \ldots, n$. In this more restrictive but democratic formulation the i-th throat carries K^i units of flux. We can still reach all possible integer values for flux numbers on the cycles $\mathcal{B}_a$.

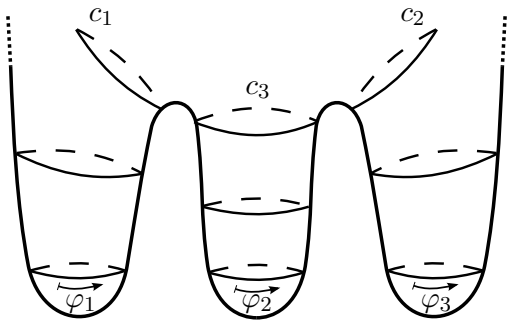

Figure 6.8: A cartoon of the $\mathcal{B}$-cycle of the triple throat, $n = 3$ and $m = 2$. The local c-axion excursions c_1, c_2 and c_3, $c_i = \mathrm{Re}\,\hat{\mathcal{G}}_i$, must be chosen so that no overall flux is generated on $\mathcal{B}$, i.e. $0 \stackrel{!}{=} c_1 + c_2 + c_3$ [18] or rather $\sum_i \hat{\mathcal{G}}^i = 0$.

excursions as measured near the entrance of the i-th throat.

We would like to interpret (a subset of) these as light physical degrees of freedom. This is motivated by the fact that there are m light axions on the other side of the conifold transition that correspond to the integrals of $C_2 - \tau B_2$ over the independent resolution 2-cycles. Indeed, the counting is correct. A consistent axionic field excursion must not induce any overall flux on *any* of the global $\mathcal{B}$-cycles (see figure 6.8). There are hence $n - m$ *no-flux* conditions, one for each linearly independent $\mathcal{B}$-cycle, leaving only m physical axions. These can be parametrized as $\hat{\mathcal{G}}^i = \sum_{I=1}^{m} p_i^I \mathcal{G}_I$ and we are led to the following conjecture:

> *The Lagrange multipliers λ_I must be promoted to m light axionic degrees of freedom, $\lambda_I \longrightarrow \frac{\mathcal{G}_I}{2\pi}$. Moreover, the z^i are promoted to n physically independent degrees of freedom.*

The normalization factor 2π is chosen such that locally in the i-th throat a shift of axionic field excursion $\mathcal{G}_I$ by 2π (or $2\pi\tau$) for some I is indistinguishable from an increase of the F_3-flux (respectively H_3-flux) on the local portion of the $\mathcal{B}$-cycle of the throat by an integer amount p_I^i.

Thus, our propoal for the superpotential is

$$W = \sum_{i=1}^{n} \left(M_i \frac{z_i}{2\pi i} \log(z_i) + M_i g^i(z) - \tau K^i z_i \right) - \sum_{I=1}^{m} \frac{\mathcal{G}_I}{2\pi} P^I + \hat{\hat{W}}_0(z)\,. \tag{6.44}$$

We find it interesting to note that the axions $\mathcal{G}^I$ now serve as the *stabilizer fields* for the combinations of the local deformation parameters that break the m CY conditions

[18] Compare this to figure 6.5: The 'no-flux' condition in the double throat setup amounts to $c_1 = -c_2$. The two axions c_1 and c_2 are actually identified, up to a sign due to different orientation of the two-sphere in the definition. This is why we only had one axion c to begin with in the 10d analysis of section 6.1.

$P^I = 0,\ I = 1, ..., m$. This form of the dynamical thraxion superpotential is fairly unique in that it preserves the set of *discrete* shift-symmetries

$$z_i \longrightarrow z_i e^{\frac{2\pi i}{M_i}\sum_I \eta_I p_i^I}\,, \quad \mathcal{G}_I \longrightarrow \mathcal{G}_I + 2\pi\eta_I\,, \quad \forall \eta \in \mathbb{C}^m : \sum_I \eta_I p_i^I \in M_i\mathbb{Z} \quad \forall i\,. \tag{6.45}$$

Our proposal for the Kähler potential is

$$K(\mathcal{G}_I, \bar{\mathcal{G}}_{\bar{I}}, T, \bar{T}, z, \bar{z}) = K_1(\mathcal{G}_I - \bar{\mathcal{G}}_{\bar{I}}, T + \bar{T}) + K_{\rm cs}(z, \bar{z})\,, \tag{6.46}$$

where $K_{\rm cs}$ is the Kähler potential (6.41) and K_1 is the Kähler potential of the m axions (and Kähler moduli T) on the other side of the conifold transition as derived in [153], and quoted in eq. (3.49).

We expect (6.41) and (6.44) to hold even when we break the CY condition $P^I \neq 0$ with the important subtlety that the domain of the holomorphic functions g^i, g_K and $\hat{\hat{W}}_0$ must be extended *beyond* complex structure moduli space. We find it reasonable to expect that such an extension *exists* although even full knowledge of the CY periods would not determine their behavior away from the moduli space. The detailed form of these functions will be of no importance in what follows. Moreover, we expect that using the potential K_1 gives an excellent approximation because the kinetic term of the axions is dominated by contributions from the UV where the deformation or resolution of the conifold plays but a tiny role. Note, that the behavior of the kinetic terms of the complex structure moduli is dominated by the logarithmic terms in (6.41). In particular, the functions $g^i(z)$ and $g_K(z)$ contribute to kinetic terms only at sub-leading order.

Since we are interested in small z_i we Taylor-expand

$$\begin{aligned} g^i(z) =& g_0^i + \sum_{j=1}^n g_1^{ij} z_j + \mathcal{O}(z^2)\,, \quad \hat{\hat{W}}(z) = g_{W,0} + \sum_{i=1}^n g_{W,1}^i z_i + \mathcal{O}(z^2)\,, \\ g_K(z) =& g_{K,0} + \sum_{i=1}^n g_{K,1}^i z_i + \mathcal{O}(z^2)\,. \end{aligned} \tag{6.47}$$

This should really be understood as a Taylor expansion in n *independent* variables z_i and makes our conjectured extension of the domain of these functions beyond the complex structure moduli space manifest.

We absorb all $\mathcal{O}(z^0)$ terms in the superpotential in the definition $\hat{W}_0 \equiv g_{W,0} + \sum_{i=1}^n M_i g_0^i$. The coefficients in (6.47) should all be viewed as independent of the flux quanta that thread the cycles of the multi throat system, and only $(g_{W,0}, g_{W,1}^i)$ depend on fluxes on other cycles.

It is clear that to obtain the effective superpotential for the $\mathcal{G}$-fields we should integrate out the local deformation parameters. Before we discuss this in full generality it is instructive to first consider the simplest case of the double throat, i.e. $n = 2$ and $m = 1$. There are two $\mathcal{A}$-cycles $\mathcal{A}^1$ and $\mathcal{A}^2$ and we choose the homology relation to be $\mathcal{A}^1 \sim \mathcal{A}^2$. Hence, there are two deformation parameters z_1 and z_2 and one axion $\mathcal{G}$. For ease of exposition we assume that of all the coefficients defined in (6.47), only $g_{W,0}$ and $g_{K,0}$ are non-vanishing, in other words, we choose $\hat{W}_0$ as well as all non-logarithmic terms in the Kähler potential to be constant. In doing so we accept an $\mathcal{O}(1)$ error in all expressions, in particular in the resulting superpotential W_{eff} for $\mathcal{G}$. This simplifying assumption will be dropped when we generalize the discussion to the multi throat case in section 6.6.3.

We must set $M_1 = M_2 \equiv M$ due to the homology relation between the shrinking cycles, and we choose $K^1 \equiv K/2 \equiv K^2$ which results in flux $K^1 + K^2 = K$ on the $\mathcal{B}$-cycle. All choices of the pair (K^1, K^2) that satisfy $K^1 + K^2 = K$ are physically equivalent to this choice and can be brought back to the symmetric choice via a linear redefinition of $\mathcal{G}$ (compare the discussion below (6.40)). The superpotential takes the form

$$W(z_1, z_2, \mathcal{G}) = \sum_{i=1}^{2} \left(\frac{z_i}{2\pi i} \log(z_i) M - \frac{1}{2} K \tau z_i \right) - \frac{\mathcal{G}}{2\pi}(z_1 - z_2) + \hat{W}_0 + \mathcal{O}(z_i^2) \,. \tag{6.48}$$

First, the F-terms F_{z_i} are given by

$$\begin{aligned} D_{z_i} W &= \partial_{z_i} W + (\partial_{z_i} K) W = \frac{\log(z_i) + 1}{2\pi i} M - \frac{K}{2}\tau \mp \frac{\mathcal{G}}{2\pi} + \mathcal{O}(z_i) \\ &= \frac{M}{2\pi i} \left(\log(z_i/z_0) \mp i\mathcal{G}/M \right) + \mathcal{O}(z_0) \,, \end{aligned} \tag{6.49}$$

where

$$z_0 = e^{-1} \exp\left(2\pi i \tau \frac{K/2}{M} \right) + \mathcal{O}\left(e^{-4\pi \frac{K/2}{g_s M}} \right) = \mathcal{O}\left(e^{-2\pi \frac{K/2}{g_s M}} \right) \,. \tag{6.50}$$

As usual, with $K/2 > g_s M$ one obtains $|z_0| \ll 1$ with universal dependence on the flux numbers. Following section 6.1 we may integrate out the local deformation parameters, which yields

$$z_1 = z_0 e^{i\mathcal{G}/M} \,, \quad z_2 = z_0 e^{-i\mathcal{G}/M} \,. \tag{6.51}$$

The effective superpotential for the axion $\mathcal{G}$ reads

$$\begin{aligned} W_{\text{eff}}(\mathcal{G}) &= 2\epsilon \left(1 - \cos(\mathcal{G}/M) \right) + W_0 + \mathcal{O}(z_0^2) \,, \\ \text{with} \quad \epsilon &\equiv M \frac{z_0}{2\pi i} \,, \quad \text{and} \quad W_0 \equiv \hat{W}_0 - \frac{z_0}{\pi i} M \,. \end{aligned} \tag{6.52}$$

This is the expression we were after. Crucially, it is consistent with the results of section 6.1: In section 6.8 we will show that $V(\mathcal{G},\bar{\mathcal{G}}) \propto |\partial_{\mathcal{G}} W(\mathcal{G})|^2$. So, if we restrict to $\mathcal{G} = c \in \mathbb{R}$, we reproduce the periodic potential (6.21) with all the correct parametric scaling properties, in particular $|\epsilon|^2 \sim |z_0|^2 \sim a_0^6$.

Note that because we have made use of the *unwarped* Kähler potential we do not reproduce the correct mass-scale of the local deformation parameters z_i. Here, this is of no importance because all degrees of freedom that are related to strongly warped regions are integrated out supersymmetrically. In particular, the potential energy induced by a non-vanishing field excursion of the field $\mathcal{G}$ receives its dominant contributions from the bulk CY where warping plays no role. Because in going from weak to strong warping, the solutions of the complex structure F-terms are left invariant [154], and because the z_i are parametrically heavier than $\mathcal{G}$ even when the appropriate red-shift factors are introduced in the scalar potential, this procedure is justified.[19]

We are now ready to expand on the conclusions we have drawn in section 6.1. First of all, the kinetic term of the full complex field $\mathcal{G}$ lives in the bulk. This implies that the mass^2 of $\mathcal{G}$ is of order $|z_0|^2 \ll 1$. Since the Kähler potential is independent of $\mathrm{Re}(\mathcal{G})$ a discrete shift-symmetry $\mathcal{G} \longrightarrow \mathcal{G} + 2\pi M$ is manifest[20], while the IR superpotential breaks the shift-symmetry corresponding to $\mathrm{Im}(\mathcal{G})$ completely.

While in principle the target space distance traversed by $\mathrm{Im}(\mathcal{G})$ can be made large, the scalar potential grows exponentially as a function thereof as is common for saxionic directions in field space. In particular, this direction in field space is of little use for (slow-roll) inflation. There is a *critical* field excursion $|\mathrm{Im}(\mathcal{G})_{\mathrm{crit}}| \lesssim 3M \log(a_0^{-1})$ beyond which one side of the double throat is entirely pulled up into the bulk CY, $z_1 \sim 1$ or $z_2 \sim 1$. Near this field excursion we no longer know the form of the potential because we work to lowest order in $|z_1|, |z_2|$. Moreover, there is a tower of warped KK-modes with masses that scale as

$$m_n^2 \sim n^2 a_0^2 \exp\left(-2|\mathrm{Im}(\mathcal{G})|/3M\right) , \tag{6.53}$$

where the warp factor $a_0^2 \sim |z_i|^{2/3}$ now depends on $\mathrm{Im}(\mathcal{G})$. Since these modes have been integrated out, the ratio $\Lambda/m_{\mathcal{G}}$ of the cutoff of the $\mathcal{G}$-EFT (i.e. the smallest KK-mass) over the mass-scale of $\mathcal{G}$, comes down as

$$\Lambda/m_{\mathcal{G}} \sim a_0^{-2} e^{-\frac{4}{3M}|\mathrm{Im}\,\mathcal{G}|} \leq a_0^{-2} \exp\left(-\frac{\phi_b}{M_{\mathrm{P}}}\right) , \tag{6.54}$$

[19] For a recent computation of the warped (i.e. physical) Kähler potential, see ref. [296]

[20] In an exact no-scale background the scalar potential even has periodicity πM. However, any no-scale breaking effects will break it to the periodicity of the superpotential.

at large field excursion, consistent with a *distance conjecture* [30, 205–207, 272, 297–299]. Here ϕ_b measures the canonical field distance from the origin along the imaginary $\mathcal{G}$-axis[21].

At strong warping the maximal allowed field excursion is $|\text{Im}(\mathcal{G})_{\text{max}}| \sim \frac{3}{2} M \log(a_0^{-1})$ before the 4d EFT description breaks down. Near this field excursion, a large fraction of the reservoir of fluxes of one of the throats has been transferred to the other one and the mass scale of the $\mathcal{G}$-field is of the same order as the warped KK-scale of the longer throat. At this point, contributions to the scalar potential from non-vanishing F-terms $D_{z_i} W$ start to play a significant role, or from a 10d point of view, the potential energy sinks down into the longer throat.

6.6.3 The general multi throat

We now wish to generalize our results to the case of an arbitrary multi-throat. For general m and n that satisfy $n-m > 0$ homology relations, there are n local deformation parameters $z_1, ..., z_n$ that have to be integrated out. We are left with an effective supergravity theory of m axions $\mathcal{G}^I$. The computational steps are analogous to the double throat case that was laid out in detail. Hence, we only state the effective axion superpotential

$$W_{\text{eff}}(\mathcal{G}^I) = -\sum_{i=1}^{n} \epsilon_i e^{i \sum_{I=1}^{m} p_I^i \mathcal{G}^I / M_i} + \hat{W}_0 \,, \tag{6.55}$$

and we have defined

$$\epsilon_i \equiv \frac{M_i}{2\pi i} z_{0,i} (1 - 2\pi \tilde{\bar{g}}_0^i \hat{W}_0 / (a M_i)) \,, \quad \tilde{g}_0^i \equiv g_0^i - \overline{g_{K,1}^i} \,, \quad a \equiv -2\text{Im}(g_{K,0}) \,, \tag{6.56}$$

and

$$z_{0,i} = e^{-1} \exp\left(-\frac{2\pi i}{M_i} \left(\sum_j M_j g_1^{ji} + g_{W,1}^i + i \tilde{\bar{g}}_0^i \hat{W}_0 / a \right) \right) e^{2\pi i \frac{K^i \tau}{M_i}} + \mathcal{O}\left(e^{-4\pi \frac{K^i}{g_s M_i}} \right) . \tag{6.57}$$

It is important to note that the $z_{0,i}$ as defined above can in general *not* be interpreted as the values of the local deformation in the vacuum. The physical local deformation parameters are given by

$$z_{\text{ph},i} \equiv z_{0,i} \exp\left(i \sum_I p_I^i \mathcal{G}^I / M_i \right) , \tag{6.58}$$

[21] See appendix D of [2] for the conversion rule between $\mathcal{G}$ and the canonical distance in field space ϕ_b. At any point in field space, $g_{\mathcal{G}\bar{\mathcal{G}}} < M_{\text{P}}^2 / M^2$. Hence, $\phi_b = \int_0^{\text{Im}\,\mathcal{G}} \text{d}\,\text{Im}\,\mathcal{G}' \sqrt{g_{\mathcal{G}\bar{\mathcal{G}}}} < \text{Im}\,\mathcal{G} M_{\text{P}} / M$.

where in the vacuum the $\mathcal{G}^I$ need not vanish in general.

6.6.4 Comments on the b-axion

In the above supergravity completion we have 'complexified' the c-axion by pairing it with the analogous b-axion. We have outlined the 10d backreaction of the b-axion already in section 6.5. Now that we have addressed the scalar potential of the b-axion quantitatively, in this section we would like to comment on a potential worry and how to resolve it: We recall that the effect of a non-vanishing field excursion of the b-axion is the creation of a pair of fluxes of the NS field strength H_3. Since both throats are filled up with H_3-fluxes already in the vacuum one should think about this process more properly as a transfer of H_3-flux from one throat to the other. Since the magnitudes of the local deformation parameters (and the associated hierarchy) are set by the ratio of local H_3-flux (on the $\mathcal{B}$-cycle) and F_3-flux (on the $\mathcal{A}$-cycle) it is clear that these will backreact when the H_3-fluxes are redistributed, see eq. (6.34).

However, it is also clear that when the local H_3-fluxes are changed, the circumference of the throat at the UV end is affected strongly. This is because it is set by the total D3-charge that is stored in the throat which is itself proportional to the amount of H_3-flux [162], compare figure 6.7. Hence, naively one might worry that such considerable change at the UV ends of the throats could lead to a large potential energy. One may convince one-self that this is not the case as follows. Starting from the supersymmetric situation we can redistribute a small amount of fluxes from one throat to the other, so that throat A has δ units of H-flux more than throat B. We can now proceed to convert the extra fluxes into a number of D3-branes by going through the Kachru-Pearson-Verlinde (KPV) transition [174]. From the UV perspective this process is only detected by a change in the throat complex structure which is a tiny perturbation far from the tip of the throat. Now we are back to an even flux distribution with a number of mobile D3-branes. These can be moved out of the throat at no cost in energy so the situation with the mobile branes should be a vacuum again. In other words, the redistribution of fluxes creates an energy density that is *only* due to the misalignment of local deformation parameters and the change of size of the throats at their UV ends does not generate an extra contribution to the potential. We reiterate that the situation is analogous to the backreaction of the c-axion with the phases of the local deformation parameters replaced by the logarithms of their magnitude.

Finally, note that in the Kähler potential (3.49) the b-axion appears explicitly, while the approximate c-axion shift symmetry is manifest. One might suspect that the small scale of the b-axion is therefore accidental due to our use of tree level supergravity. This conclusion would be incorrect: The target space manifold with Kähler metric

derived from (3.49) is shift symmetric also in the b-direction [300, 301]. In general we expect both continuous shift symmetries to be preserved to all orders in the perturbative expansion with explicit breaking only due to the superpotential (6.55). When moreover *non-perturbative* Kähler moduli dependent terms are generated in the superpotential such as the ones considered in [41], the b-axion mass is lifted to the scale of Kähler moduli stabilization while the c-axion can remain parametrically lighter. We will be more precise about this in section 6.8

6.7 The axion potential from the KS gauge theory

We have derived the axion potential via a classical computation within 10d SUGRA, and proposed a 4d SUGRA description that matches it. Since the local throats are believed to have a dual description in terms of KS gauge theories [162], it is useful to give an alternative derivation of our results on the gauge theory side of the correspondence. This will give us a further consistency check and allow us to conclude that the validity of our results is not endangered as we make the throat curvatures large in string units. Recall form section 3.6.2 that the KS gauge theory is a $SU(N+M)\times SU(N)$ gauge theory with gauge coupling constants set according to eq. (3.62). The radial running of the $\mathcal{G}$-field together with $\tau=$ const. matches the RG-running of the gauge theory coupling constants. Throughout this section (and in contrast to the preceding ones) $\mathcal{G}$ takes values in its suitable fundamental domain.

As the KS gauge theory flows to the infrared, it undergoes repeated steps of Seiberg dualities that reduce the ranks of the gauge groups according to

$$\begin{aligned} SU(N_0+M)\times SU(N_0) &\longrightarrow SU(N_1+M)\times SU(N_1) \\ &\longrightarrow \cdots \longrightarrow SU(N_k+M)\times SU(N_k)\,, \end{aligned} \tag{6.59}$$

with $N_k\equiv N-kM$, $k\in\mathbb{N}$. If we start with $N=KM$, after K steps in the duality cascade the gauge group is $SU(M)$. Since, roughly speaking, it corresponds to the first gauge group factor in $SU(M)\equiv SU(N_K+M)\times SU(N_K)$, its holomorphic scale is given by

$$\Lambda^{3M}=\mu_{\rm IR}^{3M}\exp\left(2\pi i\tau_{\rm YM}(\mu_{\rm IR})\right)=\mu_{\rm IR}^{3M}\exp(i\mathcal{G})\,, \tag{6.60}$$

where $\mu_{\rm IR}$ is the infrared scale of the throat and where we make use of the KS dictionary $\tau_{\rm YM}\simeq\mathcal{G}/2\pi$. Gaugino condensation leads to an effective ADS-superpotential [162, 169, 265]

$$W_{\rm eff}(\mathcal{G})=M\Lambda^3\sim M\mu_{\rm IR}^3\exp(2\pi i\tau_{\rm YM}/M)\sim M\mu_{\rm UV}^3\exp\left(\frac{2\pi i}{M}\left(\tau K+\frac{\mathcal{G}}{2\pi}\right)\right), \tag{6.61}$$

where we have used that the IR-scale is related to the UV-scale by

$$\mu_{\rm IR}^3 = \mu_{\rm UV}^3 \exp\left(-2\pi \frac{K}{g_s M}\right) . \tag{6.62}$$

The superpotential that we have proposed on the gravity side of the correspondence (6.55) indeed takes precisely this form,

$$W \propto \sum_{i=1}^{n} M_i A_i \exp\left(\frac{2\pi i}{M_i}\left(\tau K^i + \frac{\hat{\mathcal{G}}^i}{2\pi}\right)\right) + const\,, \tag{6.63}$$

with $\hat{\mathcal{G}}^i = \sum_{I=1}^{m} p_I^i \mathcal{G}^I$, and[22]

$$A_i \equiv \left(1 - \frac{2\pi \bar{\tilde{g}}_0^i}{a M_i}\hat{W}_0\right) \exp\left(-\frac{2\pi i}{M_i}\left(\sum_{j=1}^{n} M_j g_1^{ji} + g_{W,1}^i + i\bar{\tilde{g}}_0^i \hat{W}_0/a\right)\right) . \tag{6.64}$$

From the gauge theory perspective we should interpret the appearance of the constants g_1^{ji}, $g_{W,1}^i$, $\tilde{g}_0^i$ and $\hat{W}_0$ as a parameterization of threshold corrections near the UV cutoff.[23] Indeed, as they are taken to zero the A_i become unity.

It is now obvious that the M-fold extension of the periodicity of the c-axion is related to gaugino condensation in the KS gauge theory[24] [78, 162, 294, 304]: As usual there is a $U(1)_R$ symmetry that is broken to $\mathbb{Z}_{2M}$ by gauge theory instantons. Gaugino condensation spontaneously breaks $\mathbb{Z}_{2M} \to \mathbb{Z}_2$, so there are M gauge theory vacua. As we transform $c \to c + 2\pi$, we move from one gauge theory vacuum to the next, and the gaugino condensate (which corresponds to the local deformation parameters on the gravity side) picks up a phase $\exp(2\pi i/M)$. This is as in section 6.2 where we learned that the M different vacua are reached by dialing the RR flux quanta on the $\mathcal{B}$-cycle $Q = 0, \ldots, M-1$ (see (6.10)).

This point of view is useful for our understanding of how the discrete gauged axion shift symmetry is broken spontaneously in this case. The domain walls of the Kaloper-Sorbo description correspond to aligned gauge theory domain walls of either of the two throat gauge theories, across which the value of the respective gaugino condensates jump by factors of $e^{\pm 2\pi i/M}$. From the supergravity perspective, for a single throat this is known to be a $D5$ brane wrapped on the $\mathcal{A}$-cycle S^3 at the bottom of the throat [294]. Therefore, in our case this must be a combination of two $D5$ branes each one wrapped (with opposite orientation) on one of the two S^3s at the bottom of the throats.

[22] Note that in (6.55) we have set $M_{\rm P} = 1$. Therefore we identify $\mu_{\rm UV} \sim M_{\rm P}$.

[23] Of course these are in general functions of all other complex structure moduli that do not control the infrared regions of the throats and are frozen at a high scale.

[24] Related observations were made in the non-compact flux-less multi-node setting of [302, 303].

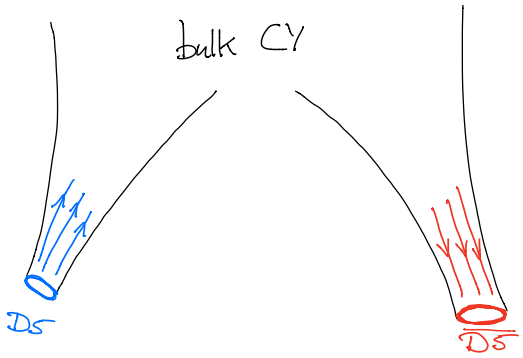

Figure 6.9: The Kaloper-Sorbo domain wall across which the c^I-axion field excursion jumps by one unit (by one *monodromy charge* unit). The homology-trivial cycle $\sum_{i=1}^{n} p_I^i \mathcal{A}_i \sim 0$ is wrapped by D5 brane(s). This configuration can be thought of as a set of n D5 branes, parallel in the non-compact directions, that wrap the internal $\mathcal{A}$-cycles in a combination of zero overall D5 charge. The corresponding domain wall for the b^I axion is obtained by replacing the D5s by NS5 branes.

For the b-axion it is an NS5 brane instead (see figure 6.9).

6.8 Kähler moduli stabilization

In this section we would like to comment on the question to what extend our results are modified when no-scale breaking due to Kähler moduli stabilization is introduced. So far we have proposed a superpotential as a function of the field $\mathcal{G}$,

$$W(\mathcal{G}) = \epsilon \left(1 - \cos\left(\mathcal{G}/M\right)\right) + W_0 \,. \tag{6.65}$$

This was argued to appropriately capture the thraxion scalar potential in purely classical ISD solutions, and in particular we have left out Kähler moduli in our discussion. First, in order to derive the thraxion scalar potential we do need to consider the Kähler moduli as they form a 'no-scale' sector together with the $\mathcal{G}$-axions. Following our conjecture of section 6.6 we make use of the type IIB $\mathcal{N} = 1$ $O3/O7$-orientifold Kähler potential as derived by Grimm and Louis [153], *on the other side of the conifold transition*, denoted $\mathcal{M}'$, where the thraxions would become part of the CY zero mode sector. We denote by $\mathcal{G}^1, ..., \mathcal{G}^{h^{1,1}_- + m}$ the $h^{1,1}_- + m$ complex axions, and by $T^1, ..., T^{h^{1,1}_+}$ the $h^{1,1}_+$ Kähler moduli. For simplicity let us set $h^{1,1}_- = 0$. The Hodge numbers are those of $\mathcal{M}$.

We specify to the simplest case with $h^{1,1}_+ = 1$ and use the Kähler potential of (3.49). Strictly speaking we must allow for at least two Kähler moduli on the other side of the conifold transition, the universal Kähler modulus and the resolution modulus. Since the latter is set to zero as we go through the transition, we only keep the universal one.

The triple intersection number matrix κ_{+ij} is negative definite, and symmetric in the indices i, j. We define $\beta^i \equiv -i(\mathcal{G} - \bar{\mathcal{G}})^i$, and raise and lower the indices using the metric $-\kappa_{+ij}$. In the basis of target space vector fields $\{\partial_T, \partial_{\mathcal{G}^i}\}$ the field space metric, its inverse and the one-form ∂K take the form

$$g_{A\bar{B}} = \frac{3}{F^2}\begin{pmatrix} 1 & -i\alpha\beta_{\bar{j}} \\ i\alpha\beta_i & -\alpha F\kappa_{+i\bar{j}} + \alpha^2\beta_i\beta_{\bar{j}} \end{pmatrix}_{A\bar{B}}, \quad g^{\bar{A}B} = \frac{F^2}{3}\begin{pmatrix} 1+\frac{\alpha}{F}\beta_k\beta^k & i\frac{1}{F}\beta^j \\ -i\frac{1}{F}\beta^{\bar{i}} & -\frac{1}{\alpha F}(\kappa_1^{-1})^{\bar{i}j} \end{pmatrix}^{\bar{A}B}, \tag{6.66}$$

and $\partial_i K = -\frac{3}{F}(1, i\beta v_i)_i^T$, where $\alpha \equiv \frac{3}{4}g_s$. As usual, the Kähler potential satisfies the no-scale relations

$$g^{-1}\cdot\partial K = -F(1,0)^T, \quad \partial K^\dagger g^{-1}\partial K = 3\,. \tag{6.67}$$

As a consequence, for a general superpotential $W = W(T, G)$ one obtains

$$V = e^K(|DW|^2 - 3|W|^2) = \frac{e^{K_0}}{F^3}(|\partial W|^2 - 2F\mathrm{Re}(\overline{W}\partial_T W))\,. \tag{6.68}$$

For the purely classical background, we have $W = W(\mathcal{G}^i)$. Hence the potential is given by

$$V = \frac{e^{K_0}}{3\alpha F^2}(-\kappa_+^{-1})^{ij}\partial_{\mathcal{G}^i}W\overline{\partial_{\mathcal{G}^j}W} \overset{m=1}{=} \frac{2|\epsilon|^2}{9M^2F^2}(-\kappa_+^{-1})^{11}|\sin(\mathcal{G}/M)|^2\,. \tag{6.69}$$

In the last equality we have set $m = 1$. For $\mathrm{Re}(\tau) = 0$, $b = 0$ this reduces to the potential of eq. (6.30). Note that if we freeze the value of the overall CY volume, and the dilaton, this potential is of the same form as one derived in rigid SUSY with flat field space metric $\propto (-\kappa_+^{-1})^{ij}$. Therefore, it possesses only Minkowski minima.[25]

Kähler moduli stabilization will distort this scalar potential as the second term on the r.h. side of (6.68) indicates. Adding a KKLT term $NAe^{-2\pi T/N}$ to the superpotential one obtains the following scalar potential

$$\begin{aligned} e^{-K_0}\cdot V =& \frac{1}{3F^2}|2\pi Ae^{-2\pi T/N}|^2(F + \alpha|\beta|^2) - \frac{4\mathrm{Im}\,\mathcal{G}}{3MF^2}\mathrm{Im}\left[2\pi Ae^{-aT}\bar{\epsilon}\sin(\mathcal{G}/M)^*\right] \\ &+ \frac{|\epsilon|^2(-\kappa_+^{-1})^{11}}{3\alpha M^2F^2}|\sin(\mathcal{G}/M)|^2 \\ &+ \frac{2}{F^2}\mathrm{Re}\left[2\pi Ae^{-2\pi T/N}\left(W_0 + Ae^{-aT} + \epsilon(1-\cos(\mathcal{G}/M))\right)^*\right]\,. \end{aligned} \tag{6.70}$$

The SUSY critical point is the same as in ordinary KKLT, with $\mathcal{G} = 0$.

This scalar potential has a rather complicated structure in particular due to the spontaneous breaking of the b-axion shift symmetry due to the presence of the seven-

[25] Strictly speaking, (pseudo-)moduli spaces of positive energy vacua are also possible [305]. But it is clear that this situation will not occur in our examples.

brane stack. In other words, the b-axion will in general be sourced by gaugino condensation. We will neglect this effect now by setting $\mathrm{Im}\,\mathcal{G} = 0$ and focus on the c-axion dependence of the scalar potential. This is simpler because the c-axion shift symmetry is preserved by the D7 brane stack. The scalar potential simplifies to become the sum of the axion-independent KKLT scalar potential and an axion dependent scalar potential of the schematic form

$$V_{\text{axion}}(c) = |z_0 \sin(c/M)|^2 + \mathrm{Re}\left[\langle\lambda\lambda\rangle \bar{z}_0 (1 - \cos(c/M))\right] . \tag{6.71}$$

First, as advertised one observes that the axionic shift symmetry is preserved in the limit $z_0 \longrightarrow 0$, but now the non-perturbative volume stabilization effects induce also a term linear in z_0. The appearance of this term can be understood from a ten-dimensional perspective: In section 6.3 we have considered the effective five-dimensional deformation field $z(r)$ and seen that a non-trivial field excursion of the axions effectively displaces the two Dirichlet-type boundary conditions at the IR ends of the throats against each other. In general, when non-perturbative ISD breaking effects are turned on in the bulk, this radial mode will be sourced also at the UV end,

$$S[z] \longrightarrow S[z] + \frac{M_{10}^8}{2} \int d^4x \int \frac{dr}{r} \left(J_{\text{UV}} \bar{z} + c.c.\right) , \tag{6.72}$$

with $J_{\text{UV}} = j \cdot r\delta(r - r_{\text{UV}})$. In our case the source j is identified with the gaugino condensate, $j \sim \langle\lambda\lambda\rangle$. From the dual KS field theory perspective the presence of such a source term is interpreted as a relevant deformation of the field theory Lagrangian, by a certain chiral operator of dimension $\Delta = 3$, listed in ref. [106].[26] The solution to the equations of motion of $z(r)$ subject to Dirichlet boundary conditions at the IR ends as given in section 6.3 is now modified by the UV source, and reads

$$z(r) = \frac{1}{4} j (r^2 - r_{\text{IR}}^2) + z_1 + \frac{1}{2} \frac{r^2 - r_{\text{IR}}^2}{r_{\text{UV}}^2 - r_{\text{IR}}^2} (z_2 - z_1) . \tag{6.73}$$

This holds in the first throat. In order to obtain the profile in the second throat, one simply exchanges $z_1 \longleftrightarrow z_2$. Plugging this back into the action, one obtains a scalar

[26] More precisely we actually consider perturbations of the conformal Klebanov-Witten theory. The KS theory is interpreted as a small perturbation of the KW theory due to the logarithmic running of five form fluxes and the warp factor. As we neglect this logarithmic running also in the throat calculation, a comparison to the KW theory using the usual AdS/CFT dictionary is appropriate.

potential

$$\begin{aligned} V(c) &= |z_1 - z_2|^2 + \mathrm{Re}\,(\bar{j}(z_1 + z_2)) + const. \\ &= 4|z_0|^2 |\sin(c/M)|^2 - 2\mathrm{Re}(j\bar{z}_0)(1 - \cos(c/M)) + const. \end{aligned} \tag{6.74}$$

This indeed reproduces the form of the c-axion scalar potential as computed from four-dimensional supergravity (6.71). Similar cross-terms are expected to be generated for *any* type of no-scale breaking, be it of perturbative type as generated by the BBHL correction of eq. (3.72), particularly relevant for moduli stabilization à la LVS [75], or of non-perturbative type as in KKLT [41]. Indeed, one can show that including the BBHL correction into the Kähler potential leads to an analogous cross term proportional to W_0. We leave the task of matching the full scalar potential as a function also of the b-axion for future work.

In the limit $|z_0| \ll |j|$ the axion mass is raised to a value of order

$$\frac{m_c}{M_P} \sim (a_0)^{\frac{3}{2}} \cdot \sqrt{\frac{m_T}{M_P}}\,, \tag{6.75}$$

where m_T is the scale of Kähler moduli stabilization. While this is a somewhat larger value than the one encountered in the classical ISD solutions it is still *exponentially* smaller than the mass scale of Kähler moduli.

Note that one might worry that already in the classical ISD background such a UV source term is present. After all the $U(1)_R$ symmetry is broken by the bulk. This is unfounded as can be seen as follows. The general solution $z(r) = c_1 + c_2 r^2$ contributes to the local strength of $U(1)_R$ breaking in the combination

$$z(r)/r^3 = c_1 r^{-3} + c_2 r^{-1}\,. \tag{6.76}$$

The first term is the *normalizable* perturbation due to the deformation of the conifold. The latter is activated only in the presence of an appropriate source in the UV. From the radial scaling we observe that this mode *grows* toward the infrared. But we know from the definition of a compactly embedded conifold region that the CY metric does not possess such a mode. Rather all geometric effects of bulk symmetry breaking fall off towards the IR end of the conifold. Therefore, this particular mode cannot be activated in an ISD background, i.e. $j|_{\mathrm{ISD}} = 0$.[27]

[27] Questions of this sort have been addressed systematically e.g. in [106, 306]. In general, the KS throat is known to not admit *any* supersymmetry breaking but ISD preserving relevant perturbations [306].

6.9 Applications

6.9.1 Thraxions on the Quintic: Drifting Monodromy

In this section we would like to give an example of a string compactification where a light thraxion can appear. Along the way we identify concrete setups in which parametrically super-Planckian racetrack-type axion periodicities are possible. We choose the CY to be the quintic three-fold as introduced in section 3.3 near its 'classical' conifold transition locus described in section 3.4. Recall that there are 16 vanishing three-cycles $\mathcal{A}^i$, $i = 1, ..., 16$. Because the solution set lies on a $\mathbb{P}^2$ submanifold of $\mathbb{P}^4$, there is precisely one homology relation among them,

$$\sum_{i=1}^{16} [\mathcal{A}^i] = 0\,. \tag{6.77}$$

Hence, we have a multi throat system with $n = 16$ and $m = 1$ so there is one light axion.

Let us give two examples that differ by choices of flux numbers. In both examples we set the coefficients A_i defined in (6.64) to unity. Generically we expect these to be of order one. Inserting $\mathcal{O}(1)$ factors below does not change the physical outcome.[28]

Example 1: A simple thraxion potential

First we dial the flux quanta as

$$M_i = (-1)^{i+1} M\,, \quad K^i = (-1)^{i+1} K\,, \tag{6.78}$$

with $K/g_s M \gg 1$. Then we have $\epsilon_i \equiv (-1)^{i+1}\epsilon$, and

$$W_{\text{eff}}(\mathcal{G}) = -16i\epsilon \sin 2\mathcal{G}/M + \hat{W}_0 = 16i\epsilon(1 - \cos 2\mathcal{G}'/M) + W_0\,, \tag{6.79}$$

with $W_0 = \hat{W}_0 - 16\epsilon$, $\mathcal{G}' = \mathcal{G} - \pi M/4$, and small $|\epsilon| \propto \exp(-2\pi K/g_s M)$. Up to the numerical pre-factor this is exactly what we found for $n = 2$ and $m = 1$.

Example 2: Drifting Monodromy

We now slightly detune the F_3 fluxes from one another:

$$M_1 = M\,, \quad M_2 = M + 1\,, \quad M_3 = -M\,, \quad M_4 = -(M + 1)\,, \tag{6.80}$$

[28] If some coefficients can be tuned parametrically smaller than others, new qualitative features might arise. We leave an investigation of this possibility to future research.

and $M_{i+4} = M_i$, with $K^i \equiv \text{sign}(M_i)|K|$, and again $K/g_s M \gg 1$. In this case

$$W_{\text{eff}}(\mathcal{G}) \approx -8iz_0(M\sin(\mathcal{G}/M) + (M+1)\sin(\mathcal{G}/(M+1))) + \hat{W}_0\,, \tag{6.81}$$

with $z_0 \sim \exp 2\pi i K\tau/M$. In addition to the previous simplification, we have also neglected order one prefactors that arise from the fact that the ratios K^i/M_i are not all exactly equal. Again, this is of no consequences for our purposes.

The superpotential (6.81) is a *racetrack*-type superpotential for $\mathcal{G}$. The axion periodicity is now given by $2\pi M(M+1)$. Crucially, this implies another M-fold extension of the axion field range *on top* of the one already discussed in the simpler examples of the double throat and the first example of this section. Clearly, one may take this even further to periodicities such as $2\pi M\cdots(M+3)$. Since we still only have to fulfill the requirement that the throats fit into the bulk CY, this implies the existence of a simple, concrete and explicit mechanism in string theory that can generate huge super-Planckian axion periodicities. In general the full periodicity of the superpotential is given by the least common multiple of the different RR flux numbers M_i. We dub this mechanism of generating a parametrically large axion monodromy *drifting monodromies* since it relies on a frequency drift within a set of several finite-order monodromy effects. This is related but different from the winding idea, where a constraint forces the effective axion on a long trajectory in a multi-axion moduli space [44, 278–281]. Here, by contrast, one may think of a single fundamental axion extended by several small, finite-order monodromy effects. The result of this can still be large as explained above. The intended outcome, namely to realize an effective large-f axion accepting a short-wavelength oscillatory potential, is of course the same (see in particular the very recent analysis of [71]).

The minima of the potential $V \propto |\partial_{\mathcal{G}} W|^2$ are located along the slice $\text{Im}\,\mathcal{G} = 0$ where it takes the form

$$\begin{aligned} V(c) &\propto \left[\cos(c/M) + \cos(c/(M+1))\right]^2\,, \quad c \equiv \text{Re}\,\mathcal{G}\,, \\ &\propto \cos^2\left(\frac{2M+1}{2M(M+1)}c\right)\cdot\cos^2\left(\frac{1}{2M(M+1)}c\right)\,, \end{aligned} \tag{6.82}$$

which has $2M+1$ distinct Minkowski vacua (see figure 6.10).

We can now compare this with the results of section 4.35 where we noticed that generic racetrack type superpotentials are in conflict with the weak gravity conjecture for axions. In our case, we note that despite the long $2\pi M(M+1)$ periodicity the scalar potential oscillates on shorter wavelengths of order $2\pi M$. This is essentially due to the rank condition (6.40) which forces us to introduce flux numbers of both signs. We have not shown in general that suppressing such shorter wavelength oscillations in order to

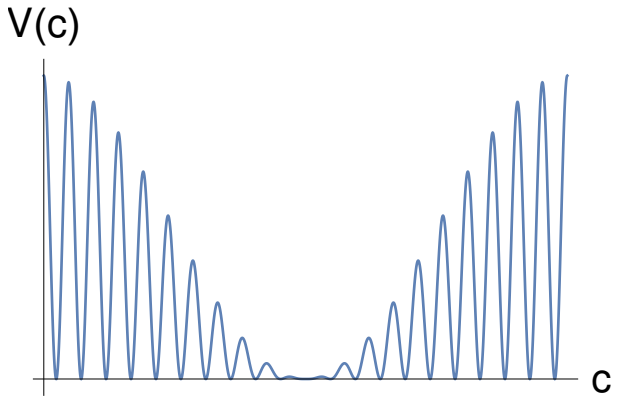

Figure 6.10: The axion potential of example 2 for the case $M = 10$. There are sub-Planckian oscillations within a long super-Planckian envelope.

produce a smooth super-Planckian axion potential is impossible. At this point we only note that the condition (6.40) presents a severe obstacle towards this. Furthermore this condition is *global* in the sense that it need not hold in a non-compact CY where gravity is decoupled, and swampland criteria are not expected to be applicable.

The examples given above also serve to illustrate that by scanning over flux numbers one may obtain a vast number of possible effective superpotentials and axionic potentials.

6.9.2 A Clash with the Weak Gravity Conjecture

In this section we would like to point out that the axion potential we have derived clashes with the WGC for axions [31] (see section 3.8.2). We have computed the axion potential via a classical supergravity calculation. However, one may equally well associate it to non-perturbative effects in the KS gauge theory (namely gaugino condensation), as argued in section 6.7. As such, (if true) the weak gravity conjecture should apply to our construction.

By comparison with an instanton induced scalar potential (3.100), we may associate an (effective) Euclidean instanton action to each of the leading exponentially suppressed terms in the axion (super)potential.[29]

$$S^i_{\text{eff}} \approx 3\log(1/a^i_0) \approx 2\pi \frac{K^i}{g_s M_i}\,. \tag{6.83}$$

As computed in appendix D of [2], in the regime where the throats marginally fit into the bulk CY, the periodicities f_{eff}/q^i of the dominant terms in the superpotential

[29] Taking the correspondence with instantons seriously, these are ($\frac{1}{2}$BPS-)instantons.

associated to each throat $i = 1, ..., n$ read

$$f_{\text{eff}}/q_i \approx \frac{2}{3}(r^i_{\text{D3}})^{1/2}\log(1/a^i_0)^{-1/2}M_{\text{P}} \approx \frac{2}{3}(r^i_{\text{D3}})^{1/2}\left(\frac{2\pi}{3}\frac{K^i}{g_s M_i}\right)^{-1/2} M_{\text{P}}\,, \tag{6.84}$$

where $r^i_{\text{D3}} = M_i K^i/N^{\text{flux}}_{\text{D3}}$ is the fraction of the total D3 brane charge of three-form fluxes which is stored in the i-th throat. Hence,

$$S^i_{\text{eff}} \cdot f_{\text{eff}}/q^i \sim 2(r^i_{\text{D3}})^{1/2}\sqrt{\log(1/a^i_0)}\, M_{\text{P}} \approx 2(r^i_{\text{D3}})^{1/2}\sqrt{\frac{2\pi}{3}\frac{K^i}{g_s M_i}}M_{\text{P}}\,. \tag{6.85}$$

In the regime $a_0 \ll 1$ (i.e. $K^i \gg g_s M_i$) the r.h. side is parametrically larger than $\mathcal{O}(1)$ so the objects that generate the relevant terms in the superpotential do not satisfy a weak gravity conjecture bound.

Of course as is always true in string theory compactifications [52] there *does* exist a tower of instantons that satisfies the weak gravity bound (3.99) but generates no monodromy.[30] It is also apparent that these instantons occupy a sub-lattice of the full charge lattice. This sub-lattice corresponds to all the possible wrapping numbers of a Euclidean D1 string. However, in our setup this sub-lattice can be made parametrically *coarse*.[31] Let us illustrate this with a concrete example: We consider a variant of the *drifting monodromies* example given in section 6.9.1, with flux numbers $M_i \in \{5, 6, 7, 8, -5, -6, -7, -8\}$. The axion decay constant is enhanced by the least common multiple of $5, 6, 7, 8$ which is 840. The instantons that satisfy (3.99) respect the periodicity of the axion *before* monodromy. Thus the possible charges take values in $840\mathbb{Z} \subset \mathbb{Z}$. Clearly, a lattice WGC is parametrically violated, while a sub-lattice WGC [62, 64] (see also [70]) is always satisfied but with parametrically coarse sub-lattice. Note that generically these instantons only give rise to sub-leading corrections to the scalar potential (if they contribute at all), compare section 6.9.5.

However, we observe that the n effective instantons that *do* give the dominant contribution to the superpotential satisfy a relation

$$S^i_{\text{eff}} \leq \frac{4}{3} r^i_{\text{D3}}(q^i\, M_{\text{P}}/f_{\text{eff}})^2\,, \quad \forall i = 1, ..., n\,. \tag{6.86}$$

Together with $S^i_{\text{eff}} > 1$, as is required for controlled expansion in powers of $e^{-S^i_{\text{eff}}}$, this *still* implies the existence of short wavelength harmonics in the superpotential. This motivates a closer look at the full spectrum of our effective instantons to which we now turn.

[30] In our case, these are Euclidean D1 strings wrapping representative S^2's in the UV, compare section 6.9.5.
[31] Hence we seem to realize explicitly the loop hole mentioned in footnote 25 of ref. [64].

6.9.3 The Spectrum of Effective Instantons

We now set aside the spectrum of instantons that satisfy the WGC but instead ask what are the properties of the effective instantons that generate the superpotential. We have written down the most dominant contributions to the superpotential (6.55) and noted that they satisfy (6.86). This condition is weaker than (3.99) because at fixed control parameter $S^i_{\rm eff} \overset{!}{>} 1$ the value of $qM_{\rm P}/f_{\rm eff}$ is constrained to be bigger only than $\sqrt{S_{\rm eff}}$ rather than $S_{\rm eff}$. Nevertheless, as a consequence the dominant effective instantons give rise to sub-Planckian wavelength oscillations in the scalar potential.

In general it is easy to see that beyond these most dominant effective instantons there exists a whole $\mathbb{Z}^n$ lattice worth of these effective instantons[32]. These correspond to the higher orders in the $|z_i|$, $i = 1, ..., n$, that we have neglected in section 6.6. Therefore, the n dominant effective instantons serve as n basis vectors of the lattice $\mathbb{Z}^n$ and a general effective instanton is labeled by an effective charge vector $\vec{k} \in \mathbb{Z}^n$. The bound (6.86) for these general effective instantons reads

$$S^{\vec{k}}_{\rm eff} \;\leq\; \frac{4}{3} M_{\rm P}^2 \sum_{i=1}^{n} |k^i|\, r^i_{\rm D3} \left(\frac{q^i}{f_{\rm eff}}\right)^2 , \tag{6.87}$$

and the r.h. side defines a (1-)norm on $\mathbb{Z}^n$.

The effective instantons can also be embedded into the full one-dimensional charge lattice $\mathbb{Z}$ of the preceding section via $\vec{k} \longmapsto \sum_{i=1}^n k^i q^i$, although they do not satisfy the WGC bound (3.99).

Again, this is perhaps best understood using the explicit example given in the preceding section. The dominant instantons have charges

$$q^i \in \left\{\pm\frac{840}{M_i}\right\}_i = \left\{\pm\frac{840}{8}, \pm\frac{840}{7}, \pm\frac{840}{6}, \pm\frac{840}{5}\right\} = \{\pm 105, \pm 120, \pm 140, \pm 168\}\,. \tag{6.88}$$

They induce harmonics in the superpotential with periodicities $f_{\rm eff}/q^i$. Due to the relation (6.86) precisely these combinations are restricted to be sub-Planckian as long as $S^i_{\rm eff} > 1$. They can be understood to occupy a coarse $105\mathbb{Z} + 120\mathbb{Z} + 140\mathbb{Z} + 168\mathbb{Z}$ sub-lattice of $\mathbb{Z}$ while the Euclidean D1-instantons satisfying the WGC occupy an even coarser sub-lattice $840\mathbb{Z} \subset \mathbb{Z}$.

6.9.4 Axion Phenomenology

We have identified a string theory axion with remarkable properties. It is parametrically lighter than the tower of states that is usually associated to strongly warped regions

[32] We consider general combinations of holomorphic and anti-holomorphic instantons.

$m_{\text{tower}} \propto a_0 M_{\text{P}}$. The axion mass can be tuned almost independently of the periodicities of the dominant oscillations in the scalar potential, since we have $m \propto a_0^3 M_{\text{P}}$, while the oscillation period f_{eff}/q of the scalar potential depends only weakly on the warp factor $f_{\text{eff}}/q \sim M_{\text{P}}/\sqrt{\log(a_0^{-1})}$. Conversely, the mass scales unusually strongly with the oscillation wavelength,

$$\frac{m^2}{M_{\text{P}}^2} \propto a_0^6 \approx e^{-2S_{\text{eff}}} \approx \exp\left(-\alpha\left(\frac{qM_{\text{P}}}{f_{\text{eff}}}\right)^2\right) , \tag{6.89}$$

with $\alpha = \mathcal{O}(1)$.

In contrast most other stringy axions usually satisfy the relation [307]

$$\frac{m^2}{M_{\text{P}}^2} \sim \exp\left(-\alpha\frac{qM_{\text{P}}}{f_{\text{eff}}}\right) , \tag{6.90}$$

As such the thraxion assumes a rather special place in the string theory landscape. This is potentially interesting for axion phenomenology. We refer the reader to [307] for a range of phenomenological applications for different axion mass scales.

We have to emphasize that at least in the simplest setups our axion is not a generic inflaton candidate because of the generic presence of dominant sub-Planckian wavelength modes in the scalar potential, *despite* the large monodromy enhancement of the effective axion decay constant.

6.9.5 Uplifting

We would like to briefly comment on some possible scenarios of uplifting to de Sitter space, using our construction. To actually implement these ideas in concrete models involves the complicated interplay of different effects. First, uplifting requires as a precondition, that a full mechanism of Kähler moduli stabilization is in place, and we have sketched the non-trivial interplay between the CY breaking potential and the no-scale breaking from Kähler moduli stabilization in section 6.8. Second, as explained in section 4.1 in order to argue for a successful uplift to de Sitter space, we need very good control over the ingredients of the uplift. Here, we will only sketch qualitative ideas how this might work. The following two scenarios are both based on the idea of adding an oscillating potential of different wavelength to the known thraxion potentials of the form (6.21) or (6.82).

We wish to look at situations where c-dependent corrections to the Kähler potential may become relevant. This is certainly the case in the regime $|W_0| \sim 1$, leading us to consider LVS-like moduli stabilization [75]. Potentially interesting corrections may

arise from Euclidean D1-brane instantons that wrap members of the family of two-spheres that vanish at the tips of the conifold. Since the cycle is trivial in homology we expect no corrections to the superpotential but at most corrections to the Kähler potential of the form

$$\delta e^{-K_1} \sim \mathcal{C} e^{-S_{\rm DBI} - i S_{\rm CS}} + c.c.\,, \tag{6.91}$$

with $\mathcal{C} = \mathcal{O}(1)$ and DBI and CS actions

$$S_{\rm DBI} = \frac{1}{g_s} \frac{{\rm Vol}(S^2)|_{\rm UV}}{2\pi\alpha'}\,, \quad \text{and} \quad S_{\rm CS} = \frac{1}{2\pi\alpha'} \int_{S^2} \left(\sum_p C_p \right) \wedge e^B|_{2-\rm form} = {\rm Re}\,\mathcal{G}\,. \tag{6.92}$$

Here, we have evaluated the DBI action on a representative sphere in the UV, i.e. in the bulk CY. This is because we expect such a representative to give the dominant contribution: As explained in App. A, there are different two-spheres at a given radial coordinate in the throat that are labeled by a $U(1)$ phase and that all share the same volume. As we scan over this phase, the corresponding integrals of C_2 at a given radial coordinate pass through their fundamental domain. Therefore, integrating over all Euclidean brane instantons on the two-spheres should cancel all contributions due to the oscillatory behavior of the correction (6.91). This is consistent with the fact that after accounting for backreaction of the phases of the throat deformations the C_2 field excursion cannot be measured in the local throats. In the analysis of section 6.6, we extended this result to ${\rm Re}(\mathcal{G})$, i.e. to $C_2 - C_0\, B_2$. In passing towards the UV, our description of the throat breaks down. In particular, we do not expect the different sphere representatives to all share the same volume. Thus, we expect non-vanishing instanton corrections.

Using ${\rm Vol}(S^2)|_{\rm UV} \gtrsim R^2_{\rm throat} \propto (g_s M K)^{1/2} \alpha'$, this leads to corrections to the scalar potential of the form

$$\delta V \lesssim e^{-\alpha \sqrt{\frac{K M}{g_s}}} \left(1 - \cos({\rm Re}\,\mathcal{G})\right), \tag{6.93}$$

with $\alpha = \mathcal{O}(1)$. Assuming that the exponentially small prefactors of the classical warping suppressed potential (6.21) (or that of example 1 of section 6.9.1) and the non-perturbative correction terms are of the same order, it is feasible that additional local minima appear in the scalar potential that could *in principle* lift to meta-stable non-supersymmetric minima, possibly even de Sitter vacua. The exponential terms are of comparable magnitude when

$$\sqrt{\frac{K}{g_s M}} \gtrsim M\,. \tag{6.94}$$

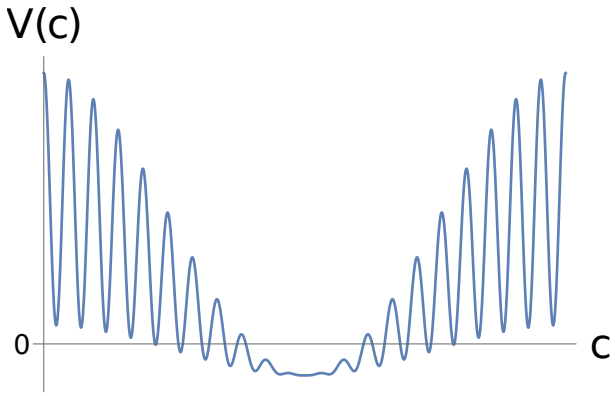

Figure 6.11: The axion potential of figure 6.10 with an additive correction $\delta V(c) \propto \text{const.} + \cos(c/f_{\text{eff}})$ that shares the periodicity of the superpotential.

In F-theory models with large Euler characteristic we do not see an immediate obstacle to realizing this.

We may turn this around and add large-wavelength corrections to shorter-wavelength oscillations such as those of the example of *drifting monodromies* given in section 6.9.1. On the large scale of $f_{\text{eff}} \sim M M_{\text{P}}$ there are several Minkowski vacua of the potential (6.82), compare figure 6.10. It is conceivable that these are uplifted to de Sitter vacua once further corrections to the potential are taken into account. This might happen automatically when the no-scale properties of the Kähler potential (3.49) get broken by perturbative or non-perturbative corrections, since we know that the existence of Minkowski vacua strongly depends on the cancellation of different terms in the scalar potential. These scalar potential corrections follow the periodicity of the superpotential, which is given by the super-Planckian decay constant f_{eff}. An optimistic sketch of this is illustrated in figure 6.11.

However, for both ideas, uplift flattening as discussed in chapter 4 is an obstacle due to the direct cross-couplings between the F-terms $F_{\mathcal{G}}$ and other no-scale breaking sources from moduli stabilization. We leave a more thorough search for de Sitter vacua using these uplifting ideas for future research.

6.10 Conclusions

In this chapter we have argued for the generic existence of a new type of ultra light axion in the flux landscape of type IIB string theory. It is part of the light spectrum whenever fluxes stabilize a CY orientifold near conifold transition locus in complex structure moduli space. We have given three independent computations of its scalar potential that all agree with each other: 1) From the ten-dimensional perspective non-

vanishing axion field excursions displace the *local* deformation parameters of distinct warped throats against each other, forcing them to violate the CY condition, while supersymmetry in the local throats is almost perfectly restored. The CY breaking potential was computed from the gradient energy of the ten-dimensional field profile that interpolates between the mis-aligned throats. 2) A four-dimensional supergravity formulation was proposed in which the thraxion fields play the roles of the stabilizer fields whose F-term conditions enforce the CY condition. After integrating out the geometric modes, one is left with an effective superpotential for the thraxions. The effective scalar potential computed from this proposal matches the ten-dimensional one. 3) In the field theory dual to the throats a non-perturbative superpotential is generated from gaugino condensation that matches the one previously proposed, indicating that the model is equally valid for both large and small throat curvatures.

This is interesting from a fundamental perspective because it exemplifies how the light spectrum of an $\mathcal{N} = 1$ flux compactification can contain CY zero modes of a pair of *distinct* CY manifolds, connected to each other via a conifold transition. As it is widely believed that stabilization near such loci in CY space occurs rather naturally [42, 163], this phenomenon in general may play an important role in the phenomenology of string compactifications. Specifically, we have argued that our axion can easily be exponentially lighter than Kähler moduli, and the scale of the potential can be far smaller than the WGC for axions would lead one to expect. We are led to conclude that the reasons why all models of large field inflation *might* fail in string theory cannot be as simple and universal as one might have hoped, while a relation similar to, but different from, the WGC for axions holds also in our class of models (see eq. (6.86)).

Furthermore, we have presented a mechanism to generate parametrically large axion decay constants by superimposing slightly detuned harmonics in the superpotential via judicious choices of flux quanta. While the scalar potential still oscillates on sub-Planckian scales in field space, it suggests that certain strong forms of the distance conjecture do not hold universally (see section 3.8.3). In particular, it seems to be possible to traverse a parametrically super-Planckian distance in the enhanced axion field space without a tower of light states decreasing their mass exponentially as the distance is traversed. This is *not* in conflict with the weaker statement given in section 3.8.3 which states that for a given model (i.e. for fixed choices of flux quanta and other data) it is impossible to traverse *arbitrarily* large distances in field space without encountering an exponentially light tower of states. There is no contradiction simply because the monodromy we encounter is always finite, though it can be made parametrically large. Similarly, our construction is not in conflict with the sub-lattice WGC but it is suggested that the populated sub-lattice in instanton charge space can

be made parametrically sparse.

It would be interesting to generalize our backreaction scheme to more complicated axion monodromy proposals such as the one of [46]. As F_3 fluxes on the $\mathcal{A}$-cycles are dual to D5 branes wrapped on the resolution two-cycles via the geometric transition [304], we have implicitly given a fully backreacted example of five-brane axion monodromy, based on the very simplest example of a double KS throat. Interestingly, in our case, the 5-brane anti-5-brane pair has relaxed to a supersymmetric ground state *without* annihilating against each other. The flux-tube that connects the two is supersymmetrized by the NS fluxes on the $\mathcal{B}$-cycle, automatically resolving the issue of brane anti-brane backreaction in the vacuum, as formulated in ref. [308]. Furthermore, while the conifold transition is the simplest type of geometric transition, it is not the only one [304]. It would be worthwhile understanding what is the light spectrum near such transition loci in general.

Finally, we have briefly commented on the possibility to generate de Sitter vacua using SUSY breaking minima of the thraxion field as the uplift. Generically, uplift flattening as discussed in section 4.1 poses an obstacle toward realizing this idea, but it would be interesting to see if de Sitter vacua can be found in our axion landscape nevertheless. We leave this issue for future research.

Chapter 7

Conclusions and outlook

It is a fascinating and fundamental question whether string theory has solutions that can describe the accelerated expansion of the universe, in particular the one we observe today, and the inflationary expansion that observation strongly suggests has happened in the very early universe [19]. We believe that it is the right time to address this question in detail, as cosmic microwave background observations are becoming sensitive enough to rule out wide ranges of models of inflation [27], and the bounds on the equation of state of dark energy become tight [112]. In this thesis we have focused on two particular aspects of this problem, the viability of models of de Sitter vacua that are the leading candidates to describe the late time accelerated expansion of our universe, and the possibility of large field inflation in the very early universe. Although the characteristic energy scales of both phenomena can be much lower than the Planck scale, it is natural to address them in string theory as a UV complete framework. This is due to the fact that the success or failure of concrete models is sensitive to a large number of Planck suppressed operators which may mediate significant backreaction effects on geometric moduli of string theory [3–5]. Moreover, the existence and structure of no-go theorems against de Sitter vacua in many weakly coupled corners of string theory suggests that such backreaction effects *generically* spoil the success of de Sitter uplifts.

In order to make concrete progress we have chosen to focus on the celebrated KKLT model [41] for de Sitter vacua in string theory. We have argued that this proposal, originally made partially within the framework of four-dimensional supergravity, survives many non-trivial ten-dimensional consistency checks. In particular, ten-dimensional tadpole cancellation conditions are fulfilled exactly [1, 110], indicating that the success of the model is *not* threatened by too strong backreaction effects.[1] Moreover, the four-dimensional conditions for unbroken supersymmetry match the ten-dimensional ones in a natural and intuitive way [5, 309]. However, we have also pointed out that the

[1] See however ref. [111] that claims the opposite.

KKLT type de Sitter vacua are always very close or even well within a regime of poor computational control, where the use of ten-dimensional supergravity as an approximation to the whole string theory becomes questionable [1]. This is closely related to the consistency requirement that the physical size of the so-called warped throats employed to engineer parametrically small uplifts has to be smaller than the overall size of the compactification space. Thus, we have pointed out a highly vulnerable point in the KKLT construction to be scrutinized further in the future. In particular, we have argued that highly non-generic compactifications could in principle evade the problems of computational control, and it would be interesting to understand if these can actually be realized in a sufficiently controlled fashion.

In the second part, motivated by the goal to realize large field inflation in string theory, we have constructed a new class of axion-like particles. These we have argued that to arise in rather generic classes of string theory solutions, namely the type IIB flux landscape. The construction displays several interesting features that make it interesting both from the phenomenological as well as the theoretical perspective: the axion mass is extremely small both in relation to the Planck as well as in relation to its decay constant. As a consequence, it can be used as a counter example to the weak gravity conjecture for axions [31], while a similar but different bound is fulfilled that precludes super-Planckian monotonic regions in the axion potential in all explicit examples that we have studied. For these examples, this prevents us from building a model of large field natural inflation on the basis of this idea. Nevertheless, parametrically super-Planckian effective decay constants are possible to the best of our understanding. This means that our model can also be used to argue against strong forms [205] of the swampland distance conjecture [30], because there is no tower of modes that becomes light as the parametrically super-Planckian axion valley is traversed. These results uncover a yet incomplete understanding regarding the *quantitative* form and validity of the aforementioned swampland conjectures, while some of their *qualitative* predictions seem to be in accord with our findings.

Furthermore, the axions we have identified play a previously unknown role as the relevant light degrees of freedom that control crucial aspects of the global geometry of a Calabi-Yau manifold stabilized in close vicinity to a conifold transition locus in complex structure moduli space: they control the relative length and orientation of distinct warped throats. Finally, our results are consistent with the gauge gravity correspondence of warped throats: in the small 't Hooft coupling regime, the axion potential can be understood to arise from the misalignment of the gaugino condensates of several confining gauge groups, while in the opposite regime it is a twisting of two or more throats against each other.

We conclude that the phenomenology of string theory solutions in general, and the *swampland program* in particular, is one of the most promising roads towards relating observable phenomena with physics at the Planck scale. It is of crucial importance to continue to pursue this direction in the future, and it is clear that new phenomena are waiting to be discovered. We are optimistic that the questions of viability of large field inflation and de Sitter cosmology in string theory that were pursued in this thesis can be settled to a sufficient degree of satisfaction in the near future.

Acknowledgments

Alexander, thank you for your great support, advice and friendship, as well as the uncounted discussions about inflation, M-theory, music, warp drives and hobbits. I am very grateful for the time you invested in my PhD, the many lessons I learned, as well as the freedom you gave me in pursuing the topics that I cared about most. Mafalda and Jonny, thank you for your friendship, for exploring the oceans together, for always finding the best pubs, and giving me valuable scientific advice. I hope we will see each other often in the future. Ander, thank you for the fun times we had together at DESY, for an interesting collaboration, and for trying to teach me some football. I am afraid I unlearned it all. Federico and Yvette, I am glad we had a shared year here at DESY and I am grateful for many discussions about the swampland and inflation, and about the proper way to make pancakes. Yvette, thank you also for the useful feedback on the introductory parts of my thesis. I want to thank Marco Zagermann for agreeing to be my co-supervisor, for carefully reading a manuscript of this thesis, and for the valuable feedback he gave me. Moreover, I want to thank my collaborators Federico Carta, Arthur Hebecker, Sascha Leonhardt, Ander Retolaza, and Thomas Van Riet for the many insightful and enjoyable discussions, and the many lessons I learned thereby. Finally, I would like to thank my colleagues at DESY for making my stay here so enjoyable. I hope you can forgive me for my poor qualities as a cookie distributor. This thesis was supported by the ERC Consolidator Grant STRINGFLATION under the HORIZON 2020 grant agreement no. 647995.

Lisa, danke für deine Liebe und Unterstützung!

Appendix A

The Axion Potential in the Local Throat

In the main text we have repeatedly made use of the fact that the C_2- and B_2-axions c and b can only enter the scalar potential that is generated in the local throat in certain combinations with the 'local complex structure' of the throat, namely the real and imaginary part of

$$M \log(z) - i\mathcal{G}\,, \tag{A.1}$$

where $\mathcal{G} = c - \tau b$, and z is the 'local complex structure'. Here, we would like to derive this without using 'local flux stabilization' as in section 6.3 but rather rely only on asymptotic properties of the KS/KT solution [78, 162]. For simplicity we will set the RR zero form to zero, i.e. $\tau = i g_s^{-1}$.

We cut off the throat at a radial coordinate $r_{\rm UV}$ and define the b-axion at that value of the radial coordinate. Since the B_2 profile runs along the radial direction [160], changing $b \longrightarrow b + \delta b$ can be realized by choosing a *different* UV-cutoff $r'_{\rm UV}$. Since the absolute value of the complex structure is defined in units of the UV-cutoff it scales as

$$z \longrightarrow e^{-\frac{\delta b}{g_s M}} z\,. \tag{A.2}$$

Since this is just a coordinate transformation from the perspective of the local KS throat, the combination $g_s M \log(|z|) + b$ cannot appear in the scalar potential that is generated within the throat. It acquires physical meaning only if the throat is cut off at fixed, finite $r_{\rm UV}$.

Similarly, in the limit $|z|/r^3 \to 0$ the RR three form takes the form

$$F_3 = 2\pi\alpha' M \left(g^5 + {\rm d}c/M\right) \wedge \omega_\Sigma\,, \tag{A.3}$$

where the on-form $g^5 = \mathrm{d}\psi + ...$ is defined in [294], and ω_Σ is the normalized harmonic 2-form of $T^{1,1}$. The field $c(x)$ transforms like a Goldstone boson under (local) coordinate transformations [294]

$$\psi \longrightarrow \psi + 2\omega(x)\,, \quad c(x) \longrightarrow c(x) - 2M\omega(x)\,. \tag{A.4}$$

Shifting along this angular direction is an isometry of the asymptotic KS solution (called $U(1)_R$). Near the IR this is not the case precisely because (by definition) the phase of the complex structure also transforms like a Goldstone boson,

$$\arg z \longrightarrow \arg z - 2\omega\,. \tag{A.5}$$

Again, in the local throat the combination $M \arg z + c$ has no physical meaning as it is eaten via the Higgs mechanism. Only when the throat is glued into the CY space at finite radial coordinate r_{UV} does the c-axion gain its independent physical meaning because the $U(1)_R$ symmetry is badly broken by the CY geometry. Putting together the real and imaginary part of $\mathcal{G} = c - \tau b$, we arrive at the conclusion that only the combination (A.1) is physical when considering a single throat. A second degree of freedom only becomes physical by finiteness of the throat, i.e. by breaking the asymptotic $U(1)_R$ symmetry. This also implies that the kinetic terms for the fields $\varphi_{1,2}$ stated in (6.28) actually take the form $[\partial(\varphi_{1,2} \pm c/M)]^2$ since they arise from local throat physics. We have disregarded some unimportant off-diagonal terms in the kinetic matrix.

Appendix B

Background on Multi Conifolds

In this appendix we discuss preliminaries that are important for section 6.6. In the derivation of the $\mathcal{B}$-cycle periods we follow Chapter 8 of [295]. We are interested in what happens when n cycles γ^i with m homology relations among them shrink at a conifold point in moduli space. The Picard-Lefschetz formula states that upon encircling a conifold point in moduli space, a three-cycle δ undergoes the monodromy [77, 310, 311]

$$\delta \longrightarrow \delta + \sum_{i=1}^{n} (\delta \cap \gamma^i)\gamma^i \,. \tag{B.1}$$

Knowing this monodromy transformation is enough to determine that

$$\int_\delta \Omega = \frac{1}{2\pi i} \sum_{i=1}^{n} (\delta \cap \gamma^i) \int_{\gamma^i} \Omega \, \log(\int_{\gamma^i} \Omega) + \text{single-valued} \,. \tag{B.2}$$

We may choose $n-m$ of the degenerating cycles as part of the integral $\mathcal{A}$-cycle basis $\mathcal{A}^i = \gamma^i$ for $i = 1, \ldots, n-m$, while the remaining m vanishing cycles are integer linear combinations $\gamma^i = \sum_{a=1}^{n-m} c_a^i \mathcal{A}^a$ for $i = n-m+1, \ldots, n$. By applying (B.1) to the cycles $\mathcal{B}_a$ one arrives at [295]

$$G^a = \int_{\mathcal{B}_a} \Omega = \frac{1}{2\pi i} z_a \log(z_a) + \frac{1}{2\pi i} \sum_{i=n-m+1}^{n} c_a^i z_i \log(z_i) + g^a(z) \,, \quad a = 1, ..., n-m \,, \tag{B.3}$$

where $g^a(z)$ are $n-m$ holomorphic functions. We have defined $z_i \equiv \sum_{a=1}^{n-m} c_a^i z_a$ for $i = n-m+1, \ldots, n$, i.e. $z_i \equiv \int_{\gamma^i} \Omega$ when applying (B.1)[1]. At frozen values of z^a, $a = n-m+1, ..., h^{2,1}+1$ the periods associated to other cycles, $G^a = \int_{\mathcal{B}_a} \Omega$, with

[1] When using a local expression for the holomorphic three-form Ω in the vicinity of smoothed conical singularity described by (6.5) one can calculate $\int_{\gamma^i} \Omega = z_i$ [304]. This identifies the z_i defined here with the local deformation parameter of the i-th throat.

$a = n-m+1, ..., h^{2,1}+1$, are holomorphic in the complex structures that parametrize the multi conifold deformations, i.e. in the z^i, with $i = 1, ..., n-m$. In what follows we denote by z^a only the multi conifold deformation parameters.

We may now evaluate the GVW superpotential $W = \int_M G_3 \wedge \Omega$ where we choose flux quanta M_a and K^a according to $G_3 = -\sum_{a=1}^{n-m}(M_a\alpha^a - \tau K^a\beta_a)$. Using 3.28 one obtains

$$\begin{aligned} W(z_a) = &\sum_{a=1}^{n-m} \frac{M_a}{2\pi i} z_a \log(z_a) + \sum_{i=n-m+1}^{n} \frac{M_i}{2\pi i} z_i \log(z_i) \\ &+ \sum_{a=1}^{n-m} M_a g^a(z) - \tau \sum_{a=1}^{n-m} K^a z_a + \hat{\hat{W}}_0(z_a)\,, \end{aligned} \tag{B.4}$$

where we have *defined* $M_i \equiv \sum_{a=1}^{n-m} c_a^i M_a$, and the holomorphic function $\hat{\hat{W}}_0(z_a)$ parametrizes the contributions from fluxes on other cycles. We may use the z_a and z_i with $i = n-m+1, ..., n$ on the same footing by interpreting our definition of the z_i as m constraint equations

$$0 = P^I \equiv \sum_{i=1}^{n} p_i^I z_i \equiv z_{n-m+I} - \sum_{a=1}^{n-m} c_a^{n-m+I} z_a\,, \quad I = 1, ..., m\,. \tag{B.5}$$

Here, the $m \times n$ matrix p_i^I is defined as

$$p_i^I = \begin{cases} -c_i^{n-m+I}\,, & i = 1, \dots, n-m\,, \\ \delta_i^{n-m+I}\,, & i = n-m+1, \dots, n\,. \end{cases} \tag{B.6}$$

We may now write the superpotential as

$$W(z_i) = \sum_{i=1}^{n} \left(M_i \frac{z_i}{2\pi i} \log(z_i) + M_i g^i(z) - \tau K^i z_i \right) + \sum_{I=1}^{m} \lambda_I P^I + \hat{\hat{W}}_0(z_i)\,, \tag{B.7}$$

with m Lagrange multipliers λ_I. The homology relations are now enforced via the F-term of the fields λ_I. In doing so, we have defined g^i to be zero for $i > n-m$.

The F_3-flux on γ^i is given by M_i. By the definition of M_i for $i = n-m+1, \dots, n$, the flux numbers automatically fulfill $\sum_{i=1}^{n} p_i^I M_i = 0$ for all I. In democratic terms, the n flux numbers M_i must be chosen in compliance with the m homology constraints $\sum_{i=1}^{n} p_i^I M_i = 0$. The H_3-flux on $\mathcal{B}_a$ is given by $K^a + \sum_{I=1}^{m} c_a^{n-m+I} K^{n-m+I}$, as this is the coefficient appearing in front of z_a. In other words the $n-m$ flux quantization conditions read $K^a + \sum_{I=1}^{m} c_a^{n-m+I} K^{n-m+I} \in \mathbb{Z}$. Note that we may transform $K^i \to$

$K^i + \sum_I \alpha_I p_i^I$ for any $\alpha \in \mathbb{C}^m$ because the superpotential is left invariant upon imposing the constraint equations, that is to say, we can undo such a transformation by also shifting the Lagrange multipliers $\lambda^I \to \lambda^I + \tau\alpha^I$. Of course, the flux quantization conditions are invariant under these shifts. Finally, the Kähler potential is given by

$$K_{\rm cs}(z_i, \bar{z}_i) = -\log\left(-i\int \Omega \wedge \bar{\Omega}\right) = -\log\left(ig_K(z) - i\overline{g_K}(z) + \sum_{a=1}^{n-m} i\bar{z}_a G^a(z) + c.c.\right) \tag{B.8}$$

$$= -\log\left(ig_K(z) - i\overline{g_K(z)} + \sum_{i=1}^{n}\left[\frac{|z_i|^2}{2\pi}\log(|z_i|^2) + i\bar{z}_i g^i(z) - iz_i\overline{g^i(z)}\right]\right), \tag{B.9}$$

where $g_K = \sum_{a=n-m+1}^{h^{2,1}+1} \overline{z_a} G^a(z)$ encodes contributions from other cycles. The g_K are holomorphic in z_a, $a = 1, ..., n-m$. Note that despite the democratic formulation, the Kähler and superpotential are strictly defined only along complex structure moduli space, where $P^I = 0$. As explained in section 6.6 we propose to extend the domain of these functions to the deformation space parametrized by *all* z_i by introducing general Taylor expansions of $g^i(z_i)$, $g_K(z_i)$ and $\hat{W}_0(z_i)$ in (6.47).

Bibliography

[1] F. Carta, J. Moritz, and A. Westphal, "Gaugino condensation and small uplifts in KKLT," arXiv:1902.01412 [hep-th].

[2] A. Hebecker, S. Leonhardt, J. Moritz, and A. Westphal, "Thraxions: Ultralight Throat Axions," *JHEP* **04** (2019) 158, arXiv:1812.03999 [hep-th].

[3] J. Moritz, A. Retolaza, and A. Westphal, "On uplifts by warped anti D3 branes," *Fortsch. Phys.* **67** no. 1-2, (2019) 1800098, arXiv:1809.06618 [hep-th].

[4] J. Moritz and T. Van Riet, "Racing through the swampland: de Sitter uplift vs weak gravity," *JHEP* **09** (2018) 099, arXiv:1805.00944 [hep-th].

[5] J. Moritz, A. Retolaza, and A. Westphal, "Toward de Sitter space from ten dimensions," *Phys. Rev.* **D97** no. 4, (2018) 046010, arXiv:1707.08678 [hep-th].

[6] **ATLAS** Collaboration, G. Aad *et al.*, "Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC," *Phys. Lett.* **B716** (2012) 1–29, arXiv:1207.7214 [hep-ex].

[7] **CMS** Collaboration, S. Chatrchyan *et al.*, "Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC," *Phys. Lett.* **B716** (2012) 30–61, arXiv:1207.7235 [hep-ex].

[8] S. L. Glashow, "Partial Symmetries of Weak Interactions," *Nucl. Phys.* **22** (1961) 579–588.

[9] A. Salam, "Weak and Electromagnetic Interactions," *Conf. Proc.* **C680519** (1968) 367–377.

[10] S. Weinberg, "A Model of Leptons," *Phys. Rev. Lett.* **19** (Nov, 1967) 1264–1266. https://link.aps.org/doi/10.1103/PhysRevLett.19.1264.

[11] G. 't Hooft and M. J. G. Veltman, "Regularization and Renormalization of Gauge Fields," *Nucl. Phys.* **B44** (1972) 189–213.

[12] **T2K** Collaboration, K. Abe *et al.*, "Indication of Electron Neutrino Appearance from an Accelerator-produced Off-axis Muon Neutrino Beam," *Phys. Rev. Lett.* **107** (2011) 041801, arXiv:1106.2822 [hep-ex].

[13] **SNO** Collaboration, Q. R. Ahmad *et al.*, "Direct evidence for neutrino flavor transformation from neutral current interactions in the Sudbury Neutrino Observatory," *Phys. Rev. Lett.* **89** (2002) 011301, arXiv:nucl-ex/0204008 [nucl-ex].

[14] **Super-Kamiokande** Collaboration, Y. Fukuda *et al.*, "Evidence for oscillation of atmospheric neutrinos," *Phys. Rev. Lett.* **81** (1998) 1562–1567, arXiv:hep-ex/9807003 [hep-ex].

[15] H. Georgi and S. L. Glashow, "Unity of All Elementary Particle Forces," *Phys. Rev. Lett.* **32** (1974) 438–441.

[16] E. Hubble, "A relation between distance and radial velocity among extra-galactic nebulae," *Proc. Nat. Acad. Sci.* **15** (1929) 168–173.

[17] **Supernova Cosmology Project** Collaboration, S. Perlmutter *et al.*, "Measurements of Omega and Lambda from 42 high redshift supernovae," *Astrophys. J.* **517** (1999) 565–586, arXiv:astro-ph/9812133 [astro-ph].

[18] **Supernova Search Team** Collaboration, A. G. Riess *et al.*, "Observational evidence from supernovae for an accelerating universe and a cosmological constant," *Astron. J.* **116** (1998) 1009–1038, arXiv:astro-ph/9805201 [astro-ph].

[19] **Planck** Collaboration, Y. Akrami *et al.*, "Planck 2018 results. I. Overview and the cosmological legacy of Planck," arXiv:1807.06205 [astro-ph.CO].

[20] A. H. Guth, "The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems," *Phys. Rev.* **D23** (1981) 347–356.

[21] A. D. Linde, "A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems," *Phys. Lett.* **B108** (1982) 389–393.

[22] A. Albrecht and P. J. Steinhardt, "Cosmology for Grand Unified Theories with Radiatively Induced Symmetry Breaking," *Phys. Rev. Lett.* **48** (1982) 1220–1223. [Adv. Ser. Astrophys. Cosmol.3,158(1987)].

[23] M. Rees, "POLARIZATION AND SPECTRUM OF THE PRIMEVAL RADIATION IN AN ANISOTROPIC UNIVERSE." *Astrophys. J., 153: L1-5(July 1968). .*

[24] M. Zaldarriaga and U. Seljak, "An all sky analysis of polarization in the microwave background," *Phys. Rev.* **D55** (1997) 1830–1840, arXiv:astro-ph/9609170 [astro-ph].

[25] M. Kamionkowski, A. Kosowsky, and A. Stebbins, "Statistics of cosmic microwave background polarization," *Phys. Rev.* **D55** (1997) 7368–7388, arXiv:astro-ph/9611125 [astro-ph].

[26] D. H. Lyth, "What would we learn by detecting a gravitational wave signal in the cosmic microwave background anisotropy?," *Phys. Rev. Lett.* **78** (1997) 1861–1863, arXiv:hep-ph/9606387 [hep-ph].

[27] **Planck** Collaboration, Y. Akrami *et al.*, "Planck 2018 results. X. Constraints on inflation," arXiv:1807.06211 [astro-ph.CO].

[28] D. Baumann and L. McAllister, *Inflation and String Theory*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 2015. arXiv:1404.2601 [hep-th]. http://www.cambridge.org/mw/academic/subjects/physics/theoretical-physics-and-mathematical-physics/inflation-and-string-theory?format=HB.

[29] C. Vafa, "The String landscape and the swampland," arXiv:hep-th/0509212 [hep-th].

[30] H. Ooguri and C. Vafa, "On the Geometry of the String Landscape and the Swampland," *Nucl. Phys.* **B766** (2007) 21–33, arXiv:hep-th/0605264 [hep-th].

[31] N. Arkani-Hamed, L. Motl, A. Nicolis, and C. Vafa, "The String landscape, black holes and gravity as the weakest force," *JHEP* **06** (2007) 060, arXiv:hep-th/0601001 [hep-th].

[32] L. Susskind, "Trouble for remnants," arXiv:hep-th/9501106 [hep-th].

[33] L. Alvarez-Gaume, P. H. Ginsparg, G. W. Moore, and C. Vafa, "An O(16) x O(16) Heterotic String," *Phys. Lett.* **B171** (1986) 155–162.

[34] J. Polchinski, "Dirichlet Branes and Ramond-Ramond charges," *Phys. Rev. Lett.* **75** (1995) 4724–4727, arXiv:hep-th/9510017 [hep-th].

[35] J. Polchinski, *String theory. Vol. 2: Superstring theory and beyond*. Cambridge University Press, 2007.

[36] A. Adams, O. DeWolfe, and W. Taylor, "String universality in ten dimensions," *Phys. Rev. Lett.* **105** (2010) 071601, arXiv:1006.1352 [hep-th].

[37] T. Kaluza, "Zum Unitätsproblem der Physik," *Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.)* **1921** (1921) 966–972, arXiv:1803.08616 [physics.hist-ph]. [Int. J. Mod. Phys.D27,no.14,1870001(2018)].

[38] O. Klein, "Quantum Theory and Five-Dimensional Theory of Relativity. (In German and English)," *Z. Phys.* **37** (1926) 895–906. [,76(1926)].

[39] K. Dasgupta, G. Rajesh, and S. Sethi, "M theory, orientifolds and G - flux," *JHEP* **08** (1999) 023, arXiv:hep-th/9908088 [hep-th].

[40] S. B. Giddings, S. Kachru, and J. Polchinski, "Hierarchies from fluxes in string compactifications," *Phys. Rev.* **D66** (2002) 106006, arXiv:hep-th/0105097 [hep-th].

[41] S. Kachru, R. Kallosh, A. D. Linde, and S. P. Trivedi, "De Sitter vacua in string theory," *Phys. Rev.* **D68** (2003) 046005, arXiv:hep-th/0301240 [hep-th].

[42] F. Denef and M. R. Douglas, "Distributions of flux vacua," *JHEP* **05** (2004) 072, arXiv:hep-th/0404116 [hep-th].

[43] D. Martinez-Pedrera, D. Mehta, M. Rummel, and A. Westphal, "Finding all flux vacua in an explicit example," *JHEP* **06** (2013) 110, arXiv:1212.4530 [hep-th].

[44] J. E. Kim, H. P. Nilles, and M. Peloso, "Completing natural inflation," *JCAP* **0501** (2005) 005, arXiv:hep-ph/0409138 [hep-ph].

[45] S. Dimopoulos, S. Kachru, J. McGreevy, and J. G. Wacker, "N-flation," *JCAP* **0808** (2008) 003, arXiv:hep-th/0507205 [hep-th].

[46] L. McAllister, E. Silverstein, and A. Westphal, "Gravity Waves and Linear Inflation from Axion Monodromy," *Phys. Rev.* **D82** (2010) 046003, arXiv:0808.0706 [hep-th].

[47] F. Marchesano, G. Shiu, and A. M. Uranga, "F-term Axion Monodromy Inflation," *JHEP* **09** (2014) 184, arXiv:1404.3040 [hep-th].

[48] A. Hebecker, S. C. Kraus, and L. T. Witkowski, "D7-Brane Chaotic Inflation," *Phys. Lett.* **B737** (2014) 16–22, arXiv:1404.3711 [hep-th].

[49] R. Blumenhagen and E. Plauschinn, "Towards Universal Axion Inflation and Reheating in String Theory," *Phys. Lett.* **B736** (2014) 482–487, arXiv:1404.3542 [hep-th].

[50] E. Palti, "On Natural Inflation and Moduli Stabilisation in String Theory," *JHEP* **10** (2015) 188, arXiv:1508.00009 [hep-th].

[51] R. Blumenhagen, "Large Field Inflation/Quintessence and the Refined Swampland Distance Conjecture," *PoS* **CORFU2017** (2018) 175, arXiv:1804.10504 [hep-th].

[52] T. Banks, M. Dine, P. J. Fox, and E. Gorbatov, "On the possibility of large axion decay constants," *JCAP* **0306** (2003) 001, arXiv:hep-th/0303252 [hep-th].

[53] T. Banks, M. Johnson, and A. Shomer, "A Note on Gauge Theories Coupled to Gravity," *JHEP* **09** (2006) 049, arXiv:hep-th/0606277 [hep-th].

[54] T. Rudelius, "On the Possibility of Large Axion Moduli Spaces," *JCAP* **1504** no. 04, (2015) 049, arXiv:1409.5793 [hep-th].

[55] M. Montero, A. M. Uranga, and I. Valenzuela, "Transplanckian axions!?" *JHEP* **08** (2015) 032, arXiv:1503.03886 [hep-th].

[56] J. Brown, W. Cottrell, G. Shiu, and P. Soler, "Fencing in the Swampland: Quantum Gravity Constraints on Large Field Inflation," *JHEP* **10** (2015) 023, arXiv:1503.04783 [hep-th].

[57] T. C. Bachlechner, C. Long, and L. McAllister, "Planckian Axions and the Weak Gravity Conjecture," *JHEP* **01** (2016) 091, arXiv:1503.07853 [hep-th].

[58] J. Brown, W. Cottrell, G. Shiu, and P. Soler, "On Axionic Field Ranges, Loopholes and the Weak Gravity Conjecture," *JHEP* **04** (2016) 017, arXiv:1504.00659 [hep-th].

[59] D. Junghans, "Large-Field Inflation with Multiple Axions and the Weak Gravity Conjecture," *JHEP* **02** (2016) 128, arXiv:1504.03566 [hep-th].

[60] T. Rudelius, "Constraints on Axion Inflation from the Weak Gravity Conjecture," *JCAP* **1509** no. 09, (2015) 020, arXiv:1503.00795 [hep-th].

[61] Y. Nakayama and Y. Nomura, "Weak gravity conjecture in the AdS/CFT correspondence," *Phys. Rev.* **D92** no. 12, (2015) 126006, arXiv:1509.01647 [hep-th].

[62] B. Heidenreich, M. Reece, and T. Rudelius, "Sharpening the Weak Gravity Conjecture with Dimensional Reduction," *JHEP* **02** (2016) 140, arXiv:1509.06374 [hep-th].

[63] D. Harlow, "Wormholes, Emergent Gauge Fields, and the Weak Gravity Conjecture," *JHEP* **01** (2016) 122, arXiv:1510.07911 [hep-th].

[64] B. Heidenreich, M. Reece, and T. Rudelius, "Evidence for a sublattice weak gravity conjecture," *JHEP* **08** (2017) 025, arXiv:1606.08437 [hep-th].

[65] M. Montero, G. Shiu, and P. Soler, "The Weak Gravity Conjecture in three dimensions," *JHEP* **10** (2016) 159, arXiv:1606.08438 [hep-th].

[66] C. Long, L. McAllister, and J. Stout, "Systematics of Axion Inflation in Calabi-Yau Hypersurfaces," *JHEP* **02** (2017) 014, arXiv:1603.01259 [hep-th].

[67] G. Shiu, P. Soler, and W. Cottrell, "Weak Gravity Conjecture and Extremal Black Hole," arXiv:1611.06270 [hep-th].

[68] A. Hebecker and P. Soler, "The Weak Gravity Conjecture and the Axionic Black Hole Paradox," *JHEP* **09** (2017) 036, arXiv:1702.06130 [hep-th].

[69] B. Heidenreich, M. Reece, and T. Rudelius, "The Weak Gravity Conjecture and Emergence from an Ultraviolet Cutoff," *Eur. Phys. J.* **C78** no. 4, (2018) 337, arXiv:1712.01868 [hep-th].

[70] S. Andriolo, D. Junghans, T. Noumi, and G. Shiu, "A Tower Weak Gravity Conjecture from Infrared Consistency," *Fortsch. Phys.* **66** no. 5, (2018) 1800020, arXiv:1802.04287 [hep-th].

[71] A. Hebecker, D. Junghans, and A. Schachner, "Large Field Ranges from Aligned and Misaligned Winding," arXiv:1812.05626 [hep-th].

[72] A. Hebecker, F. Rompineve, and A. Westphal, "Axion Monodromy and the Weak Gravity Conjecture," *JHEP* **04** (2016) 157, arXiv:1512.03768 [hep-th].

[73] A. Westphal, "de Sitter string vacua from Kahler uplifting," *JHEP* **03** (2007) 102, arXiv:hep-th/0611332 [hep-th].

[74] A. Hebecker, J. Moritz, A. Westphal, and L. T. Witkowski, "Towards Axion Monodromy Inflation with Warped KK-Modes," *Phys. Lett.* **B754** (2016) 328–334, arXiv:1512.04463 [hep-th]. [Erratum: Phys. Lett.B767,493(2017)].

[75] V. Balasubramanian, P. Berglund, J. P. Conlon, and F. Quevedo, "Systematics of moduli stabilisation in Calabi-Yau flux compactifications," *JHEP* **03** (2005) 007, arXiv:hep-th/0502058 [hep-th].

[76] P. Candelas, P. S. Green, and T. Hubsch, "Finite Distances Between Distinct Calabi-yau Vacua: (Other Worlds Are Just Around the Corner)," *Phys. Rev. Lett.* **62** (1989) 1956.

[77] B. R. Greene, D. R. Morrison, and A. Strominger, "Black hole condensation and the unification of string vacua," *Nucl. Phys.* **B451** (1995) 109–120, arXiv:hep-th/9504145 [hep-th].

[78] I. R. Klebanov and A. A. Tseytlin, "Gravity duals of supersymmetric SU(N) x SU(N+M) gauge theories," *Nucl. Phys.* **B578** .

[79] G. Obied, H. Ooguri, L. Spodyneiko, and C. Vafa, "De Sitter Space and the Swampland," arXiv:1806.08362 [hep-th].

[80] S. K. Garg and C. Krishnan, "Bounds on Slow Roll and the de Sitter Swampland," arXiv:1807.05193 [hep-th].

[81] H. Ooguri, E. Palti, G. Shiu, and C. Vafa, "Distance and de Sitter Conjectures on the Swampland," *Phys. Lett.* **B788** (2019) 180–184, arXiv:1810.05506 [hep-th].

[82] F. Denef, A. Hebecker, and T. Wrase, "de Sitter swampland conjecture and the Higgs potential," *Phys. Rev.* **D98** no. 8, (2018) 086004, arXiv:1807.06581 [hep-th].

[83] C. Roupec and T. Wrase, "de Sitter extrema and the swampland," *Fortsch. Phys.* **2018** (2018) 1800082, arXiv:1807.09538 [hep-th].

[84] J. P. Conlon, "The de Sitter swampland conjecture and supersymmetric AdS vacua," *Int. J. Mod. Phys.* **A33** no. 29, (2018) 1850178, arXiv:1808.05040 [hep-th].

[85] Y. Akrami, R. Kallosh, A. Linde, and V. Vardanyan, "The Landscape, the Swampland and the Era of Precision Cosmology," *Fortsch. Phys.* **67** no. 1-2, (2019) 1800075, arXiv:1808.09440 [hep-th].

[86] H. Murayama, M. Yamazaki, and T. T. Yanagida, "Do We Live in the Swampland?," *JHEP* **12** (2018) 032, arXiv:1809.00478 [hep-th].

[87] K. Choi, D. Chway, and C. S. Shin, "The dS swampland conjecture with the electroweak symmetry and QCD chiral symmetry breaking," *JHEP* **11** (2018) 142, arXiv:1809.01475 [hep-th].

[88] A. Hebecker and T. Wrase, "The asymptotic dS Swampland Conjecture - a simplified derivation and a potential loophole," *Fortsch. Phys.* **2018** (2018) 1800097, arXiv:1810.08182 [hep-th].

[89] B. de Wit, D. J. Smit, and N. D. Hari Dass, "Residual Supersymmetry of Compactified D=10 Supergravity," *Nucl. Phys.* **B283** (1987) 165.

[90] J. M. Maldacena and C. Nunez, "Supergravity description of field theories on curved manifolds and a no go theorem," *Int. J. Mod. Phys.* **A16** (2001) 822–855, arXiv:hep-th/0007018 [hep-th]. [,182(2000)].

[91] S. P. de Alwis, "On Potentials from fluxes," *Phys. Rev.* **D68** (2003) 126001, arXiv:hep-th/0307084 [hep-th].

[92] M. P. Hertzberg, S. Kachru, W. Taylor, and M. Tegmark, "Inflationary Constraints on Type IIA String Theory," *JHEP* **12** (2007) 095, arXiv:0711.2512 [hep-th].

[93] R. Flauger, S. Paban, D. Robbins, and T. Wrase, "Searching for slow-roll moduli inflation in massive type IIA supergravity with metric fluxes," *Phys. Rev.* **D79** (2009) 086011, arXiv:0812.3886 [hep-th].

[94] S. S. Haque, G. Shiu, B. Underwood, and T. Van Riet, "Minimal simple de Sitter solutions," *Phys. Rev.* **D79** (2009) 086005, arXiv:0810.5328 [hep-th].

[95] C. Caviezel, P. Koerber, S. Kors, D. Lust, T. Wrase, and M. Zagermann, "On the Cosmology of Type IIA Compactifications on SU(3)-structure Manifolds," *JHEP* **04** (2009) 010, arXiv:0812.3551 [hep-th].

[96] U. H. Danielsson, S. S. Haque, G. Shiu, and T. Van Riet, "Towards Classical de Sitter Solutions in String Theory," *JHEP* **09** (2009) 114, arXiv:0907.2041 [hep-th].

[97] T. Wrase and M. Zagermann, "On Classical de Sitter Vacua in String Theory," *Fortsch. Phys.* **58** (2010) 906–910, arXiv:1003.0029 [hep-th].

[98] G. Shiu and Y. Sumitomo, "Stability Constraints on Classical de Sitter Vacua," *JHEP* **09** (2011) 052, arXiv:1107.2925 [hep-th].

[99] U. H. Danielsson, S. S. Haque, P. Koerber, G. Shiu, T. Van Riet, and T. Wrase, "De Sitter hunting in a classical landscape," *Fortsch. Phys.* **59** (2011) 897–933, arXiv:1103.4858 [hep-th].

[100] F. F. Gautason, D. Junghans, and M. Zagermann, "On Cosmological Constants from alpha'-Corrections," *JHEP* **06** (2012) 029, arXiv:1204.0807 [hep-th].

[101] D. Kutasov, T. Maxfield, I. Melnikov, and S. Sethi, "Constraining de Sitter Space in String Theory," *Phys. Rev. Lett.* **115** no. 7, (2015) 071305, arXiv:1504.00056 [hep-th].

[102] D. Andriot and J. Blåbäck, "Refining the boundaries of the classical de Sitter landscape," *JHEP* **03** (2017) 102, arXiv:1609.00385 [hep-th]. [Erratum: JHEP03,083(2018)].

[103] D. Andriot, "On classical de Sitter and Minkowski solutions with intersecting branes," *JHEP* **03** (2018) 054, arXiv:1710.08886 [hep-th].

[104] R. Kallosh and A. D. Linde, "Landscape, the scale of SUSY breaking, and inflation," *JHEP* **12** (2004) 004, arXiv:hep-th/0411011 [hep-th].

[105] D. Baumann, A. Dymarsky, I. R. Klebanov, J. M. Maldacena, L. P. McAllister, and A. Murugan, "On D3-brane Potentials in Compactifications with Fluxes and Wrapped D-branes," *JHEP* **11** (2006) 031, arXiv:hep-th/0607050 [hep-th].

[106] D. Baumann, A. Dymarsky, S. Kachru, I. R. Klebanov, and L. McAllister, "D3-brane Potentials from Fluxes in AdS/CFT," *JHEP* **06** (2010) 072, arXiv:1001.5028 [hep-th].

[107] A. Dymarsky and L. Martucci, "D-brane non-perturbative effects and geometric deformations," *JHEP* **04** (2011) 061, arXiv:1012.4018 [hep-th].

[108] Y. Hamada, A. Hebecker, G. Shiu, and P. Soler, "Brane gaugino condensate in 10d," arXiv:1812.06097 [hep-th].

[109] R. Kallosh, "Gaugino Condensation and Geometry of the Perfect Square," arXiv:1901.02023 [hep-th].

[110] Y. Hamada, A. Hebecker, G. Shiu, and P. Soler, "Understanding KKLT from a 10d perspective," arXiv:1902.01410 [hep-th].

[111] F. F. Gautason, V. Van Hemelryck, T. Van Riet, and G. Venken, "A 10d view on the KKLT AdS vacuum and uplifting," arXiv:1902.01415 [hep-th].

[112] **Planck** Collaboration, N. Aghanim *et al.*, "Planck 2018 results. VI. Cosmological parameters," arXiv:1807.06209 [astro-ph.CO].

[113] A. G. Riess *et al.*, "Milky Way Cepheid Standards for Measuring Cosmic Distances and Application to Gaia DR2: Implications for the Hubble Constant," *Astrophys. J.* **861** no. 2, (2018) 126, arXiv:1804.10655 [astro-ph.CO].

[114] W. D. Kenworthy, D. Scolnic, and A. Riess, "The Local Perspective on the Hubble Tension: Local Structure Does Not Impact Measurement of the Hubble Constant," arXiv:1901.08681 [astro-ph.CO].

[115] J. M. Cline, "TASI Lectures on Early Universe Cosmology: Inflation, Baryogenesis and Dark Matter," arXiv:1807.08749 [hep-ph].

[116] C. Cheung, P. Creminelli, A. L. Fitzpatrick, J. Kaplan, and L. Senatore, "The Effective Field Theory of Inflation," *JHEP* **03** (2008) 014, arXiv:0709.0293 [hep-th].

[117] M. Sasaki, "Gauge Invariant Scalar Perturbations in the New Inflationary Universe," *Prog. Theor. Phys.* **70** (1983) 394.

[118] H. Kodama and M. Sasaki, "Cosmological Perturbation Theory," *Prog. Theor. Phys. Suppl.* **78** (1984) 1–166.

[119] V. F. Mukhanov, "Quantum Theory of Gauge Invariant Cosmological Perturbations," *Sov. Phys. JETP* **67** (1988) 1297–1302. [Zh. Eksp. Teor. Fiz.94N7,1(1988)].

[120] W. Hu, "Lecture Notes on CMB Theory: From Nucleosynthesis to Recombination," arXiv:0802.3688 [astro-ph].

[121] **BICEP2, Keck Array** Collaboration, P. A. R. Ade *et al.*, "BICEP2 / Keck Array x: Constraints on Primordial Gravitational Waves using Planck, WMAP, and New BICEP2/Keck Observations through the 2015 Season," *Phys. Rev. Lett.* **121** (2018) 221301, arXiv:1810.05216 [astro-ph.CO].

[122] **SDSS** Collaboration, M. Betoule *et al.*, "Improved cosmological constraints from a joint analysis of the SDSS-II and SNLS supernova samples," *Astron. Astrophys.* **568** (2014) A22, arXiv:1401.4064 [astro-ph.CO].

[123] G. Veneziano, "Construction of a crossing-simmetric, Regge-behaved amplitude for linearly rising trajectories," *Il Nuovo Cimento A (1965-1970)* **57** no. 1, (Sep, 1968) 190–197. https://doi.org/10.1007/BF02824451.

[124] J. Scherk and J. H. Schwarz, "Dual models for non-hadrons," *Nuclear Physics B* **81** no. 1, (1974) 118 – 144. http://www.sciencedirect.com/science/article/pii/0550321374900108.

[125] L. E. Ibanez and A. M. Uranga, *String theory and particle physics: An introduction to string phenomenology*. Cambridge University Press, 2012. http://www.cambridge.org/de/knowledge/isbn/item6563092/?site_locale=de_DE.

[126] E. D'Hoker and D. H. Phong, "Two loop superstrings. 3. Slice independence and absence of ambiguities," *Nucl. Phys.* **B636** (2002) 61–79, arXiv:hep-th/0111016 [hep-th].

[127] F. Gliozzi, J. Scherk, and D. I. Olive, "Supersymmetry, Supergravity Theories and the Dual Spinor Model," *Nucl. Phys.* **B122** (1977) 253–290.

[128] D. J. Gross, J. A. Harvey, E. J. Martinec, and R. Rohm, "The Heterotic String," *Phys. Rev. Lett.* **54** (1985) 502–505.

[129] C. M. Hull and P. K. Townsend, "Unity of superstring dualities," *Nucl. Phys.* **B438** (1995) 109–137, arXiv:hep-th/9410167 [hep-th]. [,236(1994)].

[130] E. Witten, "String theory dynamics in various dimensions," *Nucl. Phys.* **B443** (1995) 85–126, arXiv:hep-th/9503124 [hep-th]. [,333(1995)].

[131] P. K. Townsend, "The eleven-dimensional supermembrane revisited," *Phys. Lett.* **B350** (1995) 184–187, arXiv:hep-th/9501068 [hep-th]. [,265(1995)].

[132] P. Horava and E. Witten, "Heterotic and type I string dynamics from eleven-dimensions," *Nucl. Phys.* **B460** (1996) 506–524, arXiv:hep-th/9510209 [hep-th]. [,397(1995)].

[133] M. Dine, P. Y. Huet, and N. Seiberg, "Large and Small Radius in String Theory," *Nucl. Phys.* **B322** (1989) 301–316.

[134] J. Dai, R. G. Leigh, and J. Polchinski, "New Connections Between String Theories," *Mod. Phys. Lett.* **A4** (1989) 2073–2083.

[135] W. Lerche, C. Vafa, and N. P. Warner, "Chiral Rings in N=2 Superconformal Theories," *Nucl. Phys.* **B324** (1989) 427–474.

[136] K. Hori and C. Vafa, "Mirror symmetry," arXiv:hep-th/0002222 [hep-th].

[137] K. S. Narain, M. H. Sarmadi, and E. Witten, "A Note on Toroidal Compactification of Heterotic String Theory," *Nucl. Phys.* **B279** (1987) 369–379.

[138] P. H. Ginsparg, "Comment on Toroidal Compactification of Heterotic Superstrings," *Phys. Rev.* **D35** (1987) 648.

[139] M. B. Green, J. A. Harvey, and G. W. Moore, "I-brane inflow and anomalous couplings on d-branes," *Class. Quant. Grav.* **14** (1997) 47–52, arXiv:hep-th/9605033 [hep-th].

[140] Y.-K. E. Cheung and Z. Yin, "Anomalies, branes, and currents," *Nucl. Phys.* **B517** (1998) 69–91, arXiv:hep-th/9710206 [hep-th].

[141] E. Witten, "On flux quantization in M theory and the effective action," *J. Geom. Phys.* **22** (1997) 1–13, arXiv:hep-th/9609122 [hep-th].

[142] A. Sen, "Strong - weak coupling duality in four-dimensional string theory," *Int. J. Mod. Phys.* **A9** (1994) 3707–3750, arXiv:hep-th/9402002 [hep-th].

[143] K. Hori, S. Katz, A. Klemm, R. Pandharipande, R. Thomas, C. Vafa, R. Vakil, and E. Zaslow, *Mirror symmetry*, vol. 1 of *Clay mathematics monographs*. AMS, Providence, USA, 2003. http://www.claymath.org/library/monographs/cmim01.pdf.

[144] S. Kachru, A. Tripathy, and M. Zimet, "K3 metrics from little string theory," arXiv:1810.10540 [hep-th].

[145] D. Z. Freedman and A. Van Proeyen, *Supergravity*. Cambridge Univ. Press, Cambridge, UK, 2012. http://www.cambridge.org/mw/academic/subjects/physics/theoretical-physics-and-mathematical-physics/supergravity?format=AR.

[146] E. Calabi, "On Kähler Manifolds with Vanishing Canonical Class," in *Algebraic Geometry and Topology*. Princeton University Press. https://doi.org/10.1515%2F9781400879915-006.

[147] S.-T. Yau, "On the ricci curvature of a compact kähler manifold and the complex monge-ampére equation, I," *Communications on Pure and Applied Mathematics* **31** no. 3, (1978) 339–411, https://onlinelibrary.wiley.com/doi/pdf/10.1002/cpa.3160310304. https://onlinelibrary.wiley.com/doi/abs/10.1002/cpa.3160310304.

[148] M. Bodner and A. C. Cadavid, "Dimensional Reduction of Type Iib Supergravity and Exceptional Quaternionic Manifolds," *Class. Quant. Grav.* **7** (1990) 829.

[149] R. Bohm, H. Gunther, C. Herrmann, and J. Louis, "Compactification of type IIB string theory on Calabi-Yau threefolds," *Nucl. Phys.* **B569** (2000) 229–246, arXiv:hep-th/9908007 [hep-th].

[150] P. Candelas and X. C. de la Ossa, "Comments on Conifolds," *Nucl. Phys.* **B342** (1990) 246–268.

[151] A. Strominger, "Massless black holes and conifolds in string theory," *Nucl. Phys.* **B451** (1995) 96–108, arXiv:hep-th/9504090 [hep-th].

[152] F. Denef, "Les Houches Lectures on Constructing String Vacua," *Les Houches* **87** (2008) 483–610, arXiv:0803.1194 [hep-th].

[153] T. W. Grimm and J. Louis, "The Effective action of N = 1 Calabi-Yau orientifolds," *Nucl. Phys.* **B699** (2004) 387–426, arXiv:hep-th/0403067 [hep-th].

[154] S. Gukov, C. Vafa, and E. Witten, "CFT's from Calabi-Yau four folds," *Nucl. Phys.* **B584** (2000) 69–108, arXiv:hep-th/9906070 [hep-th]. [Erratum: Nucl. Phys.B608,477(2001)].

[155] F. Denef, M. R. Douglas, B. Florea, A. Grassi, and S. Kachru, "Fixing all moduli in a simple f-theory compactification," *Adv. Theor. Math. Phys.* **9** no. 6, (2005) 861–929, arXiv:hep-th/0503124 [hep-th].

[156] S. B. Giddings and A. Maharana, "Dynamics of warped compactifications and the shape of the warped landscape," *Phys. Rev.* **D73** (2006) 126003, arXiv:hep-th/0507158 [hep-th].

[157] L. Randall and R. Sundrum, "A Large mass hierarchy from a small extra dimension," *Phys. Rev. Lett.* **83** (1999) 3370–3373, arXiv:hep-ph/9905221 [hep-ph].

[158] J. M. Maldacena, "The Large N limit of superconformal field theories and supergravity," *Int. J. Theor. Phys.* **38** (1999) 1113–1133, arXiv:hep-th/9711200 [hep-th]. [Adv. Theor. Math. Phys.2,231(1998)].

[159] I. R. Klebanov and E. Witten, "Superconformal field theory on three-branes at a Calabi-Yau singularity," *Nucl. Phys.* **B536** (1998) 199–218, arXiv:hep-th/9807080 [hep-th].

[160] I. R. Klebanov and N. A. Nekrasov, "Gravity duals of fractional branes and logarithmic RG flow," *Nucl. Phys.* **B574** (2000) 263–274, arXiv:hep-th/9911096 [hep-th].

[161] N. Seiberg, "Electric - magnetic duality in supersymmetric nonAbelian gauge theories," *Nucl. Phys.* **B435** (1995) 129–146, arXiv:hep-th/9411149 [hep-th].

[162] I. R. Klebanov and M. J. Strassler, "Supergravity and a confining gauge theory: Duality cascades and chi SB resolution of naked singularities," *JHEP* **08** (2000) 052, arXiv:hep-th/0007191 [hep-th].

[163] A. Hebecker and J. March-Russell, "The Ubiquitous throat," *Nucl. Phys.* **B781** (2007) 99–111, arXiv:hep-th/0607120 [hep-th].

[164] K. Becker, M. Becker, M. Haack, and J. Louis, "Supersymmetry breaking and alpha-prime corrections to flux induced potentials," *JHEP* **06** (2002) 060, arXiv:hep-th/0204254 [hep-th].

[165] M. Dine and N. Seiberg, "Is the Superstring Weakly Coupled?," *Phys. Lett.* **162B** (1985) 299–302.

[166] E. Witten, "Nonperturbative superpotentials in string theory," *Nucl. Phys.* **B474** (1996) 343–360, arXiv:hep-th/9604030 [hep-th].

[167] R. Kallosh, A.-K. Kashani-Poor, and A. Tomasiello, "Counting fermionic zero modes on M5 with fluxes," *JHEP* **06** (2005) 069, arXiv:hep-th/0503138 [hep-th].

[168] E. Bergshoeff, R. Kallosh, A.-K. Kashani-Poor, D. Sorokin, and A. Tomasiello, "An Index for the Dirac operator on D3 branes with background fluxes," *JHEP* **10** (2005) 102, arXiv:hep-th/0507069 [hep-th].

[169] I. Affleck, M. Dine, and N. Seiberg, "Dynamical Supersymmetry Breaking in Supersymmetric QCD," *Nucl. Phys.* **B241** (1984) 493–534.

[170] J. Terning, *Modern supersymmetry: Dynamics and duality*. 2006.

[171] B. S. Acharya and C. Vafa, "On domain walls of N=1 supersymmetric Yang-Mills in four-dimensions," arXiv:hep-th/0103011 [hep-th].

[172] M. Dine and Y. Shirman, "Remarks on the racetrack scheme," *Phys. Rev.* **D63** (2001) 046005, arXiv:hep-th/9906246 [hep-th].

[173] S. Sethi, "Supersymmetry Breaking by Fluxes," *JHEP* **10** (2018) 022, arXiv:1709.03554 [hep-th].

[174] S. Kachru, J. Pearson, and H. L. Verlinde, "Brane / flux annihilation and the string dual of a nonsupersymmetric field theory," *JHEP* **06** (2002) 021, arXiv:hep-th/0112197 [hep-th].

[175] J. Blaback, U. H. Danielsson, and T. Van Riet, "Resolving anti-brane singularities through time-dependence," *JHEP* **02** (2013) 061, arXiv:1202.1132 [hep-th].

[176] I. Bena, D. Junghans, S. Kuperstein, T. Van Riet, T. Wrase, and M. Zagermann, "Persistent anti-brane singularities," *JHEP* **10** (2012) 078, arXiv:1205.1798 [hep-th].

[177] I. Bena, M. Grana, S. Kuperstein, and S. Massai, "Polchinski-Strassler does not uplift Klebanov-Strassler," *JHEP* **09** (2013) 142, arXiv:1212.4828 [hep-th].

[178] I. Bena, A. Buchel, and O. J. C. Dias, "Horizons cannot save the Landscape," *Phys. Rev.* **D87** no. 6, (2013) 063012, arXiv:1212.5162 [hep-th].

[179] I. Bena, J. Blaback, U. H. Danielsson, and T. Van Riet, "Antibranes cannot become black," *Phys. Rev.* **D87** no. 10, (2013) 104023, arXiv:1301.7071 [hep-th].

[180] D. Junghans, D. Schmidt, and M. Zagermann, "Curvature-induced Resolution of Anti-brane Singularities," *JHEP* **10** (2014) 34, arXiv:1402.6040 [hep-th].

[181] I. Bena, M. Grana, S. Kuperstein, and S. Massai, "Giant Tachyons in the Landscape," *JHEP* **02** (2015) 146, arXiv:1410.7776 [hep-th].

[182] U. H. Danielsson and T. Van Riet, "Fatal attraction: more on decaying anti-branes," *JHEP* **03** (2015) 087, arXiv:1410.8476 [hep-th].

[183] I. Bena and S. Kuperstein, "Brane polarization is no cure for tachyons," *JHEP* **09** (2015) 112, arXiv:1504.00656 [hep-th].

[184] D. Cohen-Maldonado, J. Diaz, T. van Riet, and B. Vercnocke, "Observations on fluxes near anti-branes," *JHEP* **01** (2016) 126, arXiv:1507.01022 [hep-th].

[185] G. S. Hartnett, "Localised Anti-Branes in Flux Backgrounds," *JHEP* **06** (2015) 007, arXiv:1501.06568 [hep-th].

[186] U. H. Danielsson, "Perturbative decay of anti-branes in flux backgrounds due to space time instabilities," arXiv:1502.01234 [hep-th].

[187] B. Michel, E. Mintun, J. Polchinski, A. Puhm, and P. Saad, "Remarks on brane and antibrane dynamics," *JHEP* **09** (2015) 021, arXiv:1412.5702 [hep-th].

[188] J. Polchinski, "Brane/antibrane dynamics and KKLT stability," arXiv:1509.05710 [hep-th].

[189] L. Aalsma, M. Tournoy, J. P. Van Der Schaar, and B. Vercnocke, "Supersymmetric embedding of antibrane polarization," *Phys. Rev.* **D98** no. 8, (2018) 086019, arXiv:1807.03303 [hep-th].

[190] J. Armas, N. Nguyen, V. Niarchos, N. A. Obers, and T. Van Riet, "Meta-stable non-extremal anti-branes," arXiv:1812.01067 [hep-th].

[191] S. Kachru, R. Kallosh, A. D. Linde, J. M. Maldacena, L. P. McAllister, and S. P. Trivedi, "Towards inflation in string theory," *JCAP* **0310** (2003) 013, arXiv:hep-th/0308055 [hep-th].

[192] C. Vafa, "Evidence for F theory," *Nucl. Phys.* **B469** (1996) 403–418, arXiv:hep-th/9602022 [hep-th].

[193] T. Weigand, "TASI Lectures on F-theory," arXiv:1806.01854 [hep-th].

[194] W. Taylor and Y.-N. Wang, "The F-theory geometry with most flux vacua," *JHEP* **12** (2015) 164, arXiv:1511.03209 [hep-th].

[195] E. Palti, "The Swampland: Introduction and Review," 2019. arXiv:1903.06239 [hep-th].

[196] C. Vafa and E. Witten, "Parity Conservation in QCD," *Phys. Rev. Lett.* **53** (1984) 535.

[197] R. D. Peccei and H. R. Quinn, "CP Conservation in the Presence of Instantons," *Phys. Rev. Lett.* **38** (1977) 1440–1443. [,328(1977)].

[198] N. Kaloper and L. Sorbo, "A Natural Framework for Chaotic Inflation," *Phys. Rev. Lett.* **102** (2009) 121301, arXiv:0811.1989 [hep-th].

[199] N. Kaloper, A. Lawrence, and L. Sorbo, "An Ignoble Approach to Large Field Inflation," *JCAP* **1103** (2011) 023, arXiv:1101.0026 [hep-th].

[200] G. Dvali, "Three-form gauging of axion symmetries and gravity," arXiv:hep-th/0507215 [hep-th].

[201] E. Witten, "Symmetry and Emergence," *Nature Phys.* **14** (2018) 116–119, arXiv:1710.01791 [hep-th].

[202] A. Kapustin, "Lectures on Electric-Magnetic Duality and the Geometric Langlands Program,".

[203] C. Vafa, "Geometric origin of Montonen-Olive duality," *Adv. Theor. Math. Phys.* **1** (1998) 158–166, arXiv:hep-th/9707131 [hep-th].

[204] M. Montero, "A Holographic Derivation of the Weak Gravity Conjecture," arXiv:1812.03978 [hep-th].

[205] D. Klaewer and E. Palti, "Super-Planckian Spatial Field Variations and Quantum Gravity," *JHEP* **01** (2017) 088, arXiv:1610.00010 [hep-th].

[206] T. W. Grimm, E. Palti, and I. Valenzuela, "Infinite Distances in Field Space and Massless Towers of States," *JHEP* **08** (2018) 143, arXiv:1802.08264 [hep-th].

[207] T. W. Grimm, C. Li, and E. Palti, "Infinite Distance Networks in Field Space and Charge Orbits," arXiv:1811.02571 [hep-th].

[208] G. Buratti, J. Calderón, and A. M. Uranga, "Transplanckian Axion Monodromy !?" arXiv:1812.05016 [hep-th].

[209] M. Cicoli, S. De Alwis, A. Maharana, F. Muia, and F. Quevedo, "De Sitter vs Quintessence in String Theory," *Fortsch. Phys.* **2018** (2018) 1800079, arXiv:1808.08967 [hep-th].

[210] Z. Komargodski and N. Seiberg, "From Linear SUSY to Constrained Superfields," *JHEP* **09** (2009) 066, arXiv:0907.2441 [hep-th].

[211] D. V. Volkov and V. P. Akulov, "Is the Neutrino a Goldstone Particle?," *Phys. Lett.* **46B** (1973) 109–110.

[212] K. Choi, A. Falkowski, H. P. Nilles, and M. Olechowski, "Soft supersymmetry breaking in KKLT flux compactification," *Nucl. Phys.* **B718** (2005) 113–133, arXiv:hep-th/0503216 [hep-th].

[213] O. Lebedev, H. P. Nilles, and M. Ratz, "De Sitter vacua from matter superpotentials," *Phys. Lett.* **B636** (2006) 126–131, arXiv:hep-th/0603047 [hep-th].

[214] F. Brummer, A. Hebecker, and M. Trapletti, "SUSY breaking mediation by throat fields," *Nucl. Phys.* **B755** (2006) 186–198, arXiv:hep-th/0605232 [hep-th].

[215] I. Antoniadis, E. Dudas, S. Ferrara, and A. Sagnotti, "The Volkov-Akulov-Starobinsky supergravity," *Phys. Lett.* **B733** (2014) 32–35, arXiv:1403.3269 [hep-th].

[216] S. Ferrara, R. Kallosh, and A. Linde, "Cosmology with Nilpotent Superfields," *JHEP* **10** (2014) 143, arXiv:1408.4096 [hep-th].

[217] R. Kallosh and T. Wrase, "Emergence of Spontaneously Broken Supersymmetry on an Anti-D3-Brane in KKLT dS Vacua," *JHEP* **12** (2014) 117, arXiv:1411.1121 [hep-th].

[218] E. A. Bergshoeff, K. Dasgupta, R. Kallosh, A. Van Proeyen, and T. Wrase, "$\overline{\text{D3}}$ and dS," *JHEP* **05** (2015) 058, arXiv:1502.07627 [hep-th].

[219] R. Kallosh, F. Quevedo, and A. M. Uranga, "String Theory Realizations of the Nilpotent Goldstino," *JHEP* **12** (2015) 039, arXiv:1507.07556 [hep-th].

[220] R. Kallosh, A. Linde, E. McDonough, and M. Scalisi, "de Sitter Vacua with a Nilpotent Superfield," *Fortsch. Phys.* **67** no. 1-2, (2019) 1800068, arXiv:1808.09428 [hep-th].

[221] R. Kallosh, A. Linde, E. McDonough, and M. Scalisi, "dS Vacua and the Swampland," *JHEP* **03** (2019) 134, arXiv:1901.02022 [hep-th].

[222] M. P. Garcia del Moral, S. Parameswaran, N. Quiroz, and I. Zavala, "Anti-D3 branes and moduli in non-linear supergravity," *JHEP* **10** (2017) 185, arXiv:1707.07059 [hep-th].

[223] M. Berg, M. Haack, and B. Kors, "Loop corrections to volume moduli and inflation in string theory," *Phys. Rev.* **D71** (2005) 026005, arXiv:hep-th/0404087 [hep-th].

[224] P. G. O. Freund and M. A. Rubin, "Dynamics of Dimensional Reduction," *Phys. Lett.* **97B** (1980) 233–235.

[225] A. Saltman and E. Silverstein, "A New handle on de Sitter compactifications," *JHEP* **01** (2006) 139, arXiv:hep-th/0411271 [hep-th].

[226] C. P. Burgess, A. de la Macorra, I. Maksymyk, and F. Quevedo, "Supersymmetric models with product groups and field dependent gauge couplings," *JHEP* **09** (1998) 007, arXiv:hep-th/9808087 [hep-th].

[227] J. J. Blanco-Pillado, C. P. Burgess, J. M. Cline, C. Escoda, M. Gomez-Reino, R. Kallosh, A. D. Linde, and F. Quevedo, "Racetrack inflation," *JHEP* **11** (2004) 063, arXiv:hep-th/0406230 [hep-th].

[228] J. J. Blanco-Pillado, M. A. Urkiola, and J. M. Wachter, "Racetrack Potentials and the de Sitter Swampland Conjectures," *JHEP* **01** (2019) 187, arXiv:1811.05463 [hep-th].

[229] N. Goheer, M. Kleban, and L. Susskind, "The Trouble with de Sitter space," *JHEP* **07** (2003) 056, arXiv:hep-th/0212209 [hep-th].

[230] B. Freivogel and M. Lippert, "Evidence for a bound on the lifetime of de Sitter space," *JHEP* **12** (2008) 096, arXiv:0807.1104 [hep-th].

[231] P. Agrawal, G. Obied, P. J. Steinhardt, and C. Vafa, "On the Cosmological Implications of the String Swampland," *Phys. Lett.* **B784** (2018) 271–276, arXiv:1806.09718 [hep-th].

[232] D. Junghans and M. Zagermann, "A Universal Tachyon in Nearly No-scale de Sitter Compactifications," *JHEP* **07** (2018) 078, arXiv:1612.06847 [hep-th].

[233] E. Silverstein, "(A)dS backgrounds from asymmetric orientifolds," *Clay Mat. Proc.* **1** (2002) 179, arXiv:hep-th/0106209 [hep-th].

[234] A. Maloney, E. Silverstein, and A. Strominger, "De Sitter space in noncritical string theory," in *The future of theoretical physics and cosmology: Celebrating Stephen Hawking's 60th birthday. Proceedings, Workshop and Symposium, Cambridge, UK, January 7-10, 2002*, pp. 570–591. 2002. arXiv:hep-th/0205316 [hep-th]. http://www-public.slac.stanford.edu/sciDoc/docMeta.aspx?slacPubNumber=SLAC-PUB-9228.

[235] M. Dodelson, X. Dong, E. Silverstein, and G. Torroba, "New solutions with accelerated expansion in string theory," *JHEP* **12** (2014) 050, arXiv:1310.5297 [hep-th].

[236] C. Caviezel, T. Wrase, and M. Zagermann, "Moduli Stabilization and Cosmology of Type IIB on SU(2)-Structure Orientifolds," *JHEP* **04** (2010) 011, arXiv:0912.3287 [hep-th].

[237] J. Polchinski and E. Silverstein, "Dual Purpose Landscaping Tools: Small Extra Dimensions in AdS/CFT," in *Strings, gauge fields, and the geometry behind: The legacy of Maximilian Kreuzer*, A. Rebhan, L. Katzarkov, J. Knapp, R. Rashkov, and E. Scheidegger, eds., pp. 365–390. 2009. arXiv:0908.0756 [hep-th]. http://www-public.slac.stanford.edu/sciDoc/docMeta.aspx?slacPubNumber=SLAC-PUB-13748.

[238] X. Dong, B. Horn, E. Silverstein, and G. Torroba, "Micromanaging de Sitter holography," *Class. Quant. Grav.* **27** (2010) 245020, arXiv:1005.5403 [hep-th].

[239] X. Dong, B. Horn, S. Matsuura, E. Silverstein, and G. Torroba, "FRW solutions and holography from uplifted AdS/CFT," *Phys. Rev.* **D85** (2012) 104035, arXiv:1108.5732 [hep-th].

[240] E. Silverstein, "Simple de Sitter Solutions," *Phys. Rev.* **D77** (2008) 106006, arXiv:0712.1196 [hep-th].

[241] C. P. Burgess, R. Kallosh, and F. Quevedo, "De Sitter string vacua from supersymmetric D terms," *JHEP* **10** (2003) 056, arXiv:hep-th/0309187 [hep-th].

[242] V. Balasubramanian and P. Berglund, "Stringy corrections to Kahler potentials, SUSY breaking, and the cosmological constant problem," *JHEP* **11** (2004) 085, arXiv:hep-th/0408054 [hep-th].

[243] J. P. Conlon, F. Quevedo, and K. Suruliz, "Large-volume flux compactifications: Moduli spectrum and D3/D7 soft supersymmetry breaking," *JHEP* **08** (2005) 007, arXiv:hep-th/0505076 [hep-th].

[244] M. Cicoli, A. Maharana, F. Quevedo, and C. P. Burgess, "De Sitter String Vacua from Dilaton-dependent Non-perturbative Effects," *JHEP* **06** (2012) 011, arXiv:1203.1750 [hep-th].

[245] J. Louis, M. Rummel, R. Valandro, and A. Westphal, "Building an explicit de Sitter," *JHEP* **10** (2012) 163, arXiv:1208.3208 [hep-th].

[246] M. Cicoli, D. Klevers, S. Krippendorf, C. Mayrhofer, F. Quevedo, and R. Valandro, "Explicit de Sitter Flux Vacua for Global String Models with Chiral Matter," *JHEP* **05** (2014) 001, arXiv:1312.0014 [hep-th].

[247] M. Rummel and Y. Sumitomo, "De Sitter Vacua from a D-term Generated Racetrack Uplift," *JHEP* **01** (2015) 015, arXiv:1407.7580 [hep-th].

[248] M. Cicoli, F. Quevedo, and R. Valandro, "De Sitter from T-branes," *JHEP* **03** (2016) 141, arXiv:1512.04558 [hep-th].

[249] A. P. Braun, M. Rummel, Y. Sumitomo, and R. Valandro, "De Sitter vacua from a D-term generated racetrack potential in hypersurface Calabi-Yau compactifications," *JHEP* **12** (2015) 033, arXiv:1509.06918 [hep-th].

[250] A. Retolaza and A. Uranga, "De Sitter Uplift with Dynamical Susy Breaking," *JHEP* **04** (2016) 137, arXiv:1512.06363 [hep-th].

[251] D. Gallego, M. C. D. Marsh, B. Vercnocke, and T. Wrase, "A New Class of de Sitter Vacua in Type IIB Large Volume Compactifications," *JHEP* **10** (2017) 193, arXiv:1707.01095 [hep-th].

[252] J. J. Heckman, C. Lawrie, L. Lin, and G. Zoccarato, "F-theory and Dark Energy," arXiv:1811.01959 [hep-th].

[253] J. J. Heckman, C. Lawrie, L. Lin, J. Sakstein, and G. Zoccarato, "Pixelated Dark Energy," arXiv:1901.10489 [hep-th].

[254] M. Grana, R. Minasian, M. Petrini, and A. Tomasiello, "Supersymmetric backgrounds from generalized Calabi-Yau manifolds," *JHEP* **08** (2004) 046, arXiv:hep-th/0406137 [hep-th].

[255] D. Baumann, A. Dymarsky, I. R. Klebanov, and L. McAllister, "Towards an Explicit Model of D-brane Inflation," *JCAP* **0801** (2008) 024, arXiv:0706.0360 [hep-th].

[256] B. Heidenreich, L. McAllister, and G. Torroba, "Dynamic SU(2) Structure from Seven-branes," *JHEP* **05** (2011) 110, arXiv:1011.3510 [hep-th].

[257] A. R. Frey and M. Lippert, "AdS strings with torsion: Non-complex heterotic compactifications," *Phys. Rev.* **D72** (2005) 126001, arXiv:hep-th/0507202 [hep-th].

[258] P. Koerber and L. Martucci, "From ten to four and back again: How to generalize the geometry," *JHEP* **08** (2007) 059, arXiv:0707.1038 [hep-th].

[259] P. Koerber and L. Martucci, "Warped generalized geometry compactifications, effective theories and non-perturbative effects," *Fortsch. Phys.* **56** (2008) 862–868, arXiv:0803.3149 [hep-th].

[260] F. F. Gautason, V. Van Hemelryck, and T. Van Riet, "The tension between 10D supergravity and dS uplifts," arXiv:1810.08518 [hep-th].

[261] P. G. Camara, L. E. Ibanez, and A. M. Uranga, "Flux induced SUSY breaking soft terms," *Nucl. Phys.* **B689** (2004) 195–242, arXiv:hep-th/0311241 [hep-th].

[262] M. Grana, T. W. Grimm, H. Jockers, and J. Louis, "Soft supersymmetry breaking in Calabi-Yau orientifolds with D-branes and fluxes," *Nucl. Phys.* **B690** (2004) 21–61, arXiv:hep-th/0312232 [hep-th].

[263] P. G. Camara, L. E. Ibanez, and A. M. Uranga, "Flux-induced SUSY-breaking soft terms on D7-D3 brane systems," *Nucl. Phys.* **B708** (2005) 268–316, arXiv:hep-th/0408036 [hep-th].

[264] M. Kim and L. McAllister, "Monodromy Charge in D7-brane Inflation," arXiv:1812.03532 [hep-th].

[265] G. Veneziano and S. Yankielowicz, "An Effective Lagrangian for the Pure N=1 Supersymmetric Yang-Mills Theory," *Phys. Lett.* **113B** (1982) 231.

[266] G. Shiu, G. Torroba, B. Underwood, and M. R. Douglas, "Dynamics of Warped Flux Compactifications," *JHEP* **06** (2008) 024, arXiv:0803.3068 [hep-th].

[267] T. D. Brennan, F. Carta, and C. Vafa, "The String Landscape, the Swampland, and the Missing Corner," *PoS* **TASI2017** (2017) 015.

[268] M. Kreuzer and H. Skarke, "Complete classification of reflexive polyhedra in four-dimensions," *Adv. Theor. Math. Phys.* **4** (2002) 1209–1230, arXiv:hep-th/0002240 [hep-th].

[269] M. Demirtas, C. Long, L. McAllister, and M. Stillman, "The Kreuzer-Skarke Axiverse," arXiv:1808.01282 [hep-th].

[270] A. Banlaki, A. Chowdhury, C. Roupec, and T. Wrase, "Scaling limits of dS vacua and the swampland," *JHEP* **03** (2019) 065, arXiv:1811.07880 [hep-th].

[271] **CMB-S4** Collaboration, K. N. Abazajian *et al.*, "CMB-S4 Science Book, First Edition," arXiv:1610.02743 [astro-ph.CO].

[272] R. Blumenhagen, I. Valenzuela, and F. Wolf, "The Swampland Conjecture and F-term Axion Monodromy Inflation," *JHEP* **07** (2017) 145, arXiv:1703.05776 [hep-th].

[273] T. C. Bachlechner, M. Dias, J. Frazer, and L. McAllister, "Chaotic inflation with kinetic alignment of axion fields," *Phys. Rev.* **D91** no. 2, (2015) 023520, arXiv:1404.7496 [hep-th].

[274] E. Silverstein and A. Westphal, "Monodromy in the CMB: Gravity Waves and String Inflation," *Phys. Rev.* **D78** (2008) 106003, arXiv:0803.3085 [hep-th].

[275] S. Ashok and M. R. Douglas, "Counting flux vacua," *JHEP* **01** (2004) 060, arXiv:hep-th/0307049 [hep-th].

[276] K. Dasgupta, H. Firouzjahi, and R. Gwyn, "On The Warped Heterotic Axion," *JHEP* **06** (2008) 056, arXiv:0803.3828 [hep-th].

[277] S. Franco, D. Galloni, A. Retolaza, and A. Uranga, "On axion monodromy inflation in warped throats," *JHEP* **02** (2015) 086, arXiv:1405.7044 [hep-th].

[278] M. Berg, E. Pajer, and S. Sjors, "Dante's Inferno," *Phys. Rev.* **D81** (2010) 103535, arXiv:0912.1341 [hep-th].

[279] A. Hebecker, P. Mangat, F. Rompineve, and L. T. Witkowski, "Winding out of the Swamp: Evading the Weak Gravity Conjecture with F-term Winding Inflation?," *Phys. Lett.* **B748** (2015) 455–462, arXiv:1503.07912 [hep-th].

[280] K. Choi and S. H. Im, "Realizing the relaxion from multiple axions and its UV completion with high scale supersymmetry," *JHEP* **01** (2016) 149, arXiv:1511.00132 [hep-ph].

[281] D. E. Kaplan and R. Rattazzi, "Large field excursions and approximate discrete symmetries from a clockwork axion," *Phys. Rev.* **D93** no. 8, (2016) 085007, arXiv:1511.01827 [hep-ph].

[282] R. Blumenhagen, D. Herschmann, and F. Wolf, "String Moduli Stabilization at the Conifold," *JHEP* **08** (2016) 110, arXiv:1605.06299 [hep-th].

[283] P. Candelas, P. S. Green, and T. Hubsch, "Rolling Among Calabi-Yau Vacua," *Nucl. Phys.* **B330** (1990) 49.

[284] M. Reid, "The Moduli Space of 3-Folds with K = 0 may Nevertheless be Irreducible.," *Mathematische Annalen* **278** (1987) 329–334. http://eudml.org/doc/164295.

[285] T.-m. Chiang, B. R. Greene, M. Gross, and Y. Kanter, "Black hole condensation and the web of Calabi-Yau manifolds," *Nucl. Phys. Proc. Suppl.* **46** (1996) 82–95, arXiv:hep-th/9511204 [hep-th]. [,82(1995)].

[286] A. C. Avram, M. Kreuzer, M. Mandelberg, and H. Skarke, "The Web of Calabi-Yau hypersurfaces in toric varieties," *Nucl. Phys.* **B505** (1997) 625–640, arXiv:hep-th/9703003 [hep-th].

[287] V. V. Batyrev, "Dual polyhedra and mirror symmetry for Calabi-Yau hypersurfaces in toric varieties," *J. Alg. Geom.* **3** (1994) 493–545, arXiv:alg-geom/9310003 [alg-geom].

[288] V. Batyrev and M. Kreuzer, "Constructing new Calabi-Yau 3-folds and their mirrors via conifold transitions," *Adv. Theor. Math. Phys.* **14** no. 3, (2010) 879–898, arXiv:0802.3376 [math.AG].

[289] M. Lynker and R. Schimmrigk, "Conifold transitions and mirror symmetries," *Nucl. Phys.* **B484** (1997) 562–582, arXiv:hep-th/9511058 [hep-th].

[290] V. V. Batyrev, I. Ciocan-Fontanine, B. Kim, and D. van Straten, "Conifold transitions and mirror symmetry for Calabi-Yau complete intersections in Grassmannians," *Nucl. Phys.* **B514** (1998) 640–666.

[291] P. Candelas, X. C. De La Ossa, P. S. Green, and L. Parkes, "A Pair of Calabi-Yau manifolds as an exactly soluble superconformal theory," *Nucl. Phys.* **B359** (1991) 21–74. [AMS/IP Stud. Adv. Math.9,31(1998)].

[292] T. W. Grimm and A. Klemm, "U(1) Mediation of Flux Supersymmetry Breaking," *JHEP* **10** (2008) 077, arXiv:0805.3361 [hep-th].

[293] A. Retolaza, A. M. Uranga, and A. Westphal, "Bifid Throats for Axion Monodromy Inflation," *JHEP* **07** (2015) 099, arXiv:1504.02103 [hep-th].

[294] C. P. Herzog, I. R. Klebanov, and P. Ouyang, "D-branes on the conifold and N=1 gauge / gravity dualities," in *Progress in string, field and particle theory: Proceedings, NATO Advanced Study Institute, EC Summer School, Cargese, France, June 25-July 11, 2002*, pp. 189–223. 2002. arXiv:hep-th/0205100 [hep-th]. [,383(2002)].

[295] B. R. Greene, "String theory on Calabi-Yau manifolds," in *Fields, strings and duality. Proceedings, Summer School, Theoretical Advanced Study Institute in Elementary Particle Physics, TASI'96, Boulder, USA, June 2-28, 1996*, pp. 543–726. 1996. arXiv:hep-th/9702155 [hep-th].

[296] R. Blumenhagen, D. Kläwer, and L. Schlechter, "Swampland Variations on a Theme by KKLT," arXiv:1902.07724 [hep-th].

[297] B. Heidenreich, M. Reece, and T. Rudelius, "Emergence of Weak Coupling at Large Distance in Quantum Gravity," *Phys. Rev. Lett.* **121** no. 5, (2018) 051601, arXiv:1802.08698 [hep-th].

[298] S.-J. Lee, W. Lerche, and T. Weigand, "Tensionless Strings and the Weak Gravity Conjecture," *JHEP* **10** (2018) 164, arXiv:1808.05958 [hep-th].

[299] S.-J. Lee, W. Lerche, and T. Weigand, "A Stringy Test of the Scalar Weak Gravity Conjecture," *Nucl. Phys.* **B938** (2019) 321–350, arXiv:1810.05169 [hep-th].

[300] I. Antoniadis, J.-P. Derendinger, P. M. Petropoulos, and K. Siampos, "Heisenberg symmetry and hypermultiplet manifolds," *Nucl. Phys.* **B905** (2016) 293–312, arXiv:1512.06418 [hep-th].

[301] P. Corvilain, T. W. Grimm, and D. Regalado, "Shift-symmetries and gauge coupling functions in orientifolds and F-theory," *JHEP* **05** (2017) 059, arXiv:1607.03897 [hep-th].

[302] J. J. Heckman, J. Seo, and C. Vafa, "Phase Structure of a Brane/Anti-Brane System at Large N," *JHEP* **07** (2007) 073, arXiv:hep-th/0702077 [hep-th].

[303] J. J. Heckman and C. Vafa, "Geometrically Induced Phase Transitions at Large N," *JHEP* **04** (2008) 052, arXiv:0707.4011 [hep-th].

[304] F. Cachazo, K. A. Intriligator, and C. Vafa, "A Large N duality via a geometric transition," *Nucl. Phys.* **B603** (2001) 3–41, arXiv:hep-th/0103067 [hep-th].

[305] K. A. Intriligator and N. Seiberg, "Lectures on Supersymmetry Breaking," *Class. Quant. Grav.* **24** (2007) S741–S772, arXiv:hep-ph/0702069 [hep-ph]. [Les Houches87,125(2008)].

[306] S. Gandhi, L. McAllister, and S. Sjors, "A Toolkit for Perturbing Flux Compactifications," *JHEP* **12** (2011) 053, arXiv:1106.0002 [hep-th].

[307] A. Arvanitaki, S. Dimopoulos, S. Dubovsky, N. Kaloper, and J. March-Russell, "String Axiverse," *Phys. Rev.* **D81** (2010) 123530, arXiv:0905.4720 [hep-th].

[308] J. P. Conlon, "Brane-Antibrane Backreaction in Axion Monodromy Inflation," *JCAP* **1201** (2012) 033, arXiv:1110.6454 [hep-th].

[309] L. Martucci, J. Rosseel, D. Van den Bleeken, and A. Van Proeyen, "Dirac actions for D-branes on backgrounds with fluxes," *Class. Quant. Grav.* **22** (2005) 2745–2764, arXiv:hep-th/0504041 [hep-th].

[310] E. Picard and G. Simart, *Théorie des fonctions algébriques de deux variables indépendantes*. No. Bd. 1 in PCMI collection. Gauthier-Villars, 1897.

[311] S. Lefschetz, *L'analysis situs et la géométrie algébrique*. Gauthier-Villars, Paris, 1950.

Eidesstattliche Erklärung

Hiermit versichere ich an Eides statt, die vorliegende Dissertationsschrift selbst verfasst und keine anderen als die angegebenen Hilfsmittel und Quellen benutzt zu haben.

Die eingereichte schriftliche Fassung entspricht der auf dem elektronischen Speichermedium.

Die Dissertation wurde in der vorgelegten oder einer ähnlichen Form nicht schon einmal in einem früheren Promotionsverfahren angenommen oder als ungenügend beurteilt.

Hamburg, den 13. Mai 2019 Unterschrift