Anti-de Sitter 3-dimensional gravity with torsion*

M. Blagojević†
Institute of Physics, P. O. Box 57,
11001 Belgrade, Serbia

M. Vasilić‡
Institute of Physics, P. O. Box 57,
11001 Belgrade, Serbia

Abstract

Using the canonical formalism, we study the asymptotic symmetries of the topological 3-dimensional gravity with torsion. In the anti-de Sitter sector, the symmetries are realized by two independent Virasoro algebras with classical central charges. In the simple case of the teleparallel vacuum geometry, the central charges are equal to each other and have the same value as in general relativity, while in the general Riemann-Cartan geometry, they become different.

Introduction

Three-dimensional (3d) gravity has been used as a theoretical laboratory to test some of the conceptual problems of both classical and quantum gravity [1, 2]. One can identify several particularly important achievements in the development of these ideas. Brown and Henneaux demonstrated that, under suitable asymptotic conditions, the asymptotic symmetry of 3d gravity has an extremely rich structure, described by two independent canonical Virasoro algebras with classical central charges [3]. Soon after that, Witten found that general relativity with a cosmological constant (GR_Λ) can be formulated as a Chern-Simons gauge theory [4]. The equivalence between gravity and an ordinary gauge theory was crucial for a deeper understanding of quantum gravity. Next, the discovery of the black hole solution by Bañados, Teitelboim and Zanelli, had a powerful impact on 3d gravity [5]. It turned out that the Virasoro algebra of the asymptotic symmetry plays

---

* Invited talk given at *Workshop on Quantum Gravity and Noncommutative Geometry*, Universidade Lusófona, Lisbon, 20-23 July 2004, and at *III Summer School in Modern Mathematical Physics*, Zlatibor, Serbia, 20-31 August 2004. This is a shortened version of the talk. The complete version will be published in Int. J. Mod. Phys. **A** (2005), see also the preprint gr-qc/0412072.

† e-mail address: `mb@phy.bg.ac.yu`

‡ e-mail address: `mvasilic@phy.bg.ac.yu`

a central role in our understanding of the quantum nature of black hole [6–11].

Following a widely spread belief that the dynamics of gravity is to be described by general relativity, investigations of 3d gravity have been carried out mostly in the realm of *Riemannian* geometry. However, since the early 1990s, the possibility of *Riemann-Cartan* geometry has been also explored [12–18]; it is a geometry in which both the *curvature* and the *torsion* are present as independent geometric characteristics of spacetime [19, 20]. In this way, one expects to clarify the influence of geometry on the dynamical content of spacetime.

Dynamics of a theory is determined not only by its action, but also by the asymptotic conditions. The dynamical content of asymptotic conditions is best seen in topological theories, where the non-trivial dynamics is bound to exist only at the boundary. General action for topological 3d gravity in Riemann–Cartan spacetime has been proposed by Mielke and Baekler [12, 13]. This model is our starting point for exploring the structure of 3d gravity with torsion. In particular, we shall investigate

- the existence of the black hole with torsion, and
- the asymptotic structure of 3d gravity with torsion.

We restrict ourselves to the anti-de Sitter (AdS) sector of the theory, with negative effective cosmological constant. For a particular choice of parameters, the Mielke-Baekler action leads to the *teleparallel* (Weizenböck) geometry in vacuum [21, 22, 20], defined by the requirement of vanishing curvature, which we choose as the simplest framework for studying the influence of torsion on the spacetime dynamics.

The paper is organized as follows. In Sect. 2 we introduce Riemann–Cartan spacetime as a general geometric arena for 3d gravity with torsion, and discuss the teleparallel description of gravity in vacuum. In Sect. 3 we construct the teleparallel black hole solution. Then, in Sect. 4, we introduce the concept of asymptotically AdS configuration, and show that the related asymptotic symmetry is the same as in general relativity—the conformal symmetry. In the next section, the gauge structure of the theory is incorporated into the canonical formalism by investigating the Poisson bracket algebra of the asymptotic generators. The asymptotic symmetry is characterized by two independent canonical Virasoro algebras with classical central charges, the values of which are the same as in Riemannian spacetime of general relativity. In Sect. 6 we discuss the general case of Riemann-Cartan geometry, and show that the related classical central charges are different. Finally, Sect. 7 is devoted to concluding remarks.[1]

. . .

Concluding remarks

- 3d gravity with torsion, defined by the Mielke-Baekler action, possesses the teleparallel black hole solution, a generalization of the Riemannian BTZ black hole.

[1]For the complete version of the text, see the preprint gr-qc/0412072.

▪ Assuming the AdS asymptotic conditions, the canonical asymptotic symmetry is realized by two commuting Virasoro algebras with central extensions:
− in GR_Λ and in the teleparallel theory: $c_1 = c_2 = 3\ell/2G$,
− in Riemann-Cartan theory: $c_1 \neq c_2$. The implications of this result for the quantum structure of black hole are to be explored.

References

[1] S. Deser, R. Jackiw and G 't Hooft, Three-dimensional Einstein gravity: dynamics of flat space Ann. Phys. (N.Y) **152**, 220 (1984); S. Deser and R. Jackiw, Three-dimensional cosmological gravity: dynamics of constant curvature Ann. Phys. (N.Y) **153**, 405 (1984).

[2] See also the reviews: S. Carlip, Lectures on (2+1)-dimensional gravity, J. Korean Phys. Soc. **28**, 447 (1995), preprint gr-qc/9503024; *Quantum Gravity in 2+1 Dimensions* (Cambridge University Press, Cambridge, 1998).

[3] J. D. Brown and M. Henneaux, Central Charges in the Canonical Realization of Asymptotic Symmetries: An Example from Three Dimensional Gravity, Comm. Math. Phys. **104**, 207 (1986); see also M. Henneaux, Energy-momentum, angular momentum, and superscharge in 2+1 dimensions Phys. Rev. **D29**, 2766 (1984).

[4] E. Witten, 2+1 dimensional gravity as an exactly soluble system Nucl. Phys. **B311**, 46 (1988); A. Achucarro and P. Townsend, A Chern-Simons Action For Three-Dimensional Anti-De Sitter Supergravity Theories, Phys. Lett. **B180**, 89 (1986).

[5] M. Bañados, C. Teitelboim and J. Zanelli, The Black Hole in Three-Dimensional Spacetime, Phys. Rev. Lett. **16**, 1849 (1993); M. Bañados, M. Henneaux, C. Teitelboim and J. Zanelli, Geometry of 2+1 Black Hole, Phys. Rev. **D48**, 1506 (1993).

[6] O. Coussaert, M. Henneaux and P. van Driel, The asymptotic dynamics of three-dimensional Einstein gravity with negative cosmological constant, Class. Quant. Grav. **12**, 2961 (1995).

[7] M. Bañados, Global charges in Chern-Simons theory and 2+1 black hole, Phys. Rev. **D52**, 5861 (1996).

[8] A. Strominger, Black hole entropy from Near–Horizon Microstates, JHEP **9802**, 009 (1998).

[9] J. Navaro–Salas and P. Navaro, A Note on Einstein Gravity on AdS_3 and Boundary Conformal Field Theory, Phys. Lett. **B439**, 262 (1998).

[10] M. Bañados, Notes on black holes and three-dimensional gravity, Proceedings of the VIII Mexican School on Particles and Fields, AIP Conf. Proc. **490**, 198 (1999).

[11] M. Bañados, T. Brotz and M. Ortiz, Boundary dynamics and the statistical mechanics of the 2+1 dimensional black hole, Nucl. Phys. **B545**, 340 (1999).

[12] E. W. Mielke, P. Baekler, Topological gauge model of gravity with torsion, Phys. Lett. **A156**, 399 (1991).

[13] P. Baekler, E. W. Mielke, F. W. Hehl, Dynamical symmetries in topological 3D gravity with torsion, Nuovo Cim. **B107**, 91 (1992).

[14] T. Kawai, Teleparallel theory of (2+1)-dimensional gravity, Phys. Rev. **D48**, 5668 (1993); Poincaré gauge theory of (2+1)-dimensional gravity, Phys. Rev. **D49**, 2862 (1994); Exotic black hole solution in teleparallel theory of (2+1)-dimensional gravity, Prog. Theor. Phys. **94**, 1169 (1995); A. A. Sousa, J. W. Maluf, Canonical

Formulation of Gravitational Teleparallelism in 2+1 Dimensions in Schwinger's Time Gauge, Prog. Theor. Phys. **104**, 531 (2000).

[15] A. García, F. W. Hehl, C. Heinecke and A. Macías, Exact vacuum solutions of (1+2)-dimensional Poincaré gauge theory: BTZ solution with torsion, Phys. Rev. **D67**, 124016 (2003).

[16] M. Blagojević and M. Vasilić, Asymptotic symmetries in 3D gravity with torsion, Phys. Rev. **D67**, 084032 (2003).

[17] M. Blagojević and M. Vasilić, Asymptotic dynamics in 3D gravity with torsion, Phys. Rev. **D68**, 124007 (2003).

[18] M. Blagojević and M. Vasilić, 3D gravity with torsion as a Chern-Simons gauge theory, Phys. Rev. **D68**, 104023 (2003).

[19] F. W. Hehl, Four lectures on Poincaré gauge theory, Proceedings of the 6th Course of the School of Cosmology and Gravitation: *Spin, Torsion, Rotation and Supergravity,* held at Erice, Italy, 1979, eds. P. G. Bergmann, V. de Sabbata (Plenum, New York, 1980) p. 5; E. W. Mielke, *Geometrodynemics of Gauge Fields* – On the geometry of Yang–Mills and gravitational gauge theories (Academie – Verlag, Berlin, 1987).

[20] M. Blagojević, *Gravitation and gauge symmetries* (Bristol, Institute of Physics Publishing, 2002); see also: Three lectures on Poincaré gauge theory, Proceedings of the *2nd Summer School in Modern Mathematical Physics*, Kopaonik, Yugoslavia, 1-12 September 2002, SFIN **A1**, 147 (2003) [arXiv:gr-qc/0302040].

[21] C. Møller, On the crisis in the theory of gravitation and a possible solution, Mat. Fys. Medd. Dan. Vid. Selsk. **39** No. 13 (1978) (31 pages). K. Hayashi and T. Shirafuji, New general relativity, Phys. Rev. **D19**, 3524 (1979).

[22] J. Nitsch, The macroscopic limit of the Poincaré gauge field theory of gravitation, Proceedings of the 6th Course of the School on Cosmology and Gravitation: *Spin, torsion, Rotation and Supergravity*, held at Erice, Italy, 1979, eds. P. G. Bergmann, V. de Sabbata (Plenum, New York, 1980) p. 63; F. W. Hehl, J. Nitsch and P. von der Heyde, Gravitation and the Poincaré gauge field theory with quadratic Lagrangian, *General Relativity and Gravitation – One Hundred Years after the birth of Albert Einstein*, ed. A. Held (Plenum, New York, 1980) vol. 1, p. 329.

[23] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Spacetime* (Cambridge, Cambridge Ubiversity Press, 1973).

[24] M. Henneaux and C. Teitelboim, Asymptotically Anti-de Sitter Spaces, Commun. Math. Phys. **98**, 391 (1985).

[25] L. Castellani, Symmetries of constrained Hamiltonian systems, Ann. Phys. (N.Y) **143**, 357 (1982).

[26] T. Regge and C. Teitelboim, Role of surface integrals in the Hamiltonian formulation of general relativity, Ann. Phys. (N.Y) **88**, 286 (1974).