



Breathing comoving Hubble: Initial condition and eternity in view of the trans-Planckian censorship conjecture

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ARTICLE INFO

Article history:

Received 4 December 2019

Accepted 24 April 2020

Available online 1 May 2020

Editor: A. Ringwald

ABSTRACT

In this paper we put forward the idea that the comoving Hubble horizon undergoes multiple stages of contraction (*a.k.a.* inflationary phase) and expansion. The observable inflation, that produces the CMB anisotropies and generates primordial gravitational waves, follows and is followed by multiple early and late inflations. The trans-Planckian censorship conjecture restricts the duration of each inflationary phases and determines their Hubble rates. Early inflations could start immediately after the universe emerges from the Planck era. It alleviates the initial condition problem for the lower-scale observable inflation. Late inflations collectively assist the observable inflation to accommodate the present horizon. Moreover, it makes eternal inflation possible and consistent with observations.

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1. Introduction

The paradigm of cosmic inflation was proposed to alleviate the fine-tuning problem in the initial conditions of the standard big bang model [1–4]. During inflation, the universe experiences an accelerated expansion so that its comoving Hubble horizon shrinks and physical scales are stretched. Without inflation, the present horizon would be composed of around 10^6 causally disconnected patches at the time of recombination and about 10^{27} parts at the time of nucleosynthesis with no correlated past histories. It is in direct tension with the observed large scale homogeneities from the CMB and the LSS measurements. In principle, the big bang model can be applied immediately after the universe has emerged from the Planck epoch. Then the current horizon consists of around 10^{87} isolated pieces at the Planck time. The natural expectation is that inflation instantly takes over, provides the initial kick for Hubble expansion and expands a Planck size smooth patch to make the observed universe.

Furthermore when quantum effects are considered, inflations makes genuine predictions. It amplifies vacuum quantum fluctuations in scalar and gravitational fields [5–7]. Physical modes are stretched to superhorizon scales, classicalized and freeze. They re-enter the horizon at later stages of the cosmic evolution. Therefor, inflation predicts small inhomogeneities and primordial gravitational waves on the background. The former is actually observed as temperature anisotropies on the CMB. In the last 30 years, these anisotropies have been measured with high accuracy and provide

a solid support for inflationary dynamics [8–10]. The latest results from the *Planck* satellite, in particular, favors a lower scale inflation with Hubble rate $H_{\text{inf}} \sim 10^{14}$ GeV [11]. The delayed phase of inflation, besides losing the beginning bang, reintroduce the initial condition problem [12–14]. It needs to overcome inhomogeneities propagating inward the horizon and preclude inflation. The initial smooth patch of size around H_{inf}^{-1} must span over 10^4 to 10^6 parts at the Planck time, for matter and radiation intervening epochs respectively (see [15–17] for supporting arguments).

Moreover under general assumptions, quantum fluctuations can take over classical dynamics and prevents inflation from termination [18–23]. Once inflation began it never ends completely all over. Eternal inflation can populate vacua in the string landscape by the so-called pocket universes. Although there is no direct experimental handle to this prediction, this picture of multiverse offers a solution to fine-tuning of low energy parameters.

Inflation is a quasi-de Sitter (dS) state of the universe and is best understood in terms of an effective field theory (EFT) below the Planck scale. In general, it is difficult to incorporate the dS space in a theory of quantum gravity; both technically and conceptually. For instance, there are far fewer constructions of dS solutions than anti dS/Minkowski ones within string theory. It motivates to prepare a set of criteria that every low energy EFT (that induces positive cosmological constant) must fulfill in order to have UV completion in a quantum gravity [24–26]. Examples of these conditions, called the swampland conjectures, are the dS conjecture [27] (and a milder version [28,29]) and the distant conjecture [30,31]. There are no proofs yet but they have strong support from many examples in string theory (we refer the reader to a review [32] of the vast literature).

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The swampland conjectures have important consequences for inflation as the shape of potential and the field excursion is highly constrained (the literature is too extensive to be covered here, instead we refer to the pioneering studies [33–38]). Moreover, there are arguments that eternal inflation is not allowed [39–41] or is marginally allowed but not consistent with observation [42–46] as it cannot be followed by a normal inflation.

Recently, another swampland condition, the trans-Planckian censorship conjecture (TCC), was proposed in [47]. Quantum modes with wavelength smaller than the Planck length belong to the theory of quantum gravity and cannot be described by a low energy EFT below Planck scale. In an expanding background these modes are stretched and if they extend beyond the horizon then they freeze and classicalize. If these modes subsequently re-enter the horizon, they affect classical spacetime which is described by a low energy EFT like general relativity. The TCC proposes that an EFT never classicalizes sub-Planckian modes otherwise belong to the swampland. Immediate implications follow for the present status of the universe and during primordial inflation [48]. In particular it constrains the number of e-folds or equivalently the Hubble rate (at the end) of expansion as

$$e^N \equiv \frac{a_{\text{end}}}{a_{\text{ini}}} < \frac{m_{\text{Pl}}}{H_{\text{end,inf}}}, \quad (1)$$

where a is the scale factor and $m_{\text{Pl}} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass. On the other hand, a minimum number of e-folds are needed to explain the present horizon. If we assume a standard scenario with a single inflation, followed by an epoch with EoS parameter w and Hubble parameter H_1 at the end of that epoch, then we find

$$H_{\inf}^{2-2/3(1+w)} < m_{\text{Pl}} H_0 T_0^{-1} T_{\text{rad}} H_1^{-2/3(1+w)}. \quad (2)$$

The temperature at the onset of radiation domination T_{rad} is greater than about 10 MeV to initiate nucleosynthesis. For radiation and matter domination following inflation we find $H_{\text{inf}} < 0.1 \text{ GeV}$ and $H_{\text{inf}} < 50 \text{ GeV} \cdot (10 \text{ MeV}/T_{\text{rad}})^{1/4}$, respectively (see [49–67] for further elaborations and refinements). Low scale Hubble parameter implies huge amount of fine-tuning in initial condition of inflation. The smooth patch at the onset of inflation includes 10^{17} or 10^{30} similar parts at the Planck time, for matter and radiation epochs respectively. Moreover, it implies no primordial gravitational waves will be detected in future observations.

Strong pieces of evidence imply that the universe has experienced at two stages of inflation. It is possible that the universe has followed many stages of inflation. In [56] we put forward the idea that the upper bound (2) can be relaxed multiple inflation happens

$$H_{\inf}^{2-2/3(1+w)} < m_{\text{Pl}} H_0 T_0^{-1} T_{\text{rad}} H_1^{-2/3(1+w)} \times e^{\sum_i N_i} \prod_i \left(\frac{H_i}{H_{i+1}} \right)^{2/3(1+w_i)}, \quad (3)$$

where w_i is the EoS parameters of intervening epoch i with final expansion rate H_i . The observable inflation has imprints on the CMB and the LSS observables and the following ones assist to explain the present horizon (multiple inflation was previously studied by different motivations [68–73]). Moreover, it is possible that there are epochs of inflations preceding the observable one that prepare the smooth patch and alleviate the initial condition for it to happen. Additionally, eternal inflation is permitted as the follow-up slow-roll inflations are made consistent with observations.

The initial condition and the eternity are the most debated subjects in the inflationary paradigm. In this note, we study these

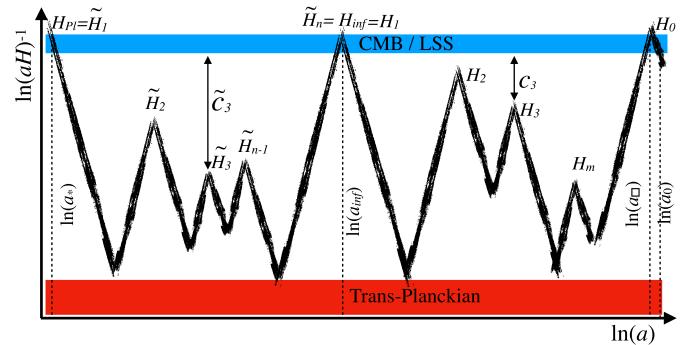


Fig. 1. A generic scenario with breathing comoving Hubble horizon. The blue band is the observational window and the red floor is provided by the TCC.

issues in the light of the TCC in the framework of multiple inflation. A generic scenario is illustrated in Fig. 1 and will be discussed in the following sections. The paper is structured as follows. In section 2 we see that a high scale observable inflation is consistent with the TCC. In section 3 we solve the initial condition problem for the observable inflation through introducing pre-inflations. In section 4 we study power-law inflations precede and follow the plateau-like observable inflation. Finally, we conclude in section 5.

2. Observable inflation at high-scale

In this section we study the case of high-scale observable inflation. First we compute the total number of e-folds needed to explain the horizon. Then, applying the TCC, we compute the Hubble rate for each inflationary epoch. In the end, we present a simple scenario.

2.1. The present horizon with multiple inflation

To address the horizon problem, the present comoving Hubble horizon be inside the comoving horizon the onset of observable inflation. If numerous inflations with scale H_i happen then we find

$$\begin{aligned}
1 &\geq \frac{a_{\text{ini},1} H_{\text{inf}}}{a_0 H_0} = \frac{a_{\text{ini},1}}{a_{\text{end},1}} \frac{a_{\text{end},1}}{a_{\text{ini},2}} \dots \frac{a_{\text{ini},m-1}}{a_{\text{end},m}} \frac{a_{\text{end},m}}{a_{\text{rad}}} \frac{a_{\text{rad}}}{a_0} \frac{H_{\text{inf}}}{H_0} \\
&= e^{-\sum_{i=1}^m N_i} \frac{T_0}{T_{\text{rad}}} \frac{H_{\text{inf}}}{H_0} \frac{a_{\text{end},m}}{a_{\text{rad}}} \\
&= e^{-\sum_{i=1}^m N_i} \frac{T_0}{T_{\text{rad}}} \frac{H_{\text{inf}}}{H_0} \left(\frac{H_{\text{rad}}}{H_m} \right)^{\frac{2}{3(1+w_m)}} \prod_{i=1}^{m-1} \left(\frac{H_{i+1}}{H_i} \right)^{\frac{2}{3(1+w_i)}}, \tag{4}
\end{aligned}$$

where index 1 indicates the observable inflation ($H_{\text{inf}} = H_1$) and m is the last one. In the third line we assume that a single stage with EoS parameter w_i intervene between two inflations. For simplicity, we assume $w_i = w$ then the total number of e-folds $N_{\text{tot}} = \sum_{i=1}^m N_i$ satisfies

$$N_{\text{tot}} \geq \ln \left(\frac{T_0}{H_0} \right) + \frac{1}{3(1+w)} \ln \left(T_{\text{rad}}^{1-3w} H_{\text{inf}}^{1+3w} m_{\text{Pl}}^{-2} \right). \quad (5)$$

2.2. The upper bound on the Hubble parameters

We parametrize the ratio of the comoving horizon at the onset if each inflation to the present horizon as

$$c_i = \frac{a_{\text{ini},i} H_i}{a_0 H_0}. \quad (6)$$

The first inflation is assumed to be the observable one. It generates the CMB anisotropies and the gravitational waves. Thus, $c_1 \leq 1$. On

the other hand, the succeeding inflations must not jeopardize the success of CMB observations which are in the range

$$10^{-3} \lesssim \frac{k}{a_0 H_0} \lesssim 1. \quad (7)$$

Therefore, $c_i \geq 10^3$ for $i = 2, \dots, m$. Moreover, if we happen to have a homogeneous patch before the onset of the first inflation, as the comoving Hubble volume never surpasses the initial one, the following inflations will follow without any further tuning.

For the final (m th) stage of inflation we find that

$$c_m = \frac{a_{\text{ini},m} H_m}{a_0 H_0} = \frac{a_{\text{ini},m}}{a_{\text{end},m}} \frac{a_{\text{end},m}}{a_{\text{rad}}} \frac{a_{\text{rad}}}{a_0} \frac{H_m}{H_0} \\ = e^{-N_m} \left(\frac{H_{\text{rad}}}{H_m} \right)^{\frac{2}{3(1+w_m)}} \frac{T_0}{T_{\text{rad}}} \frac{H_m}{H_0}, \quad (8)$$

where in second line we assumed a single stage epoch. Applying the TCC inequality, we find an upper bound on the Hubble rate H_m as

$$\left(\frac{H_m}{m_{\text{Pl}}} \right)^{2+\frac{2}{1+3w_{m-1}}} < (c_m H_0 T_0^{-1})^{\frac{3(1+w_m)}{1+3w_m}} \left(\frac{m_{\text{Pl}}}{T_{\text{rad}}} \right)^{\frac{1-3w_m}{1+3w_m}}. \quad (9)$$

For the preceding inflations for $i = m-1, m-2, \dots, 2, 1$, we find

$$c_i = \frac{a_{\text{ini},i} H_i}{a_0 H_0} = c_{i+1} \frac{a_{\text{ini},i} H_i}{a_{\text{ini},i+1} H_{i+1}} \quad (10) \\ = c_{i+1} \frac{a_{\text{ini},i}}{a_{\text{end},i}} \frac{a_{\text{end},i}}{a_{\text{ini},i+1}} \frac{H_i}{H_{i+1}} = c_{i+1} e^{-N_i} \left(\frac{H_i}{H_{i+1}} \right)^{\frac{1+3w_i}{3(1+w_i)}}.$$

Similarly applying the TCC, we find an upper on H_i

$$\left(\frac{H_i}{m_{\text{Pl}}} \right)^{2+\frac{2}{1+3w_i}} < \left[\frac{c_i}{c_{i+1}} \right]^{\frac{3(1+w_i)}{1+3w_i}} \frac{H_{i+1}}{m_{\text{Pl}}}. \quad (11)$$

In particular, in the large m limit, we find the upper bound on the observable inflation as

$$H_1 < m_{\text{Pl}} \prod_{i=1}^m \left[\frac{c_i}{c_{i+1}} \right]^{\frac{3(1+w_i)}{4+6w_i}} \left[\frac{1+3w_i}{4+6w_i} \right]^{i-1}, \quad (12)$$

where we define the parameter $c_{m+1} = 1$. To see the importance of the above bound, we consider a special case with $c_i = 10^3$ for $i = 2, \dots, m$. Assuming simple post-inflationary evolutions $w_i = w$ we find

$$H_1 < m_{\text{Pl}} (10^{-3} c_1)^{1-\left[\frac{1+3w}{4+6w}\right]^{m-1}} \quad (13)$$

2.3. A simple scenario with high-scale inflation

If we take $c_1 \approx 1$ and assume matter-domination in between epochs of inflation $w = 0$, then

$$H_1 < 10^{-3(1-4^{1-m})} m_{\text{Pl}} \rightarrow 10^{15} \text{ GeV}, \quad (14)$$

and (5) implies we need at least 48.8 e-folds. Applying (9) and (11) repeatedly we find

$$H_m < 10^{9/4} H_0^{3/4} T_0^{-3/4} T_{\text{rad}}^{-1/4} m_{\text{Pl}}^{5/4} \\ H_{m-1} < m_{\text{Pl}}^{3/4} H_m^{1/4} \\ \vdots \\ H_2 < m_{\text{Pl}}^{3/4} H_1^{1/4} \\ H_1 < m_{\text{Pl}}^{3/4} H_2^{1/4} 10^{-9/4}. \quad (15)$$

Then, the Hubble rate of the final inflation is

$$H_m < 10 \text{ TeV}, \quad (16)$$

(for $T_{\text{rad}} = 10$ MeV) and the number of e-folds is $N_m < 33.1$. Note that we would get the standard result of single inflation ($c_m = 1$) so that $H_m < 50$ GeV and $N_m < 38.4$.

The Hubble rate for the preceding one is

$$H_{m-1} < 6.1 \times 10^{13} \text{ GeV}, \quad (17)$$

and the number of e-folds is bounded as $N_{m-1} < 10.6$. So far, we get $N < 33.1 + 10.6 = 43.7$ e-folds. Note that, if this stage were the observable inflationary stage ($c_{m-1} = 1$), then we would have

$$H_{m-1} < 3.4 \times 10^{11} \text{ GeV}, \quad (18)$$

and the number of e-folds would be bounded as $N_{m-1} < 15.8$. Thus the total number of e-folds is bounded by $33.1 + 15.8 = 48.9$.

For the previous inflation we find

$$H_{m-2} < 1.7 \times 10^{17} \text{ GeV}, \quad (19)$$

and the number of e-folds $N_{m-2} < 2.6$. It violates the bound from (14). Thus, it must be the observable (the first) inflation ($c_{m-2} = c_1 = 10^3$) with

$$H_{m-2} < 9.6 \times 10^{14} \text{ GeV}, \quad (20)$$

and the number of e-folds $N_{m-2} = N_1 < 7.8$. The total number of e-folds $N < 33.1 + 10.6 + 7.8 = 51.5$ e-folds. This stage of inflation lasts for more than 7 e-folds and leave imprints on the CMB for scales spanning 3 orders of magnitude. Moreover, if the primordial gravitational waves are discovered in future measurements, then it can be accommodated in multiple-stage inflation.

3. Initial condition for the observable inflation

The scale of the observable inflation is at least three orders of magnitude below the Planck scale. If we define L_{ini} as the physical size of the smooth patch at the onset of inflation which is of order H_{inf} , and L_{Pl} the corresponding length scale at the Planck time, then

$$\frac{L_{\text{ini}} H_{\text{inf}}}{L_{\text{Pl}} H_{\text{Pl}}} = \left(\frac{H_{\text{inf}}}{H_{\text{Pl}}} \right)^{\frac{1+3\tilde{w}}{3(1+\tilde{w})}}. \quad (21)$$

In the above, \tilde{w} is the EoS parameter of the fluid which takes over the Universe's dynamics when it has emerged from the Planck era until the dawn of observable inflation. If we take $H_{\text{inf}} \sim 10^{14}$ GeV and $H_{\text{Pl}} \lesssim m_{\text{Pl}}$ then (unless $w = -1/3$) the above equation implies that the physical volume of the Universe at the Planck time encompasses at least 10^4 (for $w = 0$ and 10^6 for $w = 1/3$) Hubble spheres with radius around Planck length. This is the initial condition for the observable inflation which must be granted if it is supposed to solve the initial condition problem of hot big bang model.

This problem can be alleviated via the very mechanism that solves the horizon problem of big bang model. After the Planck era, the universe follows an inflationary epoch with Hubble parameter close to the Planck scale. However, this inflation is not observable as we cannot observe modes that leave the horizon during this period. Besides, this cannot be prolonged as constrained by the TCC. In order to have a lower scale observable inflation with initial conditions set naturally at the Planck time without any fine-tuning we expect

$$1 \leq \frac{L_{\text{ini}} H_{\text{inf}}}{L_{\text{Pl}} H_{\text{Pl}}} = \frac{a_{\text{ini}} H_{\text{inf}}}{a_{\text{Pl}} H_{\text{Pl}}}. \quad (22)$$

Several epochs of inflation can happen in between the Planck scale one and the observable one. With a more convenient notation $H_{\text{Pl}} = \tilde{H}_1$ and $a_{\text{Pl}} = a_{\text{ini},1}$, for n stages of inflation we find

$$1 \geq \frac{a_{\text{ini},1}\tilde{H}_1}{a_{\text{ini}}H_{\text{inf}}} = e^{-\sum_{i=1}^n \tilde{N}_i} \frac{\tilde{H}_1}{H_{\text{inf}}} \prod_{i=1}^{n-1} \frac{a_{\text{end},i}}{a_{\text{ini},i+1}} \frac{a_{\text{end},n}}{a_{\text{ini}}}. \quad (23)$$

Assuming a simple inter-inflationary epoch we find

$$e^{\sum_{i=1}^n \tilde{N}_i} \geq \frac{\tilde{H}_1}{H_{\text{inf}}} \left(\frac{H_{\text{inf}}}{\tilde{H}_n} \right)^{\frac{2}{3(1+\tilde{w}_n)}} \prod_{i=1}^{n-1} \left(\frac{\tilde{H}_{i+1}}{\tilde{H}_i} \right)^{\frac{2}{3(1+\tilde{w}_i)}} \rightarrow \left(\frac{\tilde{H}_1}{H_{\text{inf}}} \right)^{\frac{1+3\tilde{w}}{3(1+\tilde{w})}}, \quad (24)$$

where in the last line we assumed $w_i = w$. Therefore,

$$e^{\sum_{i=1}^n \tilde{N}_i} \gtrsim 21.5 \text{ (for } w=0 \text{) (or 100 for } w=1/3\text{)}, \quad (25)$$

and thus $\tilde{N}_{\text{tot}} \gtrsim 3$ for matter-domination in intermediate stages (we would find around 4.6 e-folds for radiation domination).

To find the Hubble rates we parametrize the ratio of comoving horizon at the onset of each inflation ($i = 1, 2, \dots, n$) to the Planck horizon as

$$\frac{a_{\text{ini},i}\tilde{H}_i}{a_{\text{ini},1}\tilde{H}_1} = \tilde{c}_i \geq 1. \quad (26)$$

Then,

$$\begin{aligned} \tilde{c}_{i+1} &= \frac{a_{\text{ini},i+1}\tilde{H}_i}{a_{\text{ini},1}\tilde{H}_1} = c_i \frac{a_{\text{ini},i+1}\tilde{H}_{i+1}}{a_{\text{ini},i}\tilde{H}_i} \\ &= c_i e^{N_i} \left(\frac{\tilde{H}_{i+1}}{\tilde{H}_i} \right)^{\frac{1+3\tilde{w}_i}{3(1+\tilde{w}_i)}}. \end{aligned} \quad (27)$$

Applying the TCC, we find a lower bound on the Hubble scale of the succeeding inflation as

$$\tilde{H}_{i+1} > \left[\frac{\tilde{c}_{i+1}}{\tilde{c}_i} \right]^{\frac{3(1+\tilde{w}_i)}{1+3\tilde{w}_i}} m_{\text{Pl}} \left(\frac{\tilde{H}_i}{m_{\text{Pl}}} \right)^{2+\frac{2}{1+3\tilde{w}_i}}. \quad (28)$$

If we assume the same EoS parameter for all inter-inflationary stages and parametrize $\tilde{H}_1 = \alpha^{-1} m_{\text{Pl}}$ ($\alpha > 1$) then we find the scale of succeeding inflation as follows

$$\tilde{H}_i > \tilde{c}_i \alpha^{-\left[\frac{4+6\tilde{w}}{1+3w}\right]^{i-1}} m_{\text{Pl}}, \quad (29)$$

where

$$\tilde{c}_i = \prod_{j=2}^i \left[\left(\frac{\tilde{c}_j}{\tilde{c}_{j-1}} \right)^{\frac{3(1+\tilde{w})}{1+3\tilde{w}}} \right]^{\frac{4+6\tilde{w}}{1+3\tilde{w}}} \quad (30)$$

The number of e-folds for each stage of inflation is bounded by

$$\tilde{N}_i < \left[\frac{4+6\tilde{w}}{1+3\tilde{w}} \right]^{i-1} \ln \alpha - \ln c \tilde{c}_i, \quad (31)$$

for some arbitrary constant $c > 1$ which turns the inequality in (29) into equality. The total number of e-folds, including the first one, is

$$\tilde{N} < \frac{\left(\frac{4+6w}{1+3w} \right)^n - 1}{1 + \frac{2-2w}{1+3w}} \ln \alpha - \ln c \tilde{c}, \quad (32)$$

where $\ln \tilde{c} = \sum_{i=2}^n \sum_{j=2}^i \frac{(3+3\tilde{w})(4+6w)^{j-2}}{(1+3w)^{j-1}} \ln \frac{\tilde{c}_j}{\tilde{c}_{j-1}}$.

3.1. A simple sequence of pre-inflations

For matter domination $\tilde{w} = 0$ and a simple model with $\tilde{c}_i \approx 1$, assuming $\alpha = \sqrt{3}$ we find $\tilde{H}_1 \approx 0.5 m_{\text{Pl}}$, $\tilde{H}_2 > 0.1 m_{\text{Pl}}$ and $\tilde{H}_3 > 10^{-4} m_{\text{Pl}}$ which touches the *Planck 2018* bound. Consequently, the TCC implies $\tilde{N}_1 < 0.5$, $\tilde{N}_2 < 2.2$, $\tilde{N}_3 < 8.8$. Therefore, in order to set the initial conditions of the observable inflation, there need to be at least 3 epochs of inflation which in total provide around 3 e-folds. For radiation domination $\tilde{w} = 1/3$ we find $\tilde{H}_2 > 0.2 m_{\text{Pl}}$ ($\tilde{N}_2 < 1.6$), $\tilde{H}_3 > 0.007 m_{\text{Pl}}$ ($\tilde{N}_3 < 4.9$). Although three inflations satisfy the lower bound (25), in order to satisfy the observational bound from non-detection of gravitational waves, we need at least one more inflation before the observable one.

4. Power-law assisting epochs of inflation

The latest results from the *Planck* satellite favors plateau-like (observable) inflation. However, as the other stages of inflations have no footprint on the CMB, they could be power-law in nature. In this section, we examine this scenario for early inflations and the late ones where the Universe is dominated by a fluid with EoS parameter $\hat{w} < -1/3$. In this case, the Hubble rate at the beginning of inflation H_{ini} is not equal to the rate at the end of inflation H_{end} . Therefore, using the dynamical equations in an expanding background, the TCC reads as follows

$$e^{N_i} \equiv \frac{a_{\text{end},i}}{a_{\text{ini},i}} < \frac{m_{\text{Pl}}}{H_{\text{end},i}} = \left(\frac{m_{\text{Pl}}}{H_{\text{ini},i}} \right)^{-\frac{2}{1+3\hat{w}_i}}. \quad (33)$$

The results of this section match the results of the previous section for $\hat{w} = -1$. It is convenient to define the following parameters

$$\gamma_i = 1 - \frac{1 + \hat{w}_i}{1 + w_i}, \quad (34)$$

$$p_i = 1 - \frac{2}{1 + 3\hat{w}_i} + \frac{2}{1 + 3w_i}, \quad (35)$$

where $1 \leq \gamma < 0$ and $p > 2$ and it can be large in the limit $\hat{w} \rightarrow -1/3$.

4.1. Inflation in early times

With power-law inflation, the minimum number of e-folds to set the initial smooth path of the observable inflation is computed as

$$\begin{aligned} e^{\sum_{i=1}^n N_i \gamma_i} &\geq \frac{\tilde{H}_{\text{ini},1}}{H_{\text{inf}}} \left(\frac{H_{\text{inf}}}{\tilde{H}_{\text{ini},n}} \right)^{\frac{2}{3(1+\tilde{w}_n)}} \\ &\times \prod_{i=1}^{n-1} \left(\frac{\tilde{H}_{\text{ini},i+1}}{\tilde{H}_{\text{ini},i}} \right)^{\frac{2}{3(1+\tilde{w}_i)}}, \end{aligned} \quad (36)$$

that can be compared to (24). For $\tilde{w}_i = w$ we find

$$N_{\text{tot}} \geq \gamma^{-1} \frac{1 + 3\tilde{w}}{3(1 + \tilde{w})} \ln \left(\frac{\tilde{H}_{\text{ini},1}}{H_{\text{inf}}} \right). \quad (37)$$

For instance, for $\hat{w} = -2/3$, in matter domination we obtain $N_{\text{tot}} > 4.6$ and in radiation domination we find $N_{\text{tot}} > 6.9$. A stronger lower bound is obtained in the limit $\hat{w} \rightarrow -1/3$ from below and $\tilde{w} \rightarrow -1/3$ from above.

To find the Hubble rates of power-law inflations, we parametrize the ratio of comoving Hubble horizons as

$$\begin{aligned} \tilde{c}_{i+1} &= \frac{a_{\text{ini},i+1} \tilde{H}_{\text{ini},i+1}}{a_{\text{ini},1} \tilde{H}_{\text{ini},1}} = c_i \frac{a_{\text{ini},i+1} \tilde{H}_{\text{ini},i+1}}{a_{\text{ini},i} \tilde{H}_{\text{ini},i}} \\ &= c_i e^{\gamma_i N_i} \left(\frac{\tilde{H}_{\text{ini},i+1}}{\tilde{H}_{\text{ini},i}} \right)^{\frac{1+3\hat{w}_i}{3(1+\hat{w}_i)}}. \end{aligned} \quad (38)$$

Therefore, using the TCC relation (33) we find

$$\tilde{H}_{\text{ini},i+1} > \left[\frac{\tilde{c}_{i+1}}{c_i} \right]^{\frac{3(1+\hat{w}_i)}{1+3\hat{w}_i}} m_{\text{Pl}} \left(\frac{\tilde{H}_{\text{ini},i}}{m_{\text{Pl}}} \right)^{p_i}. \quad (39)$$

We note that a single epoch of power-law inflation close to the Planck scale, with EoS \hat{w} can prepare the smooth patch of the observable inflation with Hubble rate 4 orders of magnitude less than the Planck scale. We parametrize $\tilde{H}_{\text{ini},i} = \alpha^{-1} m_{\text{Pl}}$ and assume $w = 0$ then

$$\hat{\omega} = -\frac{1}{3} - \frac{2 \ln \alpha}{3[\ln(m_{\text{Pl}} H_{\text{inf}}^{-1}) - 3 \ln \alpha]}, \quad (40)$$

and the number of e-folds is $N < \ln(m_{\text{Pl}} H_{\text{inf}}^{-1}) - 3 \ln \alpha$. For $\alpha = \sqrt{3}$ and $H_{\text{inf}} \sim 10^{14}$ GeV we find $N < 7.6$ (it never becomes more than 9.2) $\hat{w} = -1/3 - 2 \log \sqrt{3} / 3(4 - 3 \log \sqrt{3})$.

4.2. Late times

To accommodate the present horizon with plateau-like observable inflation followed by multiple power-law ones we find a lower bound on the number of e-folds

$$\begin{aligned} 1 &\geq e^{-\sum_{i=1}^m N_i \gamma_i} \frac{T_0}{T_{\text{rad}}} \frac{H_{\text{inf}}}{H_0} \left(\frac{H_{\text{rad}}}{H_{\text{ini},m}} \right)^{\frac{2}{3(1+w_m)}} \\ &\times \prod_{i=1}^{m-1} \left(\frac{H_{\text{ini},i+1}}{H_{\text{ini},i}} \right)^{\frac{2}{3(1+w_i)}}. \end{aligned} \quad (41)$$

For the simple case $w_i = w$ we find the total number of e-folds as $N_{\text{tot}} = N_1 + \gamma \sum_{i=2}^m N_i$. It must obey (24).

For the final (m th) stage of inflation we find

$$\begin{aligned} c_m &= \frac{a_{\text{ini},m} H_{\text{ini},m}}{a_0 H_0} = \frac{a_{\text{ini},m}}{a_{\text{end},m}} \frac{a_{\text{end},m}}{a_{\text{rad}}} \frac{a_{\text{rad}}}{a_0} \frac{H_{\text{ini},m}}{H_0} \\ &= e^{-\gamma_m N_m} \frac{T_0}{H_0} \left(\frac{H_{\text{ini},m}}{m_{\text{Pl}}} \right)^{\frac{1+3w_m}{3(1+w_m)}} \left(\frac{T_{\text{rad}}}{m_{\text{Pl}}} \right)^{\frac{1-3w_m}{3(1+w_m)}}, \end{aligned} \quad (42)$$

which after applying the TCC (33) implies

$$\left(\frac{H_{\text{ini},m}}{m_{\text{Pl}}} \right)^{p_m} < \left(c_m H_0 T_0^{-1} \right)^{\frac{3(1+w_m)}{1+3w_m}} \left(\frac{m_{\text{Pl}}}{T_{\text{rad}}} \right)^{\frac{1-3w_m}{1+3w_m}}. \quad (43)$$

The Hubble rate for preceding inflations $i = m-1, m-2, \dots, 2, 1$ are bounded by

$$\left(\frac{H_{\text{ini},i}}{m_{\text{Pl}}} \right)^{p_i} < \left[\frac{c_i}{c_{i+1}} \right]^{\frac{3(1+w_i)}{1+3w_i}} \left(\frac{H_{\text{ini},i+1}}{m_{\text{Pl}}} \right). \quad (44)$$

We recall that for the first stage $p_1 = 2 + \frac{2}{1+3w_1}$.

We can model a simple two-stage inflation to explain the horizon. The observable one with EoS parameter $\hat{w} = -1$ is followed by a power-law inflation with EoS parameter determined by (44) as

$$\hat{w} = -\frac{1}{3} - \frac{2}{3 \left[\frac{3 \ln(H_0 T_0^{-1}) + \ln(m_{\text{Pl}} T_{\text{rad}}^{-1})}{-4 \ln(m_{\text{Pl}} H_{\text{inf}}^{-1}) + 3 \ln c_2} - 3 \right]}. \quad (45)$$

The number of e-folds is bounded as

$$\begin{aligned} N_2 &< 12 \ln(m_{\text{Pl}} H_{\text{inf}}^{-1}) - 9 \ln c_2 \\ &- 3 \ln(H_0 T_0^{-1}) - \ln(m_{\text{Pl}} T_{\text{rad}}^{-1}). \end{aligned} \quad (46)$$

For $H_1 = H_{\text{inf}} \sim 10^{14}$ GeV we find that $\hat{w} = -1/3 - 7/69$. The Hubble rate H_2 is around (less than) 10^{11} GeV. The number of e-folds is $N_2 < 105.8$. Given that $N_1 < 9.2$, the total number of e-folds is bounded as $N_{\text{tot}} < 115$.

We find that a simple three stage inflation, starting from the Planck era, can happen naturally and explain the present horizon without any need of fine-tuning in initial conditions.

5. Conclusion

In this paper we revisit the framework of multiple inflationary scenario in light of the TCC. We found that high-scale inflations in the early times after the Planck era can prepare the initial conditions for the observable inflation. The observable inflation, can be of any scale and touch the *Planck 2018* bound. If it is of high scale, then by the TCC, it cannot be prolonged and thus cannot solve the initial condition problem of the big bang model. However, we found that the succeeding inflations can explain the present horizon. All inflationary stages admit to the TCC. We also studied scenarios with a plateau-like observable inflation and supplementary power-law inflations. We found that in this scenario less number of inflations are needed provided that the EoS parameter during inflation is close to the critical value for the accelerated expansion. Eternal inflation is allowed in the framework of multi-inflation. We did not present any particular model and we describe the dynamics in terms of the effective EoS parameter of each epoch.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

This work is supported by the research deputy of SUT.

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