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STUDY OF THE GENERALIZED ELECTRON EMISSION THEORY IN A SUPERCONDUCTING CAVITY*

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Abstract

Research is being conducted on field emission, thermionic emission, generalized electron emission, and electron emission from superconducting cavities. Generalized electron emission theories, which encompass field emission and thermionic emission, are currently under investigation. In field emission, electrons are emitted from metals due to a strong local electric field, while in thermionic emission, electrons are emitted due to high local temperatures. Field emission is being explored in relation to dimensions, and thermionic emission is likewise examined as a function of dimensions. The distribution of the electric field is illustrated over surface curvature. Furthermore, field emission characteristics are specifically analyzed within the context of superconducting RF cavities.

INTRODUCTION

Electron emissions from superconducting cavities can degrade the quality of superconductors in accelerator physics. Field emission phenomena have been studied and applied to superconducting cavities [1, 2]. Field emission has been investigated as a function of dimension [3], while thermionic emission has been explored in terms of dimension [4]. A unified theory for electron emission encompassing both field emission and thermionic emission has been proposed [5, 6]. Surface charge density has been studied in relation to surface curvature [7]. This research primarily reviews field emission in terms of dimension and thermionic emission as a function of dimension. Generalized electron emission is discussed, incorporating both field emission and thermionic emission. The electric field is examined with respect to conducting surface curvature. Field emission from a superconducting cavity is illustrated on the RF surface within the cavity.

FIELD EMISSION

Generalized field emission has been studied, and the generalized current density can be expressed as a function of dimension [3]:

$$J_{nD} = \frac{(2\pi)^{(n-5)/2} e m^{(n-3)/2} E_F{}^{(n-2)/2} F^2}{h^{(n-2)}(E_F+\Phi_W)\sqrt{\Phi_W}} e^{-4k\Phi_W^{3/2}/3F} \quad (1)$$

where J_{nD} is the current density for the electron emission, n represents the dimensionality of the system, Φ_W is the work function of the emitting material, E_F is the Fermi energy of the material, F is the electric field, e is the elementary charge, h is Planck's constant and m is the electron mass.

From Eq. (1), the one-dimensional current density for field emission is

$$J_{1D} = \frac{ehF^2}{4\pi^2 m(E_F+\Phi_W)\sqrt{E_F\Phi_W}} e^{-4k\Phi_W^{3/2}/3F} \quad (2)$$

From Eq. (1), the two-dimensional current density for field emission is

$$J_{2D} = \frac{eF^2}{2\sqrt{2}\pi^{3/2}\sqrt{m}(E_F+\Phi_W)\sqrt{\Phi_W}} e^{-4k\Phi_W^{3/2}/3F} \quad (3)$$

From Eq. (1), the three-dimensional current density for field emission is

$$J_{3D} = \frac{e\sqrt{E_F}F^2}{2\pi h(\Phi_W+E_F)\sqrt{\Phi_W}} e^{-4k\Phi_W^{3/2}/3F} \quad (4)$$

Field emission depends on dimensions. The field emission current density in dimensions is proportional to fundamental physical constants, which depend on dimension. The field emission current density increases with dimension. The field emission effect on electric field and work function remains consistent across all dimensions.

THERMIONIC EMISSION

Generalized thermionic emission has been investigated, and the generalized current density of thermionic emission is given by [4]:

$$J_{nD} = 2e(2\pi m)^{(n-1)/2} \frac{(k_BT)^{(n+1)/2}}{h^n} e^{-\frac{\Phi_W}{k_BT}} \quad (5)$$

where T is the absolute temperature and k_B is the Boltzmann constant.

From Eq. (5), the one-dimensional current density for thermionic emission is

$$J_{1D} = \frac{2ek_BT}{h} e^{-\frac{\Phi_W}{k_BT}} \quad (6)$$

From Eq. (5), the two-dimensional current density for thermionic emission is

$$J_{2D} = 2\sqrt{2\pi}e(m)^{1/2} \frac{(k_BT)^{3/2}}{h^2} e^{-\frac{\Phi_W}{k_BT}} \quad (7)$$

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From Eq. (5), the three-dimensional current density for thermionic emission is

$$J_{3D} = \frac{4\pi emk_B^2T^2}{h^3} e^{-\frac{\Phi_W}{k_BT}} \quad (8)$$

Thermionic emission is dimension-dependent. The current density for thermionic emission varies according to the power of temperature. Specifically, the one-dimensional current density of thermionic emission is directly influenced by temperature, while the three-dimensional current density is proportional to the square of the temperature.

UNIFIED ELECTRON EMISSION

Field emission occurs at the zero-temperature limit, where electrons undergo quantum tunneling near the Fermi energy. On the other hand, thermionic emission is observed at high temperatures and zero electric field, where electrons with energies above the work function are emitted.

A unified electron emission theory has been developed for conducting materials under arbitrary electric field and temperature conditions, in one, two, and three dimensions [5, 6]. In this theory, field emission is possible for electron energy levels below the work function.

The unified current density in three dimensions is expressed as [6]:

$$J_{Unif} = \frac{4\pi emk_BT}{h^3} [\int_0^{E_F+\Phi_W} dE \frac{4\sqrt{E}\sqrt{E_F+\Phi_W-E}}{(E_F+\Phi_W)} ln(1 + exp(-\frac{E-E_F}{k_BT}))e^{-4k(E_F+\Phi_W-E)^{3/2}/3F} + k_BTe^{-\Phi_W/k_BT}]. \quad (9)$$

This unified current density incorporates both field emission and thermionic emission, accounting for arbitrary electric field and temperature conditions. It is important to note that the unified current density is not simply an addition of the individual field emission and thermionic emission currents; rather, it accounts for the modified field emission current as described in Eq. (9).

ELECTRIC FIELD FROM SURFACE CURVATURE

When two conducting spheres are connected with a wire, they reach the same potential energy. This potential energy can be expressed as:

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r_1} = \frac{1}{4\pi\varepsilon_o}\frac{q_2}{r_2} \quad (10)$$

where V is the potential energy, r_1 is the radius of sphere 1, r_2 is the radius of sphere 2, q_1 is the charge of sphere 1, and q_2 is the charge of sphere 2, and ε_o is the permittivity of free space.
From Eq. (10), we can find the relation:

$$\frac{q_1}{r_1} = \frac{q_2}{r_2} \quad (11)$$

Electric field becomes

$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r_1^2} = \frac{\sigma_1}{\varepsilon_o} \quad (12)$$

where E_1 is the electric field of sphere 1 and σ_1 is the surface charge density of sphere 1.
From Eq. (11) and Eq. (12), we can derive

$$\frac{E_1}{E_2} = \frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1} \quad (13)$$

where E_1 and E_2 are the electric fields of spheres 1 and 2, respectively, and σ_1 and σ_2 are their surface charge densities.
Curvature, denoted by κ, is defined as the rate of change of unit tangent vector with respect to length:

$$\kappa = \frac{dT_t}{ds} \quad (14)$$

where κ is the curvature, T_t is the tangent vector, and s is the length. Curvature measures how sharply a curve is turning as it is traversed, with a straight line having zero curvature and a circle having a curvature equal to the reciprocal of its radius as:

$$\kappa = \frac{1}{r} \quad (15)$$

The Laplace equation for a conductor is

$$\nabla^2V = \frac{\partial^2V}{\partial x^2} + \frac{\partial^2V}{\partial y^2} + \frac{\partial^2V}{\partial z^2} = (k_x + k_y)\frac{\partial V}{\partial n} + \frac{\partial^2V}{\partial n^2} = 0 \quad (16)$$

The average curvature is defined as

$$k = \frac{(k_x+k_y)}{2} \quad (17)$$

From Eq. (16) and Eq. (17), the relation between curvature and potential energy is

$$2k\frac{\partial V}{\partial n} + \frac{\partial^2V}{\partial n^2} = 0 \quad (18)$$

The electric field is given by

$$F = -\frac{\partial V}{\partial n} \quad (19)$$

From Eq. (18) and Eq. (19), the electric field can be expressed with the average curvature as

$$F = F_o\exp(-2\kappa n) \quad (20)$$

From Eq. (19) and Eq. (20), the potential energy is approximately:

$$\Delta V \approx \int_{\Delta n}^{0} Fdn = \frac{F_o}{2k}[\exp(-2k\Delta n) - 1] \quad (21)$$

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where n is the normal direction on the surface.

From Eq. (21), the electric field on the surface can be expressed as [7]

$$F_o = \lim_{\Delta n \to 0} \left(\frac{2k\Delta V}{\exp(-2k\Delta n)-1}\right) \quad (22)$$

From Eq. (22), we can find the following relations:

$$\sigma = \varepsilon F_o = \lim_{\Delta n \to 0} \left(\frac{2\varepsilon k\Delta V}{\exp(-2k\Delta n)-1}\right) = -\varepsilon \frac{dV}{dn} \quad (23)$$

Indeed, local electric field and surface charge density can be determined from surface curvature. The relationship between curvature and electric field, as well as between curvature and surface charge density, allows for the derivation of these quantities from the geometry of the surface.

ELECTRON EMISSION FROM CAVITY RF SURFACE

Superconducting cavities exhibit magnetic defects in terms of magnetic field variations and are susceptible to field emission effects caused by electric field fluctuations. To ensure the production of high-quality superconducting cavities, meticulous processes such as careful electron beam welding (EBW), BCP, and HPR are employed [8]. The highest-quality superconducting cavities are devoid of magnetic defects and exhibit minimal field emission, as indicated by their high quality factor.

Field emission can originate from particles on the cavity surface and surface irregularities, with emission rates escalating due to the accumulation of various gases on the RF surface. When considering DC electric fields, Eq. (4) can be applied. In the context of accelerating ion beams, RF power is applied to superconducting cavities. Consequently, the field emission current for AC electric fields is described by [2]:

$$I = \frac{Ae\sqrt{E_F}F^2\beta^2 sin^2\omega t}{2\pi h(\Phi_W+E_F)\sqrt{\Phi_W}} e^{-4k\Phi_W^{1.5}/3F\beta sin\omega t} \quad (24)$$

where β is the field enhancement factor, A is the area, and ω is the angular frequency of the time-dependent electric field. Eq. (24) describes how the field emission current varies with time due to the oscillating electric field and the properties of the emitting surface.

The average current is defined as

$$\langle I \rangle = \frac{1}{T}\int_0^T I(t)dt \quad (25)$$

From Eq. (24) and Eq. (25), the average current is approximately derived as

$$\langle I \rangle = \frac{0.4Ae\sqrt{E_F}F^{2.5}\beta^{2.5}}{h\sqrt{k}(\Phi_W+E_F)\Phi_W^{0.75}} e^{-4k\Phi_W^{1.5}/3F\beta} \quad (26)$$

The field enhancement factor arises from surface curvature. Upon the occurrence of field emission, free electrons in the cavity are accelerated by acquiring RF energy, leading to X-ray generation. Consequently, RF power consumption increases significantly alongside field emission and X-ray production.

For future research, consideration of imaginary charge for field emission is necessary. To establish a generalized electron emission theory for superconducting cavities, the thermionic emission effect should be taken into account for AC scenarios. Additionally, the relationship between RF power consumption and X-ray generation, as well as the correlation between field emission and X-ray production, warrants further investigation.

CONCLUSION

We have investigated field emission, thermionic emission, generalized electron emission, and electron emission from superconducting cavities. Both field emission and thermionic emission are characterized in terms of dimensions, elucidating their behavior in various spatial contexts. The generalized electron emission phenomenon is illustrated, showcasing its comprehensive nature. The distribution of the electric field is depicted over surface curvature. Specific analyses of field emission characteristics are presented within the context of superconducting RF cavities. The ongoing development of generalized electron emission within RF cavities is currently underway, indicating a continuous effort to advance our understanding and application of electron emission phenomena in this field.

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