Research Methods of Hawking Radiation

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Abstract. Hawking radiation is the product of the combination of quantum field theory and general relativity, and is an important link in the study of quantum gravity. In this paper, Hawking radiation has been briefly introduced, and at the same time, several common calculation methods in the research of Hawking radiation are summarized. Readers can have a certain understanding of Hawking radiation research through this paper and can complete a simple introduction to Hawking radiation based on the introduction of calculation methods.

1.Introduction

Blackhole is an important research frontier in modern astrophysics and cosmology. Schwartz gave a vacuum solution to Einstein's equation in 1916 [1]. This solution predicted the existence of an astronomical object in the universe. This astronomical object has a "horizon" within which even light cannot escape. Wheeler named this astronomical object a black hole [2]. In the following years, black holes, as an important object in modern physics research, have been attracting people to explore, among which Hawking radiation is one of them.

Hawking radiation is a theory put forward by British physicist Hawking in 1974 [3]. This theory explains that black holes are not all black but can radiate particles from their horizon. Hawking radiation is the product of the combination of quantum field theory and general relativity, and is also an important link in the study of quantum gravity. In the following 50 years, Hawking radiation research began to develop fast. In addition, Hawking radiation skillfully combines quantum theory, gravity theory and thermodynamics together, so researchers expect to understand the dynamic behavior of quantum gravity theory through the study of Hawking radiation of black holes. In the development of Hawking radiation research, due to the need for different background space-time and different particle research, some calculation methods have emerged, which provide a great convenience for people to carry out Hawking radiation research.

This paper aims to summarize some commonly used calculation methods of Hawking radiation and include them in view of the fact that there are still new calculation methods. It is hoped that by summarizing these methods, readers can have a systematic understanding of Hawking radiation calculation methods.

2.Subject

2.1. Damour-Ruffini-Sannan method

Damour-Ruffini-Sannan method is mainly a research method for Hawking radiation of stationary black holes. In 1976 Damour and Ruffini proved Hawking radiation by analytical continuation [4]. Different from usual, their method assumes that the wave function can be extended analytically and requires the black hole to have a future horizon. Only relativistic quantum mechanics of curved space-time can prove the existence of thermal radiation of the black hole, instead of second quantization. This method does not require the black hole to have thermal equilibrium with the outside world and the collapse of the black hole, so that it can be used for all event horizons. In a further expansion, Sannan used the idea of statistical physics in 1988 to make this method not only applicable to boson, but also applicable to fermion [5]. According to Sannan's research, we can know the emergent wave's blackbody radiation spectrum as

$$N_{\omega lm} = \frac{\Gamma_{\omega lm}}{e^{8\pi M\omega}} \quad (1)$$

And because the Damour-Ruffini-Sannan method studies the radiation of each point on the black hole surface point by point, it can also study black holes with different temperatures at each point on the surface.

2.1.1. Vaidya-Bonner-de sitter space-time

Vaidya-Bonner-de Sitter space-time line elements are represented by advanced Eddington coordinates, tortoise coordinate transformation is used, and the outgoing wave is analytically extended at the outer horizon of the black hole. Then, the blackbody spectrum of Hawking radiation and Hawking radiation temperature can be obtained by Damour-Ruffini-Sannan method. Through the discussion of temperature, it can be proved that the inner and outer horizons of the black hole and the cosmic horizon of Vaidya-Bonner-de Sitter space-time cannot coincide, otherwise it will violate the third law of thermodynamics [6].

2.1.2. Quasi-static Schwarzschild Black Hole

In the quasi-static process, the apparent interfaces of Schwarzschild black holes will be split into two. When tortoise coordinates are adopted and the two horizons are extended analytically twice according to Damour-Ruffini-Sannan method, Hawking radiation of quasi-static Schwarzschild black holes can be obtained. And when the condition becomes static, Hawking radiation temperature of quasi-static Schwarzschild black holes returns to the static condition [7].

2.2. Gravitational anomaly

As early as 1977, Chistensen and Fulling first linked the conformal anomaly of quantum field with Hawking radiation, and discovered the conformal anomaly of Schwarzchild's space-time quantum field [8]. Through the constraint conditions given by this anomaly to the energy-momentum tensor, Hawking radiation can be derived near the (1+1) dimensional black hole. The anomaly of quantum field shows the contradiction between the quantization method and the symmetry of classical action, and the anomaly of global symmetry also indicates new physical content. Unfortunately, this method is too special to be generalized as a universal method.

In 2005, Robinson and Wiczek published a paper. They studied Hawking radiation of spherically symmetric black holes using anomalous methods based on quantum field theory in gradually flat space-time [9]. The anomaly is

$$\nabla_\mu T^\mu_\nu = \frac{1}{96\pi\sqrt{-g}} \epsilon^{\beta\delta}\,\partial_\delta\,\partial_\alpha \Gamma^\alpha_{\nu\beta} \quad (2)$$

The core idea of this method is that the effective field theory is formed outside the horizon. If the modes unrelated to the classic are ignored, the number of exit modes is no longer equal to the number of incident modes, resulting in asymmetry. The theory assumes that the background space-time is two-dimensional space-time, the field is mass-less, and there is no background radiation effect. For some high-dimensional space-time, it can be effectively reduced to two-dimensional space-time. The theory locates the source of the anomaly in the area near the horizon, which can simplify the equation, and the anomaly itself is a basic property of quantum field theory, so the effectiveness of the theory is worthy of our trust. In addition, according to this theory, we can only get Hawking radiation from the information at the black hole.

2.2.1. Dilaton-axion black hole
In spherically symmetric dilation-axion black holes, the influence of the dilation field can be ignored due to static background space-time, but the symmetry of the axion field cannot be ignored due to the consideration of symmetry with dilaton-axion. Using the anomalous method, the Hawking radiation at the horizon of the black hole can be obtained by eliminating the gauge anomaly and the gravitational anomaly under the condition of the scalar field and ignoring the spin field. Meanwhile, the validity of the gauge anomaly and the gravitational anomaly can be verified by using the fermion-radiation spectrum (but not limited to the fermion-radiation spectrum). We can get the result that at Hawking temperature with appropriate chemical potential, the form of energy momentum tensor current is consistent with that of two-dimensional black hole radiation [10].

2.2.2. Schwartzchild-de sitter space-time
As we all know, Schwartzchild-de sitter space-time has two visual interfaces: black hole horizon and the cosmic horizon. The scalar field in this background is discussed, if the quantum field is described by the set of two-dimensional fields (t,r) near the interface of two horizons, the gravitational anomaly problem can be discussed by two-dimensional space-time. In two-dimensional space-time, because particles lose causal connection with the outside world after entering the black hole through the black hole horizon, the incident mode can be ignored near the two horizons, and the exit mode can be regarded as particles moving to the right. Furthermore, the elimination of gravitational anomaly and gravitational anomaly near the interface of two horizons are studied. By discussing the background space-time boundary conditions, Hawking radiation is obtained near the interface of two horizons [11].

2.3. Other methods
With the follow-up research, some other research methods are also proposed. For example, tunneling theory is a concise theory to explain Hawking radiation [12]. The horizon of a black hole is regarded as a potential energy barrier, and the virtual particles inside the black hole have a probability of passing through this potential energy barrier and will further become real particles. Therefore, the temperature and entropy of a black hole can be calculated by tunneling radiation. Later, it was suggested that Hamilton-Jacobi equation of curved space-time was derived from scalar field equation of curved space-time by semi-classical method, which was used to calculate Hawking radiation greatly simplifying the research of black hole radiation [13].

2.4. Recent experiments:
Whether a physical theory is correct or not needs to be supported by corresponding experimental results. In 2019, Hawking radiation was verified by experimental simulation of black holes [14]. The simulated black hole is composed of Bose-Einstein condensates, and has reduced magnetic field noise, enhanced mechanical and thermal stability and redesigned optical system. In this experiment, Hawking radiation correlation spectrum is observed to be very consistent with thermal spectrum (Fig.1).

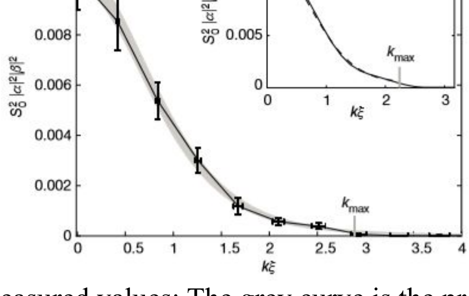

Fig.1 The black curve is the measured values; The grey curve is the predicted thermal spectrum using the Hawking temperature.

Its temperature is given by surface gravity, and the observation result is in a linear discrete state, similar to a real black hole, and the inside of the black hole is only composed of negative energy partner mode.

3. conclusion

In recent years, from the perspective of development trends, Hawking radiation research methods still maintain strong vitality. In recent years, the number of Hawking radiation research articles has also shown an obvious upward trend. Through the introduction of the above three calculation methods, we can understand that Hawking radiation has extremely strong physical research value. Each method of studying Hawking radiation shows the amazing wisdom of predecessors and the cleverness of the theoretical form of physics, and the theoretical experimental verification also shows the correctness of Hawking radiation.

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