

# Higgs fields on spinor gauge-natural bundles

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**Abstract.** We show that the Lie derivative of spinor fields is parametrized by Higgs fields defined by the kernel of a gauge-natural Jacobi morphism associated with the Einstein–Cartan–Dirac Lagrangian. In particular, the generalized Kosmann lift to the total bundle of the theory is constrained by *variational* Higgs fields on gauge-natural bundles.

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## 1. Introduction

Natural and gauge (classical) Lagrangian field theories have been framed within the geometric construction of a gauge-natural bundle, according to which classical physical fields are sections of bundles functorially associated with gauge-natural prolongations (also known as Ehresmann prolongations [2]) of principal bundles, by means of left actions of Lie groups on manifolds [1]. It is well known that, while the jet prolongation of a principal bundle is not a principal bundle, the gauge-natural prolongation of a principal bundle is provided with the structure of a principal bundle [2, 9]. We consider Lagrangian field theories which are assumed to be invariant with respect to the action of a gauge-natural group  $W_n^{(r,k)}\mathbf{G}$  defined as the semidirect product of a  $k$ -th order differential group of the base manifold with the group of  $r$ -th order  $n$ -th velocities on the structure group  $\mathbf{G}$  ( $n = \dim \mathbf{X}$  is the dimension of the basis manifold).

Within such theories there is *a priori* no natural way of relating infinitesimal gauge transformations with infinitesimal base transformations; we found that a *canonical* determination of Noether conserved quantities, without fixing any connection *a priori*, can be performed on a reduced bundle of  $W_n^{(r,k)}\mathbf{P}$  determined by the original  $W_n^{(r,k)}\mathbf{G}$ -invariant variational problem. Connections can be characterized by means of such a canonical reduction [4, 5, 13, 17, 18]. Such conserved quantities can be characterized in terms of Higgs fields on gauge principal bundles having moreover the richer structure of a gauge-natural prolongation [19].

We consider the particular case of the Einstein–Cartan–Dirac Lagrangian and we show that, being the Lie derivative of fields constrained by Jacobi equations, the Kosmann lift to the total bundle of spin-tetrads, spin-connections and spinors is associated with a *variational* Higgs field on the underlying gauge-natural principal bundle.

## 2. Gauge-natural Jacobi fields and canonically conserved quantities

Let  $J_s\mathbf{Y}$  of  $s$ -jet prolongations of (local) sections of a fibered manifold  $\pi : \mathbf{Y} \rightarrow \mathbf{X}$ , with  $\dim \mathbf{X} = n$  and  $\dim \mathbf{Y} = n + m$ . The natural fiberings  $\pi_{s-1}^s$  are *affine* fiberings inducing a

natural splitting  $J_s \mathbf{Y} \times_{J_{s-1} \mathbf{Y}} T^* J_{s-1} \mathbf{Y} = J_s \mathbf{Y} \times_{J_{s-1} \mathbf{Y}} (T^* \mathbf{X} \oplus V^* J_{s-1} \mathbf{Y})$ , which yields rising order decompositions: given a vector field  $\Xi : J_s \mathbf{Y} \rightarrow TJ_s \mathbf{Y}$ , we have a naturally induced decomposition in the sum of its *horizontal part*  $\Xi_H$  and its *vertical part*  $\Xi_V$  and analogously for the exterior differential on  $\mathbf{Y}$  we have  $(\pi_r^{r+1})^* \circ d = d_H + d_V$ , where  $d_H$  and  $d_V$  are the *horizontal* and *vertical differential*, respectively.

Naturally induced is also a *sheaf splitting*  $\mathcal{H}_{(s+1,s)}^p = \bigoplus_{t=0}^p \mathcal{C}_{(s+1,s)}^{p-t} \wedge \mathcal{H}_{s+1}^t$ , where  $\mathcal{H}_{(s,q)}^p$  and  $\mathcal{H}_s^p$  are the sheaves of *horizontal forms* with respect to the projections  $\pi_q^s$  and  $\pi_0^s$ , respectively, while  $\mathcal{C}_{(s,q)}^p \subset \mathcal{H}_{(s,q)}^p$  and  $\mathcal{C}_s^p \subset \mathcal{C}_{(s+1,s)}^p$  are *contact forms* (see *e.g.* [7, 11]); the projection on the summand of lesser contact degree  $h$  is *the horizontalization*. We set  $\Theta_s^* \doteq \ker h + d \ker h$ , where  $d \ker h$  is the sheaf generated by the corresponding presheaf. By quotienting the de Rham sequence with the contact structure so defined, we have *the Krupka variational sequence*  $0 \rightarrow \mathcal{I}\mathcal{R}_Y \rightarrow \mathcal{V}_s^*$ , where  $\mathcal{V}_s^* = \Lambda_s^*/\Theta_s^*$ . Let  $\mathcal{E}_*$  denote its differential morphisms; a section  $\lambda \in \mathcal{V}_s^n$  is a generalized Lagrangian and correspondingly a section  $E_{d\lambda} \doteq \mathcal{E}_n(\lambda) \in \mathcal{V}_s^{n+1}$  is the *generalized higher order Euler–Lagrange type morphism* associated with  $\lambda$  [11].

Let  $\mathbf{P} \rightarrow \mathbf{X}$  be a principal bundle with structure group  $\mathbf{G}$ . For  $r \leq k$  integers consider the *gauge-natural prolongation* of  $\mathbf{P}$  given by  $\mathbf{W}^{(r,k)}\mathbf{P} \doteq J_r \mathbf{P} \times_{\mathbf{X}} L_k(\mathbf{X})$ , where  $L_k(\mathbf{X})$  is the bundle of  $k$ -frames in  $\mathbf{X}$  [1, 9];  $\mathbf{W}^{(r,k)}\mathbf{P}$  is a principal bundle over  $\mathbf{X}$  with structure group  $\mathbf{W}_n^{(r,k)}\mathbf{G}$  which is the *semidirect* product with respect to the action of  $GL_k(n)$  on  $\mathbf{G}_n^r$  given by jet composition and  $GL_k(n)$  is the group of  $k$ -frames in  $\mathbb{R}^n$ . Here we denote by  $\mathbf{G}_n^r$  the space of  $(r, n)$ -velocities on  $\mathbf{G}$ . Let  $\mathbf{F}$  be a manifold and  $\zeta : \mathbf{W}_n^{(r,k)}\mathbf{G} \times \mathbf{F} \rightarrow \mathbf{F}$  be a left action of  $\mathbf{W}_n^{(r,k)}\mathbf{G}$  on  $\mathbf{F}$ . There is a naturally defined right action of  $\mathbf{W}_n^{(r,k)}\mathbf{G}$  on  $\mathbf{W}^{(r,k)}\mathbf{P} \times \mathbf{F}$  so that we get in a standard way the associated *gauge-natural bundle* of order  $(r, k)$ :  $\mathbf{Y}_\zeta \doteq \mathbf{W}^{(r,k)}\mathbf{P} \times_\zeta \mathbf{F}$ . All our considerations shall refer to a fibered manifold  $\mathbf{Y}$  which has also the structure of a gauge-natural bundle.

Functorial linearity properties of a gauge-natural lift  $\hat{\Xi}$  (for details, see *e.g.* [3, 9]) enabled us to define the *gauge-natural generalized Jacobi morphism* associated with a Lagrangian  $\lambda$  and the variation vector field  $\hat{\Xi}_V$ , *i.e.* the linear morphism  $\mathcal{J}(\lambda, \hat{\Xi}_V) \doteq E_{\lceil E_{j_s \hat{\Xi}} \rfloor d\lambda}$  [14]. The space  $\mathfrak{K} \doteq \ker \mathcal{J}(\lambda, \hat{\Xi}_V)$  defines generalized gauge-natural Jacobi equations, the solutions of which we call *generalized Jacobi vector fields* and characterize *canonical* covariant conserved quantities [13].

Induced linearity properties of the Lie derivative of sections of gauge-natural bundles characterize the form  $\omega(\lambda, \hat{\Xi}_V) \doteq -\mathcal{L}_{\hat{\Xi}} \lceil \mathcal{E}_n(\lambda)$  as a new Lagrangian defined on an extended space. It is remarkable that when  $\omega(\lambda, \hat{\Xi}_V)$  is an horizontal differential (*i.e.* a null Lagrangian) we get a conservation law which holds true along any section of the gauge natural bundle (not only along solutions of the Euler–Lagrange equations). It is also remarkable that the new Lagrangian  $\omega$ , in principle, is not gauge-natural invariant; nevertheless, its restriction  $\omega(\lambda, \mathfrak{K})$  is invariant and corresponding Noether conservation laws and Noether identities [20] can be obtained, so that a canonical determination of conserved quantities is given on a reduced bundle of  $\mathbf{W}^{(r,k)}\mathbf{P}$  determined by the original  $\mathbf{W}_n^{(r,k)}\mathbf{G}$ -invariant variational problem [15, 16]; in particular, necessary conditions for the existence of global solutions of Jacobi equations associated with the existence of canonically defined global conserved quantities can be interpreted as topological conditions for the existence of a Cartan connection on the principal bundle  $\mathbf{W}^{(r,k)}\mathbf{P}$  [18].

### 3. Spinor gauge-natural Higgs fields

In the following we shortly recall the Einstein–Cartan–Dirac theory; details can be found *e.g.* in [3, 25]. In particular, we point out the gauge-natural structure of such a theory.

On a 4-dimensional manifold admitting Lorentzian structures ( $SO(1, 3)^e$ -reductions)  $\mathbf{X}$  consider a  $SPIN(1, 3)^e$ -principal bundle  $\mathbf{\Sigma} \rightarrow \mathbf{X}$  and a bundle map inducing a spin-frame on  $\mathbf{\Sigma}$  given by  $\tilde{\Lambda} : \mathbf{\Sigma} \rightarrow L(\mathbf{X})$  defining a metric  $g$  *via* the reduced subbundle  $SO(\mathbf{X}, g) = \tilde{\Lambda}(\mathbf{\Sigma})$

of  $L(\mathbf{X})$ . A left action  $\rho$  of the group  $W^{(0,1)}SPIN(1,3)^e$  on the manifold  $GL(4, \mathbb{R})$  is given so that the associated bundle  $\Sigma_\rho \doteq W^{(0,1)}\Sigma \times_\rho GL(4, \mathbb{R})$  is a gauge-natural bundle of order  $(0,1)$ , the *bundle of spin-tetrads*  $\theta$ . Let  $\mathfrak{so}(1,3) \simeq \mathfrak{spin}(1,3)$  be the Lie algebra of  $SO(1,3)$ . One can consider the left action of  $W_4^{(1,1)}SPIN(1,3)^e$  on the vector space  $(\mathbb{R}^4)^* \otimes \mathfrak{so}(1,3)$ . The associated bundle  $\Sigma_l \doteq W^{(1,1)}\Sigma \times_l ((\mathbb{R}^4)^* \otimes \mathfrak{so}(1,3))$  is a gauge-natural bundle of order  $(1,1)$ , the *bundle of spin-connections*  $\phi$ . If  $\hat{\gamma}$  is the linear representation of  $SPIN(1,3)^e$  on the vector space  $\mathbb{C}^4$  induced by the choice of matrices  $\gamma$  we get a  $(0,0)$ -gauge-natural bundle  $\Sigma_{\hat{\gamma}} \doteq \Sigma \times_{\hat{\gamma}} \mathbb{C}^4$ , the *bundle of spinors*. A spinor connection  $\tilde{\phi}$  is defined in a standard way in terms of the spin connection. We notice that a spin (as well as spinor) connection is induced by a principal connection on  $\Sigma$ .

Within this picture, we assume that the total Lagrangian of a gravitational field interacting with spinor matter is  $\lambda = \lambda_{EC} + \lambda_D$ , where the Einstein–Cartan Lagrangian and the Dirac Lagrangian can be represented by the morphisms

$$\lambda_{EC} : \Sigma_\rho \times_{\mathbf{X}} J_1 \Sigma_l \rightarrow \wedge^4 T^* \mathbf{X}, \quad \lambda_D : \Sigma_\rho \times_{\mathbf{X}} \Sigma_l \times_{\mathbf{X}} J_1 \Sigma_{\hat{\gamma}} \rightarrow \wedge^4 T^* \mathbf{X},$$

respectively (local expressions can be found *e.g.* in [3]).

Let  $\mathfrak{k}$  be the vector bundle defined by the Jacobi equations  $\mathcal{J}(\lambda_{EC} + \lambda_D, \hat{\Xi}_V) = 0$ , where  $\hat{\Xi}$  is the gauge-natural lift to the associated total bundle of an infinitesimal principal automorphism of the principal bundle underlying the theory, *i.e.* of the  $SPIN(1,3)^e$ -principal bundle  $\Sigma \rightarrow \mathbf{X}$ . Since for each gauge-natural lift we have the well known equality  $\hat{\Xi}_V(\psi) = -\mathcal{L}_{\bar{\Xi}}\psi$ , in local fibered coordinates on the total bundle given by  $(x^\mu, \theta_\mu^a, \phi^{ab}, \phi_\mu^{ab}, \psi)$  the gauge-natural Jacobi equations read

$$(-1)^{|\sigma|} d_\sigma (d_\mu (-\mathcal{L}_{\bar{\Xi}}\psi)^{ab} (\partial_{cd}(\partial_{ab}^\mu) \lambda - \sum_{|\alpha|=0}^{s-|\mu|} (-1)^{|\mu+\alpha|} \frac{(\mu+\alpha)!}{\mu! \alpha!} d_\alpha \partial_{cd}^\alpha (\partial_{ab}^\mu \lambda))) = 0,$$

with  $0 \leq |\sigma|, |\mu| \leq 1$ ,  $d_\mu$  is the total derivative and we write for the total Lagrangian

$$\lambda \doteq \lambda_{EC} + \lambda_D = -\frac{1}{2k} \Phi_{ab} \wedge \epsilon^{ab} + \left( \frac{i\alpha}{2} (\bar{\psi} \gamma^a \nabla_a \psi - \nabla_a \bar{\psi} \gamma^a \psi) - m \bar{\psi} \psi \right) \epsilon,$$

where  $\epsilon$  is a volume density on  $\mathbf{X}$  and  $\Phi$  the curvature form of the spin-connection  $\phi$ ,  $\alpha$  and  $m$  are constants. Along  $\mathfrak{k}$ , we have  $\bar{\Xi}_v^{ab} = -\tilde{\nabla}^{[a} \xi^{b]}$  (the so-called Kosmann lift [10]), where  $\tilde{\nabla}$  is the covariant derivative with respect to the standard transposed connection on the bundle of spin-tetrads  $\Sigma_\rho$ . We remark that, since the Lie derivative of spinor fields  $\mathcal{L}_{\bar{\Xi}}\psi$  can be written in terms of  $\hat{\Xi}_{\tilde{h}}$  (the horizontal part of  $\hat{\Xi}$  *with respect to the spinor-connection*) the spinor-connection  $\tilde{\omega}$  is constrained [4, 26]. In the following we shall characterize this fact more precisely.

By an abuse of notation, we denote by  $\mathfrak{k}$  the Lie algebra of generalized Jacobi vector fields. Let now  $\mathfrak{h}$  be the Lie algebra of right-invariant vertical vector fields on  $W^{(1,1)}\Sigma$ ; the Lie algebra  $\mathfrak{k}$  is characterized as a Lie subalgebra of  $\mathfrak{h}$ ; the Jacobi morphism is self-adjoint and  $\mathfrak{k}$  is of constant rank; the split structure  $\mathfrak{h} = \mathfrak{k} \oplus \text{Im } \mathcal{J}$  is well defined and it is also reductive, being  $[\mathfrak{k}, \text{Im } \mathcal{J}] = \text{Im } \mathcal{J}$  [17].

In particular, for each  $\mathbf{p} \in W^{(1,1)}\Sigma$  by denoting  $\mathcal{S} \doteq \mathfrak{h}_\mathbf{p}$ ,  $\mathcal{R} \doteq \mathfrak{k}_\mathbf{p}$  and  $\mathcal{V} \doteq \text{Im } \mathcal{J}_\mathbf{p}$  we have the reductive Lie algebra decomposition  $\mathcal{S} = \mathcal{R} \oplus \mathcal{V}$ , with  $[\mathcal{R}, \mathcal{V}] = \mathcal{V}$ . Notice that  $\mathcal{S}$  is the Lie algebra of the Lie group  $WG \doteq W_4^{(1,1)}SPIN(1,3)^e$ . For the purposes of this note, it is sufficient to know that the Lie algebra  $\mathcal{R}$  exists and it is well defined; we shall not write down explicitly such a Lie algebra, although this question is of great interest and will be investigated extensively elsewhere.

As a consequence of the fact that  $\mathcal{R}$  is a reductive Lie algebra of  $\mathcal{S}$ , there exists an isomorphism between  $\mathcal{V} \doteq \text{Im} \mathcal{J}_{\mathbf{p}}$  and  $T\mathbf{X}$  so that  $\mathcal{V}$  turns out to be the image of an horizontal subspace. Thus we characterize a principal bundle  $\mathbf{S} \rightarrow \mathbf{X}$ , with  $\dim \mathbf{S} = \dim \mathcal{S}$  and such that  $\mathbf{X} = \mathbf{S}/\mathbf{R}$ , where  $\mathbf{R}$  is a Lie group of the Lie algebra  $\mathcal{R}$  and  $\mathcal{R} = T_{\mathbf{q}}\mathbf{S}/\mathbf{R}$ ; the principal subbundle  $\mathbf{S} \subset \mathbf{W}^{(1,1)}\Sigma$  is then a reduced principal bundle.

In the following, to simplify the notation, we shall omit the orders of a gauge-natural prolongation; in particular, denote by  $W\mathbf{G}$  a gauge-natural prolongation of a given appropriate order of the structure group of the Einstein–Cartan–Dirac theory. The Lie group  $\mathbf{R}$  of the Lie algebra  $\mathcal{R}$  is in particular a closed subgroup of  $W\mathbf{G}$  [18, 19]. We have the composite fiber bundle  $W\Sigma \rightarrow W\Sigma/\mathbf{R} \rightarrow \mathbf{X}$ , such that  $W\Sigma/\mathbf{R} = W\Sigma \times_{W\mathbf{G}} W\mathbf{G}/\mathbf{R} \rightarrow \mathbf{X}$  is a gauge-natural bundle functorially associated with  $W\Sigma \times W\mathbf{G}/\mathbf{R} \rightarrow \mathbf{X}$  by the right action of  $W\mathbf{G}$ . The left action of  $W\mathbf{G}$  on  $W\mathbf{G}/\mathbf{R}$  is in accordance with the reductive Lie algebra decomposition.

**Definition 1** We call a global section  $h : \mathbf{X} \rightarrow W\Sigma/\mathbf{R}$  a *spinor gauge-natural Higgs field*.

### 3.1. Higgs fields and the Lie derivative of spinors

Let  $\omega$  be a principal connection on  $W\Sigma$  and  $\bar{\omega}$  a principal connection on the principal bundle  $\mathbf{S}$  i.e. a  $\mathcal{R}$ -invariant horizontal distribution defining the vertical parallelism  $\bar{\omega} : V\mathbf{S} \rightarrow \mathcal{R}$  in the usual and standard way. It defines the splitting  $T_{\mathbf{p}}\mathbf{S} \simeq_{\bar{\omega}} \mathcal{R} \oplus \hat{H}_{\mathbf{p}}$ ,  $\mathbf{p} \in \mathbf{S}$ . Since  $\mathcal{R}$  is a subalgebra of the Lie algebra  $\mathcal{S}$  and  $\dim \mathbf{S} = \dim \mathcal{S}$ , it is defined a principal Cartan connection of type  $\mathcal{S}/\mathcal{R}$ , such that  $\hat{\omega}|_{V\mathbf{S}} = \bar{\omega}$ . It is a connection on  $W\Sigma = \mathbf{S} \times_{\mathbf{R}} W\mathbf{G} \rightarrow \mathbf{X}$ , thus a Cartan connection on  $\mathbf{S} \rightarrow \mathbf{X}$  with values in  $\mathcal{S}$  [5] and it splits into the  $\mathcal{R}$ -component which is a principal connection form on the  $\mathcal{R}$ -manifold  $\mathbf{S}$ , and the  $\mathcal{V}$ -component which is a displacement form [18].

A gauge-natural Higgs field, being a global section of  $\hat{H}_{\mathbf{p}}$ , with  $\mathbf{p} \in \mathbf{S}$ , is related with the displacement form defined by the  $\mathcal{V}$ -component of the Cartan connection  $\hat{\omega}$  above. The pull-back by  $h$  of the  $\mathcal{R}$  valued component of a  $\mathcal{S}$  valued principal connection  $\omega$  on  $W\Sigma$  onto the reduced subbundle  $\mathbf{S}$  is the connection form of a principal connection on  $\mathbf{S}$ . Given the composite fiber bundle  $W\Sigma \rightarrow W\Sigma/\mathbf{R} \rightarrow \mathbf{X}$ , we have the exact sequence

$$0 \rightarrow V_{W\Sigma/\mathbf{R}} W\Sigma \rightarrow VW\Sigma \rightarrow W\Sigma \times_{W\Sigma/\mathbf{R}} VW\Sigma/\mathbf{R} \rightarrow 0,$$

where  $V_{W\Sigma/\mathbf{R}} W\Sigma$  denote the vertical tangent bundle of  $W\Sigma \rightarrow W\Sigma/\mathbf{R}$ . Every connection  $\tilde{\omega}$  on the latter bundle determines a splitting  $VW\Sigma = V_{W\Sigma/\mathbf{R}} W\Sigma \oplus_{W\Sigma/\mathbf{R}} \tilde{\omega}(W\Sigma \times_{W\Sigma/\mathbf{R}} VW\Sigma/\mathbf{R})$ , by means of which we can define a vertical covariant differential as a mapping  $J^1 W\Sigma \rightarrow T^* \mathbf{X} \otimes_{W\Sigma} V_{W\Sigma/\mathbf{R}} W\Sigma$ . The covariant differential on  $W\Sigma_h$  relative to the pull-back connection  $h^*(\tilde{\omega})$  can be expressed by means of this mapping in a known way; for coordinate expressions and further details see [12].

**Remark 1** A geometric interpretation of the Kosmann lift as a reductive lift has been proposed for the definition of a  $SO(1, 3)^e$ -reductive Lie derivative of spinor fields [8]. From a variational point of view the Kosmann lift is characterized as the only gauge-natural lift satisfying the naturality condition  $\mathcal{L}_{j_{s+1}\hat{\Xi}_H} [\mathcal{L}_{j_{s+1}\hat{\Xi}_V} \lambda] \doteq 0$  equivalent with Jacobi equations. Gauge-natural Jacobi equations state that Lie derivatives of spinors coincide with the vertical parts of gauge-natural lift of principal automorphisms *lying in*  $\mathfrak{K}$ , which can be expressed through the vertical covariant differential, defined for each global section  $h$  of  $W\Sigma/\mathbf{R} \rightarrow \mathbf{X}$ ; then we can say that the Lie derivative of gauge-natural spinors is constrained and it is parametrized by a Higgs field  $h$  defined by  $\mathfrak{K}$ . This condition implies a reduction of the structure group  $W\mathbf{G}$  to  $\mathbf{R}$ . Each global section  $h$  of  $W\Sigma/\mathbf{R} \rightarrow \mathbf{X}$  affects spin and spinor connections induced functorially on the associated bundle. In particular, the Kosmann lift to the total bundle of spin-tetrads, spin-connections and spinors is constrained by *variational* Higgs fields on the spinor gauge-natural bundle.

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## References

- [1] D.J. Eck: Gauge-natural bundles and generalized gauge theories, *Mem. Amer. Math. Soc.* **247** (1981) 1–48.
- [2] C. Ehresmann: Les prolongements d'un espace fibré différentiable, *C. R. Acad. Sci. Paris* **240** (1955) 1755–1757.
- [3] L. Fatibene, M. Francaviglia: *Natural and gauge natural formalism for classical field theories. A geometric perspective including spinors and gauge theories*; Kluwer Academic Publishers, Dordrecht, 2003.
- [4] M. Ferraris, M. Francaviglia, M. Palese, E. Winterroth: Canonical connections in gauge-natural field theories, *Int. J. Geom. Methods Mod. Phys.* **5** (6) (2008) 973–988.
- [5] M. Ferraris, M. Francaviglia, M. Palese, E. Winterroth: Gauge-natural Noether currents and connection fields, *Int. J. Geom. Methods Mod. Phys.* (2011).
- [6] M. Ferraris, M. Francaviglia, M. Raiferi: Conserved Quantities from the Equations of Motion (with applications to natural and gauge natural theories of gravitation) *Class. Quant. Grav.* **20** (2003) 4043–4066.
- [7] M. Francaviglia, M. Palese, R. Vitolo: Symmetries in finite order variational sequences, *Czech. Math. J.* **52(127)** (1) (2002) 197–213.
- [8] M. Godina, P. Matteucci: Reductive  $G$ -structures and Lie derivatives, *J. Geom. Phys.* **47** (1) (2003) 66–86.
- [9] I. Kolář, P.W. Michor, J. Slovák: *Natural Operations in Differential Geometry*, (Springer–Verlag, N.Y., 1993).
- [10] Y. Kosmann: Dérivée de Lie de spineurs, *C. R. Acad. Sci. Paris Sér. A-B* **262** (1966) A289–A292; – Dérivée de Lie de spineurs. Applications, *C. R. Acad. Sci. Paris Sér. A-B* **262** (1966) A394–A397.
- [11] D. Krupka: Variational Sequences on Finite Order Jet Spaces, *Proc. Diff. Geom. and its Appl.* (Brno, 1989); J. Janyška, D. Krupka eds.; World Scientific (Singapore, 1990) 236–254.
- [12] L. Mangiarotti, G. Sardanashvily: *Connections in Classical and Quantum Field Theory*, (World Scientific, Singapore, 2000).
- [13] M. Palese, E. Winterroth: Covariant gauge-natural conservation laws, *Rep. Math. Phys.* **54** (3) (2004) 349–364.
- [14] M. Palese, E. Winterroth: Global Generalized Bianchi Identities for Invariant Variational Problems on Gauge-natural Bundles, *Arch. Math. (Brno)* **41** (3) (2005) 289–310.
- [15] M. Palese, E. Winterroth: The relation between the Jacobi morphism and the Hessian in gauge-natural field theories, *Theoret. Math. Phys.* **152**(2) (2007), 1191–1200.
- [16] M. Palese, E. Winterroth: Noether identities in Einstein–Dirac theory and the Lie derivative of spinor fields, *Proc. Diff Geom. Appl., Olomouc* (2007), World Scientific Publ., Hackensack, NJ, (2008) 643–653.
- [17] M. Palese, E. Winterroth: Lagrangian reductive structures on gauge-natural bundles, *Rep. Math. Phys.* **62** (2) (2008) 229–239.
- [18] M. Palese, E. Winterroth: Invariant variational problems and Cartan connections on gauge-natural bundles, *AIP Conf. Proc.* **1191** (2009) 160–165.
- [19] M. Palese, E. Winterroth: A variational perspective on classical Higgs fields in gauge-natural theories *Theoret. Math. Phys.*, **168**(1) (2011) 1002–1008.
- [20] E. Noether: Invariante Variationsprobleme, *Nachr. Ges. Wiss. Göttingen, Math. Phys. Kl.* **II** (1918) 235–257.
- [21] G. Sardanashvily: On the geometry of spontaneous symmetry breaking. *J. Math. Phys.* **33** (1992), no. 4, 15461549.
- [22] G. Sardanashvily: Geometry of classical Higgs fields. *Int. J. Geom. Methods Mod. Phys.* **3** (2006), no. 1, 139148.
- [23] D.J. Saunders: *The Geometry of Jet Bundles*, Cambridge Univ. Press (Cambridge, 1989).
- [24] A. Trautman: Noether equations and conservation laws, *Comm. Math. Phys.* **6** (1967) 248–261.
- [25] S. Weinberg: *Gravitation and cosmology: principles and applications of the general theory of relativity*, Wiley, New York, 1972.
- [26] E.H.K. Winterroth: *Variational derivatives of gauge-natural invariant Lagrangians and conservation laws*, PhD thesis University of Torino, 2007.