The energy spectrum of ultra-high energy cosmic rays measured at the Pierre Auger Observatory and the Telescope Array

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Abstract. In the study of cosmic rays, the measurement of the energy spectrum of the primaries is one of the main issues and provides fundamental information on the most energetic phenomena in the Universe. At ultra-high energies, beyond 10^{18} eV, the cosmic rays are studied by the two largest observatories built so far, the Pierre Auger Observatory and the Telescope Array. Both observatories are based on a hybrid design and reported a measurement of the energy spectrum using the high duty cycle of the surface detector and the calorimetric estimation of the energy scale provided by the fluorescence detector.
The differences among the reported spectra are scrutinized by a working group made by members of the Pierre Auger and Telescope Array Collaborations. The two measurements have been found well in agreement below 10^{19} eV while, at higher energies, they show an energy-dependent difference that is beyond the systematic uncertainties associated to the energy scale.
In this contribution we review the status and perspectives of the working group activities including new studies aiming at addressing the impact on the flux measurement at the highest energies of potential biases in the estimation of the shower size.

1 Introduction

Ultra-high-energy cosmic rays (UHECRs) are atomic nuclei arriving from outer space with energies beyond 10^{18} eV, the highest-energy particles observed in nature. Their energy spectrum, the differential flux of particles, is the basic experimental information because its absolute scale and its shape provide information on the acceleration mechanisms, the spatial distribution of the sources, and propagation of cosmic rays in the inter-galactic space. UHECRs arrive at the Earth very rarely, with a flux that decreases with energy, reaching less than one event per square kilometer per century at 10^{20} eV, and therefore their observation require huge detection area and long observation times.

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The Pierre Auger Observatory (Auger) and the Telescope Array (TA) are the two largest observatories of UHECRs built so far. They have been in operation fo more than a decade in the Southern and Northern hemispheres, respectively. Auger [1] is located near the small town of Malargüe in the province of Mendoza (Argentina) at a latitude of about 35.2° S. It has a surface detector (SD) of 1600 water Cherenkov detectors placed on a triangular grid with 1500 m spacing that extends over 3000 km^2. TA [2, 3] is located near Delta, Millard County, Utah, U.S. at the latitude 39.3° N. Its SD has 507 scintillation counters on a square grid with 1.2 km spacing covering an area of 700 km^2. Both observatories follow a so-called hybrid approach, because the SD measurements are combined with the ones performed by a fluorescence detector (FD). With the FD it is possible to reconstruct the longitudinal profile of the shower and to obtain a calorimetric estimation of the shower energy. In this way, it is possible to measure the energy spectrum with the high exposure of the SD and with an energy scale that, being set by the FD, it is largely independent of air shower simulations and of assumptions on hadronic interaction models.

The differences among the spectra measured at the two observatories are scrutinized by a joint working group that was formed in 2012 and that has reported its studies in the UHECR and ICRC conference series [4, 5]. In this contribution, we briefly report the results of the activities of the working group revising the details of the Auger and TA data analysis and focusing on the discrepancy between the spectra at the highest energies. In particular, we discuss new studies of potential biases in the estimation of the shower size measured with the SD that can't be corrected by the calibration performed with the FD energies.

2 The experimental methods

Despite the TA and Auger observatories having both adopted the hybrid detection technique, there are several differences in the analysis methods to determine the energy scale and to estimate the energy spectrum.

In both observatories, the signals detected by the SD stations are fitted with a lateral density function. The fit provides the position of the shower core and the lateral density function evaluated at a some distance from the core is used to get an energy estimator that is calibrated against the FD energies. This distance is 1000 m for Auger and 800 m for TA and the resulting energy estimators are called $S(1000)$ and $S(800)$, respectively. The calibration procedures against the FD energies developed by the two Collaborations are rather different as detailed below.

The Auger analysis is performed selecting showers with zenith angles $\theta < 60°$ and with energies greater than 2.5×10^{18} eV and these conditions ensure that the trigger efficiency is 100%. In this way $S(1000)$ can be corrected for attenuation effects using the empirical procedure of the so-called Constant Intensity Cut (CIC) method [6]. The resulting energy estimator, S_{38}, is the zenith-angle independent energy estimator and can be thought of as the signal, $S(1000)$, that the shower would have produced at a zenith angle of 38°. Then, S_{38} is calibrated against the FD energies using a power-law relationship $E_{\rm FD} = A\, S_{38}^{B}$ where the two parameters A and B are fitted to the data ($B \approx 1.03$). The Auger analysis is described in detail in [7].

The full trigger efficiency for the TA SD is attained at $10^{18.8}$ eV for showers with zenith angles $\theta < 55°$. Therefore, in order to estimate the shower energy below this energy threshold, the CIC method can't be used and one has to calculate the attenuation effects using Monte-Carlo (MC) simulations. For a given value of $S(800)$ and θ, a MC lookup table is used to estimate the shower energy ($E_{\rm TBL}$). The TA MC simulations use the CORSIKA software package and the showers are generated according the QGSJetII-03 hadronic interaction model assuming proton primaries. Then, $E_{\rm TBL}$ is calibrated against the FD energies ($E_{\rm FD}$) using a linear relationship among $E_{\rm TBL}$ and $E_{\rm FD}$. The calibration fit provides a normalization factor of the MC energies equal to 1/1.27. For further details on TA energy determination see [8].

A detailed analysis of the systematic uncertainties in the energy scale of TA is reported in [10] and for Auger in [11]. The results are summarized in table 1. The total uncertainty for TA and Auger is 21% and 14%, respectively, and they are almost energy independent. The main difference in the uncertainties is related to the fluorescence yield. Auger uses the recent and precise measurements of the Airfly experiment [12, 13] while TA uses the measurement of the absolute yield made by Kakimoto et al. in 1996 [14], and the wavelength spectrum measured more recently by FLASH [15]. Another important difference is related to the atmosphere, and in particular to the determination of the aerosols that in Auger are measured every hour of data taking while, in TA, the same average aerosol profile is used to reconstruct all FD events. It is worth nothing that, besides the contribution to the uncertainty being not so large, the invisible energy also plays an important role in the determination of the energy scale. The invisible energy is the non-calorimetric part of the FD energy estimation being associated to the high-energy muons and neutrinos that do not deposit their energies in the atmosphere. In TA it is estimated using MC simulations of proton showers and it amounts to about 7% of the total shower energy, while the Auger estimation is significantly larger, at the level of 14%, as it incorporates the muon number excess measured with the water Cherenkov detectors [16].

Table 1. Systematic uncertainties in the energy scale for TA [10] and Auger [11]. The mild energy dependence of the uncertainties is reported for Auger.

Systematic uncertainties in the energy scale		
	TA	Auger
Fluorescence Yield	11%	3.6%
Atmosphere	11%	3.4%–6.2%
FD Calibration	10%	9.9%
FD Reconstruction	9%	6.5%–5.6%
Invisible Energy	5%	3%–1.5%
Other contributions		5%
Total	21%	14%

The other important ingredient for the determination of the energy spectrum is the exposure. The Auger analysis is performed in an energy range in which the SD is fully efficient, and therefore the calculation of the exposure reduces to a geometrical calculation plus the knowledge of the live-time of the array. In TA, since the analysis is extended at energies where the detector is not fully efficient, the exposure related to the geometry and live-time of the array has to be folded with the trigger and reconstruction efficiency that are estimated using MC simulations. Another ingredient is the energy resolution that must be known in order to account for the distortion of the spectrum shape given by the migration of events between neighboring energy bins. Resolution corrections are in general not so large and are precisely estimated.

For this contribution, we use the Auger data set presented in [7], whose exposure is 60 400 km^2 sr yr, and the TA data set presented in [9], whose exposure ranges from

Table 2. Parameters relevant for the measurements of the energy spectrum at the Auger Observatory [7] and the Telescope Array [9].

	Telescope Array	Auger
data period	11/05/2008 – 11/05/2019	01/01/2004 – 31/08/2018
energy threshold	$10^{18.2}$ eV	$10^{18.4}$ eV
zenith angle	$\theta < 45°$	$\theta < 60°$
declination band	$-6° < \delta < 90°$	$-90° < \delta < 24.8°$
full trigger efficiency	$> 10^{18.8}$ eV	$> 10^{18.4}$ eV
exposure above full trigger efficency	$\approx 10\,000$ km^2 sr yr	$60\,400$ km^2 sr yr
number of events $E > 10^{19}$ eV (10^{20} eV)	3292 (13)	16 737 (15)
SD energy resolution (10^{19} eV – 10^{20} eV)	21% – 15%	11% – 8%
FD energy resolution	19%	7.4%
uncertainty in the energy scale	21%	14%

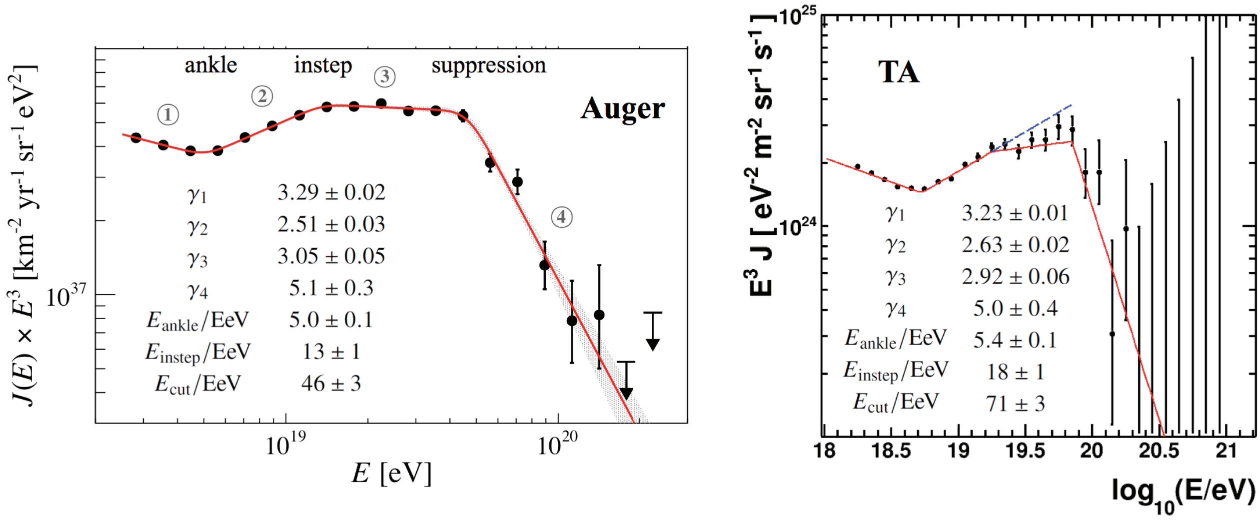

Figure 1. Energy spectrum and spectral features measured by Auger [7] and TA [9]. Only statistical uncertainties are shown. For a rough estimation of the systematic uncertainties in the energy that defines the transition points one can consider the total uncertainty in the energy scale reported in table 1 with the remark that it would be fully correlated for the different inflection points.

770 km^2 sr yr at $10^{18.2}$ eV up to $\approx 10\,000$ km^2 sr yr above $10^{18.8}$ eV. Other information and relevant parameters for the determination of the energy spectrum at the two observatories are reported in table 2.

3 Comparison of the spectra measured at the two observatories

The measurements of the energy spectrum performed by Auger [7] and TA [9] are shown in figure 1. They are presented multiplying the flux by the third power of the energy (corresponding to the central value of the energy bin $\Delta \log_{10} E = 0.1$) in order to better visualize their shape. The spectrum is characterized by both Collaborations with four broken power laws, i.e. a sequence of four power laws with fast transitions of the spectral index in three inflection points. The latter are identified as the *ankle*, the *suppression* at highest energies and the *instep*, the new steepening recently reported just above 10^{19} eV. The spectral features are in remarkable agreement with some tension that emerges at the highest energies. It is worth noting that the flux parameterization is the simplest possible model that describes well the data. A more realistic evolution of the spectral index with energy is likely more complex than simple broken power laws, as it depends by many factors, such as the production rate in the sources, the evolution of mass composition with energy and propagation effects [17]. Further statistics is needed to address more precisely the shape of the energy spectrum [7].

The two spectra are compared in the top-left panel of figure 2. The TA data points are systematically higher than the Auger ones with a discrepancy that is larger at the highest energies. Such discrepancy can be interpreted as a difference in the energy scale of the two observatories. This is because the uncertainty in the energy scale gives the dominant contribution to the uncertainty in the spectrum normalization. As a first approximation one has $\Delta J/J \sim (\gamma - 1)\,\Delta E/E$ where γ is the spectral index. With the typical uncertainties affecting the energy scale, it is likely to observe differences in the measured spectra that can be as large as several tens of percentage. Other contri-

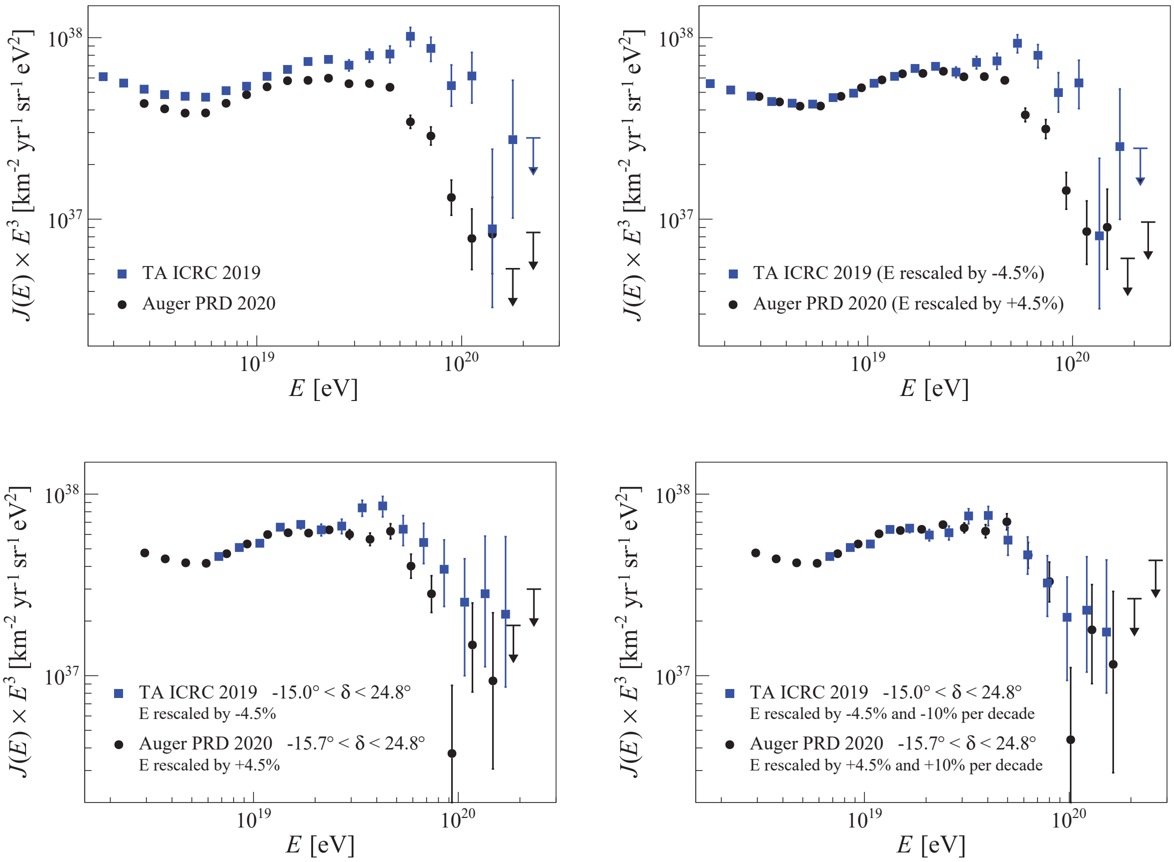

Figure 2. Energy spectrum measured by Auger [7] and TA [9] in the full declination band (upper-left), in the full band after a rescaling of the energy by an overall 9% factor (upper-right), in the common band with the same energy rescaling factor (bottom-left) and in the common band once the energy scale is furtherly rescaled in an energy dependent way by 20%/decade defined as $\Delta E/E = \left[\pm 4.5 \pm 10 \log_{10}(E/10^{19}\text{ eV})\right]\%$.

butions to the uncertainty in the flux are in general much smaller (see e.g. [7]).

In the top-right panel of figure 2 we show how the spectra can be put in agreement up to about 10^{19} eV introducing an overall energy independent energy shift of 9% (−4.5% for TA and +4.5% for Auger). The discrepancy persisting at the highest energies can be recovered only introducing an energy dependent energy shift. However, interpreting the spectral difference only as an energy scale issue may not be fully correct. This because the two detectors observe different parts of the celestial sphere and at the highest energies, where the deflection of the CRs in the galactic and inter-galactic magnetic fields are expected to be not so large, the energy spectrum may depend on the sources that fall within the field of view of the detector.

A more correct comparison of the two spectra at the highest energies is done by limiting the measurements to the declination band accessible by both observatories. This is shown in the bottom-left panel of figure 2. Here, the TA spectrum is calculated using the selection criteria adopted for the analyses aiming to search the anisotropy signals in the CRs arrival directions. This allows us to extend the zenith angle range from $\theta < 45°$ to $\theta < 55°$ and therefore to decrease the minimum detectable declination from $-6°$ to $-15°$, allowing to have a larger overlap with the Auger field of view. For this selection criteria the analysis is limited at the energies larger than $10^{18.8}$ eV. In the figure we show how the discrepancy at the highest energies persists even in the common band: from the agreement attained at $\approx 10^{19}$ eV thanks to the energy rescaling factor determined in the full band around the ankle, the difference between then flux becomes up to about 70% at 10^{20} eV. As shown in the bottom-right panel of figure 2, such energy dependent difference can be explained introducing a further energy dependent energy shift that amounts to 20% per decade (−10%/decade for TA and +10%/decade for Auger). We have verified that the same results are obtained when the analysis accounts for the different shapes of the directional exposure of the two observatories [18].

The statistical significance of the energy-dependent energy shift has been evaluated using the following method: thousands of independent spectra with the TA statistics in the common band have been generated simulating events according to the functional shape that describes the Auger spectrum (both shape and normalization) and taking into account resolution effects. For each generated spectrum we estimate the energy dependent energy shift to reach full agreement with Auger obtaining a distribution centered in 0 and with an RMS of 6%. The distribution allows us to reject the scenario in which the 20%/decade energy shift is due to a statistical fluctuation with a significance of about 3σ.

4 Toward understanding the differences between the energy scales of the two observatories

In this section we discuss the energy shifts determined in the previous section in the light of the systematic uncertainties in the energy scales. The discussion is done separately for the constant energy shift derived below 10^{19} eV and for the additional energy dependent energy shift at higher energies because, as we will see, they need an analysis of different types of systematic uncertainties.

4.1 Understanding the 9% energy shift below 10^{19} eV

The overall energy shift of 9% that brings the Auger and TA spectra in agreement in the energy region below 10^{19} eV is of course fully consistent with the uncertainties in the energy scales. It is worth noting that an even better agreement can be achieved if the fluorescence events collected at the two observatories would be reconstructed using the same model of the fluorescence yield and invisible energy (see [4] and references therein). Introducing the Airfly absolute yield in the TA reconstruction would lower the energy by 20%. Such energy shift is reduced to −14% if also all the other Airfly parameters describing the wavelength spectrum and quenching effects are introduced in the analysis. On the other hand, the invisible energy correction of Auger would increase the TA energies by 7%. Therefore, the combined effect of using the same model of fluorescence yield and invisible energy would lower the 9% energy shift to a value well below 5%. This value is surprisingly low when compared with the uncertainty in the energy scales obtained subtracting (in quadrature) the contributions from the fluorescence yield and invisible energy (from table 1 one can estimate 13% for Auger and 17% for TA). The good agreement between the energy scales suggests that the systematic uncertainties in the fluorescence events reconstruction, and in particular the ones on the absolute calibration of the telescopes, are well under control. On this respect, a remarkable test has been done by TA using a linear accelerator installed in front of one of the fluorescence telescopes [19]. The facility allows to mimic a cosmic ray air shower and it provides an effective test of the combined effect of the fluorescence yield and of the absolute calibration of the telescopes. The test performed by TA has a precision of 7.9% and has shown that data are fully consistent with the FD absolute calibration when the Airfly model is used in the analysis [20].

4.2 Understanding the additional 20%/decade energy shift determined above 10^{19} eV

The understanding of the additional 20%/decade energy shift needed to bring in agreement the spectra above 10^{19} eV is much more complicated than the one at lower energies, both due to the lack of statistics especially for hybrid events and because the two Collaborations perform the energy calibration with methods that are substantially different and therefore affected by different kinds of systematics.

As seen in Sect. 2, the Auger Collaboration calibrates the SD signal using a power law and therefore the systematics in the FD energies play a crucial role even at the higher energies. The analysis of the hybrid events benefits from enough statistics and good reconstruction performances. As seen in figure 3, Auger has almost 600 hybrid events above 10^{19} eV and the distribution of the ratio of SD to FD energies is rather narrow, with an RMS of 12% that is the result of the combined effect of the SD and FD energy resolutions (see table 2). These features guarantee that the SD energies are well aligned to the FD ones up to the highest energies: the statistical uncertainty in the calibration curve is 0.3% at 10^{19} eV to reach its maximum of about 1% at 10^{20} eV [7], therefore introducing only a small uncertainty that can't explain the difference

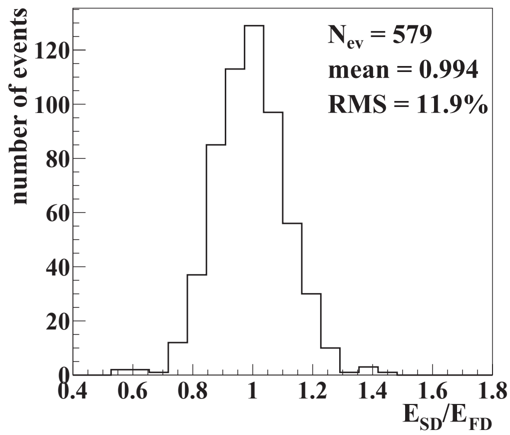

Figure 3. Ratio of the SD to FD energies for the Auger hybrid events above 10^{19} eV [7].

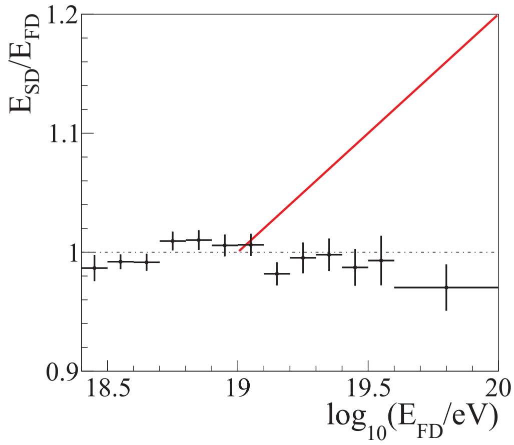

Figure 4. Average ratio of the SD to FD energies in bins of energies for the Auger hybrid events [7]. The red line shows the 20%/decade energy shift above 10^{19} eV needed to explain the difference between the Auger and TA spectra.

between the TA and Auger spectra. This is also shown in figure 3, where the 20%/decade energy shift (red line) is compared with the average ratio of the SD to FD energies calculated in bins of energy. It is then clear that for Auger it is important to address precisely the energy dependence of the systematics in the FD energies. As shown in table 1, the systematics depend only little on energy and can't explain the 20%/decade energy shift. Even in the case of the aerosols under an extreme scenario of their underestimation, the energy dependence of the bias would be rather small, below 3%/decade [21].

As seen in Sect. 2, TA calibrates the SD energy estimator through an overall normalization factor ($E_{FD} = E_{TBL}/1.27$). The study of possible energy-dependent reconstruction biases estimated comparing the SD and FD energies as a function of energy leads to a ($-1\% \pm 9\%$)/decade energy shift [21], and it is on some extent limited by the low TA hybrid statistics at higher energies. This result can't be significantly affected by the energy dependence of the FD systematics because, like in Auger, it is also small in TA [21]. Therefore, it is natural to investigate possible energy biases arising from the reconstruction of the SD energy estimator that could emerge at the higher energies as they can't be corrected by the TA calibration procedure (because it provides one factor that is the same at all energies). It is worth noting that such kinds of bias could also be relevant for Auger even though the calibration fit provides also the slope (B) of the power law ($E_{FD} = A\, S_{38}^{B}$).

The central point of the SD reconstruction is the choice of the lateral density function (LDF) and of the distance from the shower core at which the energy estimator is evaluated. Auger and TA use an LDF whose shape is fixed to a predetermined average parameterization. This because, given the large spacing of the arrays, the LDF is sampled in only few points and its shape can't be determined on an event-by-event basis (the LDF depends on how the shower has developed in the atmosphere). This causes large uncertainties in the normalization of the LDF (proxy of the shower energy) and leads to define the energy estimator using the LDF evaluated far away from the core were fluctuations of the signals are quite small (see [22] and references therein). It turns out that the fluctuations are minimal at a so-called "optimal distance" (r_{opt}) whose value primarily depends on the array geometry, with little dependence on the energy or zenith angle of the shower or choice of the LDF [22].

The LDF used by Auger is a modified Nishimura–Kamata–Greisen function (see [23] and references therein). Its shape is parametrized empirically from data as a function of $S(1000)$ and of the zenith angle analyzing those events where the LDF can be sampled with a large enough number of stations to provide a sufficiently high lever-arm. The energy estimator is evaluated at r_{opt} = 1000 m [22]. In figure 5 we present a recent cross-check of the LDF fluctuations, evaluated repeating several times the LDF fit of Auger simulated events where the slope parameters are changed in each fit according to their uncertainty. As we can see, the fluctuations are minimal at 1000 m, confirming the early results reported in [22]. The fluctuations are

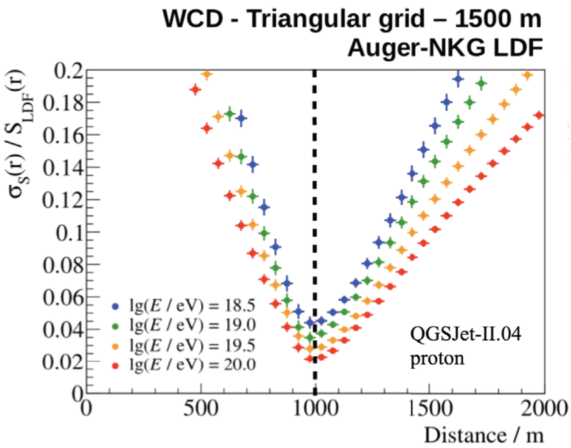

Figure 5. Fluctuations of the signal estimated with the Auger LDF arising from the uncertainty in the slope parameters.

a few percent at r_{opt} and increase to beyond 10% at 500 m from r_{opt}. Evaluating the energy estimator minimizing the uncertainties related to the LDF is important to ensure that the energy estimator is a good proxy of the shower energy. The risk in workink with larger uncertainties is that we may have an energy estimator that, once it is corrected by attenuation effects, can't be properly calibrated using a power law (something difficult to detect with the limited hybrid statistics), therefore with the risk of introducing energy biases to which the energy spectrum is very sensitive. The Auger collaboration has estimated that the uncertainties related to the $S(1000)$ reconstruction affect the measured spectrum by less than 3% [7].

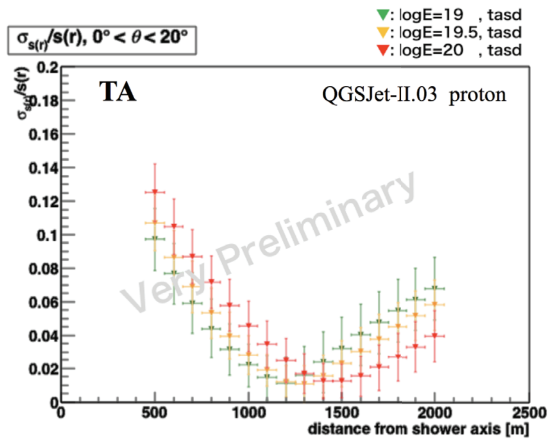

Figure 6. Fluctuations of the signal estimated with the TA LDF arising from the uncertainty in the slope parameters.

The TA LDF is the same as the one that was used by the AGASA experiment (see [8] and references therein),[1] with the slope parameter that depends only on the zenith angle. The energy estimator is obtained taking the signal at 800 m from the core because this distance minimizes the difference between the LDF of proton and iron showers. The fluctuations of the LDF for the TA events calculated changing the slope by its uncertainty are shown in

[1] Like for TA, the AGASA detectors were scintillators.

figure 6, and one can see that 800 m does not coincide with the "optimal distance" of 1300 m at which the fluctuations are minimal. The difference between the fluctuations at 1300 m and 800 m is not so large (from 2% to ≈ 6%) and therefore one can expect that the two energy estimators ($S(800)$ and $S(1300)$) provide similar performances.

In order to address this point, the TA collaboration has developed an analysis similar to the one performed by Auger. Selecting the events above 10^{19} eV where the array is fully efficient, $S(800)$ and $S(1300)$ are corrected for the attenuation effects using the CIC method (figure 7 shows the attenuation curves of $S(1300)$). The resulting zenith angle independent energy estimators are calibrated against the FD energies using a power law relationship through a χ^2 minimization (see figure 8, where S_{35} is the shower size at the zenith angle of 35°). The two energies derived from $S(800)$ and $S(1300)$ agree very well, at the 4% level as shown in figure 9, demonstrating that there is not a significant worsening of the performances of the energy reconstruction when the shower size is not estimated at $r_{\rm opt}$.

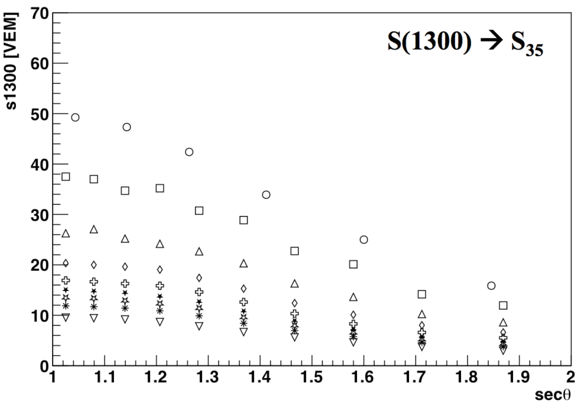

Figure 7. Attenuation curves of $S(1300)$ for different intensities.

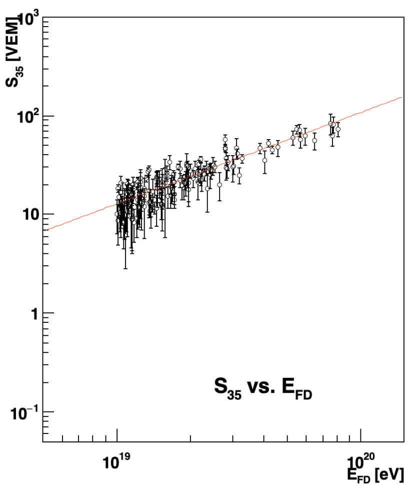

Figure 8. Energy calibration of S_{35} derived from $S(1300)$.

It has been checked that the two energy estimations are in fair agreement with the standard TA energy calculation ($E_{\rm standard}$) that is used to measure the energy spectrum. The distribution of $\ln(E_{\rm standard}/E(S_{800}))$ has a mean value of −4.2% and an RMS of 9.4%, while for

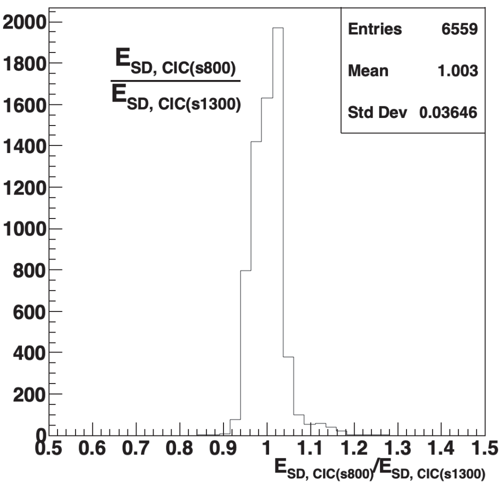

Figure 9. Comparison among the energies reconstructed starting from $S(800)$ and $S(1300)$.

$\ln(E_{\rm standard}/E(S_{1300}))$ the mean is −3.9% and the RMS is 8.2%. It is interesting to note that $E_{\rm standard}$ is obtained from S_{800} but the RMS is smaller when compared with the energy estimation performed at $r_{\rm opt}$ ($E(S_{1300})$). This demonstrates some effectiveness of the MC lookup table in providing good performances in the energy reconstruction. However, no firm conclusions useful to explain the difference between the TA and Auger spectra can be derived from the comparison of $E_{\rm standard}$ with the CIC based energy estimations, as the performances of their energy calibration are limited by the small hybrid statistics above 10^{19} eV and resolution effects (see figure 10).

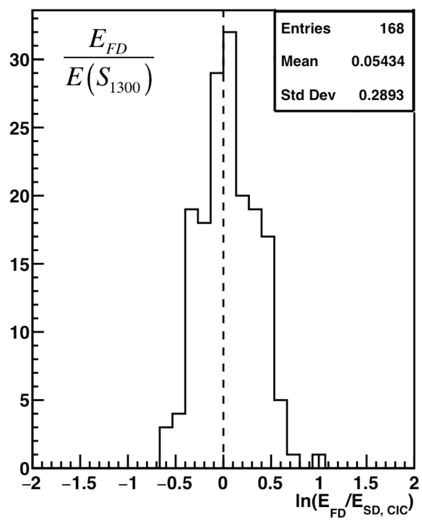

Figure 10. Ratio of the SD to FD energies for the TA hybrid events above 10^{19} eV. The SD energy is estimated from $S(1300)$.

We conclude this section showing in figure 11 another measurement of the energy spectrum obtained using the Auger SD events inclined at large zenith angles ($\theta > 60°$) [24]. These events require a completely different and more complex reconstruction technique [25] with respect to the one used for the vertical events with $\theta < 60°$ [23]. In fact, at large zenith angles, the signals are dominated by muons that make long paths in the atmosphere and the geomagnetic field, separating laterally positively and negatively charged particles, destroys the circular symmetry of the LDF making impossible to define an energy estimator like $S(1000)$. The energy estimator is a normalization factor of simulated muon density maps that are parametrized as function of the zenith and azimut angles of the shower. In this way the attenuation effects are estimated using MC simulations, contrary to what is done for $S(1000)$ where a data-driven approach (CIC) is used. The normalization factor is then fitted to the data and, like for S_{38}, it is calibrated against the FD energies using a power law relationship. As can be seen in figure 11, the Auger spectra obtained with inclined and vertical events agree rather well and this suggests that the systematic uncertainties related to the shower size reconstruction are well under control.

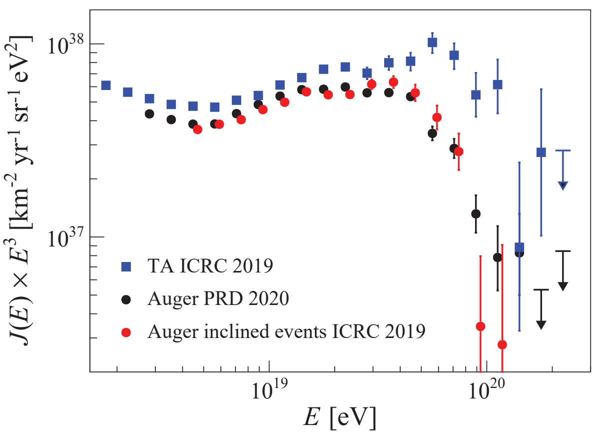

Figure 11. Auger energy spectrum obtained from the events inclined at large zenith angles ($\theta > 60°$) [24] compared with the measurements discussed in this proceeding.

5 Summary and future perspectives

We have reviewed and compared the results of the energy spectra measured at the Pierre Auger Observatory and at the Telescope Array. It has been shown that there is a good agreement among the spectral features, including the *instep* one that has been discovered only recently, modulo a difference in the energy scales that can be inferred from the different normalization of the fluxes.

The energy offset needed to explain the differences in the flux up to 10^{19} eV is 9.5% and it is fully consistent with the uncertainties in the energy scales. Moreover, the offset can be significantly reduced to a value below 5% if the two experiments would use the same model for the fluorescence yield and invisible energy. The fluorescence yield and the invisible energy can be considered as a sort of external parameters of the reconstruction of FD events and the choice of the particular model by the two Collaborations is sometimes made for historical reasons. The convergence toward the use of the same models in both experiments would have obvious benefits for both the Collaborations and the community.

The understanding of the offset between the spectra at energies above 10^{19} eV is much more complicated. In addition to the 9.5% energy shift, it is necessary to introduce an energy dependent energy shift of 20%/decade. The analysis is performed in the declination band visible by both experiments ensuring that the observed differences are caused by instrumental effects and are not of astrophysical origin. We have excluded that the energy dependent energy shift is due to a statistical fluctuation with a significance of 3σ. An even higher significance has been obtained with the larger data set used by the joint WG on the study of the arrival directions of CRs [26].

We have presented an in-depth discussions of the systematic uncertainties without finding anything useful to explain the 20%/decade energy shift. The matter is very complicated because the energy dependence of the systematic uncertainties can be related to subtle details of the event reconstruction and of the calibration procedure, details that are on some extent difficult to understand and to quantify.

We have shown that in the case of Auger it is very important to understand the energy dependence of the FD systematics. The calibration fit is performed using a power law relationship and the analysis benefits of a large enough hybrid statistics and the good energy resolution with the result that the SD energies are well aligned to the FD ones. The energy dependence of the FD uncertainties are in general quite small and can't explain the 20%/decade energy shift.

We have seen that the TA SD energy estimator is the shower energy obtained from a MC lookup table and for its calibration it is enough to consider only an energy-independent rescaling factor. By construction, the procedure can't ensure a perfect alignment of the SD energies to the FD ones over the full energy range and therefore the energy dependence of the FD systematics are less important in comparison to the Auger analysis. We have verified that potential non linearity effects in the SD energy estimator arising from the fact that the shower size is not estimated at the distance from the core that minimizes the uncertainties in the LDF are quite small. The test has been performed using the data-driven approach of Auger (CIC and energy calibration with a power law) and the resulting energies have been found in fairly agreement with the ones obtained using the MC lookup table. However, no firm conclusions useful to understand the 20%/decade energy shift can be drawn as the performances of the energy calibration fit are on some extent limited by the small TA hybrid statistics and resolution effects.

Finally, for the first time we have discussed another measurement of the spectrum performed by Auger with the events inclined at large zenith angles. The measurement agree quite well with the one performed using the vertical events (zenith angles < 60°) used for the comparison with the TA spectrum. The reconstruction of vertical and inclined events are quite different and have a different sensitivity to several physical and instrumental effects (e.g. different sensitivity of the energy estimator to the primary mass and no saturated stations in inclined events). Therefore, the good consistency between the spectra provides an indication that the systematics related to the SD reconstruction are well under control.

The joint WG is strongly engaged into continue the activities to understand the differences between the spectra. This work is of primary importance because the combination of Auger and TA data allows us to study UHECRs with full sky coverage, and a proper combination of the data requires the cross-calibration of the energy scales and therefore the understanding of their systematic uncertainties.

An interesting perspective to continue and improve the joint activity is to make the comparison in the common band including the Auger spectrum obtained with the inclined events. This will allow the common band to be extended up to the declination of 44.8° (from the current 24.8°) and therefore to increase significantly both the Auger and TA statistics. The statistics can be also increased adding more years of data taking. Concerning the energy scale it is important that the Collaborations will refine their study of the energy dependence of the systematic uncertainties associated to both the FD ans SD reconstruction. Another interesting perspective is to refine the techniques used for the energy calibration fit in order to address more precisely the consistency between SD and FD energies and to facilitate the comparison of the energy scales. On a longer time scale, a significant improvement of the joint analyses will be possible thanks to the current upgrade of the observatories, with the scintillators of AugerPrime [27] that are installed on the top of the water Cherenkov detectors, and with the larger exposure of TA×4 [28].

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