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REVIEW OF MAD-X FOR FCC-EE STUDIES*

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Abstract

The design of the e^+e^- Future Circular Collider (FCC-ee) challenges the requirements on optics codes such as MAD-X in terms of accuracy, consistency, and performance. Traditionally, MAD-X uses a transport formalism by expanding the transfer map about the origin up to second order to compute optics functions and synchrotron radiation (SR) integrals in the TWISS and EMIT modules. Conversely, the TRACK module uses symplectic maps to propagate particles. These approaches solve the same problem using different approximations, resulting in some cases in a mismatch between the models used for tracking and for optics and hence in the results when SR is taken into account. While in a machine like LHC these differences are not relevant, for the FCC-ee, given the size of the machine, the enormous number of elements and the high-energies to be reached the SR effect has to be taken into account. For instance, a "tapering" strategy for the magnets that matches the tunes for optics would mismatch the tune in tracking and vice versa. In this paper, we report about the implementation on MAD-X of an advanced method to close the gap between the maps used for optics and tracking in view of FCC-ee studies.

INTRODUCTION

The FCC-ee is an e^+e^- circular collider with an energy in the centre of mass ranging from 45.6 GeV (Z pole) up to 175-182.5 GeV ($t\bar{t}$ threshold) with a circumference of 91 km [1]. MAD-X [2] is one of the codes used to perform optics design for the FCC-ee ring, e.g. for the optics tuning simulations [3, 4].

Energy loss due to synchrotron radiation (SR) and energy gain in RF cavities introduce orbit and optics (beta-beating, tune, chromaticity) errors because the effect of the magnets on the beam depends on local energy deviations. One possible method to mitigate this, is to adjust the strength of the main magnets (dipoles, quadrupoles, sextupoles, octupoles) to the local beam energy [5, 6]. This method is called "tapering" [7] and it has been implemented in MAD-X in view of FCC-ee studies.

MAD-X has evolved from MAD8 [8], keeping the transport matrix method for the TWISS module which is used for optics calculation, while adopting symplectic tracking in the TRACK module which is used for particles tracking (not to be confused with the PTC-TRACK module [9]). It became evident that the differences in accuracy of the TWISS and TRACK calculation is an important aspect to consider. In the work presented in this paper, we have compared different existing versions of MAD-X (5.07.00 and 5.09.00), each one with different methods to calculate the tapered transfer matrix, with a new version named "NewQ+NewS", in which we have implemented a new method to compute the tapering. In particular, we have evaluated the impact on the tunes on the FCC-ee lattice.

TAPERED LATTICE PROPERTIES

In general, the "tapering" procedure transforms the magnet strengths to preserve $x(s)$ and $y(s)$ when $p_t \neq 0$ with $p_t = \frac{E-E_0}{P_0 c}$. This also implies that $x'(s) = \frac{\partial H}{\partial p_x} \approx \frac{p_x}{1+\delta}$ and $y'(s) = \frac{\partial H}{\partial p_y} \approx \frac{p_y}{1+\delta}$ are preserved, where H is the magnet's Hamiltonian and the approximation holds on straight paths. Writing the on-momentum linear transport equation of coordinates x, p_x for the magnet (for example dipole, quadrupole or sextupole), we obtain:

$$x^f = R_{11}(k, p_t = 0)x^i + R_{12}(k, p_t = 0)p_x^i \,,$$
$$x'^f = p_x^f = R_{21}(k, p_t = 0)x^i + R_{22}(k, p_t = 0)p_x^i \,,$$

where R is the magnet's transfer matrix, x^f, p_x^f are the final coordinates, x^i, p_x^i are the initial coordinates and we use $p_x = x'$ when $p_t = 0$. Similarly, for off-momentum particles due to SR and after "tapering", we obtain:

$$\tilde{x}^f = R_{11}(\tilde{k}, \tilde{p}_t)x^i + R_{12}(\tilde{k}, \tilde{p}_t)(1+\tilde{\delta})\frac{\tilde{p}_x^i}{1+\tilde{\delta}} \,,$$
$$\tilde{x}'^f = \frac{\tilde{p}_x^f}{1+\tilde{\delta}} = \frac{R_{21}(\tilde{k}, \tilde{p}_t)}{1+\tilde{\delta}}\tilde{x}^i + R_{22}(\tilde{k}, \tilde{p}_t)\frac{\tilde{p}_x^i}{1+\tilde{\delta}} \,,$$

where $\tilde{x}, \tilde{x}' = \frac{\tilde{p}_x}{1+\delta}$ are the coordinates in the tapered lattice. By imposing $x = \tilde{x}$ and $x' = \tilde{x}'$, one gets the following conditions on the R-matrix:

$$\tilde{R}_{11} = R_{11}(\tilde{k}, \tilde{p}_t) = R_{11}(k, 0)$$
$$\tilde{R}_{12} = R_{12}(\tilde{k}, \tilde{p}_t) = \frac{R_{12}(k, 0)}{1+\delta}$$
$$\tilde{R}_{21} = R_{21}(\tilde{k}, \tilde{p}_t) = R_{21}(k, 0)(1+\delta)$$
$$\tilde{R}_{22} = R_{22}(\tilde{k}, \tilde{p}_t) = R_{22}(k, 0) \,,$$

and by extension the phase advance,

$$\cos\mu_x = \frac{R_{11} + R_{22}}{2} = \frac{\tilde{R}_{11} + \tilde{R}_{22}}{2} = \cos\tilde{\mu}_x \,.$$

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This will be used to test whether the "tapering" procedure fulfils the goal to obtain the same phase advance with and without SR. Similar conditions can also be obtained for the vertical plane.

LINEAR OPTICS CALCULATIONS WITH MOMENTUM OFFSETS

SR introduces p_t offsets in the closed orbit. MAD-X TWISS computes linear transfer matrix terms as:

$$R_{ij}(k,\vec{Z}) = \frac{\partial M_i(k,\vec{Z})}{\partial Z_j} \approx R_{ij}(k,\vec{0}) + \sum_l T_{ijl}(k,\vec{0})Z_l, \quad (1)$$

with $T_{ijl} = \frac{1}{2}\frac{\partial^2 M_i}{\partial Z_j Z_l}$, where $\vec{Z}$ represents the 6 particle coordinates (x, p_x, y, p_y, t, p_t) and k the parameters of the map such as normalized strength, length, δ_s, and so on. In particular, $\vec{Z} = \vec{0}$ is the closed orbit. This implies that R computed by TWISS is correct to first order in the closed orbit coordinates. MAD-X uses this approximation in the TWISS module for dipoles and quadrupoles [10]. An exact calculation of the R matrix has been implemented in version 5.09.00 only for solenoid and drift types of elements by using the EXACT option.

In this paper, we propose a new method of calculation of the tapered matrix in MAD-X for quadrupoles and sextupoles. The orbit tracking with tapered transfer matrices both in the TRACK and TWISS modules has also been reviewed. In TRACK, only the orbit of the particles is propagated by using symplectic maps, while in TWISS, the orbit and its first and second order derivatives are tracked by using expanded maps up to second order. Tests on the calculation of tunes were conducted in the FCC-ee $t\bar{t}$ optics V.22 [11] with two different versions of MAD-X and with the new "NewQ+NewS" version (see Table 1). Given the fact that TRACK does not compute the phase advance directly, we tracked the coordinates around the closed to orbit to find the one-turn map by finite differences, and obtained the tunes through the eigenvalues.

MAD-X 5.07.00

In this version, the following approximation is used to calculate the off-momentum transfer map:

$$R_{ij}(k, p_t) = R_{ij}(k, 0) + T_{ij6}(k, 0)p_t\,. \quad (2)$$

For example, in a thin quadrupole, assuming $\beta_0 = 1$, we get: $R_{21}(k, 0) = -k$ and $T_{216}(k, 0) = +k$. The relation (2) used to determine k_{new} for "tapering" is:

$$k_{\text{new}} = \frac{k}{1 - p_t}. \quad (3)$$

This approximated expression is used to compensate exactly the effect of the energy deviation, which gave the desired results in TWISS after "tapering". However, it generates an inaccurate model in TRACK, which performs tracking via symplectic transfer maps. This effect is shown in Table 1 comparing FCC-ee $t\bar{t}$ optics V.22 tunes without and with "tapering". We can see that for MAD-X 5.07.00 the results with and without radiation match in both the TWISS and EMIT modules, but not for the TRACK module.

MAD-X 5.09.00

In this version, the equation used for "tapering" is (still assuming $\beta_0 = 1$ for simplicity):

$$R_{21} = \frac{k_{new}}{1 + p_t} \Leftrightarrow k_{new} = k(1 + p_t)\,. \quad (4)$$

This recipe improves the tune difference with and without "tapering" in the TRACK module, at the expense of TWISS, which now suffers from this approximated calculation. The EXACT option, used in the drifts, improves the TWISS accuracy as expected, but not fully, hinting that additional elements should have the EXACT option implemented, such as quadrupoles, sextupoles and dipoles.

NewQ+NewS

The method investigated in this work for quadrupoles and sextupoles consists of a transformation of coordinates from $Z \to \bar{Z} \to Z$:

$$\begin{array}{ccccc} Z & \longrightarrow & \text{TWISS} & \longrightarrow & Z, E, R, T \\ \downarrow & & & & \uparrow \\ \bar{Z} & \longrightarrow & \text{TWISS} & \longrightarrow & \bar{Z}, \bar{E}, \bar{R}, \bar{T} \end{array} \quad (5)$$

such that $p_t = 0$ and reuse the same TWISS calculation (without approximation) to compute the terms $\bar{Z}, \bar{E}, \bar{R}, \bar{T}$ used to transport the optics, where the matrix E corresponds to the initial kick, which is a constant part of the map:

$$Z_i^f = E_i + \sum_j \left(R_{ij} + \sum_l T_{ijl} Z_l \right) Z_j. \quad (6)$$

We recall the definition of the canonical coordinates used in MAD-X and the scaled magnetic field:

$$P_s = P_0(1 + \delta_s) \qquad E_s = \beta_s \gamma_s m_0 c^2 \quad (7)$$

$$k_n = \frac{q}{P_0}\frac{\partial B_y^n}{\partial x^n} \qquad \bar{k}_{n,s} = \frac{k_n}{1 + \bar{\delta}_s} \quad (8)$$

$$p_{x,y} = \frac{P_{x,y}}{P_s} \qquad p_t = \frac{E - E_s}{cP_s} \quad (9)$$

$$1 + \delta = \sqrt{p_t^2 + 2p_t/\beta_s + 1} \qquad t = \frac{s(1 + \eta\delta_s)}{\beta_s} - cT\,, \quad (10)$$

where E_s and P_s is the reference energy and momentum of the particle, respectively. In order to obtain $\bar{p}_t = 0$, we choose a $\bar{\delta}_s$ such that $\bar{P}_s = P$ and we obtain:

$$\bar{P}_s = P_0(1 + \bar{\delta}_s) = P = P_s(1 + \delta) = P_0(1 + \delta_s)(1 + \delta) \quad (11)$$

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Table 1: Comparison of tunes for FCC-ee $t\bar{t}$ optics V.22, calculated with different MAD-X versions with the TWISS, EMIT and TRACK modules

	RAD=OFF			TAPER=ON EXACT=OFF			TAPER=ON EXACT=ON		
	Q_x	Q_y	Q_s	Q_x	Q_y	Q_s	Q_x	Q_y	Q_s
5.07.00									
TWISS	0.2240	0.3600	-	0.2240	0.3593	-	-	-	-
EMIT	0.2240	0.3600	0.101	0.2240	0.3600	0.0815	-	-	-
TRACK	0.2240	0.3600	0.101	0.1797	0.3942	0.0833	-	-	-
5.09.00									
TWISS	0.2240	0.3600	-	0.2153	0.3509	-	0.2168	0.3523	-
EMIT	0.2240	0.3600	0.101	0.2153	0.3515	0.0815	0.2168	0.3530	0.0815
TRACK	0.2240	0.3600	0.101	0.2236	0.3588	0.0815	0.2236	0.3588	0.0815
NewQ+NewS									
TWISS	0.2240	0.3600	-	0.2153	0.3509	-	0.2175	0.3527	-
EMIT	0.2240	0.3600	0.101	0.2153	0.3515	0.0815	0.2175	0.3534	0.0815
TRACK	0.2240	0.3600	0.101	0.2236	0.3588	0.0815	0.2236	0.3588	0.0815

implying

$$(1+\bar{\delta}_s) = (1+\delta_s)(1+\delta)\,, \tag{12}$$

and therefore :

$$\bar{x},\bar{y} = x, y \tag{13}$$

$$\bar{p}_{x,y} = \frac{P_{x,y}}{\bar{P}_s} = \frac{p_{x,y}P_s}{\bar{P}_s} = \frac{p_{x,y}}{1+\delta}, \tag{14}$$

$$\bar{p}_t = \frac{E-\bar{E}_s}{c\bar{P}_s} = 0\,, \tag{15}$$

Finally, we redefine E_i, R_{ij}, T_{ijk} as a function of $\bar{E}_i, \bar{R}_{ij}, \bar{T}_{ijk}$:

$$E_i = Z_i^f(Z_1, ..., Z_6 = 0) = Z_i(\bar{Z}_i = \bar{E}_i) \tag{16}$$

$$R_{ij} = \frac{\partial Z_i^f}{\partial Z_j} = \frac{\partial Z_i^f}{\partial \bar{Z}_i^f}\frac{\partial \bar{Z}_i^f}{\partial \bar{Z}_j}\frac{\partial \bar{Z}_j}{\partial Z_j} = \frac{\partial Z_i^f}{\partial \bar{Z}_i^f}\frac{\partial \bar{Z}_j}{\partial Z_j}\bar{R}_{ij} \tag{17}$$

$$T_{ijl} = \frac{\partial^2 Z_i^f}{\partial Z_j Z_l} = \frac{\partial Z_i^f}{\partial \bar{Z}_i^f}\frac{\partial \bar{Z}_j}{\partial Z_j}\frac{\partial \bar{Z}_l}{\partial Z_l}\bar{T}_{ijl}, \tag{18}$$

where

$$\frac{\partial \bar{Z}_i}{\partial Z_i} = \begin{cases} 1 & \text{for } i = 1,3,5 \\ \frac{1}{1+\delta} & \text{for } i = 2,4,6\,. \end{cases} \tag{19}$$

For instance, for R, this implies:

$$R_{11} = \bar{R}_{11} \qquad R_{12} = \frac{\bar{R}_{12}}{1+\delta} \tag{20}$$

$$R_{21} = \bar{R}_{21}(1+\delta) \qquad R_{22} = \bar{R}_{22} \tag{21}$$

$$R_{33} = \bar{R}_{33} \qquad R_{34} = \frac{\bar{R}_{34}}{1+\delta} \tag{22}$$

$$R_{43} = \bar{R}_{43}(1+\delta) \qquad R_{44} = \bar{R}_{44} \tag{23}$$

$$R_{16} = \frac{\bar{R}_{16}}{1+\delta} \qquad R_{26} = \bar{R}_{26} \tag{24}$$

$$R_{36} = \frac{\bar{R}_{36}}{1+\delta} \qquad R_{46} = \bar{R}_{46} \tag{25}$$

$$R_{55} = \bar{R}_{55} \qquad R_{56} = \frac{\bar{R}_{56}}{1+\delta}\,. \tag{26}$$

When using this approach, there is a small improvement as shown in Table 1. Indeed, the results with radiation and the EXACT option (column 3) for the modules TWISS and EMIT are closer to the ones without radiation (column 1) than in the older MAD-X version. The results in EMIT are a benchmark for our model, since the calculation of the tapered transfer matrix are the same in TWISS and EMIT, but adding the longitudinal plane in the latter module. We see that without SR, the results are correct, and those are the working points we are trying to reach when SR is ON. Since on single elements the version "NewQ+NewS" works as expected, the difference in tune with TRACK is expected to come from the approximations in the dipoles and their edges, where the new method has not been implemented.

CONCLUSION AND NEXT STEPS

In the FCC-ee lattice we observed discrepancies between the TWISS and TRACK MAD-X calculation modules, in particular leading to significant differences in the working point when SR is added to the model. We conclude that the difference is due to second order p_t errors in the TWISS calculation since the TWISS calculation gives perfectly tapered lattices when magnets are corrected only to first order in p_t, while TRACK does not return the correct working point. Conversely, when magnets are tapered using the exact equation, the TRACK model gives more accurate working points. We used an improved off-momentum calculation of TWISS in drift, quadrupole and sextupoles, to obtain a small reduction in the discrepancies which is promising. The results, however, indicate that an improved TWISS calculation is also needed for dipoles and their edges.

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