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sous la direction de Ignatios ANTONIADIS

présentée par

Chrysoula MARKOU

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DOCTEUR DE LA SORBONNE UNIVERSITÉ

Sujet de la thèse :

Supersymétrie $\mathcal{N} = 2$: réalisations non–linéaires, brisure spontanée et dimensions supplémentaires

soutenue le 29 juin 2018

devant le jury composé de :

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I – Introduction

For a few decades, supersymmetry has been subjected to extensive theoretical and phenomenological research in the field of high energy physics. Initially discovered as a possible non–trivial extension of the Poincaré symmetry of interacting quantum field theories, it has been met with considerable enthusiasm largely due to its several appealing characteristics, such as the facts that it offers a stabilization of the hierarchy problem and that it predicts the existence of new elementary particles, some of which hold the status of dark matter particle candidates. Other than that, it has been been found to be a necessary ingredient of the worldsheet construction of consistent string theories, with string theory, as a framework, attracting significant scientific attention in its own right as a possible theory of quantum gravity. Nevertheless, as of today, there has been no experimental evidence that supersymmetry exists in nature. In this regard, new insights might be gained via a re–examination of the breaking of supersymmetry, especially at low energy scales, given that the Large Hadron Collider at CERN has already probed the range of several TeV.

Spontaneously broken $\mathcal{N}=1$ supersymmetry is nonlinearly realized at low energies. A simple way to realize such a scenario is to consider a chiral superfield X with components (x, G, F), which correpond to a complex scalar, a fermion and an auxiliary complex scalar respectively. At low energies, x becomes supermassive and decouples from the spectrum, or, equivalently, a nilpotent constraint is imposed on X [1, 2, 3, 4]

$$X^2 = 0\,, \tag{I.1}$$

whose solution is

$$x \sim \frac{G^2}{F}\,. \tag{I.2}$$

The constraint (I.1) thus eliminates x as a function of G, so that the final spectrum consists of a single physical field, the fermion G. A general Lagrangian for the constrained X can then be written with use of a Kähler potential and a superpotential that are quadratic and linear in X respectively. Upon substituting for F via its equation of motion, one obtains a Lagrangian for G that is on–shell [1, 5] the Volkov–Akulov Lagrangian [6] and (the action corresponding to) it is invariant under the nonlinear transformation of G, which is a remnant of the $\mathcal{N}=1$ invariance; G is thus identified with the Goldstino particle. To generalize this description, supersymmetry is said to be nonlinearly realized when there is a linear combination of fermions of the original theory that transforms nonlinearly and the fermionic and the bosonic degrees of freedom of the final spectrum are unequal in number, as is the case in the example of the constrained X.

Nonlinear realizations of supersymmetry have underwent substantial study in recent years. To begin with, to accommodate couplings of X to other incomplete $\mathcal{N}=1$ multiplets, further contraints involving X and the latter have been proposed [4, 7], all of which have been shown to arise from a single constraint imposed on X and another chiral multiplet, upon suitable choice of the latter [7]. However, the constraint (I.1) does not universally describe the Goldstino multiplet at low energies: cases in which the Goldstino chiral multiplet satisfies a weaker constraint, such as $X^3 = 0$, in the presence of matter multiplets, have been discovered [8, 9, 10].

Furthermore, the constraint (I.1) has been used to embed the Starobinsky model of $R+R^2$ gravity [11], whose linearized version [12] is interestingly a model for single–field inflation currently favoured by PLANCK data, in $\mathcal{N} = 1$ supergravity [13], in such a way that $\mathcal{N} = 1$ supersymmetry is nonlinearly realized during the inflationary phase [14]. In addition, in [15], we couple the nilpotent Goldstino multiplet to $\mathcal{N} = 1$ supergravity, which gives rise to the super–Brout–Englert–Higgs mechanism, and show that the geometric formulation of the coupling is pure $\mathcal{N} = 1$ supergravity with its chiral curvature superfield $\mathcal{R}$ satisfying the constraint

$$(\mathcal{R} - \lambda)^2 = 0\,, \tag{I.3}$$

where λ is an appropriate parameter, the special case $\lambda = 0$ of which was used in [14]. We also show how the nilpotent constraint arises in a class of modified supergravity models of the $f(\mathcal{R})$ type at the low energy limit.

Another instance of nonlinear supersymmetry finds itself in particular vacua of type I string theory, when D–branes are combined with anti–orientifold planes. In such setups there exist a massless Goldstino in the open string spectrum but not the superpartners of brane excitations [16, 17, 18, 19], so that supersymmetry is nonlinearly realized without taking the low–energy limit. The string–theoretic origin of various $\mathcal{N} = 1$ superfield constraints in terms of component fields has been widely under investigation, see for example [20, 21, 22, 23], but a complete classification has not yet been given.

Nonlinear realizations may also appear in the case of $\mathcal{N} = 2$ supersymmetry. Due to the $SU(2)$–automorphism of its algebra, $\mathcal{N} = 2$ may be viewed as a set of two $\mathcal{N} = 1$ supersymmetries. If $\mathcal{N} = 2$ is broken at two different energy scales, then one has to consider first the partial breaking $\mathcal{N} = 2 \to \mathcal{N} = 1$. This breaking is induced by means of "electric" [24] *and* "magnetic" Fayet–Iliopoulos terms introduced at the Lagrangian level of a theory of an $\mathcal{N} = 2$ Maxwell multiplet $\mathcal{W}$ [25]. In terms of $\mathcal{N} = 1$ superfields, the components of $\mathcal{W}$ are a chiral multiplet X and a spinor multiplet W, which is the field–strength of a vector multiplet; X and W transform to each other via the second $\mathcal{N} = 1$ supersymmetry. The coefficient of the magnetic FI term is a constant parameter that appears as

- a deformation of the superfield transformations under the second (upon an $SU(2)$–rotation) $\mathcal{N} = 1$ supersymmetry, such that the closure of the corresponding algebra is maintained [26, 27, 28]
- a deformation of $\mathcal{W}$ itself, yielding a deformed Maxwell multiplet $\mathcal{W}_{def}$ [29].

The first $\mathcal{N} = 1$ supersymmetry remains intact, so that the final spectrum is organized in multiplets of it, while the second $\mathcal{N} = 1$ is nonlinearly realized, with the corresponding Goldstino identified with the fermion whose transformation in the vacuum is proportional to the deformation parameter. It is thus reasonable to refer collectively to a set of two $\mathcal{N} = 1$ supersymmetries, one of which is linearly and the other nonlinearly realized, as nonlinear

$\mathcal{N}=2$ supersymmetry. In the context of string theory, nonlinear $\mathcal{N}=2$ supersymmetry appears in the case of D–branes in an $\mathcal{N}=2$ bulk.

At this point, it is important to highlight a qualitative difference between the breaking $\mathcal{N}=1 \rightarrow \mathcal{N}=0$ and the partial breaking $\mathcal{N}=2 \rightarrow \mathcal{N}=1$. In the first case, the breaking is generated by the vacuum expectation values of auxiliary fields, which can be either the complex scalar F of a chiral multiplet, or the real scalar D of a vector multiplet [30]. In the second case, F, its conjugate, and D also appear as the auxiliary fields of the Maxwell superfield $\mathcal{W}$, but in the form of an $SU(2)$–triplet. Due to this restriction, to generate the partial breaking, magnetic parameters have to be introduced by hand and *cannot* be absorbed in vacuum expectation values of the auxiliary fields. Moreover, there are three possible $\mathcal{N}=1$ multiplets in which the Goldstino may reside: apart from the chiral and the vector [25, 26, 31], it may also be part of the linear multiplet L [32], which additionaly contains a real scalar and the field–strength of an antisymmetric tensor and, together with a chiral superfield Φ, is naturally part of an $\mathcal{N}=2$ single–tensor multiplet $\mathcal{Z}$ [33, 34, 35, 36].

Now let us return to the partial breaking with the use of $\mathcal{W}$. At low energies, X becomes supermassive and decouples from the spectrum, or, equivalently, a nilpotent constraint is imposed on $\mathcal{W}_{def}$ [28]

$$\mathcal{W}_{def}^2 = 0\,, \tag{I.4}$$

whose lowest component is in fact a constraint of the type (I.1). The solution of (I.4) determines X as a function of superspace derivatives of W [26] and the Lagrangian of the remaining W contains, interestingly, the Born–Infeld Lagrangian of a D3–brane with gauge coupling and tension that depend on the electric and the magnetic parameters respectively [26, 28]. Upon coupling the constrained $\mathcal{W}_{def}$ to an $\mathcal{N}=2$ single–tensor multiplet, it has been discovered [29] that a novel version of the super–Brout–Englert–Higgs mechanism *without* gravity takes place, via which W becomes massive by absorbing the linear multiplet and the Born–Infeld Lagrangian aquires a coefficient depending on Φ. Importantly, since $\mathcal{W}$ and $\mathcal{Z}$ have opposite chiralities under the second $\mathcal{N}=1$ supersymmetry, the coupling is formulated with the use of a "long" representation $\widehat{\mathcal{Z}}$ of the single–tensor mutiplet, which admits a gauge transformation with another Maxwell multiplet playing the role of the corresponding gauge parameter [29].

In [37], motivated by the above results, we formulate a new partial breaking mechanism by means of a single–tensor multiplet, or, upon dualization via a Legendre transformation, a hypermultiplet with a shift symmetry. Note that the hypermultiplet does not enjoy an (off–shell) superfield description in the standard $\mathcal{N}=2$ superspace that we use, but instead in harmonic superspace [38, 39]. We also formulate the mechanism with the use of several single–tensor multiplets. In addition, we study the most general magnetic deformations of $\mathcal{W}$ and $\mathcal{Z}$ and derive a simple criterion as to when partial breaking occurs. We determine the infinite–mass limit in which a nilpotent constraint is imposed on $\mathcal{Z}_{def}$

$$\mathcal{Z}_{def}^2 = 0\,, \tag{I.5}$$

whose lowest component is again a constraint of the type (I.1). To couple $\mathcal{Z}_{def}$ to the Maxwell multiplet, we construct a new and "long" representation $\widehat{\mathcal{W}}$ of the latter, which admits two gauge transformations whose gauge parameters are two other independent single–tensor multiplets. Moreover, we show that deformations of long representations can only induce total or no breaking, but never partial, which implies that the only couplings relevant to nonlinear $\mathcal{N}=2$ supersymmetry are $\mathcal{N}=2$ superspace integrals of

$$\mathcal{W}_{def}\widehat{\mathcal{Z}} \quad , \quad \mathcal{Z}_{def}\widehat{\mathcal{W}} \tag{I.6}$$

As the former was explored in [29], we study the latter and find that it results again in a super–Brout–Englert–Higgs mechanism without gravity, which comes with the subtlety that the real auxiliary field D in $\mathcal{W}$ is replaced by the divergence of a vector field in $\widehat{\mathcal{W}}$. Finally, we investigate further $\mathcal{N}=2$ contraints that describe incomplete matter multiplets.

These advances could also be relevant to the partial breaking in $\mathcal{N}=2$ supergravity. In particular, the pure $\mathcal{N}=2$ supergravity multiplet consists of the graviton, two gravitinos and a vector commonly referred to as the graviphoton. It has namely the following spin content

$$(2, 3/2, 3/2, 1) \tag{I.7}$$

so that after the partial breaking the spectrum consists of the (massless) $\mathcal{N}=1$ pure supergravity multiplet

$$(2, 3/2) \tag{I.8}$$

and a massive spin–3/2 $\mathcal{N}=1$ multiplet

$$(3/2, 1, 1, 1/2) \tag{I.9}$$

which has become massive by combining with a vector, two fermions and two scalars [40]. These Goldstone modes are precisely the degrees of freedom of the $\mathcal{N}=1$ chiral and vector multiplets. As would–be Goldstone bosons, the scalars are associated with two shift symmetries. These observations have led to the *on–shell* formulation of the partial breaking in $\mathcal{N}=2$ supergravity, with the use of (at least) a Maxwell multiplet and a hypermultiplet that has two commuting isometries; the breaking is induced by means of a gauging, that is $U(1)$ in the minimal case, of the isometries of the hypermultiplet scalar manifold [41, 42, 43] and more recently [44, 45, 46]. At the moment, we are investigating [47] the off–shell generalization of these results, as well as the interactions of the Goldstone degrees of freedom of the massive spin–3/2 multiplet, for which the progress made in [29, 37] might be of use.

A $U(1)$ gauging, but in the simplest case of the $U(1)$ subgroup of the $SU(2)$–automorphism of $\mathcal{N}=2$ and not necessarily of the isometries of a scalar manifold, may induce partial breaking in supergravity in five dimensions. We are interested in this possibility in light of the recently proposed clockwork mechanism [48, 49, 50], further devoloped in [51, 52, 53, 54] followed by a growing literature in regard to applications, which is a novel way of generating an exponential scale hierarchy. In its continuum version, the clockwork spacetime has five dimensions, and, interestingly, its metric is identical to the metric of a 5D spacetime, in which a real scalar has a dilaton coupling to five–dimensional gravity and a runaway potential, and its background value is linear in the extra dimension. Remarkably, this linear dilaton model is a 5D toy model [55] of the holographic dual [56, 57, 58, 59] of 6D Little String Theory [60, 61], which is obtained from type IIB string theory in the limit

$$g_S \to 0 \, , \tag{I.10}$$

where g_S is the string coupling, namely the exponential of the dilaton background value. Since the holographic dual preserves bulk spacetime supersymmetry, the effective supergravity of the 5D toy model must be in the simplest scenario $\mathcal{N}=2$, $D=5$.

This is precisely the subject matter of [62], where we show that gauged $\mathcal{N} = 2$, $D = 5$ supergravity can accomodate the 5D linear dilaton model, see also [63]. In particular, we consider pure $\mathcal{N} = 2$, $D = 5$ supergravity [64, 65, 66] coupled to one vector multiplet, which contains a vector in 5D, a symplectic fermion $SU(2)$–doublet and a real scalar [67]. The gauging of the $U(1)$ subrgoup of $SU(2)$ generates a scalar potential, as well as fermion masses and interactions [68, 69]. We find that the potential of the real scalar is precisely the runaway dilaton potential for a specific choice of the parameters of the model and demonstrate that supersymmetry is partially broken in the linear dilaton background, such that $\mathcal{N} = 1$ is preserved on 4D slices (namely at fixed values of the extra dimension) of the 5D spacetime; we also give the final expression of the total Lagrangian. The linear dilaton coefficient is now proportional to the $U(1)$ gauge coupling g and, using the results of [55], according to which the Kaluza–Klein spectrum of the relevant 5D fields exhibits a mass gap followed by a near continuum, we observe that the mass gap is also proportional to g. Moreover, in [70], in view of phenomenological applications, we compactify the extra dimension and introduce one brane at each of the boundaries, one of which may accomodate the Standard Model. Interestingly, we find that the presence of the branes is compatible with the direction of the $\mathcal{N} = 1$ supersymmetry that remains unbroken after the gauging and does not break it further.

To conclude, the research conducted towards this thesis may be found in the following publications

- I. Antoniadis and C. Markou, *The coupling of Non-linear Supersymmetry to Supergravity*, Eur. Phys. J. C **75** (2015) no.12, 582,

  which is given as is in the appendix F, as part of it was developed during the Master's thesis of C. Markou

- I. Antoniadis, J. P. Derendinger and C. Markou, *Nonlinear $\mathcal{N} = 2$ global supersymmetry*, JHEP **1706** (2017) 052,

  which corresponds to (the whole or large part of) sections III.6, III.7, IV.2, IV.3, IV.4, IV.5, IV.6, as well as appendices B and C of the present manuscript

- I. Antoniadis, A. Delgado, C. Markou and S. Pokorski, *The effective supergravity of Little String Theory*, Eur. Phys. J. C **78** (2018) no.2, 146

  which corresponds to section V.5

and in two pieces of yet unpublished work

- I. Antoniadis, J. P. Derendinger and C. Markou, *in preparation*

- I. Antoniadis, A. Delgado, C. Markou and S. Pokorski, *in preparation*

  which corresponds to section V.6 and V.7.

II – Résumé détaillé en Français

Au cours des dernières décennies, la supersymétrie a fait l'objet de recherches théoriques et phénoménologiques approfondies dans le domaine de la physique des hautes énergies. Initialement découverte comme une extension possible et non–banale de la symétrie de Poincaré des théories quantiques et interagissantes des champs, elle a rencontré un enthousiasme considérable, en grand partie à cause de ses plusieurs caracteristiques attrayantes, comme les faits qu'elle offre une stabilisation du problème de la hiérarchie et prédit l'existence de particules élémentaires nouvelles, dont certaines ont le statut de candidates à la matière noire. En dehors de cela, elle s'est révélée comme un ingrédient nécessaire, au niveau de la surface d'univers, de la construction des théories des cordes cohérentes, alors que la théorie des cordes elle-même a attiré une grande attention scientifique comme une théorie possible de la gravité quantique. Néanmoins, à ce jour, il n'y a eu aucune preuve expérimentale indiquant que la supersymétrie existe dans la Nature. À cet égard, de nouvelles perspectives pourraient être tirées par un réexamen de la brisure de la supersymétrie, en particulier aux basses énergies, étant donné que le grand collisionneur de hadrons du CERN a déjà sondé la gamme d'énergie de plusieurs TeV.

La supersymétrie $\mathcal{N} = 1$ spontanément brisée est non–linéairement réalisée aux basses énergies. Une façon simple de réaliser un tel scénario est de considérer un superchamp chiral X des composantes (x, G, F), qui correspondent à un scalaire complexe, un fermion et un scalaire complexe auxiliaire respectivement. Aux basses énergies, x devient supermassif et découple du spectre, ou, en équivalence, une contrainte nilpotente est imposée à X [1, 2, 3, 4]

$$X^2 = 0\,, \tag{II.1}$$

dont la solution est

$$x \sim \frac{G^2}{F}\,. \tag{II.2}$$

La contrainte (I.1) élimine donc x comme une fonction du G, de manière à ce que le spectre final consiste en un seul champ physique, le fermion G. Un Lagrangien général pour le X contraint peut être écrit en utilisant un potentiel de Kähler et un superpotentiel qui sont quadratique et linéaire en X respectivement. En remplaçant F en utilisant son équation du mouvement, on obtient un Lagrangien pour G qui est sur la couche de masse [1, 5] le Lagrangien de Volkov–Akulov [6] et (l'action qui correspond à) il est invariant sous la transformation non–linéaire de G, qui est un vestige de l'invariance $\mathcal{N} = 1$; par conséquent, G est identifié au Goldstino. Pour généraliser cette description, la supersymétrie est dite être non–linéairement réalisée quand il y a une combinaison linéaire de fermions de la théorie originale qui se transforme non–linéairement et les degrés de liberté fermioniques et bosoniques du spectre final ne sont pas égaux, comme c'est le cas dans l'exemple du X contraint.

Des réalisations non–linéaires de la supersymétrie ont été récemment soumises à d'importantes études. Tout d'abord, pour décrire des couplages entre X et d'autres multiplets incomplets de $\mathcal{N}=1$, des contraintes supplémentaires comprenant X et les dernières proposées [4, 7] sont toutes constituées d'une seule contrainte imposée à X et un autre multiplet chiral choisi de façon approprié [7]. Cependant, la contrainte (II.1) ne décrit pas toujours le multiplet du Goldstino aux basses énergies: des cas dans lesquels le multiplet chiral du Goldstino satisfait une contrainte plus faible, comme $X^3 = 0$, en présence de multiplets de matière, ont été découverts [8, 9, 10].

En plus, la contrainte (II.1) a été utilisée pour incorporer le modèle de Starobinsky de la gravité $R + R^2$ [11], dont la version linéarisée [12] sert curieusement comme un modéle de l'inflation en un seul champ actuellement favorisé par le données de PLANCK, dans la supergravité $\mathcal{N}=1$ [13], de telle manière que la supersymétrie $\mathcal{N}=1$ soit non–linéairement réalisée pendant la phase de l'inflation [14]. En outre, dans [15], nous couplons le multiplet nilpotent du Goldstino avec la supergravité $\mathcal{N}=1$, qui donne lieu à la version supersymétrique du mécanisme de Brout–Englert–Higgs, et nous montrons que la formulation géométrique du couplage est la supergravité $\mathcal{N}=1$ pure, où le superchamp chiral $\mathcal{R}$ de la courbure satisfait la contrainte

$$(\mathcal{R} - \lambda)^2 = 0\,, \tag{II.3}$$

où λ est un paramètre approprié, dont le cas particulier $\lambda = 0$ a été utilisé dans [14]. Nous montrons aussi comment la contrainte nilpotente survient dans une classe de modèles de la supergravité $f(\mathcal{R})$ modifiée dans la limite de basse énergie.

Un autre exemple de la supersymétrie non–linéaire se trouve aux vides particuliers dans la théorie des cordes de type I, quand des D–branes sont combinées aux plans anti–orientifolds. Dans de telles configurations, il existe un Goldstino (sans masse) dans le spectre des cordes ouvertes, mais pas les superpartenaires des excitations des branes [16, 17, 18, 19], de sorte que la supersymétrie soit non–linéairement réalisée sans prendre la limite de basse énergie. L'origine, dans la théorie des cordes, de differents $\mathcal{N}=1$ superchamps contraintes en termes des composantes a été examinée comme par exemple dans [20, 21, 22, 23], mais une classification complète est toujours absente de la littérature.

Les réalisations non–linéaires apparaissent aussi dans le cas de la supersymétrie $\mathcal{N}=2$. Grâce à l'automorphisme $SU(2)$ de son algèbre, $\mathcal{N}=2$ peut être perçue comme une collection de deux $\mathcal{N}=1$ supersymétries. Si $\mathcal{N}=2$ est brisée à deux échelles différentes, on doit d'abord considérer la brisure partielle $\mathcal{N}=2 \to \mathcal{N}=1$. Cette brisure est induite par le mécanisme APT [25] au moyen de termes de Fayet–Iliopoulos "électriques" [24] *et* "magnétiques" introduits au niveau du Lagrangien d'une théorie d'un multiplet $\mathcal{N}=2$ de Maxwell $\mathcal{W}$ [24, 71]. En termes de superchamps de $\mathcal{N}=1$, les composantes du $\mathcal{W}$ sont un multiplet chiral X et un multiplet spinoriel W, qui contient (la courbure de jauge) le champ de Maxwell; X et W se transforment sous la deuxième supersymétrie $\mathcal{N}=1$ de paramètre η_α comme décrit dans [28]

$$\begin{aligned}
\delta^* X &= \sqrt{2} i\, \eta W \quad , \quad \delta^* \overline{X} = \sqrt{2} i\, \overline{\eta}\overline{W} \\
\delta^* W_\alpha &= \sqrt{2} i \left[\tfrac{1}{4} \eta_\alpha \overline{D}^2 \overline{X} + i (\sigma^\mu \overline{\eta})_\alpha\, \partial_\mu X \right] \\
\delta^* \overline{W}_{\dot\alpha} &= \sqrt{2} i \left[\tfrac{1}{4} \overline{\eta}_{\dot\alpha} D^2 X - i (\eta \sigma^\mu)_{\dot\alpha} \partial_\mu \overline{X} \right] .
\end{aligned} \tag{II.4}$$

En plus, le superchamp de $\mathcal{N}=2$

$$\mathcal{W}(y,\theta,\widetilde{\theta}) = X(y,\theta) + \sqrt{2}i\,\widetilde{\theta}W(y,\theta) - \frac{1}{4}\widetilde{\theta}^2\overline{D}^2\overline{X}(y,\theta)\,, \tag{II.5}$$

où θ_α et $\widetilde{\theta}_\alpha$ sont les cordonnées de Grassmann de la première et de la deuxième supersymétrie $\mathcal{N}=1$ respectivement, contient les degrés de liberté du multiplet de Maxwell. Un Lagrangian général pour $\mathcal{W}$ peut être écrit en utilisant une fonction holomorphe $\mathcal{F}(\mathcal{W})$ comme

$$\begin{aligned}\mathcal{L}_{Max} &= \tfrac{1}{2}\int d^2\theta\, d^2\widetilde{\theta}\,\mathcal{F}(\mathcal{W}) + \text{h.c.}\\ &= \tfrac{1}{2}\int d^2\theta \left(\tfrac{1}{2}\mathcal{F}''(X)W^2 - \tfrac{1}{4}\mathcal{F}'(X)\overline{D}^2\overline{X}\right) + \text{h.c.}\end{aligned} \tag{II.6}$$

où $\mathcal{F}_X = \partial\mathcal{F}/\partial X$.

Selon le mécanisme APT, pour effectuer la brisure partielle, il faut ajouter des termes de Fayet–Iliopoulos électriques m^2X et magnétiques $M^2\mathcal{F}_X$

$$\mathcal{L}_{Max,def} = \tfrac{1}{2}\int d^2\theta \left[\tfrac{1}{2}\mathcal{F}_{XX}WW - \tfrac{1}{4}\mathcal{F}_X\overline{DD}\,\overline{X} + m^2X - iM^2\mathcal{F}_X\right] + \text{h.c.} \tag{II.7}$$

où m^2 et M^2 sont de paramètres complexes. Curieusement, l'action correspondant à (II.7) n'est pas invariante sous (II.4), mais sous les transformations *deformées* de la deuxième supersymétrie $\mathcal{N}=1$ [26, 28]

$$\begin{aligned}\delta^*X &= \sqrt{2}\,i\,\eta W\\ \delta^*W_\alpha &= -\sqrt{2}M^2\,\eta_\alpha + \sqrt{2}\,i\left[\tfrac{1}{4}\eta_\alpha\overline{DD}\,\overline{X} + i(\sigma^\mu\overline{\eta})_\alpha\,\partial_\mu X\right],\end{aligned} \tag{II.8}$$

En parallèle, (II.7) peut être écrit comme

$$\mathcal{L} = \frac{1}{2}\int d^2\theta\left[\int d^2\widetilde{\theta}\,\mathcal{F}(\mathcal{W}_{def}) + m^2X\right] + \text{h.c.} \tag{II.9}$$

en utilisant le superchamp $\mathcal{W}$ *déformé*

$$\mathcal{W}_{def} = X + \sqrt{2}i\,\widetilde{\theta}W + \widetilde{\theta}\widetilde{\theta}\left[-iM^2 - \frac{1}{4}\overline{DD}\,\overline{X}\right]. \tag{II.10}$$

Le coefficient du terme magnétique de FI est donc un paramètre constant qui apparaît comme

- une déformation des transformations des superchamps sous la deuxième supersymétrie $\mathcal{N}=1$ (en général sous une rotation de $SU(2)$), de telle manière que l'algèbre supersymétrique correspondante soit toujours fermée [26, 27, 28]
- une déformation de $\mathcal{W}$ lui-même [29].

La première supersymétrie $\mathcal{N}=1$ reste intacte, afin que le spectre final soit organisé en multiplets, tandis que la deuxième $\mathcal{N}=1$ est non-linéairement realisée, et le Goldstino correspondant est identifié au fermion dont la transformation dans le vide est proportionnelle au paramètre de la déformation. Il est alors raisonnable d'appeler collectivement la collection de deux supersymétries $\mathcal{N}=1$, dont une est linéairement et l'autre non–linéairement realisée, comme de la supersymétrie $\mathcal{N} = 2$ non–linéaire. Dans le cadre de la théorie de cordes, la supersymétrie $\mathcal{N} = 2$ non–linéaire apparaît dans le cas de D–branes dans un “bulk” de $\mathcal{N} = 2$.

À ce stade, il est important de souligner une différence qualitative entre la brisure $\mathcal{N} = 1 \to \mathcal{N} = 0$ et la brisure partielle $\mathcal{N} = 2 \to \mathcal{N} = 1$. Dans le premier cas, la brisure est générée par des valeurs d'espérance du vide de champs auxiliaires, qui peuvent être le champ complexe F d'un multiplet chiral, ou le scalaire réel D d'un multiplet vectoriel [30]. Dans le deuxième cas, F, son conjugué et D apparaissent aussi comme les champs auxiliaires du superchamp de Maxwell $\mathcal{W}$, mais sous forme d'un triplet de $SU(2)$. À cause de cette restriction, pour générer la brisure partielle, des paramètres magnétiques doivent être introduits à la main et ne peuvent *pas* être absorbés dans des valeurs d'espérance du vide des champs auxiliaires. En plus, il y a trois multiplets $\mathcal{N} = 1$ possibles dans lesquels le Goldstino peut se trouver: en dehors du chiral et du vectoriel [25, 26, 31], il peut faire partie du multiplet réel et linéaire L [32], qui est défini par les contraintes

$$L = \overline{L} \quad , \quad \overline{D}^2 L = D^2 L = 0 \,. \tag{II.11}$$

Outre le Goldstino, L contient un scalaire réel et la courbure d'un tenseur antisymétrique et, avec un superchamp chiral Φ, fait naturellement partie d'un multiplet simple–tenseur $\mathcal{Z}$ de $\mathcal{N} = 2$ [33, 34, 35, 36, 29].

Sous la deuxième supersymétrie, Φ et L transforment comme

$$\begin{aligned} \delta^* \Phi = \sqrt{2} i \, \overline{\eta} \overline{D} L \quad &, \quad \delta^* \overline{\Phi} = \sqrt{2} i \, \eta D L \\ \delta^* L = -\tfrac{i}{\sqrt{2}} &(\eta D \Phi + \overline{\eta} \overline{D} \overline{\Phi}) \,. \end{aligned} \tag{II.12}$$

En plus, le superchamp de $\mathcal{N} = 2$ construit par Φ et L est

$$\mathcal{Z}(z, \theta, \overline{\overline{\theta}}) = \Phi(z, \theta) + \sqrt{2} i \, \overline{\overline{\theta}} \overline{D} L(z, \theta) - \frac{1}{4} \overline{\overline{\theta}}^2 \overline{D}^2 \overline{\Phi}(z, \theta), \tag{II.13}$$

pour lequel un Lagrangien général peut être écrit comme

$$\begin{aligned} \mathcal{L}_{ST} &= \int d^2\theta \, d^2\overline{\overline{\theta}} \, \mathcal{G}(\mathcal{Z}) + \text{h.c.} \\ &= \int d^2\theta \left(\tfrac{1}{2} \mathcal{G}''(\Phi) (\overline{D} L)(\overline{D} L) - \tfrac{1}{4} \mathcal{G}'(\Phi) \overline{D}^2 \overline{\Phi} \right) + \text{h.c.} \end{aligned} \tag{II.14}$$

en utilisant une fonction holomorphe $\mathcal{G}(\mathcal{Z})$.

Revenons maintenant à la brisure partielle en utilisant $\mathcal{W}$. Aux basses énergies, X devient supermassif et découple du spectre ou, de manière équivalente, une contrainte nilpotente est imposée à $\mathcal{W}_{def}$ [28]

$$\mathcal{W}_{def}^2 = 0 \,, \tag{II.15}$$

ou, en composantes,

$$\begin{aligned} X W_\alpha = X^2 = 0 \\ WW - \tfrac{1}{2} X \overline{DDX} - 2i \, M^2 \, X = 0 \,. \end{aligned} \tag{II.16}$$

On peut remarquer qu'en multipliant la deuxième égalité de (II.16) avec W_α ou X, on obtient la première égalité. La solution de (II.16) détermine X comme une fonction des dérivés de superespace de W [26] et le Lagrangien de W qui reste contient, de manière intéressante, le Lagrangien de Born–Infeld d'une D3–brane, dont le couplage de jauge et la tension dépendent des paramètres électriques et magnétiques respectivement [26, 28]. Sur un couplage de $\mathcal{W}_{def}$ contraint avec un multiplet simple–tenseur de $\mathcal{N}=2$, il a été découvert [29] qu'une version nouvelle du mécanisme de super–Brout–Englert–Higgs *sans* la gravité a lieu, par lequel W devient massif en absorbant le multiplet linéaire et le Lagrangien de Born–Infeld acquiert un coefficient qui dépend de Φ. Notablement, étant donné que $\mathcal{W}$ et $\mathcal{Z}$ sont de chiralités opposées sous la deuxième supersymétrie $\mathcal{N}=1$, le couplage est formulé en utilisant une représentation "longue" $\widehat{\mathcal{Z}}$ du multiplet simple–tenseur, qui a une transformation de jauge dont le paramètre est un autre multiplet de Maxwell [29]. En composantes,

$$\widehat{\mathcal{Z}} = Y + \sqrt{2}\,\widetilde{\theta}\chi - \widetilde{\theta}\widetilde{\theta}\left[\frac{i}{2}\Phi + \frac{1}{4}\overline{DD}\,\overline{Y}\right], \tag{II.17}$$

où Y est un superchamp chiral et χ_α et un superchamp spinoriel lié au L par

$$L = D\chi - \overline{D}\overline{\chi}\,. \tag{II.18}$$

Dans [37], inspirés par ces résultats, nous formulons un mécanisme nouveau de la brisure partielle en utilisant un multiplet simple–tenseur. Le Lagrangien prend la forme

$$\begin{aligned} \mathcal{L}_{ST,def} &= i\int d^2\theta d^2\overline{\theta}\left[-L^2(W_\Phi - \overline{W}_{\overline{\Phi}}) + \overline{\Phi}W - \Phi\overline{W}\right] \\ &\quad + \int d^2\theta\left[\widetilde{m}^2\Phi + \widetilde{M}^2\,W\right] + \text{h.c.} \end{aligned} \tag{II.19}$$

où $\widetilde{m}^2$ et $\widetilde{M}^2$ sont des paramètres complexes et $\mathcal{G}'(\Phi) = iW(\Phi)$. L'action correspondant à (II.19) est invariante sous la première, linéairement réalisée, supersymétrie $\mathcal{N}=1$ et sous les transformations *déformées*

$$\begin{aligned} \delta^*\Phi &= \sqrt{2}i\,\overline{\eta}\overline{D}L \quad , \quad \delta^*\overline{\Phi} = \sqrt{2}i\,\eta DL \\ \delta^*L &= \sqrt{2}\,\widetilde{M}^2\,(\overline{\theta\eta} + \theta\eta) - \tfrac{i}{\sqrt{2}}(\eta D\Phi + \overline{\eta D\Phi}) \end{aligned} \tag{II.20}$$

de la deuxième $\mathcal{N}=1$ qui est donc non–linéairement réalisée. Il faut noter que la déformation de la transformation d'un seul L a été trouvée [32] en intervertissant la chiralité des transformations déformées d'un seul W. En analysant le vide de (II.20), nous trouvons dans ce cas que

$$\widetilde{M}^2 \neq 0 \neq \widetilde{m}^2 \quad , \quad W_{\Phi\Phi} \neq 0\,, \tag{II.21}$$

la supersymétrie $\mathcal{N}=2$ est partiellement brisée et Φ devient massif, tandis que L reste sans masse. Par conséquent, nous observons une correspondence claire entre

$$\Big(X, \mathcal{F}_X(X)\Big) \text{ et } \Big(\Phi, W(\Phi)\Big)\,, \tag{II.22}$$

sous une inversion de la chiralité de Lorentz qui relie W_α à $\overline{D}_{\dot\alpha}L$. Nous présentons aussi la généralisation du mécanisme en utilisant plusieurs multiplets simple–tenseur.

Après dualisation par une transformation de Legendre, nous obtenons la théorie duale d'un hypermultiplet, contenant Φ et un autre superchamp chiral T, qui a une symétrie de shift de T

$$\mathcal{L}_{dual} = \int d^2\theta\left[-\frac{i}{2}W_\Phi(\overline{D}\widetilde{\mathcal{H}}_T)(\overline{D}\widetilde{\mathcal{H}}_T) - \frac{i}{4}W\overline{DD\Phi} + \widetilde{m}^2\Phi + \widetilde{M}^2 W\right] + \text{h.c.} \tag{II.23}$$

dont l'action est invariante sous les transformations déformées

$$\begin{aligned} \delta^*\Phi &= -\sqrt{2}i\,\overline{\eta D}\,\widetilde{\mathcal{H}}_T \quad , \quad \delta^*\overline{\Phi} = -\sqrt{2}i\,\eta D\,\widetilde{\mathcal{H}}_T\,, \\ \delta^*\widetilde{\mathcal{H}}_T &= -\sqrt{2}\,\widetilde{M}^2\,(\overline{\theta\eta} + \theta\eta) + \tfrac{i}{\sqrt{2}}(\eta D\Phi + \overline{\eta D\Phi})\,, \end{aligned} \tag{II.24}$$

où

$$\widetilde{\mathcal{H}}_T = -\frac{i}{2}\frac{T+\overline{T}}{W_\Phi - \overline{W}_{\overline{\Phi}}}\,. \tag{II.25}$$

Nous remarquons que l'hypermultiplet n'a pas de description en superchamps (hors de la couche de masse) dans le superespace ordinaire de $\mathcal{N}=2$ que nous utilisons, mais dans le superespace harmonique [38, 39].

En plus, nous étudions les déformations les plus générales de $\mathcal{W}$

$$\mathcal{W}_{nl} \;\; = \;\; A^2\theta\theta + B^2\tilde{\theta}\tilde{\theta} + 2\Gamma\theta\tilde{\theta} \tag{II.26}$$

et de $\mathcal{Z}$

$$\mathcal{Z}_{nl} = \widetilde{A}^2\,\theta\theta + \overline{\widetilde{B}}^2\,\overline{\tilde{\theta}\tilde{\theta}}\,, \tag{II.27}$$

où les paramètres $A^2, \widetilde{A}^2, \ldots$ sont en général complexes, et nous en déduisons que la brisure partielle peut avoir lieu à condition que les relations

$$\Gamma = \pm AB \tag{II.28}$$

et

$$\widetilde{A}^2 = 0 \quad \text{ou} \quad \widetilde{B}^2 = 0 \tag{II.29}$$

soient satisfaites. Nous déterminons la limite de masse infinie dans laquelle une contrainte nilpotente est imposée au $\mathcal{Z}_{def}$

$$\mathcal{Z}_{def}^2 = 0\,, \tag{II.30}$$

ou en composantes

$$\Phi = -\frac{2(\overline{D}L)(\overline{D}L)}{4\overline{\widetilde{B}}^2 - \overline{DD\Phi}} \qquad \Longrightarrow \qquad \Phi\,\overline{D}_{\dot\alpha}L = \Phi^n = 0 \quad (n \geq 2)\,. \tag{II.31}$$

Pour coupler $\mathcal{Z}_{def}$ au multiplet de Maxwell, nous construisons une représentation nouvelle et "longue" $\widehat{\mathcal{W}}$ de ce dernier

$$\widehat{\mathcal{W}} = U + \sqrt{2}\,\overline{\tilde{\theta}}\overline{\Omega} - \overline{\tilde{\theta}}^2\left[\frac{i}{2}X + \frac{1}{4}\overline{D}^2\overline{U}\right], \tag{II.32}$$

où U est un superchamp chiral et Ω_α est un superchamp chiral spinoriel

$$\overline{\Omega}_{\dot\alpha} = \overline{D}_{\dot\alpha}\mathbb{L} \tag{II.33}$$

qui est lié à $W_\alpha = -\frac{1}{4}\overline{D}^2 D_\alpha V$ par

$$V = 2(\mathbb{L} + \overline{\mathbb{L}})\,. \tag{II.34}$$

Nous notons que $\mathcal{W}$ est lié à $\widehat{\mathcal{W}}$ par

$$\mathcal{W} = -\frac{i}{2}\,\overline{\overline{D}}^2\widehat{\mathcal{W}} + \frac{i}{2}\,\overline{D}^2\overline{\widehat{\mathcal{W}}} \tag{II.35}$$

et que $\widehat{\mathcal{W}}$ a *deux* transformations de jauge dont les paramètres sont d'autres superchamps simple–tenseur indépendants. Nous montrons que des déformations des représentations longues peuvent induire une brisure totale mais pas une partielle, ce qui implique que les couplages pertinents de la supersymétrie $\mathcal{N} = 2$ non–linéaire sont des intégrales dans le superespace $\mathcal{N} = 2$ des

$$\mathcal{W}_{def}\widehat{\mathcal{Z}} \quad , \quad \mathcal{Z}_{def}\widehat{\mathcal{W}}\,. \tag{II.36}$$

Comme le premier a été exploré dans [29], nous étudions le dernier et nous trouvons qu'il donne lieu de nouveau à un mécanisme de super–Brout–Englert–Higgs sans gravité. En particulier, le couplage

$$\epsilon_{\mu\nu\rho\sigma}H^{\nu\rho\sigma}A^{\mu}\,, \tag{II.37}$$

entre le tenseur antisymétrique $b_{\mu\nu}$ de courbure $H_{\mu\nu\rho}$ du L et le champ A_μ de jauge du multiplet de Maxwell, donne une masse à A_μ. Le spectre final consiste en un multiplet vectoriel qui absorbe le multiplet linéaire en devenant massif, et un multiplet chiral qui reste sans masse. Contrairement à [29], il y a une subtilité concernant l'équation du mouvement du champ auxiliaire dans la représentation longue du multiplet de Maxwell, parce que le champ auxiliaire réel D de $\mathcal{W}$ est remplacé par la divergence d'un champ vectoriel dans $\widehat{\mathcal{W}}$. Enfin, nous étudions d'autres contraintes $\mathcal{N} = 2$ qui décrivent des multiplets incomplets de matière.

Ces avances pourraient être utiles pour la brisure partielle de la supergravité $\mathcal{N} = 2$. Le multiplet de la supergravité $\mathcal{N} = 2$ pure comprend le graviton, deux gravitinos et un vecteur connu comme le graviphoton. Il a donc le contenu de spin suivant

$$(2, 3/2, 3/2, 1)\,, \tag{II.38}$$

de sorte qu'après la brisure partielle le spectre contienne le multiplet de la supergravité $\mathcal{N} = 1$ pure (sans masse)

$$(2, 3/2) \tag{II.39}$$

et un multiplet de $\mathcal{N} = 1$ de spin–3/2 massif

$$(3/2, 1, 1, 1/2) \tag{II.40}$$

qui est devenu massif par une combinaison d'un vecteur, deux fermions et deux scalaires [40]. Ces modes de Goldstone sont associés à deux symétries de shift. Ces observations ont conduit à la formulation *sur la couche de masse* de la brisure partielle dans la supergravité $\mathcal{N} = 2$, un utilisant (au moins) un multiplet de Maxwell et un hypermultiplet qui a deux isométries commutatives; un jaugement, qui est $U(1)$ dans le cas minimal, des isométries de la variété des scalaires de l'hypermultiplet génère la brisure partielle [41, 42, 43] et plus récemment [44, 45, 46]. Actuellement, nous explorons [47] la généralisation hors de la couche de masse de ces résultats, ainsi que les interactions des degrés de liberté de Goldstone du multiplet du spin–3/2 massif, pour laquelle le progrès réalisé dans [29, 37] pourrait être pertinent.

Un jaugement $U(1)$, mais dans le cas minimal du sous–groupe $U(1)$ de l'automorphisme $SU(2)$ de $\mathcal{N} = 2$ et pas forcement des isométries d'une variété de scalaires, peut donner lieu à la brisure partielle en cinq dimensions. Nous nous sommes intéressés à cette possibilité à la lumière d'un mécanisme nouveau qui génère une hiérarchie des échelles exponentielle, le mécanisme du "clockwork", récemment proposé [48, 49, 50] et ensuite développé dans [51, 52, 53, 54], suivi par une littérature de plus en plus abondante en ce qui concerne les applications. Dans sa version dans le continu, l'espace–temps du clockwork a cinq dimensions, et, de façon intéressante, sa métrique est identique à la métrique d'un espace–temps 5D, dans lequel un scalaire réel a un couplage du dilaton à la gravité en cinq dimensions et un potentiel "runaway", et sa valeur moyenne est une fonction linéaire de la dimension supplémentaire. Remarquablement, ce modèle du dilaton linéaire est un modèle 5D [55] du dual holographique [56, 57, 58, 59] de la "Little String Theory" en six dimensions, [60, 61], qui est obtenue par la théorie de cordes du type IIB dans la limite

$$g_S \to 0\,, \tag{II.41}$$

où g_S est le couplage de cordes, à savoir l'exponentielle de la valeur moyenne du dilaton Φ. Le modèle 5D est décrit par le Lagrangien

$$e^{-1}\mathcal{L}_{LST} = e^{-\frac{\sqrt{3}\Phi}{M_5^{3/2}}}\Big(\frac{1}{2}M_5^3\mathcal{R} + \frac{3}{2}(\partial\Phi)^2 - \Lambda\Big)\,, \tag{II.42}$$

où M_5 est la masse de Planck en cinq dimensions et Λ est une constante. Puisque le dual holographique préserve la supersymétrie du "bulk", la supergravité effective du modèle 5D doit être dans le cas minimal $\mathcal{N}=2$, $D=5$.

Ceci est précisement le sujet de [62], dans lequel nous montrons que la supergravité $\mathcal{N}=2$, $D=5$ jaugée peut incorporer le modèle 5D du dilaton, voir aussi [63]. En particulier, nous considérons la supergravité $\mathcal{N}=2$, $D=5$ pure [64, 65, 66] couplée à un multiplet vectoriel qui contient un vecteur en cinq dimensions, un spineur symplectique qui est un doublet λ_i de $SU(2)$ et un scalaire réel [67]. Le jaugement du sous–groupe $U(1)$ de $SU(2)$ génère un potentiel scalaire, ainsi que des masses de fermions et des termes d'interactions [68, 69]. Nous trouvons que le potentiel du scalaire réel est exactement le potentiel du dilaton de (II.42) (après une transformation conforme) pour un choix approprié des paramètres et nous montrons que la supersymétrie est partiellement brisée, parce que, à cause de la valeur moyenne du dilaton linéaire, les composantes de λ_i transforment comme

$$\begin{aligned}\widetilde{\delta}(\lambda_1 - i\Gamma^5\lambda_2) &= 0\\ \widetilde{\delta}(\lambda_1 + i\Gamma^5\lambda_2) &\sim \epsilon_2 - i\Gamma^5\epsilon_1\,,\end{aligned} \tag{II.43}$$

où ϵ_i est le paramètre de la supersymétrie $\mathcal{N}=2$. Par conséquent, la combinaison $\lambda_1 + i\Gamma^5\lambda_2$ correspond au Goldstino et la combinaison orthogonale correspond à la supersymétrie $\mathcal{N}=1$ qui reste intacte en quatre dimensions. Nous donnons aussi l'expression finale du Lagrangien total.

Le coefficient du dilaton linéaire est proportionnel au couplage g de jauge de $U(1)$ et, en utilisant les résultats de [55], selon lesquels le spectre de Kaluza–Klein des champs 5D démontre un écart de masse suivi approximativement par un continu, nous observons que l'écart de masse est aussi proportionnel à g. La totalité des zéro modes forment les multiplets de $\mathcal{N}=1$ suivants:

- Le multiplet de la supergravité $\mathcal{N} = 1$ pure (sans masse), qui contient le graviton 4D et la combinaison linéaire $\psi_\mu^1 - i\Gamma^5\psi_\mu^2$ des gravitinos en quatre dimensions.
- Un multiplet massif de $\mathcal{N} = 1$ de spin–3/2, dont la masse est controlée par g. Il consiste en la combinaison linéaire orthogonale des gravitinos $\psi_\mu^1 + i\Gamma^5\psi_\mu^2$ en quatre dimensions, qui obtient une masse en absorbant le Goldstino $\lambda^1 + i\Gamma^5\lambda^2$, ainsi que deux champs de spin–1 et un de spin 1/2 massifs.
- Les degrés de liberté qui restent se trouvent dans un multiplet de spin–1 et un de spin–1/2 de $\mathcal{N} = 1$ sans masse.

Finalement, pour des applications phénoménologiques, dans [70] nous effectuons la compactification de la dimension supplémentaire y sur un orbifold $S^1/\mathbb{Z}_2$, aux points fixes duquel nous introduisons deux branes de tensions V_1 et V_2, à savoir à $y = 0$ et à $y = L$ respectivement. La partie bosonique de l'action sans les termes qui contiennent des champs de matière est

$$\begin{aligned} S_{dil} &= \int d^5x \Big[\sqrt{-g}e^{-\sqrt{3}\Phi}\Big(\tfrac{1}{2}\mathcal{R} + \tfrac{3}{2}(\partial\Phi)^2 - \Lambda\Big) \\ &\quad - \sqrt{-g_1}e^{-\alpha_1\Phi}V_1\delta(y) - \sqrt{-g_2}e^{-\alpha_2\Phi}V_2\delta(y-L)\Big] \end{aligned} \tag{II.44}$$

où les paramètres α_1, α_2 sont en général arbitraires. Nous trouvons que le Modèle Standard peut être introduit sur les deux branes et que la présence des branes est compatible avec la direction de la supersymétrie $\mathcal{N} = 1$ qui reste intacte après le jaugement et ne la brise pas.

III – Linear $\mathcal{N}=2$, $D=4$ global supersymmetry

III.1 $\mathcal{N}=2$ superspace

For a large part of the present thesis it will be convenient to write an $\mathcal{N}=2$ superfield in terms of two $\mathcal{N}=1$ superfields; we thus start by explaining the construction in question, following the approach of [28, 29]. $\mathcal{N}=1$ supersymmetry is generated by the fermionic conserved charge Q_α and its conjugate $\overline{Q}_{\dot\alpha}$ that satisfy the anticommutation relation

$$\{Q_\alpha, \overline{Q}_{\dot\alpha}\} = -2i\,(\sigma^\mu)_{\alpha\dot\alpha}\partial_\mu\,. \tag{III.1}$$

Some useful $\mathcal{N}=1$ superspace identities may be found in appendix A. Consider now two $\mathcal{N}=1$ superfields V_x, $x=1,2$. Under $\mathcal{N}=1$ supersymmetry, which relates the components of V_1 (or V_2) to each other, the superfields transform as

$$\delta V_x = (\epsilon Q + \bar\epsilon\overline{Q})V_x\,, \tag{III.2}$$

where ϵ is the $\mathcal{N}=1$ supersymmetry parameter. Moreover, one may define the covariant derivatives (that act as $\mathcal{N}=1$ superspace differential operators)

$$D_\alpha \equiv \partial_\alpha - i(\sigma^\mu\bar\theta)_\alpha\partial_\mu \quad , \quad \overline{D}_{\dot\alpha} \equiv \bar\partial_{\dot\alpha} - i(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu\,, \tag{III.3}$$

where

$$\partial_\alpha \equiv \frac{\partial}{\partial\theta^\alpha} \quad , \quad \bar\partial_{\dot\alpha} \equiv \frac{\partial}{\partial\bar\theta^{\dot\alpha}} \quad , \quad \partial_\mu \equiv \frac{\partial}{\partial x^\mu} \tag{III.4}$$

which satisfy the anticommutation relation

$$\{D_\alpha, \overline{D}_{\dot\alpha}\} = -2i\,(\sigma^\mu)_{\alpha\dot\alpha}\partial_\mu \tag{III.5}$$

while

$$\{Q_\alpha, D_\beta\} = \{\overline{Q}_{\dot\alpha}, \overline{D}_{\dot\beta}\} = \{Q_\alpha, \overline{D}_{\dot\beta}\} = 0\,. \tag{III.6}$$

The relations (III.6) allow us then to define a second $\mathcal{N}=1$ supersymmetry generated by D_α and $\overline{D}_{\dot\alpha}$, under which V_x transform as

$$\delta^* V_1 = (a\,\eta D - \bar a\,\bar\eta\overline{D})V_2 \quad , \quad \delta^* V_2 = -(b\,\eta D - \bar b\,\bar\eta\overline{D})V_1\,, \tag{III.7}$$

where a, b are complex numbers and η is the parameter of the second $\mathcal{N}=1$ supersymmetry. Using the closure relation

$$[\delta_1^*, \delta_2^*]V_1 = -2i\,(\eta_1\sigma^\mu\bar\eta_2 - \eta_2\sigma^\mu\bar\eta_1)\partial_\mu V_1 = \Big(\eta_1\{D,\overline{D}\}\bar\eta_2 - \eta_2\{D,\overline{D}\}\bar\eta_1\Big)V_1\,, \tag{III.8}$$

one finds that $a\bar{b}=1$. For convenience, we set

$$a=-\frac{i}{\sqrt{2}} \quad , \quad b=-\sqrt{2}i\,. \tag{III.9}$$

Note that in the above construction the $SU(2)$–covariance that arises as an automorphism of $\mathcal{N}=2$ supersymmetry is no longer manifest.

Now let us turn to the superspace expression of $\mathcal{N}=2$ superfields. $\mathcal{N}=2$ superspace [72] is defined by the coordinates $(x^\mu, \theta^i, \overline{\theta}^i)$, where x^μ is the standard four–dimensional spacetime coordinate as in (III.4) and θ^i is an $SU(2)$–doublet the contains two Grassmann coordinates. Setting

$$\theta^1=\theta \quad , \quad \theta^2=\widetilde{\theta}\,, \tag{III.10}$$

we may view θ and $\widetilde{\theta}$ as the Grassmann coordinates corresponding to the first and to the second $\mathcal{N}=1$ supersymmetry respectively, which comprise the full $\mathcal{N}=2$. Note that, in a similar fashion, the $\mathcal{N}=2$ supersymmetry parameter may be thought of as an $SU(2)$–doublet consisting of ϵ and η. One may then define the covariant derivatives

$$\widetilde{D}_\alpha \equiv \widetilde{\partial}_\alpha - i(\sigma^\mu\overline{\widetilde{\theta}})_\alpha\partial_\mu \quad , \quad \overline{\widetilde{D}}_{\dot\alpha} \equiv \overline{\widetilde{\partial}}_{\dot\alpha} - i(\widetilde{\theta}\sigma^\mu)_{\dot\alpha}\partial_\mu\,, \tag{III.11}$$

where

$$\widetilde{\partial}_\alpha \equiv \frac{\partial}{\partial\widetilde{\theta}^\alpha} \quad , \quad \overline{\widetilde{\partial}}_{\dot\alpha} \equiv \frac{\partial}{\partial\overline{\widetilde{\theta}}^{\dot\alpha}}\,, \tag{III.12}$$

as well as the coordinates

$$\begin{aligned} y^\mu &\equiv x^\mu - i\theta\sigma^\mu\overline{\theta} - i\widetilde{\theta}\sigma^\mu\overline{\widetilde{\theta}} \quad , \quad \overline{y}^\mu = x^\mu + i\theta\sigma^\mu\overline{\theta} + i\widetilde{\theta}\sigma^\mu\overline{\widetilde{\theta}} \\ z^\mu &\equiv x^\mu - i\theta\sigma^\mu\overline{\theta} + i\widetilde{\theta}\sigma^\mu\overline{\widetilde{\theta}} \quad , \quad \overline{z}^\mu = x^\mu + i\theta\sigma^\mu\overline{\theta} - i\widetilde{\theta}\sigma^\mu\overline{\widetilde{\theta}}\,. \end{aligned} \tag{III.13}$$

It is thus straightforward to see that

$$\overline{D}_{\dot\alpha}y^\mu = \overline{\widetilde{D}}_{\dot\alpha}y^\mu = 0 \quad , \quad \overline{D}_{\dot\alpha}z^\mu = \widetilde{D}_\alpha z^\mu = 0\,, \tag{III.14}$$

so that y^μ is chiral under both $\mathcal{N}=1$ supersymmetries, while z^μ is chiral under the first and antichiral under the second $\mathcal{N}=1$ supersymmetry. Consequently, we can write the following types of $\mathcal{N}=2$ superfields using $\mathcal{N}=2$ superspace:

- the "CC" superfield $\mathcal{Z}$, that is chiral under both $\mathcal{N}=1$ supersymmetries

$$\mathcal{Z}=\mathcal{Z}(y,\theta,\widetilde{\theta}) \quad , \quad \overline{D}_{\dot\alpha}\mathcal{Z} = \overline{\widetilde{D}}_{\dot\alpha}\mathcal{Z} = 0 \tag{III.15}$$

- the "CA" superfield $\mathcal{U}$, that is chiral under the first and antichiral under the second $\mathcal{N}=1$ supersymmetry

$$\mathcal{U}=\mathcal{U}(z,\theta,\overline{\widetilde{\theta}}) \quad , \quad \overline{D}_{\dot\alpha}\mathcal{U} = \widetilde{D}_\alpha\mathcal{U} = 0 \tag{III.16}$$

as well as their conjugate superfields, that are AA and AC respectively. These $\mathcal{N}=2$ superfields can be expanded in $\widetilde{\theta}$ or $\overline{\widetilde{\theta}}$ according to their chirality and the components of these expansions are $\mathcal{N}=1$ superfields transforming to each other via (III.7).

III.2 $\mathcal{N}=1$ superfields

In the following, we briefly review several (independent) $\mathcal{N}=1$ superfields, mainly to set our conventions. They will be promoted to $\mathcal{N}=1$ superfields belonging to $\mathcal{N}=2$ superfields later on. As a reference we use [73] unless stated otherwise. Chiral $\mathcal{N}=1$ superspace is defined by the coordinates $(\tilde{y}^\mu, \theta)$, where

$$\tilde{y}^\mu \equiv x^\mu - i\theta\sigma^\mu\overline{\theta}\,. \tag{III.17}$$

All degrees of freedom ("d.o.f.") given are counted off–shell.

- The chiral superfield, $4_B + 4_F$

A (left-handed) chiral superfield X is defined by the constraint

$$\overline{D}_{\dot{\alpha}} X = 0 \tag{III.18}$$

which yields the following expansion in terms of component fields

$$X(\tilde{y},\theta) = x(\tilde{y}) + \sqrt{2}\theta\kappa(\tilde{y}) - \theta^2 F(\tilde{y})\,, \tag{III.19}$$

where x and F are complex scalars (x is not to be confused with the superspace coordinate x^μ and F is the auxiliary) and κ_α is a left–handed Weyl spinor. The supersymmetry transformations are

$$\begin{aligned} \delta x &= \sqrt{2}\epsilon\kappa \\ \delta\kappa_\alpha &= -\sqrt{2}F\epsilon_\alpha - \sqrt{2}i(\sigma^\mu\overline{\epsilon})_\alpha\partial^\mu x \\ \delta F &= -\sqrt{2}i(\partial_\mu\kappa\sigma^\mu\overline{\epsilon})\,. \end{aligned} \tag{III.20}$$

- The complex linear superfield [74], $12_B + 12_F$

The complex linear superfield $\mathbb{L}$ is defined by the constraint

$$\overline{D}^2\mathbb{L} = 0\,, \tag{III.21}$$

which yields the following expansion in terms of component fields

$$\begin{aligned} \mathbb{L}(x,\theta,\overline{\theta}) &= \Phi(x,\theta,\overline{\theta}) - \overline{\theta}\overline{\omega}(x) - \theta\sigma^\mu\overline{\theta}\,\mathbb{V}_\mu(x) + \tfrac{i}{2}\theta^2\overline{\theta}\overline{\lambda}(x) \\ &\quad -\tfrac{i}{2}\overline{\theta}^2\,\theta\sigma^\mu\partial_\mu\overline{\omega}(x) + \tfrac{i}{2}\theta^2\overline{\theta}^2\partial^\mu\mathbb{V}_\mu(x)\,, \end{aligned} \tag{III.22}$$

where Φ is an $\mathcal{N}=1$ chiral superfield, ω_α and λ_α are Weyl spinors and $\mathbb{V}_\mu$ is a complex vector, which at this point is *not* necessarily related to a gauge symmetry.

- The vector superfield, $8_B + 8_F \to 4_B + 4_F$

The vector or real superfield V is defined by the constraint

$$V(x,\theta,\overline{\theta}) = \overline{V}(x,\theta,\overline{\theta}) \tag{III.23}$$

and its gauge transformation is given by a chiral superfield ϕ:

$$\delta_g V = \phi + \overline{\phi}\,, \tag{III.24}$$

under which the real vector A_μ, that is the $\theta\sigma^\mu\overline{\theta}$ component of V, transforms as

$$\delta_g A_\mu = -i\partial_\mu(z-\overline{z})\,, \tag{III.25}$$

where z is the lowest component of ϕ. In the Wess–Zumino ("WZ") gauge, the expansion of V reads

$$V(x,\theta,\overline{\theta}) = \theta\sigma^\mu\overline{\theta}\,A_\mu(x) + i\theta^2\,\overline{\theta\lambda}(x) - i\overline{\theta}^2\,\theta\lambda(x) + \frac{1}{2}\theta^2\overline{\theta}^2\,D(x)\,, \tag{III.26}$$

where λ_α is a Weyl spinor and D is an auxiliary real scalar, yielding 4_B+4_F in total.

- The chiral spinor superfield [74, 75], $8_B+8_F \to 4_B+4_F$

A chiral spinor superfield χ_α is a left–handed Weyl spinor superfield that satisfies the constraint

$$\overline{D}_{\dot\alpha}\chi_\alpha = 0 \tag{III.27}$$

and its gauge transformation is given by a vector superfield Π:

$$\delta_g\chi_\alpha = -\frac{i}{4}\overline{D}^2 D_\alpha\Pi\,, \tag{III.28}$$

under which the real antisymmetric tensor $b_{\mu\nu}$, that is the θ component of χ_α, transforms as

$$\delta_g b_{\mu\nu}(x) = 2\partial_{[\mu}\Lambda_{\nu]}(x)\,, \tag{III.29}$$

where $\Lambda_\mu(x)$ is the vector field of Π. In a choice of gauge similar to the WZ gauge, the expansion of χ_α reads

$$\chi_\alpha(\widetilde{y},\theta) = -\frac{1}{4}\theta_\alpha C(\widetilde{y}) + \frac{1}{4}(\theta\sigma^\mu\overline{\sigma}^\nu)_\alpha\,b_{\mu\nu}(\widetilde{y}) + \frac{i}{2}\theta^2\,\varphi_\alpha(\widetilde{y})\,, \tag{III.30}$$

where C is a real scalar and φ_α is a left–handed Weyl spinor, yielding 4_B+4_F in total. Note that (III.30) contains *no* auxiliary fields.

As an example, consider the chiral spinor superfield W_α, commonly utilized in order to write a kinetic Lagrangian for V. W_α is defined as

$$W_\alpha = -\frac{1}{4}\overline{D}^2 D_\alpha V\,, \tag{III.31}$$

which is gauge invariant under (III.24). Note, however, that W_α does *not* admit a gauge transformation of the form (III.28) in the case of abelian gauge theories, but using (III.31), (III.28) can be written as

$$\delta_g\chi_\alpha = iW_\alpha\,, \tag{III.32}$$

so that the gauge transformation of the spinor superfield χ_α is controlled by another, gauge–invariant, spinor superfield that is related to an abelian vector multiplet. Notice also that, due to (III.31), W_α satisfies the Bianchi identity

$$DW = \overline{DW}\,, \tag{III.33}$$

which does not hold for a generic χ_α. In the WZ gauge, the expansion of W_α reads

$$W_\alpha(\widetilde{y},\theta) = -i\lambda_\alpha(\widetilde{y}) + \theta_\alpha D(\widetilde{y}) - \frac{i}{2}(\theta\sigma^\mu\overline{\sigma}^\nu)_\alpha F_{\mu\nu}(\widetilde{y}) - \theta^2(\sigma^\mu\partial_\mu\overline{\lambda}(\widetilde{y}))_\alpha\,, \tag{III.34}$$

where

$$F_{\mu\nu}(\widetilde{y}) \equiv \partial_\mu A_\nu(\widetilde{y}) - \partial_\nu A_\mu(\widetilde{y})\,. \tag{III.35}$$

In this gauge, W_α contains $4_B + 4_F$ as V.

- The real linear superfield [74, 75], $4_B + 4_F$

The real linear superfield L is defined by the constraints

$$L = \overline{L} \quad , \quad \overline{D}^2 L = D^2 L = 0\,, \tag{III.36}$$

which yield the following expansion in terms of component fields

$$\begin{aligned} L(x,\theta,\overline{\theta}) \;\; = \;\; & C(x) + i\theta\varphi(x) - i\overline{\theta}\overline{\varphi}(x) + \theta\sigma^\mu\overline{\theta}\,\upsilon_\mu(x) \\ & + \tfrac{1}{2}\theta^2\partial_\mu\varphi(x)\sigma^\mu\overline{\theta} + \tfrac{1}{2}\overline{\theta}^2\theta\sigma^\mu\partial_\mu\overline{\varphi}(x) + \tfrac{1}{4}\theta^2\overline{\theta}^2\Box C(x) \end{aligned} \tag{III.37}$$

where C is a real scalar, φ_α is a Weyl spinor and υ_μ is a real vector satistfying

$$\partial^\mu\upsilon_\mu = 0\,, \tag{III.38}$$

which is a direct consequence of (III.36). Notice that the expansion (III.37) contains *no* auxiliary fields. Let us also note that, up to a constant, the solution of (III.38) is

$$\upsilon_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial^\nu b^{\rho\sigma} \equiv \frac{1}{6}\epsilon_{\mu\nu\rho\sigma}H^{\nu\rho\sigma}\,, \tag{III.39}$$

where $b_{\mu\nu}$ is a real antisymmetric tensor. This implies that $H_{\mu\nu\rho}$ is the field–strength of $b_{\mu\nu}$ and is invariant under the gauge variation (III.29). Importantly, using the constraints (III.27) and (III.36), the real linear superfield can be written in terms of a spinor superfield as

$$L = D\chi - \overline{D}\overline{\chi}\,. \tag{III.40}$$

Obviously, L is gauge invariant under (III.28).

III.3 The Maxwell multiplet

The Maxwell or gauge multiplet [24, 71] of $\mathcal{N} = 2$, $D = 4$ supersymmetry contains a complex scalar x, a fermion $SU(2)$–doublet λ_i and a gauge field A_μ. $SU(2)$–indices are lowered and raised using the conditions

$$\lambda_i = \varepsilon_{ji}\lambda^j \quad , \quad \lambda^i = \varepsilon^{ij}\lambda_j \tag{III.41}$$

and their position indicates the chirality of the spinors, conventionally set as

$$\lambda_i = P_L\lambda_i \quad , \quad \lambda^i = P_R\lambda^i\,. \tag{III.42}$$

Note that spinor indices are here implicit. On–shell counting of the degrees of freedom (d.o.f.) yields 4_B+4_F, but off–shell gives 5_B+8_F, which means that the off–shell representation of the multiplet must contain three auxiliary bosonic d.o.f. The latter can be viewed as components of an $SU(2)$–triplet, which we denote by the real vector $\vec{Y}$ or Y^x, where $x = 1,2,3$. Equivalently, the auxiliary d.o.f. can be viewed as components of symmetric 2×2 matrices Y^{ij} given by

$$Y^{ij} = \vec{\tau}^{ij}\cdot\vec{Y}\,, \tag{III.43}$$

that satisfy the "reality" condition

$$Y^{ij} \equiv \varepsilon^{ik}\varepsilon^{jl}Y_{kl} = (Y_{ij})^*\,, \tag{III.44}$$

with

$$\vec{\tau}^{ij} = \varepsilon^{ik}\vec{\tau}_k{}^j \equiv i\varepsilon^{ik}\vec{\sigma}_k{}^j = \vec{\tau}^{ji}\,, \tag{III.45}$$

where $\vec{\sigma}_i{}^j$ are the standard traceless hermitian $SU(2)$ Pauli matrices, rendering the $\vec{\tau}_i{}^j$ traceless and anti–hermitian.

The Maxwell multiplet has a superfield expression in terms of $\mathcal{N}=1$ superfields using $\mathcal{N}=2$ superspace. To see this, we follow [28]. In particular, consider an $\mathcal{N}=1$ chiral X and an $\mathcal{N}=1$ chiral spinor superfield W_α given by

$$W_\alpha = -\frac{1}{4}\overline{D}^2 D_\alpha V_2\,, \tag{III.46}$$

where V_2 is a vector superfield, so that W_α is gauge invariant under

$$\delta_g V_2 = \Lambda_c + \overline{\Lambda}_c\,, \tag{III.47}$$

where Λ_c is a chiral superfield. Now suppose that X and W_α are related under a second $\mathcal{N}=1$ supersymmetry as follows

$$\begin{aligned}\delta^* X &= \sqrt{2}i\,\eta W \quad , \quad \delta^*\overline{X} = \sqrt{2}i\,\overline{\eta}\overline{W}\\ \delta^* W_\alpha &= \sqrt{2}i\left[\tfrac{1}{4}\eta_\alpha\overline{D}^2\overline{X} + i(\sigma^\mu\overline{\eta})_\alpha\,\partial_\mu X\right]\\ \delta^*\overline{W}_{\dot\alpha} &= \sqrt{2}i\left[\tfrac{1}{4}\overline{\eta}_{\dot\alpha}D^2X - i(\eta\sigma^\mu)_{\dot\alpha}\partial_\mu\overline{X}\right].\end{aligned} \tag{III.48}$$

Then, X and W_α form the $\mathcal{N}=2$ Maxwell multiplet, provided that X is invariant under a gauge transformation involving the superpartner of Λ_c under the second $\mathcal{N}=1$ supersymmetry. Using (III.7) and (III.48), one finds that

$$X = \frac{1}{2}\overline{D}^2 V_1\,, \tag{III.49}$$

where V_1 is a vector superfield. X is then invariant under the gauge transformation

$$\delta_g V_1 = \Lambda_l\,, \tag{III.50}$$

where Λ_l is a real linear superfield. Note that Λ_c and Λ_l form an $\mathcal{N}=2$ single–tensor multiplet that will be the subject of the next section.

Moreover, the $\mathcal{N}=2$ superfield

$$\mathcal{W}(y,\theta,\widetilde{\theta}) = X(y,\theta) + \sqrt{2}i\,\widetilde{\theta}W(y,\theta) - \frac{1}{4}\widetilde{\theta}^2\overline{D}^2\overline{X}(y,\theta)\,, \tag{III.51}$$

is gauge invariant

$$\delta_g \mathcal{W} = 0 \tag{III.52}$$

under (III.47) and (III.50), it contains $8_B + 8_F$ d.o.f. and is CC. In particular, it contains the degrees of freedom of the Maxwell multiplet upon requiring

$$\begin{aligned} \mathcal{W} &= \sqrt{2}\theta\kappa + \sqrt{2}\widetilde{\theta}\lambda - \theta^2 F - \widetilde{\theta}^2\overline{F} + \sqrt{2}i\,\theta\widetilde{\theta}D + \ldots \\ &\overset{!}{=} \sqrt{2}\,\theta^i\lambda_i - \theta^i\theta^j Y_{ij} + \ldots \end{aligned} \tag{III.53}$$

and thus making the identification

$$\begin{aligned} \lambda_1 = \kappa \quad &, \quad \lambda_2 = \lambda \\ Y_{11} = F \quad , \quad Y_{22} = \overline{F} \quad &, \quad Y_{12} = \tfrac{1}{i\sqrt{2}}D\,. \end{aligned} \tag{III.54}$$

A general Lagrangian for the Maxwell multiplet can be written with the use of a holomorphic function $\mathcal{F}(\mathcal{W})$ as

$$\begin{aligned} \mathcal{L}_{Max} &= \tfrac{1}{2}\int d^2\theta\, d^2\widetilde{\theta}\,\mathcal{F}(\mathcal{W}) + \text{h.c.} \\ &= \tfrac{1}{2}\int d^2\theta \left(\tfrac{1}{2}\mathcal{F}''(X)W^2 - \tfrac{1}{4}\mathcal{F}'(X)\overline{D}^2\overline{X}\right) + \text{h.c.} \end{aligned} \tag{III.55}$$

To compute (III.55) in terms of component fields, we first calculate

$$W^2 = -\lambda^2 - 2i\,\theta\lambda\,D + \lambda\sigma^\mu\overline{\sigma}^\nu\theta\,F_{\mu\nu} + \theta^2\left(D^2 - \frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \frac{i}{2}F_{\mu\nu}\widetilde{F}^{\mu\nu} + 2i\,\lambda\sigma^\mu\partial_\mu\overline{\lambda}\right), \tag{III.56}$$

where

$$\widetilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\kappa\lambda}F_{\kappa\lambda} \tag{III.57}$$

and

$$\frac{1}{4}\overline{D}^2\overline{X} = \overline{F} + \sqrt{2}i\,\theta\sigma^\mu\partial_\mu\overline{\kappa} + \theta^2\,\Box\overline{x}\,. \tag{III.58}$$

We then find that

$$\begin{aligned} \mathcal{L}_{Max} = &\tfrac{1}{2}\mathcal{F}''(x)\partial_\mu x\partial^\mu\overline{x} - \tfrac{1}{8}\mathcal{F}''(x)F_{\mu\nu}F^{\mu\nu} - \tfrac{i}{8}\mathcal{F}''(x)F_{\mu\nu}\widetilde{F}^{\mu\nu} \\ &+\tfrac{i}{2}\mathcal{F}''(x)(\lambda\sigma^\mu\partial_\mu\overline{\lambda} + \kappa\sigma^\mu\partial_\mu\overline{\kappa}) + \tfrac{1}{2}\mathcal{F}''(x)\left(F\overline{F} + \tfrac{1}{2}D^2\right) \\ &+\tfrac{1}{4}\mathcal{F}'''(x)\left(F\lambda^2 + \overline{F}\kappa^2 + \tfrac{i}{\sqrt{2}}D\,\kappa\lambda\right) \\ &+\tfrac{\sqrt{2}}{8}\mathcal{F}'''(x)\,\lambda\sigma^\mu\overline{\sigma}^\nu\kappa\,F_{\mu\nu} + \tfrac{1}{8}\mathcal{F}''''(x)\,\kappa^2\lambda^2 + \text{h.c.} \end{aligned} \tag{III.59}$$

where we have explicitly written the four–fermion term and we have used the fact that

$$\mathcal{F}'(x)\Box\overline{x} = -\mathcal{F}''(x)\,\partial_\mu x\partial^\mu\overline{x} + \text{tot. deriv.} \tag{III.60}$$

In order to break supersymmetry, one may in principle add the following terms to $\mathcal{L}_{Max}$

$$\mathcal{L}_{FI} = \frac{1}{2}m^2\int d^2\theta\, X + \text{h.c.} + \xi\int d^2\theta d^2\overline{\theta}\, V_2\,, \tag{III.61}$$

where m^2 and ξ are a complex and a real parameter respectively, which we will be referring to collectively as Fayet–Iliopoulos terms. The corresponding action is by construction invariant under $\mathcal{N}=2$ supersymmetry. Note also that, using (III.49), the FI terms (III.61) can be rewritten as

$$\mathcal{L}_{FI} = \int d^2\theta d^2\overline{\theta}\Big(-2\,\text{Re}(m^2)\,V_1 + \xi V_2\Big) + \text{tot. deriv.} \tag{III.62}$$

III.4 The single–tensor multiplet

The single–tensor multiplet [33, 34, 35, 36] of $\mathcal{N}=2,\ D=4$ supersymmetry contains an antisymmetric tensor $b_{\mu\nu}$, three real scalar d.o.f. and two Weyl spinors ψ_α and ϕ_α, so that 2_B auxiliary d.o.f. are needed in the off-shell construction. In terms of $\mathcal{N}=1$ superfields, the totality of these d.o.f. is contained in a chiral superfield Φ with expansion

$$\Phi = z + \sqrt{2}\theta\psi - \theta^2 f \tag{III.63}$$

and in a real linear (gauge–invariant) superfield L whose expansion is given in (III.37). Notice that ψ_α and ϕ_α do *not* form an $SU(2)$–doublet. Now suppose that Φ and L are related under a second $\mathcal{N}=1$ supersymmetry as follows

$$\begin{aligned} \delta^*\Phi = \sqrt{2}i\,\overline{\eta}\overline{D}L \quad , \quad \delta^*\overline{\Phi} = \sqrt{2}i\,\eta DL \\ \delta^* L = -\tfrac{i}{\sqrt{2}}(\eta D\Phi + \overline{\eta}\overline{D}\overline{\Phi})\,. \end{aligned} \tag{III.64}$$

A general $\mathcal{N}=2$ Lagrangian for Φ and L can then be written as

$$\mathcal{L}_{ST} = \int d^2\theta d^2\overline{\theta}\,\mathcal{H}(L,\Phi,\overline{\Phi})\,, \tag{III.65}$$

where $\mathcal{H}$ is a real function that must be a solution of the Laplace equation [34]

$$\mathcal{H}_{LL} + 2\mathcal{H}_{\Phi\overline{\Phi}} = 0\,, \tag{III.66}$$

where $\mathcal{H}_{LL} \equiv \frac{\partial^2}{\partial L^2}\mathcal{H}$ and $\mathcal{H}_{\Phi\overline{\Phi}} \equiv \frac{\partial^2}{\partial\Phi\partial\overline{\Phi}}\mathcal{H}$, for the action corresponding to (III.65) to be invariant under (III.64). In [37] we use such an example

$$\mathcal{H}(L,\Phi,\overline{\Phi}) = -L^2\left(iW_\Phi - i\overline{W}_{\overline{\Phi}}\right) + i\overline{\Phi}W - i\Phi\overline{W}\,, \tag{III.67}$$

where W is a function of Φ so that $W_\Phi \equiv \frac{dW}{d\Phi}$. Then (III.65) becomes

$$\mathcal{L}_{ST} = \int d^2\theta\left[\frac{i}{2}W_\Phi(\overline{D}L)(\overline{D}L) - \frac{i}{4}W\overline{D}^2\overline{\Phi}\right] + \text{h.c.} \tag{III.68}$$

Moreover, the $\mathcal{N}=2$ superfield constructed from Φ and L [29]

$$\mathcal{Z}(z,\theta,\overline{\tilde{\theta}}) = \Phi(z,\theta) + \sqrt{2}i\,\overline{\tilde{\theta}}\overline{D}L(z,\theta) - \frac{1}{4}\overline{\tilde{\theta}}^2\,\overline{D}^2\overline{\Phi}(z,\theta) \tag{III.69}$$

contains $8_B + 8_F$ d.o.f. and is CA. A general Lagrangian for $\mathcal{Z}$ can then be written with the use of a holomorphic function $\mathcal{G}(\mathcal{Z})$ as

$$\begin{aligned}\mathcal{L}_{ST} &= \int d^2\theta\, d^2\widetilde{\overline{\theta}}\, \mathcal{G}(\mathcal{Z}) + \text{h.c.} \\ &= \int d^2\theta \left(\tfrac{1}{2}\mathcal{G}''(\Phi)(\overline{D}L)(\overline{D}L) - \tfrac{1}{4}\mathcal{G}'(\Phi)\overline{D}^2\overline{\Phi}\right) + \text{h.c.}\end{aligned} \tag{III.70}$$

so that

$$\mathcal{G}'(\Phi) = iW\,. \tag{III.71}$$

To compute (III.70) in terms of components fields, we first calculate (in chiral coordinates)

$$\begin{aligned}\overline{D}_{\dot\alpha}L &= i\overline{\varphi}_{\dot\alpha} - (\theta\sigma^\mu)_{\dot\alpha}(\upsilon_\mu + i\partial_\mu C) - \theta^2(\partial_\mu\varphi\sigma^\mu)_{\dot\alpha} \\ (\overline{D}L)(\overline{D}L) &= -\overline{\varphi}^2 - 2i(\upsilon_\mu + i\partial_\mu C)\,\theta\sigma^\mu\overline{\varphi} \\ &\quad -\theta^2\Big[2i\,(\partial_\mu\varphi)\sigma^\mu\overline{\varphi} + (\upsilon_\mu + i\partial_\mu C)(\upsilon^\mu + i\partial^\mu C)\Big] \\ \tfrac{1}{4}\overline{D}^2\overline{\Phi} &= \overline{f} + \sqrt{2}i\,\theta\sigma^\mu\partial_\mu\overline{\psi} + \theta^2\,\Box\overline{z}\,.\end{aligned} \tag{III.72}$$

We then find that

$$\begin{aligned}\mathcal{L}_{ST} = \;& \mathcal{G}''(z)\,\partial_\mu z\partial^\mu\overline{z} - \tfrac{1}{2}\mathcal{G}''(z)\,(\upsilon_\mu + i\partial_\mu C)(\upsilon^\mu + i\partial^\mu C) \\ & + i\mathcal{G}''(z)\left(\psi\sigma^\mu\partial_\mu\overline{\psi} - (\partial_\mu\varphi)\sigma^\mu\overline{\varphi}\right) + \mathcal{G}''(z)\,f\overline{f} \\ & + \tfrac{1}{2}\mathcal{G}'''(z)\,(f\overline{\varphi}^2 + \overline{f}\psi^2) \\ & + \tfrac{i}{\sqrt{2}}\mathcal{G}'''(z)\,(\upsilon_\mu + i\partial_\mu C)\psi\sigma^\mu\overline{\varphi} + \tfrac{1}{4}\mathcal{G}''''(z)\,\psi^2\overline{\varphi}^2 + \text{h.c.}\end{aligned} \tag{III.73}$$

Note that there is no term that couples ψ_α to φ_α as in (III.59), as the single–tensor multiplet contains one real auxiliary d.o.f. less than the Maxwell multiplet. Other than that, (III.73) can be obtained from (III.59) by performing a chirality inversion on φ_α and by replacing

$$\begin{aligned}\mathcal{F}(x) &\to 2\mathcal{G}(z) \\ \tfrac{1}{2}F_{\mu\nu}(F^{\mu\nu} + i\widetilde{F}^{\mu\nu}) &\to (\upsilon_\mu + i\partial_\mu C)(\upsilon^\mu + i\partial^\mu C)\,.\end{aligned} \tag{III.74}$$

Furthermore, analogously to (III.61), in [37] we use the superpotential

$$\mathcal{L} = \widetilde{m}^2 \int d^2\theta\,\Phi + \text{h.c.} \tag{III.75}$$

where $\widetilde{m}^2$ is a complex parameter, that may be added to L_{ST}. Obviously the action corresponding to (III.75) is invariant under the first $\mathcal{N} = 1$ supersymmetry. It is also invariant under the second $\mathcal{N} = 1$, since, using (III.64),

$$\delta^*\widetilde{m}^2 \int d^2\theta\,\Phi + \text{h.c.} = \widetilde{m}^2 \int d^2\theta\,\sqrt{2}i\,\overline{\eta}\overline{D}L + \text{h.c.} = \text{tot. deriv.} \tag{III.76}$$

where we have used the first of the expressions (III.72).

Interestingly, there exists an alternative formulation of the single–tensor multiplet, in terms of the chiral superfield Φ and an $\mathcal{N}=1$ spinor superfield χ_α, with the latter admitting the gauge transformation (III.28)

$$\delta_g \chi_\alpha = -\frac{i}{4}\overline{D}^2 D_\alpha \widehat{V}_2 \equiv i W_{\alpha,g}\,, \tag{III.77}$$

where $\widehat{V}_2$ is a vector superfield, under which Φ is of course gauge invariant. In particular, using (III.40) and (III.64), one finds the second $\mathcal{N}=1$ transformations [34]

$$\begin{aligned} \delta^*\chi_\alpha &= -\tfrac{i}{\sqrt{2}}\Phi\,\eta_\alpha \\ \delta^*\Phi &= 2\sqrt{2}i\left[\tfrac{1}{4}\,\overline{D}^2\overline{\eta\chi} + i\partial_\mu\chi\sigma^\mu\overline{\eta}\right]. \end{aligned} \tag{III.78}$$

However, additional degrees of freedom are needed in this representation [29]. More specifically, upon checking the closure relation (III.8) on χ_α, one finds

$$\begin{aligned} [\delta_1^*,\delta_2^*]\chi_\alpha &= -2i\,(\eta_1\sigma^\mu\overline{\eta}_2 - \eta_2\sigma^\mu\overline{\eta}_1)\partial_\mu\chi_\alpha \\ &\quad + \tfrac{i}{2}\overline{D}^2 D_\alpha\Big(i\eta_1\theta\,\overline{\eta}_2\overline{\chi} - i\overline{\eta}_1\overline{\theta}\,\eta_2\chi - i\eta_2\theta\,\overline{\eta}_1\overline{\chi} + i\overline{\eta}_2\overline{\theta}\,\eta_1\chi\Big)\,. \end{aligned} \tag{III.79}$$

The existence of the second term in (III.79) implies that the algebra does not close on χ_α. Note also that $i\eta_1\theta\,\overline{\eta}_2\overline{\chi} - i\overline{\eta}_1\overline{\theta}\,\eta_2\chi - i\eta_2\theta\,\overline{\eta}_1\overline{\chi} + i\overline{\eta}_2\overline{\theta}\,\eta_1\chi$ is a vector superfield, which means that the algebra closes on χ_α up to a gauge transformation of the latter. To restore the closure, one may add a chiral superfield Y, with the second $\mathcal{N}=1$ transformations becoming

$$\begin{aligned} \delta^* Y &= \sqrt{2}\,\eta\chi\,, \\ \delta^*\chi_\alpha &= -\tfrac{i}{\sqrt{2}}\Phi\,\eta_\alpha - \tfrac{\sqrt{2}}{4}\eta_\alpha\,\overline{D}^2\,\overline{Y} - \sqrt{2}i(\sigma^\mu\overline{\eta})_\alpha\partial_\mu Y\,, \\ \delta^*\Phi &= 2\sqrt{2}i\left[\tfrac{1}{4}\,\overline{D}^2\overline{\eta\chi} + i\partial_\mu\chi\sigma^\mu\overline{\eta}\right]\,. \end{aligned} \tag{III.80}$$

Note that this does not affect the transformation of L in (III.64), since

$$D^\alpha\Big(-\frac{\sqrt{2}}{4}\eta_\alpha\,\overline{D}^2\,\overline{Y} + \frac{\sqrt{2}}{2}\overline{\eta}^{\dot\alpha}\overline{D}_{\dot\alpha}D_\alpha Y\Big) + \text{h.c.} = 0\,. \tag{III.81}$$

In addition, the superfields $(Y,\ \chi_\alpha,\ \Phi)$ can be viewed as components of an $\mathcal{N}=2$ CC superfield $\widehat{\mathcal{Z}}$ with expression [29]

$$\widehat{\mathcal{Z}} = Y + \sqrt{2}\,\widetilde{\theta}\chi - \widetilde{\theta}\widetilde{\theta}\left[\frac{i}{2}\Phi + \frac{1}{4}\overline{DD}\,\overline{Y}\right], \tag{III.82}$$

which, as we point out in [37], is related to $\mathcal{Z}$ via

$$\mathcal{Z} = -\frac{i}{2}\widetilde{D}^2\widehat{\mathcal{Z}} + \frac{i}{2}\overline{\widetilde{D}}^2\overline{\widehat{\mathcal{Z}}}\,. \tag{III.83}$$

Moreover, it is natural to expect that the superpartner of $iW_{\alpha,g}$ under the second $\mathcal{N}=1$ supersymmetry is a chiral superfield X_g; X_g and $iW_{\alpha,g}$ form then an $\mathcal{N}=2$ Maxwell superfield $\mathcal{W}_g$ that acts a gauge parameter and generates the gauge trasformation of $\widehat{\mathcal{Z}}$ [29], namely

$$\delta_g \widehat{\mathcal{Z}} = \mathcal{W}_g \,, \tag{III.84}$$

or, in components,

$$\delta_g Y = X_g \quad , \quad \delta \chi_\alpha = i W_{\alpha,g} \quad , \quad \delta_g \Phi = 0 \,. \tag{III.85}$$

There exist thus two $\mathcal{N}=2$ superfields that describe the single–tensor multiplet: the gauge invariant $\mathcal{Z}$ and the gauge varying $\widehat{\mathcal{Z}}$.

III.5 Dualization and the hypermultiplet

An antisymmetric tensor $b_{\mu\nu}$ whose field–strength v_μ satisfies the constraint (III.38) can be dualized to a real scalar that has a shift symmetry. To see this, let us write a general gauge–invariant Lagrangian $\mathscr{L}$ for $b_{\mu\nu}$

$$\mathscr{L} = \mathcal{L}(v_\mu) - \sigma(x)\partial_\mu v^\mu = \mathcal{L}(v_\mu) + v^\mu \partial_\mu \sigma + \text{tot. deriv.} \tag{III.86}$$

where $\mathcal{L}$ is a function of v_μ and $\sigma(x)$ is a real scalar field that plays the role of a Lagrange multiplier that imposes the constraint (III.38). The e.o.m. of v_μ is

$$\frac{\partial \mathcal{L}}{\partial v^\mu} = -\partial_\mu \sigma \,, \tag{III.87}$$

which in principle can be solved to yield $v_\mu = v_\mu(\partial\sigma)$. Substituting this solution in $\mathscr{L}$, one obtains the Legendre transform $\widetilde{\mathcal{L}} = \widetilde{\mathcal{L}}(\partial\sigma)$ of $\mathcal{L} = \mathcal{L}(v_\mu)$ that depends solely on $\partial_\mu\sigma$. Consequently, σ is massless and $\widetilde{\mathcal{L}}$ enjoys a shift symmetry

$$\delta_{shift}\, \sigma(x) = c \,, \tag{III.88}$$

where c is a real constant.

This duality is also true at the $\mathcal{N}=1$ level [34], namely a real linear superfield L that contains the field-strength v_μ of $b_{\mu\nu}$ can be dualized to a chiral superfield that contains a real scalar with a shift symmetry instead. In particular, consider a general Lagrangian $\mathcal{L}_L$ for L

$$\mathcal{L}_L = \int d^2\theta d^2\overline{\theta}\, \mathcal{L}(L) \,, \tag{III.89}$$

where $\mathcal{L}$ is a function of L. $\mathcal{L}_L$ can be written as

$$\mathcal{L}_L = \int d^2\theta d^2\overline{\theta} \left[\mathcal{L}(V) - (Q + \overline{Q})V\right] , \tag{III.90}$$

where V and Q are a vector and a chiral superfield respectively. Note that (III.90) is invariant under the shift symmetry

$$\delta_{shift} Q = ic \,, \tag{III.91}$$

where c is a real constant. The e.o.m. of Q and its conjugate is

$$\overline{D}^2 V = D^2 V = 0 \,, \tag{III.92}$$

so that Q and its conjugate play the role of Lagrange multipliers that impose the second of the constraints (III.36) on V, thus rendering it a (real) linear superfield. Alternatively, the e.o.m. of V is

$$\mathcal{L}'(V) = Q + \overline{Q}\,, \tag{III.93}$$

which in principle can be solved to yield $V = V(Q+\overline{Q})$. Substituting this solution in (III.90), one obtains the Legendre transform $\widetilde{\mathcal{L}} = \widetilde{\mathcal{L}}(Q+\overline{Q})$ of $\mathcal{L} = \mathcal{L}(V)$. $\widetilde{\mathcal{L}}(Q+\overline{Q})$ enjoys a shift symmetry under which one of the real d.o.f. of Q transforms as in (III.88) and is massless.

Similarly, a single–tensor multiplet with kinetic Lagrangian controlled by $\mathcal{H}(L,\Phi,\overline{\Phi})$ can be dualized to a hypermultiplet with one shift symmetry [34]. A generic hypermultiplet [24, 76] contains four scalars and two spin–1/2 fermions, that can be seen as components of two $\mathcal{N}=1$ chiral superfields T and Φ. It does not, however, admit an off–shell description in standard $\mathcal{N}=2$ superspace, there is namely no $\mathcal{N}=2$ superfield analogous to (III.69) that corresponds to the hypermultiplet. This issue is associated with the fact that the $\mathcal{N}=2$ algebra allows for the existence of central charges. Nevertheless, there exists an off–shell formulation for the hypermlutiplet in harmonic or projective superspace [38, 39]; yet this implementation comes with the drawback that one has to introduce an infinite number of auxiliary fields – we will not consider this case in the present thesis.

In regard to the dualization, the Legendre transform of $\mathcal{H}(L,\Phi,\overline{\Phi})$ is

$$\widetilde{\mathcal{H}}(T+\overline{T},\Phi,\overline{\Phi}) = \mathcal{H}(V,\Phi,\overline{\Phi}) - (T+\overline{T})V\,, \tag{III.94}$$

where T and its conjugate play the role of Lagrange multipliers, with

$$\mathcal{H}_V(V,\Phi,\overline{\Phi}) = T + \overline{T}\,. \tag{III.95}$$

$\widetilde{\mathcal{H}}(T+\overline{T},\Phi,\overline{\Phi})$ is then invariant under a shift symmetry of T and the kinetic Lagrangian is

$$\begin{aligned}
\int d^2\theta d^2\overline{\theta}\,\widetilde{\mathcal{H}} &= -\tfrac{1}{4}\int d^2\theta\left[\widetilde{\mathcal{H}}_{\overline{T}}(T+\overline{T},\Phi,\overline{\Phi})\overline{D}^2\overline{T} + \widetilde{\mathcal{H}}_{\overline{\Phi}}(T+\overline{T},\Phi,\overline{\Phi})\overline{D}^2\overline{\Phi}\right] \\
&\quad + \cdots + \text{h.c.} \\
&= -\int d^2\theta\left[\widetilde{\mathcal{H}}_{\overline{T}}(T+\overline{T},\Phi,\overline{\Phi})\,\theta^2\Box\overline{q}_1 + \widetilde{\mathcal{H}}_{\overline{\Phi}}(T+\overline{T},\Phi,\overline{\Phi})\,\theta^2\Box\overline{q}_2\right] \\
&\quad + \cdots + \text{h.c.} \\
&= -\left[\widetilde{\mathcal{H}}_{\overline{T}}(q_1) + \widetilde{\mathcal{H}}_{\overline{T}}(q_2)\right]\theta^2\Box\overline{q}_1 - \left[\widetilde{\mathcal{H}}_{\overline{\Phi}}(q_1) + \widetilde{\mathcal{H}}_{\overline{\Phi}}(q_2)\right]\theta^2\Box\overline{q}_2 \\
&\quad + \cdots + \text{h.c.} \\
&= \widetilde{\mathcal{H}}_{T\overline{T}}\,\partial_\mu q_1\partial^\mu\overline{q}_1 + \widetilde{\mathcal{H}}_{\Phi\overline{T}}\,\partial_\mu q_2\partial^\mu\overline{q}_1 \\
&\quad + \widetilde{\mathcal{H}}_{T\overline{\Phi}}\,\partial_\mu q_1\partial^\mu\overline{q}_2 + \widetilde{\mathcal{H}}_{\Phi\overline{\Phi}}\,\partial_\mu q_2\partial^\mu\overline{q}_2 + \cdots + \text{h.c.}
\end{aligned} \tag{III.96}$$

where q_1 and q_2 denote the lowest components of T and Φ respectively and the dots stand for terms not containing kinetic terms for q_1 and q_2. We thus identify the Hessian matrix $\widetilde{\mathcal{H}}''$ with the Kähler metric of the scalar manifold, namely the manifold whose coordinates are the scalars q_1 and q_2.

To find the analogue of (III.66) in this case, we first note that the solution of (III.95) is

$$V = V(T + \overline{T}, \Phi, \overline{\Phi})\,, \tag{III.97}$$

with

$$V_T \equiv \frac{\partial V}{\partial T} \quad , \quad V_{\overline{T}} = \frac{\partial V}{\partial \overline{T}} \quad , \quad V_\Phi \equiv \frac{\partial V}{\partial \Phi} \quad , \quad V_{\overline{\Phi}} = \frac{\partial V}{\partial \overline{\Phi}}\,. \tag{III.98}$$

Then (III.95) yields

$$V_\Phi = -\frac{\mathcal{H}_{V\Phi}}{\mathcal{H}_{VV}} \quad , \quad V_{\overline{\Phi}} = -\frac{\mathcal{H}_{V\overline{\Phi}}}{\mathcal{H}_{VV}} \quad , \quad V_T = V_{\overline{T}} = \frac{1}{\mathcal{H}_{VV}}\,. \tag{III.99}$$

Moreover,

$$\begin{aligned} \widetilde{\mathcal{H}}_T &= \mathcal{H}_V V_T - V - (T + \overline{T}) V_T \\ \widetilde{\mathcal{H}}_\Phi &= \mathcal{H}_\Phi + \mathcal{H}_V V_\Phi - (T + \overline{T}) V_\Phi\,, \end{aligned} \tag{III.100}$$

so that

$$\begin{aligned} \widetilde{\mathcal{H}}_{T\overline{T}} = -\tfrac{1}{\mathcal{H}_{VV}} \quad &, \quad \widetilde{\mathcal{H}}_{\Phi\overline{\Phi}} = \mathcal{H}_{\Phi\overline{\Phi}} - \tfrac{\mathcal{H}_{V\Phi}\mathcal{H}_{V\overline{\Phi}}}{\mathcal{H}_{VV}} \\ \widetilde{\mathcal{H}}_{T\overline{\Phi}} = \tfrac{\mathcal{H}_{V\overline{\Phi}}}{\mathcal{H}_{VV}} \quad &, \quad \widetilde{\mathcal{H}}_{\Phi\overline{T}} = \tfrac{\mathcal{H}_{V\Phi}}{\mathcal{H}_{VV}}\,. \end{aligned} \tag{III.101}$$

Consequently, the determinant of the Kähler metric is given by

$$\det(\widetilde{\mathcal{H}}'') \equiv \widetilde{\mathcal{H}}_{T\overline{T}}\widetilde{\mathcal{H}}_{\Phi\overline{\Phi}} - \widetilde{\mathcal{H}}_{T\overline{\Phi}}\widetilde{\mathcal{H}}_{\Phi\overline{T}} = -\frac{\mathcal{H}_{\Phi\overline{\Phi}}}{\mathcal{H}_{VV}}\,, \tag{III.102}$$

so, using (III.66), one finds that [34]

$$\det(\widetilde{\mathcal{H}}'') = \frac{1}{2}\,, \tag{III.103}$$

which is a nonlinear 2nd order PDE of complex Monge–Ampère type. As a result, supersymmetry imposes the constraint (III.103) on $\widetilde{\mathcal{H}}$, or, equivalently, the constraint (III.66) on $\mathcal{H}$. Moreover, the Ricci tensor of the Kähler manifold is given by

$$R_{I\overline{J}} = -\partial_I \partial_{\overline{J}} \Big[\log\det(\widetilde{\mathcal{H}}'') \Big]\,, \tag{III.104}$$

where ∂_I ($\partial_{\overline{J}}$) denotes derivation with respect to T or Φ ($\overline{T}$ or $\overline{\Phi}$). Using (III.103), one finds that

$$R_{I\overline{J}} = 0\,, \tag{III.105}$$

so the conclusion is that the scalar manifold is Ricci–flat. This property of the scalar manifold of hypermultiplets in global supersymmetry is universal [77].

Finally, in [37] we use the particular example (III.67), so, with the replacement $L \to V$, we have that

$$\mathcal{H}_V = -2iV\,(W_\Phi - \overline{W}_{\overline{\Phi}})\,, \tag{III.106}$$

so (III.95) yields

$$V = \frac{i}{2}\frac{T + \overline{T}}{W_\Phi - \overline{W}_{\overline{\Phi}}}\,. \tag{III.107}$$

Substituting (III.107) in (III.94), we find that

$$\widetilde{\mathcal{H}}(T + \overline{T}, \Phi, \overline{\Phi}) = -\frac{i}{4}\frac{(T + \overline{T})^2}{W_\Phi - \overline{W}_{\overline{\Phi}}} + i\overline{\Phi}W - i\Phi\overline{W}\,. \tag{III.108}$$

III.6 A new Maxwell superfield

In this section, the formulation of a new $\mathcal{N}=2$ superfield that describes the Maxwell multiplet is presented, which is part of: I. Antoniadis, J. P. Derendinger and C. Markou, *Nonlinear $\mathcal{N}=2$ global supersymmetry*, JHEP **1706** (2017) 052.

The CC Maxwell superfield may be written as

$$\mathcal{W} = -\frac{i}{2}\,\overline{\widetilde{D}}^2\widehat{\mathcal{W}} + \frac{i}{2}\,\overline{D}^2\overline{\widehat{\mathcal{W}}}\,, \tag{III.109}$$

where $\widehat{\mathcal{W}}$ is a CA superfield. It is obvious that the relation (III.109) respects the chirality of $\mathcal{W}$. Generically

$$\widehat{\mathcal{W}} = U + \sqrt{2}\,\overline{\widetilde{\theta}}\overline{\Omega} - \overline{\widetilde{\theta}}^2\left[\frac{i}{2}\widehat{X} + \frac{1}{4}\overline{D}^2\overline{U}\right], \tag{III.110}$$

where U and $\widehat{X}$ are chiral $\mathcal{N}=1$ superfields and $\overline{\Omega}_{\dot\alpha}$ an $\mathcal{N}=1$ chiral spinor superfield. Using (III.109), the components of $\widehat{\mathcal{W}}$ will be written in terms of the components of $\mathcal{W}$. First, note that (the right–handed) $\overline{\Omega}_{\dot\alpha}$ can be written as

$$\overline{\Omega}_{\dot\alpha} = \overline{D}_{\dot\alpha}\mathbb{L} \quad , \quad \Omega_\alpha = -D_\alpha\overline{\mathbb{L}}\,, \tag{III.111}$$

where $\mathbb{L}$ is a complex linear $\mathcal{N}=1$ superfield. We then find

$$\begin{aligned} \tfrac{1}{4}\,\overline{\widetilde{D}}^2\widehat{W} &= \tfrac{i}{2}\widehat{X} + \tfrac{1}{4}\overline{D}^2\overline{U} + \sqrt{2}i\,\widetilde{\theta}\sigma^\mu\partial_\mu\overline{\Omega} + \widetilde{\theta}^2\Box U \\ \tfrac{1}{4}\,\overline{D}^2\overline{\widehat{W}} &= \tfrac{1}{4}\overline{D}^2\overline{U} + \tfrac{\sqrt{2}}{4}\,\widetilde{\theta}\overline{D}^2\Omega - \widetilde{\theta}^2\Big[-\tfrac{i}{8}\overline{D}^2\overline{\widehat{X}} + \tfrac{1}{16}\overline{D}^2D^2U\Big]\,. \end{aligned} \tag{III.112}$$

Substituting (III.112) in (III.109) and comparing with (III.51), we make the identifications

$$\begin{aligned} \widehat{X} &\overset{!}{=} X \\ 2(\sigma^\mu\partial_\mu\overline{\Omega})_\alpha + \tfrac{i}{2}\overline{D}^2\Omega_\alpha &\overset{!}{=} iW_\alpha\,. \end{aligned} \tag{III.113}$$

Using (III.5), (III.46) and (III.111), the second of equations (III.113) takes the form

$$-\frac{1}{2}\,\overline{D}^2D_\alpha(\mathbb{L}+\overline{\mathbb{L}}) \overset{!}{=} -\frac{1}{4}\,\overline{D}^2D_\alpha V_2\,, \tag{III.114}$$

which yields

$$V_2 = 2(\mathbb{L}+\overline{\mathbb{L}})\,, \tag{III.115}$$

which, using (III.22) and (III.26), gives

$$A_\mu = -4\,\mathrm{Re}\,\mathbb{V}_\mu \quad , \quad D = -4\partial^\mu\,\mathrm{Im}\,\mathbb{V}_\mu\,. \tag{III.116}$$

Moreover, the components of $\widehat{\mathcal{W}}$ transform under the second $\mathcal{N}=1$ supersymmetry as

$$\begin{aligned} \delta^*U &= \sqrt{2}\,\overline{\eta\Omega}, \\ \delta^*\overline{\Omega}_{\dot\alpha} &= -\frac{i}{\sqrt{2}}\Big[X\,\overline{\eta}_{\dot\alpha} + i\,\overline{D}_{\dot\alpha}(\eta DU + \overline{\eta DU})\Big], \\ \delta^*X &= 2\sqrt{2}i\Big[\frac{1}{4}\overline{DD\eta\Omega} - i\,\eta\sigma^\mu\partial_\mu\overline{\Omega}\Big]\,. \end{aligned} \tag{III.117}$$

In addition, using (III.109), (III.113) and (III.115), we find that $\mathcal{W}$ is invariant under the following gauge transformation of $\widehat{\mathcal{W}}$

$$\delta_g \widehat{\mathcal{W}} = \mathcal{Z}_g \,, \tag{III.118}$$

where $\mathcal{Z}_g$ is a CA single–tensor superfield with components $(\Phi_g,\, iL_g)$ that acts as a gauge parameter and generates the transformation. In terms of component fields, (III.118) acts as

$$\delta_g U = \Phi_g \quad , \quad \delta_g \overline{\Omega}_{\dot\alpha} = i\overline{D}_{\dot\alpha} L_g \quad , \quad \delta_g X = 0 \,. \tag{III.119}$$

However, using (III.111) and (III.115), we observe that V_2 does not transform under (III.119), since

$$\delta_g \mathbb{L} = iL_g \quad \Rightarrow \quad \delta_g V_2 = 0 \,, \tag{III.120}$$

which is not (III.47). This means that the transformation (III.119) is not to be identified with the standard gauge transformation of the Maxwell multiplet. To find the latter, we first derive the transformations (III.117) in terms of the fields $(U,\, \mathbb{L},\, V_1)$:

$$\begin{aligned} \delta^* U &= \sqrt{2}\,\overline{\eta D}\,\mathbb{L} \\ \delta^* \mathbb{L} &= \frac{i}{\sqrt{2}}\,\overline{\eta D}\, V_1 + \frac{1}{\sqrt{2}}\,(\eta D\, U + \overline{\eta D\, U}) \\ \delta^* V_1 &= -\frac{i}{\sqrt{2}}\,(\eta D + \overline{\eta}\overline{D})\, 2(\mathbb{L} + \overline{\mathbb{L}}) \,. \end{aligned} \tag{III.121}$$

Consequently, the combination of the transformations (III.47), (III.50) and (III.119) is given by

$$\delta_g U \;=\; \Phi_g \quad , \quad \delta_g \mathbb{L} \;=\; \tfrac{1}{2}\Lambda_c \; + iL_g \quad , \quad \delta_g V_1 \;=\; \Lambda_l \,. \tag{III.122}$$

Therefore, we conclude that the Maxwell multiplet admits two gauge variations that are generated by two independent single–tensor multiplets: the standard Maxwell gauge transformations generated by (Λ_l, Λ_c), which leave *both* $\mathcal{W}$ and $\widehat{\mathcal{W}}$ invariant, and another gauge transformation generated by $\mathcal{Z}_g$, which leaves only $\mathcal{W}$ invariant. Finally, note that, in the gauge $U = 0$, the second $\mathcal{N} = 1$ algebra closes on $\mathbb{L}$

$$\begin{aligned} [\delta_1^*, \delta_2^*]\,\mathbb{L} \;&=\; -2i\,(\eta_1\sigma^\mu\overline{\eta}_2 - \eta_2\sigma^\mu\overline{\eta}_1)\,\partial_\mu \mathbb{L} \\ &\quad -i\Big[i(\overline{\eta_2 D}\,\eta_1 D - \overline{\eta_1 D}\,\eta_2 D)\overline{\mathbb{L}} - i(\eta_2 D\,\overline{\eta_1 D} - \eta_1 D\,\overline{\eta_2 D})\mathbb{L}\Big] \end{aligned} \tag{III.123}$$

up to a gauge transformation of $\mathbb{L}$, since $i(\overline{\eta_2 D}\,\eta_1 D - \overline{\eta_1 D}\,\eta_2 D)\overline{\mathbb{L}} - i(\eta_2 D\,\overline{\eta_1 D} - \eta_1 D\,\overline{\eta_2 D})\mathbb{L}$ is a real linear superfield. Thus, very similarly to the construction of the single–tensor multiplet in terms of $(Y,\, \chi_\alpha,\, \Phi)$, the formulation of the Maxwell multiplet in terms of X and Ω_α necessitates the use of U. To summarize, the Maxwell multiplet can also be described by a CA superfield $\widehat{\mathcal{W}}$, alternatively to the CC $\mathcal{W}$.

III.7 The BF interaction

An antisymmetric tensor $b_{\mu\nu}$ may interact with a gauge field A_μ via the welll–known BF term

$$\epsilon^{\mu\nu\kappa\lambda} b_{\mu\nu} F_{\kappa\lambda}\,, \tag{III.124}$$

that is obviously invariant under the gauge transformation (III.29). Note that, upon dualization of $b_{\mu\nu}$ to a real scalar with a shift symmetry, the coupling (III.124) becomes a Chern–Simons term multiplied by the derivative of the scalar. The $\mathcal{N}=1$ supersymmetrization of the BF term can be written [28, 29] in terms of the supefields L and V_2 as

$$\begin{aligned} -g\int d^2\theta d^2\overline{\theta}\, LV_2 &= \tfrac{1}{2}g\Big(-CD+\lambda\varphi+\overline{\lambda}\overline{\varphi}-v_\mu A^\mu\Big) \\ &= \tfrac{1}{2}g\Big(-CD+\lambda\varphi+\overline{\lambda}\overline{\varphi}-\tfrac{1}{4}\epsilon^{\mu\nu\kappa\lambda}b_{\mu\nu}F_{\kappa\lambda}\Big)+\text{tot. deriv.} \end{aligned} \tag{III.125}$$

or, using (III.31) and (III.40), in terms of the χ_α and W_α

$$-g\int d^2\theta d^2\overline{\theta}\, LV_2 \;=\; g\int d^2\theta\,\chi W+\text{h.c.}+\text{tot.deriv.} \tag{III.126}$$

The BF term is thus accompanied by fermion interactions at the $\mathcal{N}=1$ level. Recalling that L and W_α are gauge invariant, note that both descriptions are gauge–invariant under (III.24) and (III.28), since

$$\int d^2\theta d^2\overline{\theta}\, L\delta_g V_2 = -\frac{1}{4}\int d^2\theta\, \overline{D}^2(L\Lambda_c)+\text{h.c.}+\text{tot. deriv.}=\text{tot. deriv.} \tag{III.127}$$

or, equivalently,

$$\int d^2\theta\, W\delta_g\chi+\text{h.c.}=-\frac{i}{4}\int d^2\theta\, W\overline{D}^2 D\Pi+\text{h.c.}=\text{tot. deriv.} \tag{III.128}$$

where we have performed a partial integration with respect to the derivative $\overline{D}_{\dot\alpha}$ and its conjugate.

The $\mathcal{N}=2$ supersymmetrization of the BF term can be written in terms of the CC superfields $\mathcal{W}$ and $\widehat{\mathcal{Z}}$ [28, 29]

$$\begin{aligned} \mathcal{L}_{BF} &= ig\int d^2\theta\, d^2\widetilde{\theta}\,\mathcal{W}\widehat{\mathcal{Z}}+\text{h.c.} \\ &= g\int d^2\theta\Big(\tfrac{1}{2}X\Phi+\chi W\Big)+\text{h.c.}+\text{tot. deriv.} \\ &= -\tfrac{1}{2}g(xf+\overline{x}\overline{f}+zF+\overline{z}\overline{F}+\kappa\psi+\overline{\kappa}\overline{\psi}) \\ &\quad -\tfrac{1}{8}g\,\epsilon_{\mu\nu\rho\sigma}B^{\mu\nu}F^{\rho\sigma}-\tfrac{1}{2}gCD+g\lambda\varphi+g\overline{\lambda}\overline{\varphi}+\text{tot. deriv.} \end{aligned} \tag{III.129}$$

since

$$ig\int d^2\theta d^2\overline{\theta}\,(Y\overline{X}+X\overline{Y})+\text{h.c.}=0\,. \tag{III.130}$$

Alternatively, in [37] we use the CA superfields $\widehat{\mathcal{W}}$ and $\mathcal{Z}$ as

$$
\begin{aligned}
\mathcal{L}_{BF} &= ig\int d^2\theta\, d^2\bar{\tilde{\theta}}\,\widehat{\mathcal{W}}\mathcal{Z} + \text{h.c.} \\
&= g\int d^2\theta \left(\tfrac{1}{2}X\Phi + \overline{\Omega}\overline{D}L\right) + ig\int d^2\theta d^2\overline{\theta}\,(U\overline{\Phi} + \Phi\overline{U}) + \text{h.c.} + \text{tot. deriv.} \\
&= g\int d^2\theta \left(\tfrac{1}{2}X\Phi + \overline{D}\mathbb{L}\overline{D}L\right) + \text{h.c.} + \text{tot. deriv.} \\
&= -\tfrac{1}{2}g(xf + \overline{x}\overline{f} + zF + \overline{z}\overline{F} + \kappa\psi + \overline{\kappa}\overline{\psi}) \\
&\quad -\tfrac{1}{8}g\,\epsilon_{\mu\nu\rho\sigma}B^{\mu\nu}F^{\rho\sigma} + 2g\,C\partial_\mu\,\text{Im}\,\mathbb{V}^\mu + g\lambda\varphi + g\overline{\lambda\varphi} \\
&\quad -g\,\partial_\mu\varphi\sigma^\mu\overline{\omega} - g\,\omega\sigma^\mu\partial_\mu\overline{\varphi} + \text{tot. deriv.}
\end{aligned}
\tag{III.131}
$$

where we have used that

$$ig\int d^2\theta d^2\overline{\theta}\,(U\overline{\Phi} + \Phi\overline{U}) + \text{h.c.} = 0\,. \tag{III.132}$$

The terms of the last line of (III.131) do not appear in (III.129) simply because, in computing (III.131), we have used the full expansion (III.22) of $\mathbb{L}$, while, in computing (III.129), we have used the gauge–fixed expansion (III.34) of W_α.

Consequently, the supersymmetrization of the BF term (at the action level) does not depend on U and Y. Note also that the actions corresponding to (III.131) and (III.129) are gauge invariant under (III.118) and (III.84) respectively, since, for any pair of CC Maxwell multiplets $\mathcal{W}_1$, $\mathcal{W}_2$ and for any pair of CA single–tensor multiplets $\mathcal{Z}_1$, $\mathcal{Z}_2$, the terms

$$\text{Im}\int d^2\theta\int d^2\tilde{\theta}\,\mathcal{W}_1\mathcal{W}_2 \qquad \text{and} \qquad \text{Im}\int d^2\theta\int d^2\bar{\tilde{\theta}}\,\mathcal{Z}_1\mathcal{Z}_2$$

are total derivatives.

IV – Nonlinear $\mathcal{N}=2$, $D=4$ global supersymmetry

IV.1 The APT mechanism

Initially, it was thought that global $\mathcal{N}=2$ supersymmetry cannot be partially broken to $\mathcal{N}=1$. The standard argument [78] for this no–go theorem was that the relation

$$H = \sum_{\alpha} Q^2_{\alpha i}\,, \tag{IV.1}$$

where H is the Hamiltonian of the theory, which is a direct consequence of the $\mathcal{N}=2$ supersymmetry algebra (without central charges)

$$\{Q_{\alpha i}, \overline{Q}^j_{\dot\alpha}\} = 2\delta^j_i \sigma^\mu_{\alpha\dot\alpha} P_\mu \tag{IV.2}$$

with generators $Q_{\alpha i}$, where $i=1,2$ is the $SU(2)$ index, implies that, if one $\mathcal{N}=1$ supersymmetry is unbroken, then the other $\mathcal{N}=1$ must remain unbroken as well, since

$$Q_{\alpha 1}|0\rangle = 0 \quad \Rightarrow \quad H|0\rangle = 0 \quad \Rightarrow \quad Q_{\alpha 2}|0\rangle = 0\,. \tag{IV.3}$$

A way to circumvent the no–go theorem in question is to realize that the algebra (IV.2) is not the algebra of partially broken $\mathcal{N}=2$ supersymmetry. To see this, recall that the conserved charges $Q_{\alpha i}$ are determined by the supercurrents $\mathcal{J}_{\alpha i\,\mu}$ as

$$Q_{\alpha i} = \int d^3\vec{x}\, \mathcal{J}_{\alpha i\,0}\,, \tag{IV.4}$$

a fact that offers an alternative expression for (IV.2), upon integrating over infinite volume

$$\{\overline{Q}^j_{\dot\alpha}, \mathcal{J}_{\alpha i\,\nu}\} = 2\delta^j_i \sigma^\mu_{\alpha\dot\alpha} T_{\mu\nu} + \text{S.T.} \tag{IV.5}$$

where $T_{\mu\nu}$ is the energy–momentum tensor of the theory and S.T. stands for Schwinger terms that do not appear in (IV.2) due to the integration. Notice now that one may modify (IV.5) as

$$\{\overline{Q}^j_{\dot\alpha}, \mathcal{J}_{\alpha i\,\nu}\} = 2\delta^j_i \sigma^\mu_{\alpha\dot\alpha} T_{\mu\nu} + \sigma_{\nu\alpha\dot\alpha} C^j_i + \text{S.T.} \tag{IV.6}$$

since $T_{\mu\nu} + C\eta_{\mu\nu}$ is conserved as $T_{\mu\nu}$ is, namely the theory does not have a unique energy–momentum tensor. However, integration of the term $\sigma_{\nu\alpha\dot\alpha}C^j_i$ yields an infinity in (IV.2). These observations where first made in [79, 80], where also the first instance of a realization of (IV.6) was discovered in the context of string theory, in which $\mathcal{N}=2$ is partially broken on a four–dimensional membrane propagating in six dimensions.

Moreover, the APT model [25] was the first mechanism discovered that implements the partial breaking using superspace techniques and a single Maxwell multiplet, but not string theory. In particular, it was shown that the FI terms (III.61) are not sufficient to induce the partial breaking; one needs to further add the term

$$M^2 \mathcal{F}_X\,, \tag{IV.7}$$

where M^2 is a complex parameter and $\mathcal{F}_X = \mathcal{F}'(X)$, such that, using (III.55) and (III.61), the total Lagrangian for a single Maxwell multiplet becomes

$$\begin{aligned}\mathcal{L}_{Max,def} \;&=\; \tfrac{1}{2}\int d^2\theta\, \left[\tfrac{1}{2}\mathcal{F}_{XX}WW - \tfrac{1}{4}\mathcal{F}_X\overline{DD}\,\overline{X} + m^2X - iM^2\mathcal{F}_X\right] \\ &\quad +\text{h.c.} + \xi\int d^2\theta d^2\overline{\theta}\, V_2\,. \end{aligned}\tag{IV.8}$$

Interestingly, the action corresponding to (IV.8) is not invariant under (III.48), but it is under the *deformed* second $\mathcal{N}=1$ supersymmetry variations [26, 28]

$$\begin{aligned}\delta^* X \;&=\; \sqrt{2}\,i\,\eta W \\ \delta^* W_\alpha \;&=\; -\sqrt{2}M^2\,\eta_\alpha + \sqrt{2}\,i\left[\tfrac{1}{4}\eta_\alpha\overline{DD}\,\overline{X} + i(\sigma^\mu\overline{\eta})_\alpha\,\partial_\mu X\right]\,,\end{aligned}\tag{IV.9}$$

according to which the fermion λ_α that resides in W_α transforms nonlinearly and is thus identified with the Goldstino. Partial breaking arises if the following conditions hold: the theory is interacting, $\mathcal{F}_{XXX}\neq 0$, $M^2\neq 0\neq m^2$ and $\xi=0$, there is namely an $\mathcal{N}=1$ vacuum and the final spectrum consists of a massless $\mathcal{N}=1$ vector and a massive $\mathcal{N}=1$ chiral multiplet.

Alternatively, the APT model was written as [25]

$$\mathcal{L}_{\mathcal{A}} = \frac{i}{4}\int d^2\theta\, d^2\widetilde{\theta}[\mathcal{F}(\mathcal{A}) - \mathcal{A}_D\mathcal{A}] + \frac{1}{2}(\vec{E}\cdot\vec{\Omega} + \vec{M}\cdot\vec{Y}_D) + \text{h.c.} \tag{IV.10}$$

where $\mathcal{A}$ is a general $\mathcal{N}=2$ chiral superfield and $\mathcal{A}_D$ is an $\mathcal{N}=2$ chiral superfield that satisfies the reducing constraint

$$(\epsilon_{ij}D^i\sigma_{\mu\nu}D^j)^2\mathcal{A}_D = -96\,\Box\overline{\mathcal{A}}_D\,. \tag{IV.11}$$

namely $\mathcal{A}_D$ is a Maxwell superfield. $\mathcal{A}_D$ plays the role of a Lagrange multiplier and its equation of motion imposes (F.11) on $\mathcal{A}$, namely $\mathcal{A}$ becomes a Maxwell superfield. $\vec{\Omega}$ and $\vec{Y}_D$ correspond to the auxiliary fields of $\mathcal{A}$ and $\mathcal{A}_D$ respectively and, by construction, the $\vec{Y}_D$ satisfy the reality condition (III.44), while the $\vec{\Omega}$ generically do *not*. The terms containing $\vec{E}$ and $\vec{M}$, which are three–component vectors (with complex and real parameters as components respectively), are Fayet–Iliopoulos terms for $\mathcal{A}$ and $\mathcal{A}_D$ that have been added for the partial breaking. The latter occurs for non–zero $\vec{E}$ and $\vec{M}$ and in that case one can show that [42]

$$C_i^j \sim V\delta_i^j + \vec{\sigma}_i^j\cdot(\text{Re}\,\vec{E}\times\vec{M})\,, \tag{IV.12}$$

where V is the scalar potential of the theory. The APT model thus realizes precisely the algebra (IV.6).

In the remaining sections of the present chapter, the material presented is part of: I. Antoniadis, J. P. Derendinger and C. Markou, *Nonlinear $\mathcal{N}=2$ global supersymmetry*, JHEP **1706** (2017) 052, with occasional references to other literature.

IV.2 A new partial breaking mechanism

In the following, we present a new mechanism that implements the partial breaking of global $\mathcal{N}=2$ supersymmetry by utilizing a single–tensor multiplet. First, let us consider a generic $\mathcal{N}=1$ function W of the chiral superfield Φ of the single–tensor multiplet. Then under the second $\mathcal{N}=1$ supersymmetry transformations (III.64), we have that

$$\delta^* \int d^2\theta\, W(\Phi) = \sqrt{2}i \int d^2\theta\, W_\Phi\, \overline{\eta}\overline{D}L\,, \tag{IV.13}$$

which is not a total derivative unless $W(\Phi)$ is linear in Φ. Moreover, we observe that the following $\mathcal{N}=1$ equalities hold

$$\overline{DD}\,(\overline{\theta\eta}\,L) = -2\,\overline{\eta D}\,L = \overline{DD}\,(\overline{\theta\eta}L + \theta\eta L)\,, \tag{IV.14}$$

so that, up to a total derivative, (IV.13) can be written as

$$\delta^* \int d^2\theta\, W(\Phi) + \text{h.c.} = 2\sqrt{2}i \int d^2\theta d^2\overline{\theta}\,\left[W_\Phi - \overline{W}_{\overline{\Phi}}\right](\eta\theta + \overline{\eta\theta})L\,. \tag{IV.15}$$

We now add the generic generic superpotential

$$\widetilde{M}^2\, W(\Phi)\,, \tag{IV.16}$$

where $\widetilde{M}^2$ is a complex parameter, to (III.68) and (III.75), so that the Lagrangian for the single–tensor multiplet takes the form

$$\begin{aligned}
\mathcal{L}_{ST,def} &= \int d^2\theta \left[\frac{i}{2}W_\Phi\,(\overline{D}L)(\overline{D}L) - \frac{i}{4}W\,\overline{DD}\,\overline{\Phi} + \widetilde{m}^2\Phi + \widetilde{M}^2\,W\right] + \text{h.c.} \\
&= i\int d^2\theta d^2\overline{\theta}\left[-L^2(W_\Phi - \overline{W}_{\overline{\Phi}}) + \overline{\Phi}W - \Phi\overline{W}\right] \\
&\quad + \int d^2\theta \left[\widetilde{m}^2\Phi + \widetilde{M}^2\,W\right] + \text{h.c.}
\end{aligned} \tag{IV.17}$$

The action corresponding to (IV.17) is then invariant under the first, linearly realized, $\mathcal{N}=1$ supersymmetry as well as under a second $\mathcal{N}=1$ that is nonlinearly realized. The transformations corresponding to the latter are a deformed version of (III.64), namely

$$\begin{aligned}
\delta^*\Phi &= \sqrt{2}i\,\overline{\eta}\overline{D}L \quad , \quad \delta^*\overline{\Phi} = \sqrt{2}i\,\eta DL \\
\delta^*\,L &= \sqrt{2}\,\widetilde{M}^2\,(\overline{\theta\eta} + \theta\eta) - \tfrac{i}{\sqrt{2}}(\eta D\Phi + \overline{\eta D\Phi})\,.
\end{aligned} \tag{IV.18}$$

Note that according to (IV.18)

$$\delta^*\overline{D}_{\dot\alpha}L = -\sqrt{2}\,\widetilde{M}^2\overline{\eta}_{\dot\alpha} + \sqrt{2}i\left[\frac{1}{4}\overline{\eta}_{\dot\alpha}\overline{DD\Phi} - i(\eta\sigma^\mu)_{\dot\alpha}\,\partial_\mu\Phi\right]\,, \tag{IV.19}$$

which implies that the spinor $\overline{\varphi}_{\dot\alpha}$ in $\overline{D}_{\dot\alpha}L$ transforms nonlinearly and is thus to be identified with the Goldstino. Moreover, $\delta^*\Phi$ is not deformed, since

$$\delta^*_{def}\,\mathcal{L}_{ST} = -i\sqrt{2}\,\widetilde{M}^2\int d^2\theta\, W_\Phi\,\overline{\eta D}L + \text{h.c.} = -\widetilde{M}^2\,\delta^*\int d^2\theta\, W(\Phi) + \text{h.c.} \qquad \text{(IV.20)}$$

Notice that $\mathcal{L}_{ST,def}$ depends on two complex numbers, the deformation parameter $\widetilde{M}^2$ and the parameter $\widetilde{m}^2$ in the linear $\mathcal{N}=2$ superpotential. In fact, the transformations (IV.18) for the $\mathcal{N}=1$ linear multiplet were first found by performing a chirality switch on the deformed transformations of the $\mathcal{N}=1$ Maxwell multiplet [32]. For an alternative derivation of (IV.17), see the appendix B.

We now analyze the vacuum of the theory. The terms of (IV.17) involving auxiliary fields are given by

$$\begin{aligned}\mathcal{L}_{aux.} \;=\;& i(W_\Phi - \overline{W}_{\overline{\Phi}})f\overline{f} - \widetilde{m}^2 f - \widetilde{M}^2 W_\Phi f - \overline{\widetilde{m}}^2\overline{f} - \overline{\widetilde{M}}^2\,\overline{W}_{\overline{\Phi}}\overline{f} \\ & -\frac{i}{2}W_{\Phi\Phi}[\overline{f}\,\psi\psi + f\,\overline{\varphi\varphi}] + \frac{i}{2}\overline{W}_{\overline{\Phi\Phi}}[f\,\overline{\psi\psi} + \overline{f}\,\varphi\varphi] \;=\; -V + \mathcal{L}_{ferm.}\end{aligned} \qquad \text{(IV.21)}$$

so that the following scalar potential is generated

$$V = \frac{1}{i(W_\Phi - \overline{W}_{\overline{\Phi}})}\left|\widetilde{m}^2 + \widetilde{M}^2\,W_\Phi\right|^2, \qquad \text{(IV.22)}$$

to which L does not contribute. Moreover, the fermion mass terms are given by

$$\begin{aligned}\mathcal{L}_{ferm.} \;=\;& -\frac{1}{2}\widetilde{M}^2\,W_{\Phi\Phi}\,\psi\psi - \frac{1}{2}\left[\widetilde{m}^2 + \widetilde{M}^2\,W_\Phi\right]\frac{W_{\Phi\Phi}}{W_\Phi - \overline{W}_{\overline{\Phi}}}\,\psi\psi + \text{h.c.} \\ & -\frac{1}{2}\left[\widetilde{m}^2 + \widetilde{M}^2\,W_\Phi\right]\frac{\overline{W}_{\overline{\Phi\Phi}}}{W_\Phi - \overline{W}_{\overline{\Phi}}}\,\varphi\varphi + \text{h.c.}\end{aligned} \qquad \text{(IV.23)}$$

We thus distinguish the following cases:

1. $\widetilde{M}^2 = \widetilde{m}^2 = 0$: $\mathcal{N}=2$ supersymmetry remains intact and linearly realized and all fields are massless.

2. $\widetilde{M}^2 = 0$, $\widetilde{m}^2 \neq 0$, $W_{\Phi\Phi} = 0$ (this last equality implies that the theory is canonical, namely free): the potential is an irrelevant constant $V \sim |\widetilde{m}|^4$ so that, again, $\mathcal{N}=2$ supersymmetry remains intact and linearly realized and all fields are massless.

3. $\widetilde{M}^2 = 0$, $\widetilde{m}^2 \neq 0$, $W_{\Phi\Phi} \neq 0$ (namely the theory is not free): $\mathcal{N}=2$ breaks completely to $\mathcal{N}=0$ with

$$\langle f\rangle = -\frac{\overline{\widetilde{m}}^2}{2\,\text{Im}\langle W_\Phi\rangle}. \qquad \text{(IV.24)}$$

   The theory has a vacuum state if $\langle W_{\Phi\Phi}\rangle = 0$ has a solution, fermions remain then massless and the splitting of scalar masses is controlled by $\langle W_{\Phi\Phi\Phi}\rangle$.

4. $\widetilde{M}^2 \neq 0$, $\widetilde{m}^2 = 0$, $W_{\Phi\Phi} \neq 0$: again $\mathcal{N}=2$ breaks completely to $\mathcal{N}=0$ with

$$\langle f\rangle = -\frac{\overline{\widetilde{M}}^2\langle W_\Phi\rangle}{2\,\text{Im}\langle W_\Phi\rangle}. \qquad \text{(IV.25)}$$

5. $\widetilde{M}^2 \neq 0 \neq \widetilde{m}^2$, $W_{\Phi\Phi} \neq 0$: The minimum of the potential (IV.22) is at

$$\langle W_\Phi \rangle = -\frac{\widetilde{m}^2}{\widetilde{M}^2} \quad , \quad \langle f \rangle = 0 \,. \tag{IV.26}$$

The linear superfield L remains massless, while the canonically normalized mass of Φ is given by

$$\mathcal{M}_\Phi^2 = \widetilde{M}^2 \overline{\widetilde{M}}^2 \left| \frac{\langle W_{\Phi\Phi} \rangle}{2\,\mathrm{Im}\langle W_\Phi \rangle} \right|^2 . \tag{IV.27}$$

Note that the scalar kinetic metric is $-2\,\mathrm{Im}\langle W_\Phi \rangle$, which imposes $\mathrm{Im}\langle W_\Phi \rangle < 0$. We conclude that $\mathcal{N}=2$ breaks *partially* to $\mathcal{N}=1$. In principle, Φ can acquire a very large mass by adjusting $\langle W_{\Phi\Phi} \rangle$ and decouple from the massless L.

Finally, comparing the Lagrangian (IV.8) to (IV.17), as well as and the deformed transformations (IV.9) to (IV.18), we observe that there is clearly a correspondence between

$$\Big(X, \mathcal{F}_X(X)\Big) \text{ and } \Big(\Phi, W(\Phi)\Big) , \tag{IV.28}$$

upon a Lorentz chirality inversion relating W_α to $\overline{D}_{\dot\alpha} L$. Notice, however, that, due to the absence of auxiliary fields in L, there is no "electric" FI term in (IV.17) analogous to the ξD term in (IV.8).

Since the single–tensor multiplet can be dualized to a hypermultiplet with a shift symmetry, we now perform the dualization to find a partial breaking mechanism that makes use of a single hypermultiplet. We thus use the Legendre transform (III.94), with $\mathcal{H}(V, \Phi, \overline{\Phi})$ given by (III.67) with V in the place of L, so that equation (III.95) takes the form

$$T + \overline{T} = \mathcal{H}_V = -2iV(W_\Phi - \overline{W}_\Phi) \,. \tag{IV.29}$$

Using (IV.29), the first of equations (III.100) is written as

$$\widetilde{\mathcal{H}}_T = -V = -\frac{i}{2} \frac{T + \overline{T}}{W_\Phi - \overline{W}_{\overline{\Phi}}} \,. \tag{IV.30}$$

The dual hypermultiplet theory is then

$$\begin{aligned} \mathcal{L}_{dual} &= i \int d^2\theta d^2\overline{\theta} \left[-\frac{1}{4} \frac{(T+\overline{T})^2}{W_\Phi - \overline{W}_{\overline{\Phi}}} + W\overline{\Phi} - \overline{W}\Phi \right] + \int d^2\theta \left[\widetilde{m}^2 \Phi + \widetilde{M}^2\, W \right] + \text{h.c.} \\ &= \int d^2\theta \left[-\frac{i}{2} W_\Phi (\overline{D}\widetilde{\mathcal{H}}_T)(\overline{D}\widetilde{\mathcal{H}}_T) - \frac{i}{4} W \overline{DD}\overline{\Phi} + \widetilde{m}^2 \Phi + \widetilde{M}^2 W \right] + \text{h.c.} \end{aligned} \tag{IV.31}$$

up to a total derivative. Since the superpotential depends on Φ only, the auxiliary component f_T of T does not contribute to the scalar potential. Its field equation

$$(W_\Phi - \overline{W}_{\overline{\Phi}}) f_T - (T + \overline{T}) W_{\Phi\Phi} f_\Phi = 0 \tag{IV.32}$$

is actually the $\theta\theta$ component of the duality relation (IV.29). The ground state in the partially broken phase is again given by the relations (IV.26) with, in addition due to (IV.32),

$$\langle f_T \rangle = 0 \,. \tag{IV.33}$$

To find the analogue of (IV.18), notice first that, on–shell, the relations (IV.29) and (IV.30) take the form

$$\mathcal{H}_L = T + \overline{T} \quad , \quad \widetilde{\mathcal{H}}_T = -L\,, \tag{IV.34}$$

and are consistent using the field equations for L and T,

$$\overline{DD}D_\alpha \mathcal{H}_L = 0 \quad , \quad \overline{DD}\widetilde{\mathcal{H}}_T = 0\,, \tag{IV.35}$$

as integrability conditions. The $\mathcal{N} = 1$ theory (IV.31) is then also invariant, up to a superspace derivative, under a second, nonlinearly realized, $\mathcal{N} = 1$ supersymmetry with transformations

$$\begin{aligned} \delta^*\Phi = -\sqrt{2}i\,\eta\overline{D}\,\widetilde{\mathcal{H}}_T \quad , \quad \delta^*\overline{\Phi} = -\sqrt{2}i\,\eta D\,\widetilde{\mathcal{H}}_T\,, \\ \delta^*\widetilde{\mathcal{H}}_T = -\sqrt{2}\,\widetilde{M}^2\,(\overline{\theta\eta} + \theta\eta) + \tfrac{i}{\sqrt{2}}(\eta D\Phi + \overline{\eta D\Phi})\,. \end{aligned} \tag{IV.36}$$

This can be shown either by direct check, or by substituting for L via the second of the duality relations (IV.34) in the deformed transformations (IV.18) of the single–tensor multiplet. Notice that T appears implicitly in (IV.36) via $\widetilde{\mathcal{H}}_T$. Note also that the second of equations (IV.35) guarantees that $\delta^*\widetilde{\mathcal{H}}_T$ and $\delta^*\Phi$ are a linear and a chiral superfield respectively.

IV.3 The deformed Maxwell multiplet

In the previous sections, we have seen that a necessary condition for the partial breaking is the appearance of a specific type of a FI coefficient, that can also be viewed as a deformation parameter of the transformations corresponding to the broken and nonlinearly realized supersymmetry. However, in [29] it was shown that this deformation parameter can also be thought of as a deformation of the CC Maxwell superfield $\mathcal{W}$ itself. Here we will study the most general deformations of $\mathcal{W}$ that result in partial breaking. In particular, consider $\mathcal{W}$ deformed by

$$\mathcal{W}_{nl} \quad = \quad A^2\theta\theta + B^2\widetilde{\theta}\widetilde{\theta} + 2\Gamma\theta\widetilde{\theta} \tag{IV.37}$$

where A, B, Γ are complex parameters, such that

$$\mathcal{W}_{def} = \mathcal{W} + \mathcal{W}_{nl}\,. \tag{IV.38}$$

Indeed, (IV.37) are the most general deformations when it comes to the breaking, since the fermions of $\mathcal{W}$ are its θ^i components, which implies that the deformations should be introduced as the highest components of W_{def}, were it for a fermion to be identified with a Goldstino.

Identifying

$$\mathcal{W}_{nl} \overset{!}{=} (Y_2 + iY_1)\,\theta\theta + (Y_2 - iY_1)\,\widetilde{\theta}\widetilde{\theta} - 2iY_3\,\theta\widetilde{\theta}\,, \tag{IV.39}$$

which yields

$$\vec{Y} = \left(-\frac{i}{2}[A^2 - B^2], \frac{1}{2}[A^2 + B^2], i\Gamma\right)\,, \tag{IV.40}$$

we find that, under $\mathcal{N} = 2$, the fermions of $\mathcal{W}_{def}$ transform as

$$\begin{aligned} \delta\kappa_\alpha \quad &= \quad \sqrt{2}(A^2\epsilon_\alpha + \Gamma\eta_\alpha) + \ldots \\ \delta\lambda_\alpha \quad &= \quad \sqrt{2}(B^2\eta_\alpha + \Gamma\epsilon_\alpha) + \ldots \end{aligned} \tag{IV.41}$$

where the dots stand for terms not involving the deformation parameters. By requiring that one and only one linear combination of fermions transforms nonlinearly, we find that $\mathcal{N}=2$ is partially broken to $\mathcal{N}=1$ under the condition that

$$\Gamma = \pm AB\,, \tag{IV.42}$$

for which $\mathcal{W}_{nl} = (A\theta \pm B\widetilde{\theta})^2$ and

$$\delta(B\kappa_\alpha \mp A\lambda_\alpha) = 0\,. \tag{IV.43}$$

If (IV.42) holds, then the triplet (IV.40) does *not* satisfy the reality condition (III.44), which implies that the deformation parameters cannot be absorbed in the vacuum expectation values of F and D. This means that the vacuum expectation values of the auxiliary fields *cannot* induce partial breaking.

Using an $SU(2)$ rotation, one may set $0 = Y_3 = i\Gamma$. Partial breaking occurs then in the following equivalent cases:

- if $A=0$, for which λ_α is identified with the Goldstino, or
- if $B=0$, for which κ_α is identified with the Goldstino.

Without loss of generality, we only consider the first case in which [29]

$$\mathcal{W}_{def} = X + \sqrt{2}i\,\widetilde{\theta}W + \widetilde{\theta}\widetilde{\theta}\left[B^2 - \frac{1}{4}\overline{DD}\,\overline{X}\right]\,, \tag{IV.44}$$

for which (III.55) and (III.61) give

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\int d^2\theta\left[\int d^2\widetilde{\theta}\,\mathcal{F}(\mathcal{W}_{def}) + m^2 X\right] + \text{h.c.} + \xi\int d^2\theta d^2\overline{\theta}\,V_2\\ &= \frac{1}{4}\int d^2\theta\left[\mathcal{F}_{XX}WW - \frac{1}{2}\mathcal{F}_X\overline{DD}\,\overline{X} + 2m^2X + 2B^2\mathcal{F}_X\right] + \text{h.c.}\end{aligned} \tag{IV.45}$$

Note that this is precisely the case (IV.8), with $B^2 = -iM^2$.

Since $\langle F\rangle = \langle D\rangle = 0$, the mass terms of the fermion κ_α in X are

$$-\frac{B^2}{4}\langle\mathcal{F}_{XXX}\rangle\,\kappa\kappa - \frac{\overline{B}^2}{4}\langle\overline{\mathcal{F}_{XXX}}\rangle\,\overline{\kappa\kappa}$$

and the mass of the canonically normalized X is

$$\mathcal{M}_X = \frac{B^2\,\langle\mathcal{F}_{XXX}\rangle}{2\,\text{Re}\langle\mathcal{F}_{XX}\rangle}\,. \tag{IV.46}$$

X can thus decouple from W_α in the infinite–mass limit

$$\langle\mathcal{F}_{XXX}\rangle \to \infty \quad , \quad \text{Re}\langle\mathcal{F}_{XX}\rangle = \text{constant}\,. \tag{IV.47}$$

Notice that the requirement in (IV.47), which is due to the fact that $\text{Re}\,\mathcal{F}_{XX}$ is the kinetic metric, disproves the claim made in [81].The field equation of X in the limit (IV.47) yields the constraint

$$WW - \frac{1}{2}X\overline{DDX} + 2\,B^2\,X = 0\,, \tag{IV.48}$$

which was first given in [26]. Multiplying (IV.48) by W_α or X gives

$$XW_\alpha = X^2 = 0\,, \tag{IV.49}$$

so that, in $\mathcal{N} = 2$ language, the constraint (IV.48) is equivalent to [29]

$$\mathcal{W}^2_{def} = 0\,. \tag{IV.50}$$

The solution of (IV.48), and thus of (IV.50), was first given in [26]. In our conventions, it is

$$X = -\frac{W^2}{2B^2}\left[1 - \overline{D}^2\left(\frac{\overline{W}^2}{4B^4 + a + 4B^4\sqrt{1 + \frac{a}{2B^4} + \frac{b^2}{16B^8}}}\right)\right], \tag{IV.51}$$

where

$$a = \frac{1}{2}(D^2W^2 + \overline{D}^2\overline{W}^2)\,,\; b = \frac{1}{2}(D^2W^2 - \overline{D}^2\overline{W}^2)\,. \tag{IV.52}$$

The bosonic part of lagrangian (IV.45) then takes the form

$$\mathcal{L}|_{bos} \;=\; 8m^2B^2\Big(1 - \sqrt{1 - \tfrac{1}{B^4}(-F_{\mu\nu}F^{\mu\nu} + 2D^2) - \tfrac{1}{4B^8}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2}\,\Big)\,. \tag{IV.53}$$

The equation of motion for D is then

$$D = 0\,, \tag{IV.54}$$

and, substituting back into (IV.53), one arrives at [26], [28]

$$\begin{aligned}\mathcal{L}|_{bos} &= 8m^2B^2\Big(1 - \sqrt{1 + \tfrac{1}{B^4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{4B^8}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2}\,\Big)\\ &= 8m^2B^2\Big(1 - \sqrt{-\det\left(\eta_{\mu\nu} - \tfrac{\sqrt{2}}{B^2}F_{\mu\nu}\right)}\,\Big)\,.\end{aligned} \tag{IV.55}$$

Note that if $\xi \neq 0$ in (III.61), the equation of motion for D becomes

$$-\tfrac{2}{B^4}D^2 \;=\; -\tfrac{\xi^2}{\xi^2 + 2\cdot 16^2m^4}\Big(1 + \tfrac{1}{B^4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{4B^8}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2\Big)\,, \tag{IV.56}$$

and substituting back to (IV.53), the latter becomes

$$\begin{aligned}\mathcal{L}|_{bos} &= 8m^2B^2\left(1 - \Big(1 + \tfrac{\xi^2}{4\cdot 8\cdot 16m^4}\Big)\sqrt{1 - \tfrac{1}{B^4}(-F_{\mu\nu}F^{\mu\nu} + 2D^2) - \tfrac{1}{4B^8}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2}\right)\\ &= 8m^2B^2\Big(1 - \sqrt{1 + \tfrac{\xi^2}{8^3m^4}}\,\sqrt{-\det\left(\eta_{\mu\nu} - \tfrac{\sqrt{2}}{B^2}F_{\mu\nu}\right)}\,\Big)\,.\end{aligned} \tag{IV.57}$$

Comparing with (IV.55), we observe that the only difference is that the prefactor of the Born–Infeld Lagrangian becomes dependent on ξ.

IV.4 The deformed single–tensor multiplet

We now repeat the procedure described in the previous section for the single–tensor multiplet. Again, the fermions of $\mathcal{Z}$ are its θ^i components, so

$$\mathcal{Z}_{def} = \mathcal{Z} + \mathcal{Z}_{nl} \tag{IV.58}$$

with

$$\mathcal{Z}_{nl} = \widetilde{A}^2\,\theta\theta + \overline{\widetilde{B}}^2\,\overline{\widetilde{\theta}}\overline{\widetilde{\theta}}\,, \tag{IV.59}$$

where $\widetilde{A}^2$ and $\widetilde{B}^2$ are complex parameters. Interestingly, there are only two, and not three deformation parameters, as opposed to (IV.37) in the case of the Maxwell multiplet, because $\theta_\alpha\overline{\widetilde{\theta}}_{\dot\alpha}$ is a space–time vector. This can be traced back to the fact that $\mathcal{Z}$ has two real auxiliary d.o.f. and not an $SU(2)$–triplet, as $\mathcal{W}$. The fermions of $\mathcal{Z}_{def}$ transform then as

$$\begin{aligned}\delta\psi_\alpha &= \sqrt{2}\,(\widetilde{A}^2 - f)\,\epsilon_\alpha + \dots\\ \delta\varphi_\alpha &= -\sqrt{2}\,(\widetilde{B}^2 + f)\,\eta_\alpha + \dots\end{aligned} \tag{IV.60}$$

As will become clear below after the analysis of the scalar potential, partial breaking occurs in the following two cases:

- $\widetilde{B}^2 = 0$, for which ψ the Goldstino, or
- $\widetilde{A}^2 = 0$, for which φ is the Goldstino.

Finally, just as in the case of the Maxwell multiplet, a vacuum expectation value of f *cannot* induce partial breaking.

A generic lagrangian for $\mathcal{Z}_{def}$ is

$$\mathcal{L} = \int d^2\theta\,\left[\int d^2\overline{\widetilde{\theta}}\,\mathcal{G}(\mathcal{Z}_{def}) + \widetilde{m}^2\,\Phi\right] + \text{h.c.} \tag{IV.61}$$

and contains the following terms

$$\begin{aligned}\mathcal{L}_{lin.,aux.} &= [\mathcal{G}''(z) + \overline{\mathcal{G}}''(\overline{z})]\,\overline{f}f + \left[\tfrac{1}{2}\mathcal{G}'''(z)[\overline{f}\,\psi\psi + f\,\overline{\varphi\varphi}] - \widetilde{m}^2 f\right]\\ &\quad -\mathcal{G}''(z)\left[\overline{\widetilde{B}}^2 f + \widetilde{A}^2\,\overline{f} + \widetilde{A}^2\overline{\widetilde{B}}^2\right] - \tfrac{1}{2}\mathcal{G}'''(z)\left[\overline{\widetilde{B}}^2\,\psi\psi + \widetilde{A}^2\,\overline{\varphi\varphi}\right] + \text{h.c.}\end{aligned} \tag{IV.62}$$

The e.o.m. of f is thus

$$2\,[\mathrm{Re}\,\mathcal{G}''(z)]\,\overline{f} = \mathcal{G}''(z)\overline{\widetilde{B}}^2 + \overline{\mathcal{G}}''(\overline{z})\overline{\widetilde{A}}^2 + \widetilde{m}^2 - \frac{1}{2}\mathcal{G}'''(z)\overline{\varphi\varphi} - \frac{1}{2}\overline{\mathcal{G}}'''(\overline{z})\overline{\psi\psi}\,, \tag{IV.63}$$

so that the scalar potential and the fermion bilinear terms read respectively

$$\begin{aligned}V(z,\overline{z}) &= \frac{1}{2\,\mathrm{Re}\,\mathcal{G}''}\left|\widetilde{B}^2\,\overline{\mathcal{G}}'' + \widetilde{A}^2\,\mathcal{G}'' + \overline{\widetilde{m}}^2\right|^2 + 2\,\mathrm{Re}[\widetilde{A}^2\overline{\widetilde{B}}^2\,\mathcal{G}''],\\ \mathcal{L}_{ferm.} &= \frac{1}{2}\psi\psi\Big[\frac{\mathcal{G}'''}{2\,\mathrm{Re}\,\mathcal{G}''}(\overline{\widetilde{B}}^2\,\mathcal{G}'' + \overline{\widetilde{A}}^2\,\overline{\mathcal{G}}'' + \widetilde{m}^2) - \overline{\widetilde{B}}^2\,\mathcal{G}'''\Big] + \text{h.c.}\\ &\quad +\frac{1}{2}\varphi\varphi\Big[\frac{\overline{\mathcal{G}}'''}{2\,\mathrm{Re}\,\mathcal{G}''}(\overline{\widetilde{B}}^2\,\mathcal{G}'' + \overline{\widetilde{A}}^2\,\overline{\mathcal{G}}'' + \widetilde{m}^2) - \overline{\widetilde{A}}^2\,\mathcal{G}'''\Big] + \text{h.c.}\end{aligned} \tag{IV.64}$$

and do not depend on the real scalar C, which corresponds to a flat direction of the potential.

We then distinguish the following cases:

1. $\widetilde{A}\widetilde{B} = 0$: then in an $\mathcal{N} = 1$ vacuum we must have

$$\langle V \rangle = 0 \quad \Rightarrow \quad \langle \overline{\widetilde{B}}^2 \mathcal{G}'' + \overline{\widetilde{A}}^2 \overline{\mathcal{G}}'' + \widetilde{m}^2 \rangle = 0 \tag{IV.65}$$

which, using (IV.63), gives

$$\langle f \rangle = 0 \,. \tag{IV.66}$$

To have partial breaking, $\widetilde{A}$ and $\widetilde{B}$ cannot be equal to zero simultaneously, which requires that $\widetilde{m}^2 \neq 0$, given that $\langle \mathcal{G}'' \rangle \neq 0$. Without loss of generality, we choose $\widetilde{A} = 0$, $\widetilde{B} \neq 0$, in which case the mass terms of z and ψ_α are

$$2\langle \mathrm{Re}\, \mathcal{G}'' \rangle \Big[\mathcal{M}_\Phi \overline{\mathcal{M}}_\Phi \, z\overline{z} - \frac{1}{2} \mathcal{M}_\Phi \psi\psi - \frac{1}{2} \overline{\mathcal{M}}_\Phi \, \overline{\psi\psi} \Big],$$

so that the mass of Φ is

$$\mathcal{M}_\Phi = \frac{\overline{\widetilde{B}}^2 \langle \mathcal{G}''' \rangle}{2\langle \mathrm{Re}\, \mathcal{G}'' \rangle} \,. \tag{IV.67}$$

Note that this is precisely the case (IV.17), with $\overline{\widetilde{B}}^2 = -i\widetilde{M}^2$; in this particular case

$$\begin{aligned} \mathcal{L} &= \int d^2\theta \left[\int d^2\overline{\widetilde{\theta}} \, \mathcal{G}(\mathcal{Z}_{def}) + \widetilde{m}^2 \, \Phi \right] \\ &= \int d^2\theta \left[\tfrac{1}{2} \, \mathcal{G}_{\Phi\Phi} \, (\overline{D}L)(\overline{D}L) - \tfrac{1}{4} \mathcal{G}_\Phi \, \overline{DD} \, \overline{\Phi} - i \, \widetilde{M}^2 \, \mathcal{G}_\Phi + \widetilde{m}^2 \, \Phi \right] + \mathrm{h.c.} \end{aligned} \tag{IV.68}$$

2. $\widetilde{A}\widetilde{B} \neq 0$: then the first of equations (IV.64) implies that in general

$$\langle V \rangle \neq 0 \,, \tag{IV.69}$$

so $\mathcal{N} = 2$ is totally broken.

Using the deformed single–tensor superfield, we can also present a mechanism in which several single–tensor multiplets induce the partial breaking. In particular, let us consider a set of N deformed multiplets $\mathcal{Z}^a_{def}$, where $a = 1, \ldots, N$, namely

$$\mathcal{Z}^a_{def} = \Phi^a + \sqrt{2} i \, \overline{\widetilde{\theta}} \overline{D} L^a - \frac{1}{4} \overline{\widetilde{\theta}\widetilde{\theta}} \Big[4i(\widetilde{M}^a)^2 + \overline{DD} \, \overline{\Phi}^a \Big] \,. \tag{IV.70}$$

The action corresponding to the Lagrangian

$$\begin{aligned} \mathcal{L} &= \int d^2\theta \int d^2\overline{\widetilde{\theta}} \, \mathcal{G}(\mathcal{Z}^a_{def}) + \mathrm{h.c.} \\ &= \int d^2\theta \Big[\frac{1}{2} \mathcal{G}_{ab} (\overline{D}L^a)(\overline{D}L^b) - \frac{1}{4} \mathcal{G}_a \overline{DD\Phi}^a - i(\widetilde{M}^a)^2 \, \mathcal{G}_a + \widetilde{m}_a^2 \, \Phi^a \Big] + \mathrm{h.c.}\,, \end{aligned} \tag{IV.71}$$

where

$$\mathcal{G}_a = \frac{\partial}{\partial \Phi^a} \mathcal{G}(\Phi^c) \,, \qquad \mathcal{G}_{ab} = \frac{\partial^2}{\partial \Phi^a \partial \Phi^b} \mathcal{G}(\Phi^c) \,,$$

is invariant under the generalization of transformations (IV.18)

$$\begin{aligned} \delta^*\Phi^a &= \sqrt{2}i\,\overline{\eta D}L^a \\ \delta^* L^a &= \sqrt{2}(\widetilde{M}^a)^2(\theta\eta+\overline{\theta\eta}) - \tfrac{i}{\sqrt{2}}(\eta D\Phi + \overline{\eta D\Phi}) . \end{aligned} \tag{IV.72}$$

If $\widetilde{m}_a^2 \neq 0 \neq (\widetilde{M}^b)^2$, partial breaking occurs with the analogue of (IV.65) being

$$-\,i\langle\mathcal{G}_{ab}\rangle(\widetilde{M}^b)^2 + \widetilde{m}_a^2 = 0 . \tag{IV.73}$$

In this vacuum, the mass matrix of Φ^a is

$$\mathcal{M}_{ab} = -\frac{i}{2}\langle \mathrm{Re}\,\mathcal{G}^{-1}_{ac}\rangle\langle\mathcal{G}_{bcd}\rangle(\widetilde{M}^d)^2, \tag{IV.74}$$

and is controlled by the third derivatives of $\mathcal{G}$.

Now let us turn back to the case of one deformed $\mathcal{Z}_{def}$, which we would like to investigate in the infinite–mass limit. Due to (IV.67), this limit is

$$\mathcal{G}_{zzz}(\langle z\rangle)\to\infty \quad , \quad \mathrm{Re}\,\mathcal{G}_{zz}(\langle z\rangle) = \text{constant} , \tag{IV.75}$$

analogously to the limit (IV.47) for the Maxwell multiplet. In this limit,

$$\mathcal{G}_{zz}(\Phi)\sim\mathcal{G}_{zzz}(\langle z\rangle)[\Phi-\langle z\rangle] \quad , \quad \mathcal{G}_{zzz}(\Phi)\sim\mathcal{G}_{zzz}(\langle z\rangle) \tag{IV.76}$$

so that the e.o.m. of Φ

$$\mathcal{G}_{\Phi\Phi}(\Phi)\left(-\tfrac{1}{4}\overline{DD\Phi}+\overline{\widetilde{B}}^2\right)+\tfrac{1}{2}\mathcal{G}_{\Phi\Phi\Phi}(\Phi)(\overline{D}L)(\overline{D}L)+\widetilde{m}^2=0 \tag{IV.77}$$

becomes

$$\frac{1}{2}\Phi\overline{DD\Phi}-(\overline{D}L)(\overline{D}L) = 2\overline{\widetilde{B}}^2\,\Phi , \tag{IV.78}$$

where we have made use of the redefinition $\Phi-\langle z\rangle\to\Phi$. Note that (IV.78), which was first given in [32], does not depend on the function $\mathcal{G}$. The constraint (IV.78) takes the form

$$\Phi = -\frac{2(\overline{D}L)(\overline{D}L)}{4\overline{\widetilde{B}}^2-\overline{DD\Phi}} \qquad\Longrightarrow\qquad \Phi\,\overline{D}_{\dot\alpha}L=\Phi^n=0 \quad (n\geq 2). \tag{IV.79}$$

In $\mathcal{N}=2$ language, (IV.79) is equivalent to

$$\mathcal{Z}_{def}^2 = 0 , \tag{IV.80}$$

since

$$\mathcal{Z}_{def}^2=\Phi^2+2\sqrt{2}i\,\Phi\,\overline{\widetilde{\theta}}\overline{D}L-\overline{\widetilde{\theta}\widetilde{\theta}}\left[\frac{1}{2}\Phi\overline{DD\Phi}-(\overline{D}L)(\overline{D}L)-2\overline{\widetilde{B}}^2\,\Phi\,.\right], \tag{IV.81}$$

Note that the constraint (IV.78) transforms as a total derivative under the nonlinearly realized supersymmetry:

$$\delta^*\left[\frac{1}{2}\Phi\overline{DD\Phi}-(\overline{D}L)(\overline{D}L)-2\overline{\widetilde{B}}^2\,\Phi\right] = -2\sqrt{2}\,\partial_\mu(\eta\sigma^\mu\overline{D}L\Phi) . \tag{IV.82}$$

Following [26, 32], we now give the solution $\Phi = \Phi(\overline{D}L)$ of the constraint (IV.78) or equivalently of (IV.80). In our conventions, it is

$$\Phi = -\tfrac{1}{2\widetilde{B}^2}\Big[(\overline{D}L)^2 - \overline{D}^2\Big(\tfrac{(DL)^2(\overline{D}L)^2}{4\widetilde{B}^4+\widetilde{a}+4\widetilde{B}^4\sqrt{1+\frac{\widetilde{a}}{2\widetilde{B}^4}+\frac{\widetilde{b}^2}{16\widetilde{B}^8}}}\Big)\Big]\,, \qquad \text{(IV.83)}$$

where we have assumed that $\widetilde{B}$ is real for simplicity and

$$\begin{aligned} \widetilde{a} &= \tfrac{1}{2}\Big(\overline{D}^2[(DL)^2] + D^2[(\overline{D}L)^2]\Big) = \overline{\widetilde{a}} \\ \widetilde{b} &= \tfrac{1}{2}\Big(\overline{D}^2[(DL)^2] - D^2[(\overline{D}L)^2]\Big) = -\overline{\widetilde{b}}\,. \end{aligned} \qquad \text{(IV.84)}$$

Due to (IV.80), only if $\mathcal{G}$ has linear dependence on $\mathcal{Z}$ will it contribute to (IV.61). However,

$$\int d^2\theta\, d^2\overline{\theta}\, \mathcal{Z} + \text{h.c.} \sim \int d^2\theta\, \Big(\widetilde{B}^2 - \frac{1}{4}\overline{DD}\,\overline{\Phi}\Big) + \text{h.c.} = \text{tot. deriv.} \qquad \text{(IV.85)}$$

Consequently, *only the FI term contributes* to (IV.61), which takes the form

$$\begin{aligned} \mathcal{L} &= \widetilde{m}^2 \int d^2\theta\, \Phi + \text{h.c.} \\ &= -\tfrac{\widetilde{m}^2}{2\widetilde{B}^2}\int d^2\theta\, (\overline{D}L)^2\Big[1 - \overline{D}^2\Big(\tfrac{(DL)^2}{4\widetilde{B}^4+\widetilde{a}+4\widetilde{B}^4\sqrt{1+\frac{\widetilde{a}}{2\widetilde{B}^4}+\frac{\widetilde{b}^2}{16\widetilde{B}^8}}}\Big)\Big] + \text{h.c.} \end{aligned} \qquad \text{(IV.86)}$$

We also compute that

$$\widetilde{a}|_{bos} = 4\,(\upsilon^2 - (\partial C)^2) \quad , \quad \widetilde{b}|_{bos} = -8i\,\upsilon\cdot\partial C\,, \qquad \text{(IV.87)}$$

so that

$$\begin{aligned} \mathcal{L}|_{bos} &= \widetilde{m}^2\widetilde{B}^2\Big(1 - \sqrt{1 + \tfrac{2}{\widetilde{B}^4}(\upsilon^2 - (\partial C)^2) - \tfrac{4}{\widetilde{B}^8}(\upsilon\cdot\partial C)^2}\,\Big) \\ &= \widetilde{m}^2\widetilde{B}^2\Bigg(1 - \sqrt{1 - \tfrac{2}{\widetilde{B}^4}\Big(\tfrac{1}{6}H_{\mu\nu\rho}H^{\mu\nu\rho} + \partial_\mu C\partial^\mu C\Big) - \tfrac{1}{9\widetilde{B}^8}(\epsilon_{\mu\nu\rho\sigma}H^{\nu\rho\sigma}\partial^\mu C)^2}\,\Bigg)\,. \end{aligned} \qquad \text{(IV.88)}$$

which is the analogue of (IV.57).

IV.5 Interactions

We begin by summarizing and extending what we have explained so far in regard to the partial breaking of $\mathcal{N} = 2$ global supersymmetry. The Maxwell and the single–tensor multiplets enjoy two representations in $\mathcal{N} = 2$ superspace: the "short" superfields

$$\mathcal{W} \quad , \quad \mathcal{Z} \qquad \text{(IV.89)}$$

that are CC and CA respectively and contain $8_B + 8_F$ d.o.f. each, and the "long" superfields

$$\widehat{\mathcal{W}} \quad , \quad \widehat{\mathcal{Z}} \qquad \text{(IV.90)}$$

that are CA and CC respectively and contain $16_B + 16_F$ d.o.f. each. With the use of gauge variations one can remove half of the components of (IV.90) and obtain (IV.89).

	Long	Short	Gauge variation
Maxwell:	$\widehat{\mathcal{W}}$	$\mathcal{W}$	$\delta\,\widehat{\mathcal{W}}=\mathcal{Z}_g\,,\ \delta\,\mathcal{W}=0$
Single–tensor:	$\widehat{\mathcal{Z}}$	$\mathcal{Z}$	$\delta\,\widehat{\mathcal{Z}}=\mathcal{W}_g\,,\ \delta\,\mathcal{Z}=0$

The Goldstino of the broken and nonlinearly realized $\mathcal{N}=1$ supersymmetry may belong either to a deformed Maxwell multiplet $\mathcal{W}_{def}$, or to a deformed single–tensor multiplet $\mathcal{Z}_{def}$, which are CC and CA respectively. At the low–energy limit, X and Φ decouple from the spectrum of the respective theories, which is expressed via the nilpotent constraints

$$\mathcal{W}_{def}^2=0 \quad , \quad \mathcal{Z}_{def}^2=0\,. \tag{IV.91}$$

It is then natural to consider interactions of $\mathcal{W}_{def}$ and $\mathcal{Z}_{def}$, which might also prove to be useful in the context of partial breaking of $\mathcal{N}=2$ *local* supersymmetry, as this implementation necessitates the use of *two* $\mathcal{N}=2$ multiplets [40].

However, due to the fact that $\mathcal{W}_{def}$ and $\mathcal{Z}_{def}$ have opposite chirality under the second $\mathcal{N}=1$ supersymmetry, an (invariant) $\mathcal{N}=2$ interaction term for the two cannot be written. A way around this argument would be to deform the long multiplets and consider interaction terms of the type

$$\int d^2\theta\, d^2\widetilde{\theta}\,\mathcal{W}_{def}\widehat{\mathcal{Z}}_{def} \tag{IV.92}$$

and

$$\int d^2\theta\, d^2\overline{\widetilde{\theta}}\,\widehat{\mathcal{W}}_{def}\mathcal{Z}_{def}\,. \tag{IV.93}$$

Yet the long superfields cannot be deformed in such a way that they describe a single Goldstino, namely they cannot be used for the partial breaking. To see this, consider for example a general deformation $\mathcal{W}_{nl}$ of the short Maxwell superfield. Then (III.109), which relates the short to the long superfield, implies that

$$\widehat{W}_{nl}=-\frac{i}{2}A^2\theta^2\overline{\widetilde{\theta}}^2=-\frac{i}{2}\overline{B}^2\theta^2\overline{\widetilde{\theta}}^2 \quad , \quad \Gamma=0\,, \tag{IV.94}$$

which *violates* the condition (IV.42) for the partial breaking. One can show that a similar argument holds for the single–tensor multiplet. We conclude that we can only consider interactions of the type

$$\int d^2\theta\, d^2\widetilde{\theta}\,\mathcal{W}_{def}\widehat{\mathcal{Z}}\,, \tag{IV.95}$$

and

$$\int d^2\theta\, d^2\overline{\widetilde{\theta}}\,\widehat{\mathcal{W}}\mathcal{Z}_{def}\,. \tag{IV.96}$$

The first case (IV.95) has been studied in [29]. In terms of $\mathcal{N}=1$ superfields, it is given by (III.129) plus additional terms that now depend on Y, because of the existence of the deformation parameters:

$$\begin{aligned}\mathcal{L}_{nl} &= ig\int d^2\theta\int d^2\widetilde{\theta}\,\mathcal{W}_{def}\widehat{\mathcal{Z}}+\text{h.c.}\\ &= \mathcal{L}_{BF}+ig\int d^2\theta\left[B^2Y-\sqrt{2}\,\Gamma\,\theta\chi-A^2\,\theta\theta\left(\frac{i}{2}\Phi+\frac{1}{4}\overline{DDY}\right)\right]+\text{h.c.}\end{aligned} \tag{IV.97}$$

For the particular case of partial breaking in which $A = \Gamma = 0$, one obtains

$$\mathcal{L}_{nl} = g\int d^2\theta \left[\frac{1}{2}\Phi X + \chi W + iB^2 Y\right] + \text{h.c.} \tag{IV.98}$$

Notice that the variation $\sqrt{2}iB^2\eta\chi$ of iB^2Y under the second $\mathcal{N} = 1$ supersymmetry is cancelled by the deformed part $-\sqrt{2}iB^2\eta_\alpha$ of the variation of W_α under the same $\mathcal{N} = 1$. The full Lagrangian for $\mathcal{W}_{def}$ and $\widehat{\mathcal{Z}}$ includes, apart from (IV.97), the kinetic terms (III.65) and (III.55)

$$\mathcal{L}_{kin.} = \int d^2\theta d^2\overline{\theta}\,\mathcal{H}(L,\Phi,\overline{\Phi}) + \frac{1}{2}\int d^2\theta \int d^2\widetilde{\theta}\,\mathcal{F}(\mathcal{W}_{def}) + \text{h.c.} \tag{IV.99}$$

as well as the FI terms (III.61). At the infinite–mass limit (IV.47), due the constraint (IV.50), the dependence of the total Lagrangian on $\mathcal{F}$ disappears

$$\begin{aligned}\mathcal{L}_{tot} \;&=\; \int d^2\theta d^2\overline{\theta}\,\mathcal{H}(L,\Phi,\overline{\Phi}) + \xi\int d^2\theta d^2\overline{\theta}\,V_2 + \tfrac{1}{2}m^2\int d^2\theta\,X \\ &\quad + g\int d^2\theta\left[\tfrac{1}{2}\Phi X + \chi W + iB^2Y\right] + \text{h.c.}\end{aligned} \tag{IV.100}$$

$\mathcal{L}_{tot}$ depends then only on the function $\mathcal{H}$. The resulting theory has a linear $\mathcal{N} = 1$ as well as a second nonlinear $\mathcal{N} = 1$ supersymmetry. Upon inserting the solution $X = X(W^2)$ (IV.51) of the constraint (IV.50), it has been discovered that a new version of the super–Brout–Englert–Higgs mechanism but, importantly, *without* gravity, is in operation: the final spectrum consists of a massive $\mathcal{N} = 1$ vector multiplet, as the massless vector multiplet W_α has absorbed the linear multiplet as a consequence of the BF interaction, and the massless $\mathcal{N} = 1$ chiral multiplet Φ [29].

As a final comment, notice that for $B \neq 0$ (so as to have partial breaking), the equation of motion of Y is inconsistent. One can get around this problem by using $l > 1$ deformed Maxwell multiplets instead of one, as then the e.o.m. of Y^a would take the form of a tadpole–like condition

$$g_a B_a^2 = 0 \quad , \quad a = 1, ..., l\,, \tag{IV.101}$$

where g_a would be the coupling of each BF interaction. This is in agreement with the claim made in [82, 83], namely that one cannot couple hypermultiplets to a single Maxwell multiplet in a theory with partial breaking induced by the latter.

We now turn to the second type of interaction (IV.96). In terms of $\mathcal{N} = 1$ superfields, it is given by (III.131) plus additional terms that now depend on U, because of the existence of the deformation parameters:

$$\begin{aligned}\mathcal{L}_{nl} \;&=\; ig\int d^2\theta\int d^2\overline{\widetilde{\theta}}\,\widehat{\mathcal{W}}\,\mathcal{Z}_{def} + \text{h.c.} \\ &=\; \mathcal{L}_{BF} + ig\int d^2\theta\left[\overline{\widetilde{B}}^2 U - \widetilde{A}^2\,\theta\theta\left(\frac{i}{2}X + \frac{1}{4}\overline{DDU}\right)\right] + \text{h.c.}\end{aligned} \tag{IV.102}$$

In the particular case of partial breaking in which $\widetilde{A} = 0$, we obtain

$$\mathcal{L}_{nl} = g\int d^2\theta\left[\frac{1}{2}\Phi X + (\overline{D}L)(\overline{D}\mathbb{L}) + i\overline{\widetilde{B}}^2 U\right] + \text{h.c.} \tag{IV.103}$$

Notice that the variation $\sqrt{2}i\,\overline{\widetilde{B}}^2\,\overline{\eta D}\,\mathbb{L}$ of $i\overline{\widetilde{B}}^2 U$ under the second $\mathcal{N}=1$ is cancelled by the deformed part $-\sqrt{2}i\,\overline{\widetilde{B}}^2\,\overline{\eta}_{\dot{\alpha}}$ of the variation of $\overline{D}_{\dot{\alpha}}L$ under the same $\mathcal{N}=1$ supersymmetry. The full Lagrangian for $\mathcal{W}$ and $\mathcal{Z}_{def}$ includes then, apart from (IV.103), the kinetic terms (III.55) and (III.70) as well as the superpotential (III.75) and an FI term for V_2

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{nl} + \left[\tfrac{1}{2}\int d^2\theta\int d^2\widetilde{\theta}\mathcal{F}(\mathcal{W}) + \int d^2\theta\int d^2\overline{\widetilde{\theta}}\,\mathcal{G}(\mathcal{Z}) + \int d^2\theta\,\widetilde{m}^2\Phi\right] + \text{h.c.} \\ &\quad +\xi\int d^2\theta d^2\overline{\theta}\,V_2\,.\end{aligned} \tag{IV.104}$$

At the infinite–mass limit (IV.75), due to the constraint (IV.80), $\mathcal{G}$ does not contribute to (IV.104) due to (IV.85), so (IV.104) depends on a single function $\mathcal{F}$. Moreover, inserting the solution (IV.83) of the constraint (IV.80), the bosonic part of (IV.104) becomes

$$\begin{aligned}\mathcal{L}_{bos} &= \tfrac{1}{2}\int d^2\theta\int d^2\widetilde{\theta}\,\mathcal{F}(\mathcal{W})|_{bos} + \text{h.c.} - 2\xi\,\partial^\mu\,\text{Im}\,\mathbb{V}_\mu \\ &\quad +2g\Big(-\tfrac{1}{26}\epsilon_{\mu\nu\rho\sigma}H^{\nu\rho\sigma}A^\mu + C\,\partial^\mu\,\text{Im}\,\mathbb{V}_\mu - \widetilde{B}^2\,\text{Im}\,F_U\Big) \\ &\quad +(g\,\text{Re}\,x + 2\widetilde{m}^2)\widetilde{B}^2 \\ &\qquad\cdot\left(1-\sqrt{1-\tfrac{2}{\widetilde{B}^4}\Big(\tfrac{1}{6}H_{\mu\nu\rho}H^{\mu\nu\rho}+\partial_\mu C\partial^\mu C\Big)-\tfrac{1}{9\widetilde{B}^8}(\epsilon_{\mu\nu\rho\sigma}H^{\nu\rho\sigma}\partial^\mu C)^2}\right),\end{aligned} \tag{IV.105}$$

where $\widetilde{B}$ has been assumed to be real and F_U is the auxiliary field of U. Notice that the Lagrangian (IV.88) has acquired a field–dependent coefficient $(g\,\text{Re}\,x + 2\widetilde{m}^2)\widetilde{B}^2$ as its analogue, the Born–Infeld Lagrangian, does in the case that the interaction term is (IV.95) [29].

The solution of the e.o.m. of F of X is $F=0$. Moreover, the equation of motion for the auxiliary field $\text{Im}\,\mathbb{V}_\mu$ is

$$\partial_\mu(16\,\text{Re}\,\mathcal{F}_{xx}\,\partial^\nu\,\text{Im}\,\mathbb{V}_\nu + 2g\,C) = 0\,, \tag{IV.106}$$

whose solution is

$$16\,\text{Re}\,\mathcal{F}_{xx}\,\partial^\nu\,\text{Im}\,\mathbb{V}_\nu + 2g\,C = -\lambda\,, \tag{IV.107}$$

where λ is in principle an arbitrary integration constant. There is, however, a subtlety: the second of equations (III.116)

$$D = -4\partial^\mu\,\text{Im}\,\mathbb{V}_\mu\,, \tag{IV.108}$$

that replaces the real auxiliary d.o.f. of the short multiplet with the divergence of a vector field in the long Maxwell superfield, implies that one has to impose that that the e.o.m. of D and $\mathbb{V}_\mu$ are compatible. Since the e.o.m. of the former is

$$2\,\text{Re}\,\mathcal{F}_{xx}\,D^2 + (2\xi - 2g\,C)D = 0, \tag{IV.109}$$

with solution

$$4\,\text{Re}\,\mathcal{F}_{xx}\,D + 2\xi - 2g\,C = 0, \tag{IV.110}$$

we make the identification

$$\lambda = 2\xi\,, \tag{IV.111}$$

referring the reader to appendix C for more details. The scalar potential of the theory is then

$$V = \frac{1}{32\,\mathrm{Re}\,\mathcal{F}_{xx}}(2g\,C - 2\xi)^2\,, \tag{IV.112}$$

whose supersymmetric vacuum is at

$$< C >= \frac{\xi}{g}\,. \tag{IV.113}$$

In this vacuum, x corresponds to a flat direction of the potential and is massless. The mass $\mathcal{M}^2_{C,can}$ that the (canonically normalized) C aquires is then

$$\mathcal{M}^2_{C,can} = \frac{1}{4}\frac{1}{\mathrm{Re}\,\mathcal{F}_{xx}}\,\frac{g^2\widetilde{B}^2}{2g\;\mathrm{Re}\,x + 4\widetilde{m}^2}\,. \tag{IV.114}$$

Moreover, the interaction term $-\frac{1}{12}g\,\epsilon_{\mu\nu\rho\sigma}H^{\nu\rho\sigma}A^\mu$ generates a mass term for A_μ for which

$$\mathcal{M}^2_{A_\mu,can} = \mathcal{M}^2_{C,can}\,. \tag{IV.115}$$

We thus discover that, similarly to the case of the interaction (IV.95), there a mechanism analogous to the super–Brout–Englert–Higgs effect without gravity in operation, with the final spectrum of the theory consisting of a massive $\mathcal{N}=1$ vector multiplet, since the vector multiplet W_α has absorbed the Goldstino linear multiplet, and of the massless $\mathcal{N}=1$ chiral multiplet X.

As a final comment, note that the equation of motion of U is inconsistent as was that of Y previously. Again, this problem can be solved by coupling the long Maxwell multiplet(s) to at least two short and deformed single–tensor multiplets. However, there is no reason to identify the imaginary part of the auxiliary field of U with a four–form field as was done for Y in [29]. To see this, recall that the gauge variation (III.85) of Y is

$$\delta_g Y = X_g = -\frac{1}{2}\overline{DD}\Delta'\,, \tag{IV.116}$$

where Δ' is a real superfield since X_g is part of a Maxwell multiplet, while the gauge variation (III.122) of U is

$$\delta_g U = \Phi_g\,, \tag{IV.117}$$

where Φ_g is part of a single–tensor multiplet and is thus *not* necessarily identified with $\overline{DD}\Delta''$, where Δ'' is a real superfield.

IV.6 General constraints

In nonlinear $\mathcal{N}=1$ supersymmetry, which in the simplest case is realized at low energies via a nilpotent constraint imposed on a chiral superfield

$$X^2 = 0\,, \tag{IV.118}$$

several constraints involving X and other $\mathcal{N}=1$ superfields have been proposed [4, 7], which we give in the table below for reference. In view of generalizing these $\mathcal{N}=1$ constraints, in what follows we study constraints involving the $\mathcal{N}=2$ Goldstino multiplet and other incomplete multiplets of $\mathcal{N}=2$ nonlinear supersymmetry.

Constraint	Field eliminated
$XQ=0$	complex scalar
$X\overline{Q}=$ chiral	fermion
$X(Q-\overline{Q})=0$	fermion and one real dof
$XV=0,\ XW_\alpha=0$	gaugino
$X\overline{X}L=0$	real scalar
$X\overline{D}_{\dot\alpha}L=0$	fermion
$X\overline{X}(D_\alpha\chi_\beta+D_\beta\chi_\alpha)=0$	tensor

IV.6.1 The goldstino in the Maxwell multiplet

Let us consider the case in which the Goldstino is in a deformed Maxwell multiplet $\mathcal{W}_0$, given by (IV.44)

$$\mathcal{W}_0=X_0+\sqrt{2}i\,\tilde\theta W_0+\tilde\theta\tilde\theta\left[B^2-\frac{1}{4}\overline{DD}\,\overline{X}_0\right], \tag{IV.119}$$

which satisfies the constraint

$$\mathcal{W}_0^2=0\quad\Rightarrow\quad X_0=-2\,\frac{W_0W_0}{4B^2-\overline{DD}\overline{X}_0}\,. \tag{IV.120}$$

To describe an incomplete $\mathcal{N}=2$ vector multiplet with nonlinear supersymmetry, we consider the $\mathcal{N}=2$ constraint

$$\mathcal{W}_0\mathcal{W}_1=0\,, \tag{IV.121}$$

where $\mathcal{W}_1$ is an *undeformed* short Maxwell superfield with expansion

$$\mathcal{W}_1=X_1+\sqrt{2}i\,\tilde\theta W_1-\frac{1}{4}\tilde\theta\tilde\theta\,\overline{DD}\,\overline{X}_1\,. \tag{IV.122}$$

The constraint (IV.121) then yields the following set of equations

$$\begin{aligned} X_0X_1 &= 0\,,\\ X_0W_{1\alpha}+X_1W_{0\alpha} &= 0\,,\\ X_1B^2-\tfrac{1}{4}\overline{DD}(X_0\overline{X}_1+X_1\overline{X}_0)+W_0W_1 &= 0\,. \end{aligned} \tag{IV.123}$$

We now use (IV.120) and the identity

$$(W_0W_1)W_{0\alpha}=-\frac{1}{2}(W_0W_0)W_{1\alpha} \tag{IV.124}$$

to solve the second of equations (IV.123), which yields

$$X_1=-4\,\frac{W_0W_1}{4B^2-\overline{DD}\overline{X}_0}+h\,W_0W_0\,, \tag{IV.125}$$

where h is a chiral superfield. This expression verifies the first eq. (IV.123) for all h and the third eq. (IV.123) if

$$h = -2\,\frac{\overline{DD}X_1}{(4B^2 - \overline{DD}X_0)^2} \tag{IV.126}$$

and thus

$$X_1 = -4\,\frac{W_0W_1}{4B^2 - \overline{DD}X_0} - 2\,\frac{\overline{DD}X_1}{(4B^2 - \overline{DD}X_0)^2}\,W_0W_0\,. \tag{IV.127}$$

One may further use the solution (IV.51) for X_0 and solve (IV.127) to obtain X_1 as a function of W_0, W_1 and their derivatives; the constraint (IV.121) thus eliminates X_1.

Interestingly, the constraint $\mathcal{W}_0^2 = \mathcal{W}_0\mathcal{W}_1 = 0$ is a particular case of the system of equations

$$d_{abc}\mathcal{W}_b\mathcal{W}_c = 0 \quad ; \quad a,b,c = 1,\ldots,l \tag{IV.128}$$

introduced in [84, 85] to obtain coupled DBI (Dirac–Born–Infeld) actions. In eqs. (IV.128), all $\mathcal{W}_a$ are in general deformed with different deformation parameters B_a and the constants d_{abc} are totally symmetric. The set of constraints (IV.120) and (IV.121) is obtained from (IV.128) in the case of two $\mathcal{N} = 2$ vector multiplets with $d_{000} = d_{001} = 1$ and all other d's vanishing.

Next we consider the constraint

$$\mathcal{W}_0\mathcal{Z} = 0\,, \tag{IV.129}$$

where $\mathcal{Z}$ is a short single–tensor superfield. However, due to the fact that $\mathcal{W}_0$ and $\mathcal{Z}$ have opposite chiralities under the second $\mathcal{N} = 1$ supersymmetry, (IV.129) leads to an overconstrained system of equations. We thus turn to the constraint

$$\mathcal{W}_0\widehat{\mathcal{Z}} = 0\,, \tag{IV.130}$$

where $\widehat{\mathcal{Z}}$ is a long single–tensor superfield with expansion given by (III.82). Equation (IV.130) then leads to the system

$$\begin{aligned} X_0Y &= 0\,, \\ X_0\chi_\alpha + iYW_{0\alpha} &= 0\,, \\ YB^2 - \tfrac{i}{2}\,\Phi X_0 - \tfrac{1}{4}\overline{DD}(X_0\overline{Y} + Y\overline{X}_0) - i\,W_0\chi &= 0\,, \end{aligned} \tag{IV.131}$$

which, following the same steps as before, yield

$$Y = 4i\,\frac{W_0\chi}{4B^2 - \overline{DD}X_0} - 2\,\frac{2i\Phi + \overline{DDY}}{(4B^2 - \overline{DD}X_0)^2}\,W_0W_0\,, \tag{IV.132}$$

which again one may solve to eliminate $Y = Y(W_0, \chi, \Phi)$.

One can also check if the expression (IV.132) is covariant under the gauge variation (III.84). Under the latter, the expression (IV.132) becomes

$$X_g = -4\,\frac{W_0W_g}{4B^2 - \overline{DD}X_0} - 2\,\frac{\overline{DD}X_g}{(4B^2 - \overline{DD}X_0)^2}\,W_0W_0\,, \tag{IV.133}$$

which, using (IV.127), is actually the consequence of

$$\mathcal{W}_0\mathcal{W}_g = 0\,, \tag{IV.134}$$

that is the variation of (IV.130) under (III.84). The expression (IV.132) is thus invariant only under the reduced gauge transformations (III.84) subject to the constraint (IV.134), which are not sufficient to eliminate all unphysical components of $\widehat{\mathcal{Z}}$.

Alternatively, let us rewrite (IV.130) and (IV.134) as the gauge–invariant constraints

$$\mathcal{W}_0(\widehat{\mathcal{Z}}-\mathcal{W}_g)=\mathcal{W}_0\mathcal{W}_g=0\,, \tag{IV.135}$$

where $\mathcal{W}_g$ can be eliminated by a gauge transformation (III.84). One can then choose $Y-X_g=0$ and use eq. (IV.132) to eliminate $\chi-iW_g$ in terms of the $\mathcal{N}=1$ chiral superfield Φ:

$$\chi_\alpha - iW_{g\alpha} = \frac{\Phi}{(4B^2-\overline{DDX_0})}\,W_{0\alpha}\,. \tag{IV.136}$$

In the physically–relevant linear superfield L however, W_g disappears:

$$L = D\chi-\overline{D\chi} = D(\chi-iW_g)-\overline{D}(\overline{\chi}-i\overline{W}_g)\,,$$

since W_g verifies the Bianchi identity.

IV.6.2 The goldstino in the single–tensor multiplet

Now let us consider the case in which the Goldstino is in a deformed single–tensor superfield $\mathcal{Z}_0$, given by

$$\mathcal{Z}_0 = \Phi_0+\sqrt{2}i\,\overline{\tilde{\theta}D}L_0+\overline{\tilde{\theta}\tilde{\theta}}\left[\overline{\tilde{B}}^2-\frac{1}{4}\overline{DD}\,\overline{\Phi}_0\right]\,, \tag{IV.137}$$

which satisfies (IV.80)

$$\mathcal{Z}_0^2=0 \quad , \quad \Phi_0=-2\,\frac{(\overline{D}L_0)(\overline{D}L_0)}{4\overline{\tilde{B}}^2-\overline{DD\Phi}_0}\,. \tag{IV.138}$$

To describe another incomplete $\mathcal{N}=2$ single–tensor multiplet with nonlinear supersymmetry, we consider the $\mathcal{N}=2$ constraint

$$\mathcal{Z}_0\mathcal{Z}_1=0\,, \tag{IV.139}$$

where $\mathcal{Z}_1$ is an *undeformed* short single–tensor superfield with expansion given by

$$\mathcal{Z}_1=\Phi_1+\sqrt{2}i\,\overline{\tilde{\theta}D}L_1-\frac{1}{4}\overline{\tilde{\theta}\tilde{\theta}}\,\overline{DD}\,\overline{\Phi}_1\,. \tag{IV.140}$$

Following the same steps as before, as well as the identity

$$(\overline{D}L_0\overline{D}L_1)\overline{D}_{\dot\alpha}L_0=-\frac{1}{2}\,(\overline{D}L_0\overline{D}L_0)\overline{D}_{\dot\alpha}L_1\,, \tag{IV.141}$$

we find

$$\Phi_1=-4\,\frac{\overline{D}L_0\overline{D}L_1}{4\overline{\tilde{B}}^2-\overline{DD\Phi}_0}-2\,\frac{\overline{DD\Phi}_1}{(4\overline{\tilde{B}}^2-\overline{DD\Phi}_0)^2}\overline{D}L_0\overline{D}L_0\,, \tag{IV.142}$$

which one may solve to eliminate the chiral component Φ_1 in terms of L_1 and the goldstino multiplet L_0. Note that the constraints (IV.138) and (IV.139) can be generalised to a system of equations

$$\tilde{d}_{abc}\mathcal{Z}_b\mathcal{Z}_c=0 \quad ; \quad a,b,c=1,\ldots,l\,, \tag{IV.143}$$

Constraint	Fields eliminated
$\mathcal{W}_0^2 = \mathcal{W}_0\mathcal{W}_1 = 0$	X_0 and X_1
$\mathcal{W}_0^2 = \mathcal{W}_0(\widehat{\mathcal{Z}} - \mathcal{W}_g) = \mathcal{W}_0\mathcal{W}_g = 0$	X_0 and χ_α (Y: gauge–fixed)
$\mathcal{Z}_0^2 = \mathcal{Z}_0\mathcal{Z}_1 = 0$	Φ_0 and Φ_1
$\mathcal{Z}_0^2 = \mathcal{Z}_0(\widehat{\mathcal{W}} - \mathcal{Z}_g) = \mathcal{Z}_0\mathcal{Z}_g = 0$	Φ_0 and Ω_α (U: gauge–fixed)

in analogy with the system (IV.128), where $\widetilde{d}_{abc}$ are totally symmetric constants, in order to obtain a coupled action of incomplete single–tensor multiplets.

Finally, we consider the constraint

$$\mathcal{Z}_0\widehat{\mathcal{W}} = 0\,, \tag{IV.144}$$

where $\widehat{\mathcal{W}}$ is a long Maxwell superfield given by (III.110), and, using the same procedure as before, we obtain

$$U = 4i\,\frac{\overline{D}L_0\overline{D}\mathbb{L}}{4\overline{\widetilde{B}}^2 - \overline{DD\Phi_0}} - 2\,\frac{2iX + \overline{DDU}}{(4\overline{\widetilde{B}}^2 - \overline{DD\Phi_0})^2}\overline{D}L_0\overline{D}L_0\,, \tag{IV.145}$$

which eliminates U. Using the same reasoning as before, one can show that the solution (IV.145) is invariant under the reduced gauge variation (III.118) satisfying the constraint

$$\mathcal{Z}_0\mathcal{Z}_g = 0\,. \tag{IV.146}$$

Following the same procedure as for the solution of the constraint (IV.130), one can use the *full* gauge invariance to set $U = 0$. Eq. (IV.145) can then be used to eliminate $\overline{\Omega}_{\dot\alpha} = \overline{D}_{\dot\alpha}\,\mathbb{L}$ in terms of the $\mathcal{N} = 1$ chiral superfield X:

$$\overline{D}_{\dot\alpha}\,\mathbb{L} = \frac{X}{4\overline{\widetilde{B}}^2 - \overline{DD\Phi_0}}\overline{D}_{\dot\alpha}\,L_0\,. \tag{IV.147}$$

Consequently, the constraint equation (IV.147) is invariant under the transformation of $\mathbb{L}$ under the standard gauge transformations included in (III.122). In addition, the physically–relevant $V = 2(\mathbb{L} + \overline{\mathbb{L}})$ in invariant under the gauge ambiguity (III.118).

As a final comment, we have not found constraints that keep L of $\mathcal{Z}$ or W_α of $\mathcal{W}$.

V – Extra dimensions

V.1 Pure $\mathcal{N}=2$, $D=5$ supergravity

In this section, we review the pure $\mathcal{N}=2$, $D=5$ supergravity [64, 65, 66]. The pure $\mathcal{N}=2$, $D=5$ supergravity multiplet consists of the graviton e_M^m (that is a fünfbein), the $SU(2)$–gravitino doublet ψ_M^i and the graviphoton A_M that admits a gauge transformation. The 5D spacetime metric g_{MN} is written as

$$g_{MN} = e_M^m \eta_{mn} e_N^n \,, \tag{V.1}$$

so that $M, N, \ldots$ are coordinate (curved) indices, $m, n, \ldots$ are frame (tangent) indices and η_{mn} is the 5D Minkowski metric. Gamma matrices carry frame indices so that

$$\Gamma^M = (e_M^m)^{-1}\Gamma^m \,. \tag{V.2}$$

$SU(2)$–indices i are lowered and raised using the same conditions (III.41) as in global $\mathcal{N}=2$, $D=4$ supersymmetry

$$\lambda_i = \varepsilon_{ji}\lambda^j \quad , \quad \lambda^i = \varepsilon^{ij}\lambda_j \tag{V.3}$$

where λ_i is a symplectic Majorana spinor (with implicit spinor index), namely it satisfies the condition

$$\overline{\lambda}^i = \lambda^{iT} C \,, \tag{V.4}$$

where C is the charge conjugation matrix, with

$$\text{Dirac conjugate: } \overline{\lambda}^i \equiv \lambda_i^\dagger \Gamma^0 \quad , \quad \text{charge conjugation: } C\Gamma^\mu C^{-1} = (\Gamma^\mu)^T \,. \tag{V.5}$$

All spinors of $\mathcal{N}=2$, $D=5$ supergravity including the gravitini are symplectic Majorana spinors. As such, they have *no* chirality, so that the position of the $SU(2)$–indices does not indicate chirality, unlike the $\mathcal{N}=2$, $D=4$ case.

The pure $\mathcal{N}=2$, $D=5$ supergravity Lagrangian is then written as

$$\begin{aligned} e^{-1}\mathcal{L} &= \tfrac{1}{2\kappa^2}\left[R(\omega) - \overline{\psi}^i_M \Gamma^{MNP} D_N \psi_{Pi}\right] - \tfrac{1}{4}F_{MN}F^{MN} \\ &\quad + \tfrac{\kappa e^{-1}}{6\sqrt{6}} \epsilon^{MNP\Sigma\Lambda} F_{MN} F_{P\Sigma} A_\Lambda \\ &\quad - \tfrac{3i}{8\kappa\sqrt{6}}\left(\overline{\psi}^i_M \Gamma^{MNP\Sigma}\psi_{Ni}F_{P\Sigma} + 2\overline{\psi}^{Mi}\psi_i^N F_{MN}\right) + 4\text{–gravitino terms} \end{aligned} \tag{V.6}$$

where κ is the gravitational coupling related to the 5D Planck mass M_5 via

$$\kappa^2 = \frac{1}{M_5^3} \tag{V.7}$$

and F_{MN} is the field strength of the graviphoton

$$F_{MN} = \partial_M A_N - \partial_N A_M \,. \tag{V.8}$$

Moreover,

$$e = \det(e^m_M) \tag{V.9}$$

and $\omega_{Mmn}(e)$ is the spacetime spin connection defined by

$$\omega_M{}^{mn}(e) \equiv 2e^{[mN}e_{[N,M]}{}^{n]} + e^{m\Lambda}e^{nP}e_{[\Lambda,P]}{}^{l}e_{Ml} \tag{V.10}$$

and the Ricci scalar R is defined by

$$R \equiv e^{Mn}R_{Mn} \equiv e^{Mn}e^{Nm}R_{MNmn}\,, \tag{V.11}$$

where the Riemann tensor is defined by

$$R_{MNmn} \equiv \partial_M\omega_{Nmn}(e) + \omega_{Mm}{}^{l}(e)\omega_{Nln}(e) - (M \leftrightarrow N)\,. \tag{V.12}$$

Finally,

$$D_N\psi_P = \partial_N\psi_P + \frac{1}{4}\omega_{Nmn}(e)\Gamma^{mn}\psi_P\,. \tag{V.13}$$

The action corresponding to (V.6) is invariant under the following $\mathcal{N}=2$ supersymmetry transformations

$$\begin{aligned} \delta e^m_M &= \tfrac{1}{2}\bar{\epsilon}^i\Gamma^m\psi_{Mi} \\ \delta\psi_{Mi} &= D_M(\widehat{\omega})\epsilon_i + \tfrac{i\kappa}{4\sqrt{6}}(\Gamma^{NP}_M - 4\delta^N_M\Gamma^P)\widehat{F}_{NP}\epsilon_i \\ \delta A_M &= \tfrac{i\sqrt{6}}{4\kappa}\,\overline{\psi}^i_M\epsilon_i\,, \end{aligned} \tag{V.14}$$

where a hat over a quantity signifies that the respective quantity has been supercovariantized, namely

$$\begin{aligned} \widehat{\omega}_{Mmn} &\equiv \omega_{Mmn}(e) - \tfrac{1}{4}\left(\overline{\psi}^i_n\Gamma_M\psi_{mi} + 2\overline{\psi}^i_M\Gamma_{[n}\psi_{m]i}\right) \\ \widehat{F}_{MN} &\equiv F_{MN} + \tfrac{i\sqrt{6}}{4\kappa}\,\overline{\psi}^i_{[M}\psi_{N]i} \\ D_M(\widehat{\omega})\epsilon_i &= \partial_M\varepsilon_i + \tfrac{1}{4}\widehat{\omega}_{Mmn}(e)\Gamma^{mn}\epsilon_i\,. \end{aligned} \tag{V.15}$$

V.2 $\mathcal{N}=2,\ D=5$ Maxwell–Einstein supergravity

Following [67], we now review $\mathcal{N}=2$, $D=5$ Maxwell–Einstein supergravity. The $\mathcal{N}=2$, $D=5$ Maxwell multiplet contains a real scalar ϕ, a fermion $SU(2)$– (symplectic) doublet λ^i and a gauge field A_μ (other than the graviphoton). The field content of $\mathcal{N}=2$, $D=5$ Maxwell–Einstein supergravity, namely pure $\mathcal{N}=2$, $D=5$ supergravity coupled to n vector multiplets, is thus

$$\{e^m_M, \psi^i_M, A^I_M, \lambda^a_i, \phi^x\}\,, \tag{V.16}$$

where $x=1,\ldots,n$ and $a=1,\ldots,n$. The notation A^I_M refers collectively to all gauge fields, namely $I=0,1,\ldots,n$, where $I=0$ corresponds to the graviphoton. The scalars $\{\phi^x\}$ can be seen as coordinates of a Riemannian manifold $\mathcal{M}$ with metric g_{xy}, while the fermions $\{\lambda_i^a\}$ transform as a vector of $SO(N)$, that is the tangent space group of $\mathcal{M}$. We thus have

$$g_{xy} = f_x^a \delta_{ab} f_y^b \,, \tag{V.17}$$

where f_x^a is the n–bein, so that $x, y, \ldots$ are coordinate indices and $a, b, \ldots$ are frame indices. The spin connection ${\Omega_x}^{ab}$ of $\mathcal{M}$ is a solution of the equation

$$f^a_{[x,y]} + {\Omega_{[y}}^{ab} f^b_{x]} = 0 \tag{V.18}$$

and the Riemann tensor K_{xyzu} of $\mathcal{M}$ is defined by

$$K_{xyab} \equiv \Omega_{yab,x} + \Omega_{xac}\Omega_{ycb} - (x \leftrightarrow y) \,. \tag{V.19}$$

The Lagrangian of $\mathcal{N} = 2$, $D = 5$ Maxwell–Einstein supergravity is then written as

$$\begin{aligned} e^{-1}\mathcal{L}_{ME} &= \tfrac{1}{2\kappa^2}\Big[R(\omega) - \overline{\psi}^i_M \Gamma^{MNP} D_N \psi_{Pi}\Big] - \tfrac{1}{4} G_{IJ} F^I_{MN} F^{MNJ} \\ &\quad -\tfrac{1}{2}\overline{\lambda}^{ia}(\not{D}\delta^{ab} + {\Omega_x}^{ab}\not{\partial}\phi^x)\lambda_i^b - \tfrac{1}{2} g_{xy}(\partial_\mu\phi^x)(\partial^\mu\phi^y) \\ &\quad -\tfrac{i}{2}\overline{\lambda}^{ia}\Gamma^M\Gamma^N\psi_{Mi}\, f^a_x \partial_N\phi^x + \tfrac{1}{4} h^a_I\,\overline{\lambda}^{ia}\Gamma^M\Gamma^{\Lambda P}\psi_{Mi}\, F^I_{\Lambda P} \\ &\quad +\tfrac{i\kappa}{4}\Phi_{Iab}\,\overline{\lambda}^{ia}\Gamma^{MN}\lambda^b_i\, F^I_{MN} + \tfrac{\kappa e^{-1}}{6\sqrt{6}} C_{IJK}\,\epsilon^{MNP\Sigma\Lambda} F^I_{MN} F^J_{P\Sigma} A^K_\Lambda \\ &\quad -\tfrac{3i}{8\kappa\sqrt{6}} h_I \left(\overline{\psi}^i_M \Gamma^{MNP\Sigma}\psi_{Ni} F_{P\Sigma} + 2\overline{\psi}^{Mi}\psi^N_i F_{MN}\right) \\ &\quad +4\text{–fermion terms}\,, \end{aligned} \tag{V.20}$$

where G_{IJ}, h_I, h_I^a and Φ_{Iab} are generic functions of the scalars ϕ^x, with G_{IJ} being symmetric and Φ_{Iab} being symmetric over the last two indices. Moreover, C_{IJK} is a totally symmetric constant, so as for the action corresponding to (V.20) to be gauge invariant under

$$\delta A^I_M = \partial_M \theta^I(x) \,, \tag{V.21}$$

where $\{\theta^I\}$ are real scalars. The action is also invariant, under several conditions, under the $\mathcal{N} = 2$ supersymmetry transformations

$$\begin{aligned} \delta e^m_M &= \tfrac{1}{2}\bar{\epsilon}^i \Gamma^m \psi_{Mi} \\ \delta\psi_{Mi} &= D_M(\widehat{\omega})\epsilon_i + \tfrac{i\kappa}{4\sqrt{6}}(\Gamma^{NP}_M - 4\delta^N_M\Gamma^P)\widehat{F}_{NP}\epsilon_i \\ &\quad -\tfrac{\kappa^2}{12}\Gamma_{MN}\epsilon^j\,\overline{\lambda}^b_i\Gamma^N\lambda^b_j + \tfrac{\kappa^2}{48}\Gamma_{MNP}\epsilon^j\,\overline{\lambda}^b_i\Gamma^{NP}\lambda^b_j \\ &\quad +\tfrac{\kappa^2}{6}\epsilon^j\,\overline{\lambda}^b_i\Gamma_M\lambda^b_j - \tfrac{\kappa^2}{12}\Gamma^N\epsilon^j\,\overline{\lambda}^b_i\Gamma_{MN}\lambda^b_j \\ \delta A^I_M &= -\tfrac{1}{2} h^I_a\,\bar{\epsilon}^i\Gamma_M\lambda^a_i + \tfrac{i\sqrt{6}}{4\kappa} h^I\,\overline{\psi}^i_M\epsilon_i\,, \\ \delta\lambda^a_i &= -\tfrac{i}{2} f^a_x(\widehat{\not{\partial}}\phi^x)\epsilon_i - \tfrac{i}{2}{\Omega_x}^{ab} f^x_c\bar{\epsilon}^j\lambda^c_j\lambda^b_i + \tfrac{1}{4} h^a_I\,\Gamma^{MN}\epsilon_i\widehat{F}^I_{MN} \\ &\quad -\tfrac{i\kappa}{4\sqrt{6}}\Big[-3\epsilon^j\,\overline{\lambda}^b_i\lambda^c_j + \Gamma_M\epsilon^j\,\overline{\lambda}^b_i\Gamma^M\lambda^c_j + \tfrac{1}{2}\Gamma_{MN}\epsilon^j\,\overline{\lambda}^b_i\Gamma^{MN}\lambda^c_j\Big]T^{abc} \\ \delta\phi^x &= \tfrac{i}{2} f^x_a\,\bar{\epsilon}^i\lambda^a_i\,, \end{aligned} \tag{V.22}$$

where T_{xyz} is a totally symmetric tensor that generically depends on the scalars ϕ^x and

$$(\widehat{\partial}_M\phi)^x = \partial_M\phi^x - \frac{i}{2} f_b^x\, \overline{\psi}_M^j \lambda_j^b\,. \tag{V.23}$$

Note that now

$$\widehat{F}_{MN} \equiv F_{MN} + \frac{i\sqrt{6}}{4\kappa} h^I\, \overline{\psi}_{[M}^i \psi_{N]i} + h_a^I\, \overline{\psi}_{[M}^j \Gamma_{N]} \lambda_j^a\,. \tag{V.24}$$

The algebraic conditions are the following

$$\begin{aligned} h_x^I h_I = h_{Ix} h^I = 0 \quad &, \quad h_x^I h_y^J G_{IJ} = g_{xy} \\ G_{IJ} = h_I h_J + h_I^x h_J^y g_{yx} &= h_I h_J + h_I^x h_{Jx} \end{aligned} \tag{V.25}$$

and

$$\begin{aligned} h^I h_I &= 1 \quad , \quad h_x^I h_I^y = \delta_x^y \\ \Phi_{Ixy} &= \sqrt{\tfrac{2}{3}}\left(\tfrac{1}{4} g_{xy} h_I + T_{xyz} h_I^z\right) \\ C_{IJK} &= \tfrac{5}{2} h_I h_J h_K - \tfrac{3}{2} G_{(IJ} h_{K)} + T_{xyz} h_I^x h_J^y h_k^z \end{aligned} \tag{V.26}$$

while the differential conditions are

$$\begin{aligned} h_{I,x} = \sqrt{\tfrac{2}{3}} h_{Ix} \quad &, \quad h_{,x}^I = -\sqrt{\tfrac{2}{3}} h_x^I \\ h_{Ix;y} = \sqrt{\tfrac{2}{3}}\left(g_{xy} h_I + T_{xyz} h_I^z\right) \quad &, \quad h_{x;y}^I = -\sqrt{\tfrac{2}{3}}\left(g_{xy} h^I + T_{xyz} h^{Iz}\right), \end{aligned} \tag{V.27}$$

where $,x = \frac{\partial}{\partial\phi^x}$ and "$;x$" denotes covariant differentiation with respect to the Christoffel connection Γ_{xy}^z of $\mathcal{M}$. Note that equations (V.25), together with the first of equations (V.26), imply that the indices I are raised and lowered with the use of G_{IJ}, namely

$$h^I G_{IJ} = h_J \quad , \quad h_x^I G_{IJ} = h_{Jx}\,. \tag{V.28}$$

These conditions imply that $\mathcal{M}$ is embedded in an $(n+1)$–dimensional Riemann manifold $\mathcal{E}$ with coordinates $X^I = X^I(\phi^x,\mathcal{N})$, where $\mathcal{N}$ is a real scalar function. The equation

$$\ln\mathcal{N} = k\,, \tag{V.29}$$

where k is a constant, defines a family of hypersurfaces $\mathcal{M}_k$ of $\mathcal{E}$. Upon making the identification (strictly speaking on $\mathcal{M}$ only)

$$h_I \stackrel{!}{=} \alpha n_I \quad , \quad h^I \stackrel{!}{=} \beta X^I\,, \tag{V.30}$$

where α and β are real parameters, $X_{,x}^I$ is a vector tangent to $\mathcal{M}$ and n_I is the vector normal to $\mathcal{M}_k$ given by

$$n_I = \partial_I \ln\mathcal{N}\,, \tag{V.31}$$

the conditions

$$h_x^I h_I = h_{Ix} h^I = 0 \quad , \quad h_x^I h_I^y = \delta_x^y \tag{V.32}$$

of (V.25) and (V.26) can be interpreted as the orthonormality relations

$$X_{,x}^I n_I = 0 \quad , \quad \frac{3}{2}\beta^2\, X_{,x}^I X^{J,y} = \delta_x^y \tag{V.33}$$

that arise naturally on $\mathcal{E}$, where the first of equations (V.27) has been used. The equation

$$h^I h_I = 1 \tag{V.34}$$

of (V.26) is then rewritten as

$$X^I \partial_I \ln \mathcal{N} = (\alpha\beta)^{-1} \quad \Rightarrow \quad X^I \mathcal{N}_I = (\alpha\beta)^{-1}\mathcal{N}\,, \tag{V.35}$$

where $\mathcal{N}_I = \partial_I \mathcal{N}$, $\mathcal{N}_{IJ} = \partial_I \partial_J \mathcal{N}$, etc, which implies that

$$\mathcal{N}(\lambda X^I) = \lambda^{\frac{1}{\alpha\beta}} \mathcal{N}(X^I)\,, \tag{V.36}$$

where λ is a real parameter, namely (V.34) imposes the constraint that $\mathcal{N}$ be a homogeneous function of order $(\alpha\beta)^{-1}$. Differentiating (V.35) with respect to X^I, one further deduces the indentities

$$X^I \mathcal{N}_{IJ} = \Big[(\alpha\beta)^{-1} - 1\Big]\mathcal{N}_J \tag{V.37}$$

and

$$X^I \mathcal{N}_{IJK} = \Big[(\alpha\beta)^{-1} - 2\Big]\mathcal{N}_{JK} \tag{V.38}$$

Upon differentiating equation (V.35) one obtains

$$(\beta X^I)\Big(-\frac{\alpha}{\beta}\partial_{IJ} \ln \mathcal{N}\Big) = \alpha n_J \quad \Rightarrow \quad h^I\Big(-\frac{\alpha}{\beta}\partial_{IJ} \ln \mathcal{N}\Big) = h_J \tag{V.39}$$

and comparing (V.39) to (V.28) the following identification can be made

$$G_{IJ} \stackrel{!}{=} -\frac{\alpha}{\beta}\partial_{IJ} \ln \mathcal{N} \tag{V.40}$$

Consequently, on $\mathcal{M}$

$$ds^2 = G_{IJ} dX^I dX^J = G_{IJ} X^I_{,x} X^J_{,y}\, d\phi^x d\phi^y \stackrel{!}{=} g_{xy} d\phi^x d\phi^y\,, \tag{V.41}$$

namely g_{xy} is the pull–back of G_{IJ}, which, using the second of equations (V.25) as well as the second of equations (V.27), yields

$$\frac{2}{3}\beta^{-2}\, G_{IJ} h^I_x h^J_y \stackrel{!}{=} G_{IJ} h^I_x h^J_y \quad \Rightarrow \quad \beta^2 = \frac{2}{3}\,. \tag{V.42}$$

Note that this requirement can also be seen from the second of equations (V.33).

Moreover, $\mathcal{N}$ can be restricted even further. In particular, it is straightforward to calculate the Christoffel connection Γ^K_{IJ} corresponding to (V.40)

$$\Gamma_{IJK} = -\frac{\alpha}{2\beta}\,\partial_{IJK} \ln \mathcal{N}\,. \tag{V.43}$$

Using this result and the last of equations (V.26), as well as (V.27), (V.39) and (V.43), it can be shown that on $\mathcal{M}$

$$C_{IJK} = \frac{\alpha}{2\beta^2}\bigg[\mathcal{N}_{IJK} + 9\Big(\alpha\beta - \frac{1}{3}\Big)\Big(\mathcal{N}_{(IJ}\mathcal{N}_{K)} + \Big(\alpha\beta - \frac{2}{3}\Big)\mathcal{N}_I \mathcal{N}_J \mathcal{N}_K\Big)\bigg]\,. \tag{V.44}$$

Multiplying (V.44) with $X^I X^J X^K$ we find

$$\begin{aligned} C_{IJK}X^I X^J X^K &= \tfrac{\alpha}{2\beta^2}(\alpha\beta)^{-1}\Big((\alpha\beta)^{-1}-1\Big)\Big((\alpha\beta)^{-1}-2\Big)\mathcal{N} \\ &\quad +\tfrac{9\alpha}{2\beta^2}(\alpha\beta)^{-2}\Big((\alpha\beta)^{-1}-1\Big)\Big(\alpha\beta-\tfrac{1}{3}\Big)\mathcal{N}^2 \\ &\quad +\tfrac{9\alpha}{2\beta^2}(\alpha\beta)^{-3}\Big(\alpha\beta-\tfrac{1}{3}\Big)\Big(\alpha\beta-\tfrac{2}{3}\Big)\mathcal{N}^3\,. \end{aligned} \tag{V.45}$$

We may now redefine

$$X^I \to \lambda X^I \tag{V.46}$$

in (V.45) and, using (V.36), one finds that

$$\alpha\beta = \frac{1}{3} \quad , \quad \mathcal{N} = \beta^3 C_{IJK}X^I X^J X^K\,, \tag{V.47}$$

namely $\mathcal{N}$ is restricted to be a cubic polynomial function.

In addition, using the last of equations (V.26), as well as equations (V.27), one finds a differential constraint imposed on T_{xyz}

$$T_{xyz;u} = \frac{\sqrt{6}}{2}\Big[g_{(xy}g_{z)u} - 2T_{(xy}{}^{w}T_{z)w}\Big]\,. \tag{V.48}$$

Note that equation (V.47) implies that the only freedom one has in writing an $\mathcal{N}=2$, $D=5$ Maxwell–Einstein theory is the choice of the constants C_{IJK}: once they are determined, all other functions can be computed, starting from (V.47) and using (V.40), (V.30), (V.41), (V.48), etc. Furthermore, the integrability condition

$$K_{xyzu} = \frac{4}{3}\Big[g_{x[u}g_{z]y} + T_{x[u}{}^{w}T_{z]yw}\Big] \tag{V.49}$$

can be derived from (V.48). At the same time, K_{xyzu} can be calculated for the family $\mathcal{M}_k$ using (V.40) and (V.43). The result is identical to equaton (V.141) for $k=0$, which means that $\mathcal{M}$ is defined by the equation

$$\mathcal{N} = 1 \quad \Rightarrow \quad \beta^3 C_{IJK}X^I X^J X^K = 1\,, \tag{V.50}$$

where we have used (V.47). Finally, let us note that, from the superconformal point of view, the coordinate X^0 of the embedding manifold $\mathcal{E}$ may be viewed as the real scalar contained in an additional vector multiplet that is added to the n physical vector multiplets as a compensator [86, 87]. The constraint (V.50) then eliminates X^0 as a function of the physical scalars ϕ^x and renders $\mathcal{M}$ a very special real manifold.

V.3 The gauging

A particular case of (V.48) is

$$T_{xyz;w} = 0 \tag{V.51}$$

which, using (V.141), implies that

$$K_{xyzu;w} = 0\,, \tag{V.52}$$

namely that $\mathcal{M}$ is a locally symmetric space. Note that then the differential constraint (V.48) becomes algebraic

$$T_{(xy}{}^{w}T_{z)w} = \frac{1}{2}g_{(xy}g_{z)u}\,. \tag{V.53}$$

In what follows we will only consider the case that $\mathcal{M}$ is a locally symmetric space.

Gauged Maxwell–Einstein supergravity arises when one promotes one or all Killing symmetries of $\mathcal{M}$, generated by Killing vectors k^x, under which the scalars transform as

$$\delta_I\phi^x = \theta k^x(\phi)\,, \tag{V.54}$$

where θ is a parameter, to gauge symmetries. We now review the $U(1)$–gauging of [68, 69] and refer the reader to [88, 89] for the coupling to tensor multiplets and more general gaugings. To gauge a $U(1)$ subgroup of $SU(2)$, that is the automorphism group of the $\mathcal{N}=2$ supersymmetry algebra just as in the $D=4$ case, one first chooses the respective gauge field A_M as a linear combination of the gauge fields A^I_M of the theory

$$A_M = \upsilon_I A^I_M\,, \tag{V.55}$$

where υ_I is a generic constant vector. The gauge transformation of the scalars then becomes

$$\delta_g\phi^x = \theta(x)k^x(\phi) = -\theta(x)\phi^x \tag{V.56}$$

and the $U(1)$–covariant derivatives are defined by

$$\begin{aligned}\mathcal{D}_M\phi^x &= \partial_M\phi^x - gA_Mk^x = \partial_M\phi^x + g\,\upsilon_IA^I_M\phi^x\\ \mathcal{D}_M\lambda^{ai} &= D_M\lambda^{ai} - g\kappa^2A_M\tfrac{\partial k^a}{\partial\phi^b}\,\delta^{ij}\lambda^b_j = D_M\lambda^{ai} + g\kappa^2\,\upsilon_IA^I_M\,\delta^{ij}\lambda^a_j\,,\end{aligned} \tag{V.57}$$

where g is the $U(1)$ coupling constant.

The next step is thus to replace all derivatives by the respective $U(1)$–covariant derivatives. To maintain the invariance of the total action under $\mathcal{N}=2$ supersymmetry, it is also necessary to add the following terms to (V.20)

$$\begin{aligned}e^{-1}\mathcal{L}' \;=\; & -\tfrac{g^2}{\kappa^4}\,P - \tfrac{i\sqrt{6}}{8}\tfrac{g}{\kappa^3}\,\overline{\psi}^i_M\Gamma^{MN}\psi^j_N\delta_{ij}\,P_0\\ & -\tfrac{1}{\sqrt{2}}\tfrac{g}{\kappa^2}\,\overline{\lambda}^{ia}\Gamma^M\psi^j_M\delta_{ij}\,P_a + \tfrac{i}{2\sqrt{6}}\tfrac{g}{\kappa}\,\overline{\lambda}^{ia}\lambda^{jb}\delta_{ij}\,P_{ab}\,,\end{aligned} \tag{V.58}$$

namely a scalar potential and fermion mass terms arise, as well as the following terms to (V.22)

$$\begin{aligned}\delta'\psi_{Mi} &= \tfrac{i}{2\kappa\sqrt{6}}gP_0\,\Gamma_M\epsilon_{ji}\delta^{jk}\epsilon_k\\ \delta'\lambda^a_i &= \tfrac{1}{\kappa^2\sqrt{2}}gP^a\,\epsilon_{ji}\delta^{jk}\epsilon_k\,,\end{aligned} \tag{V.59}$$

where P, P_0, P_a and P_{ab} are functions of the scalars ϕ^x that are subject to the conditions given below

$$\begin{aligned}\upsilon_I &= \tfrac{1}{2}P_0h_I + \tfrac{1}{\sqrt{2}}P^ah_{Ia}\\ P_{ab} &= \tfrac{1}{2}\delta_{ab}P_0 + 2\sqrt{2}\,T_{abc}P^c\\ P &= -P_0^2 + P_aP^a\end{aligned} \tag{V.60}$$

Using the first constraint of (V.25) in the first of (V.60), one finds

$$P_0 = 2h^I \upsilon_I \quad , \quad P^a = \sqrt{2}\, h^{Ia} \upsilon_I \, . \tag{V.61}$$

Note that P_0 and P_a can be viewed as the components of the (constant) vector υ_I expanded in the (ϕ–dependent) basis defined by (h_I, h_{Ia}). Moreover, upon differentiating the first of equations (V.61) with respect to x and using the second of equations (V.27), the value of b from equation (V.42) as well as the second of equations (V.61), one finds a differential constraint imposed on P_0:

$$P_{0,x} = -\sqrt{2}\beta\, P_x \, . \tag{V.62}$$

Similarly, by taking the covariant derivative of the second of equations (V.61) with respect to x, one finds a differential constraint imposed on P_x:

$$P_{x;y} = -\beta \left(\frac{1}{\sqrt{2}} g_{xy} P_0 + T_{xyz} P^z \right) \tag{V.63}$$

Taking the covariant derivative of (V.62) and using (V.63) one finds a differential constraint on P_0 that does not depend on P^x:

$$P_{0,x;y} + \beta T_{xy}{}^{z} P_{0,z} - \beta^2 g_{xy} P_0 = 0 \, , \tag{V.64}$$

which one may solve to find the value of P_0 and then substitute in (V.62) in order to determine P_x.

V.4 Little String Theory and the linear dilaton

We now summarize the main properties of Little String Theories [60, 61, 56, 57, 58, 59] (see also the reviews [90, 91]). 6–dimensional Little String Theory ("LST") is the theory that arises in the limit in which $NS5$–branes decouple from bulk dynamics. Such an example appears in type IIB string theory in 10 dimensions, when one considers a stack consistiting of N $NS5$–branes in the limit

$$g_S \to 0 \, , \tag{V.65}$$

where

$$g_S = e^{\Phi_0} \tag{V.66}$$

is the string coupling that is identified with the exponential of the expectation value Φ_0 of the dilaton Φ. Indeed, closed string amplitudes are proportional to g_S, so that they approach 0 in the limit (V.65). Interestingly, the limit (V.65) is not a low–energy limit: it is taken with the energy scale E being kept constant with respect to the (finite) string scale $M_S = 1/l_S$, where l_S is the string length. Note also that the presence of the $NS5$–branes breaks $\mathcal{N} = 2$ supersymmetry partially in the bulk. Perhaps remarkably, LST shares properties with both string theories and field theories, some of which are the following:

- It is **non–gravitational**.

The $4D$ gravitational coupling approaches zero in the limit (V.65), since in type IIB

$$M_{Pl} \sim \frac{M_S}{g_S} \stackrel{g_S \to 0}{\longrightarrow} \infty\,, \tag{V.67}$$

where M_{Pl} is the $4D$ Planck mass. Upon compactification of the extra six dimensions, the relation (V.67) takes the form

$$M_{Pl}^2 = \frac{1}{g_S^2} M_S^8\, V_6 \stackrel{g_S \to 0}{\longrightarrow} \infty\,, \tag{V.68}$$

where V_6 is the internal volume of the extra dimensions. In principle, it is possible to have $M_S \sim$ TeV and $V_6 \sim$ TeV^{-6}, so that LST may offer an alternative framework in regard to a possible solution of the hierarchy problem, without postulating the existence of large extra dimensions.

- It is **non–local**.

In particular, an LST compactified on a torus is T–dual to another LST, since T–duality commutes with the limit (V.65). To see this, recall that T–duality on a type II string theory compactified on a circle of radius R acts as

$$R \to \frac{l_S^2}{R}\,, \tag{V.69}$$

which is independent of g_S and thus of the limit (V.65). Due to T–duality, LST may couple to more than one gravitational backgrounds after toroidal compactification, as there is not a unique energy–momentum tensor.

Other than that, LST exhibits a **Hagedorn density** of states at high energies

$$\rho(E) \sim E^{\alpha} e^{\beta_H E}\,, \tag{V.70}$$

where α is a negative parameter that is proportional to the volume of the $NS5$–brane and the Hagedorn temperature T_H is given by

$$T_H \equiv \frac{1}{\beta_H} = \frac{M_S}{2\pi\sqrt{N}}\,. \tag{V.71}$$

- It is **interacting** for $N > 1$.

The gauge coupling g_N corresponding to the low–energy ($E \ll M_S$) $U(N)$ gauge theory on the $NS5$–branes is given by

$$\frac{1}{g_N^2} = M_S^2 \tag{V.72}$$

and is thus independent of g_S and consequently of the limit (V.65) (note, however, that g_N depends on the geometric moduli upon compactification). One way to see this is to perform an S–duality transformation

$$g_S \to \frac{1}{g_S} \quad , \quad l_S \to \sqrt{g_S}\, l_S \tag{V.73}$$

on the $U(N)$ gauge coupling g_D of a stack of D–branes

$$\frac{1}{g_D^2} = \frac{M_S^2}{g_S}\,. \tag{V.74}$$

M_S is the thus the only parameter of LST. At $E \sim M_S$, the (perturbative) gauge theory description on the $NS5$–branes breaks down, so new d.o.f. are expected at this scale.

- By studying the near–horizon geometry of the stack of $NS5$–branes in the limit

  (V.65), it can be shown [56] that its holographic dual is type II string theory on

$$\mathbb{R}^{5,1} \times \mathbb{R}_y \times S_N^3\,, \tag{V.75}$$

where $\mathbb{R}^{5,1}$ is the 6D Minkowski spacetime corresponding to the worldvolume of the $NS5$-branes, $\mathbb{R}_y$ is the infinite real line parametrized by a coordinate y in which the dilaton is **linear** with

$$\Phi = -\frac{1}{\sqrt{N} l_S} y \tag{V.76}$$

and S_N^3 is the three-sphere of radius $\sqrt{N}\, l_S$. Note that, on the three–sphere parametrized by the angular coordinates, the corresponding superconformal field theory is a level $k = N - 2$ (for $N > 1$) Wess–Zumino–Witten model on the $S_N^3 \simeq SU(2)_k$ group manifold. Moreover, the topology (V.75) can be thought of as that of an infinite "throat", with the $NS5$–branes located at $y \to -\infty$. Consequently, due to (V.66) and (F.66), the theory becomes strongly coupled in the vicinity of the $NS5$–branes and weakly coupled far away from the latter, namely at $y \to +\infty$. To treat the strong coupling singularity at $y \to -\infty$, as well as the resulting breakdown of perturbation theory, the topology may be restricted to that of a semi–infinite cigar (of circumference β_H), upon replacing the infinite throat with the former, namely removing the strong coupling region [92]. The cigar may be thought of as being connected to a "Standard model brane", located at its tip, where the extra dimension y has been cut and where g_S takes its maximal value, and an asymptotically flat "Planck brane', where g_S approaches 0. To conclude, given that LST is non–local, strongly coupled and does not admit a Lagrangian description, its holographic dual is a means of studying indirectly its properties, without facing the aforementioned difficulties.

- It has a **distinct Kaluza–Klein graviton spectrum**.

The phenomenology of LST has been studied [55], see also [93, 94], in a $5D$ toy model of its holographic dual, in which there is only one (infinite) extra dimension y, in which the dilaton is linear. Note that, to explore the Planck mass hierarchy, y has to be compactified on a $S^1/\mathbb{Z}_2$ orbifold, at the fixed points of which the SM and Planck branes have to be introduced. In the string frame, the bulk Lagrangian of the dilaton–gravity system in the 5D toy model is

$$e^{-1}\mathcal{L}_{LST} = e^{-\frac{\sqrt{3}\Phi}{M_5^{3/2}}}\Big(\frac{1}{2}M_5^3\mathcal{R} + \frac{3}{2}(\partial\Phi)^2 - \Lambda\Big)\,, \tag{V.77}$$

where Λ is a constant. The linear dilaton background in this setup produces a KK graviton spectrum in which, under specific assumptions, there is a $\sim$ TeV mass gap followed by

~ 30 GeV–seperated narrow resonances. Note that this phenomenology significantly differs from that of the Antoniadis–Arkani-Hamed–Dimopoulos–Dvali model [95, 96] of large extra dimensions, were the KK spectrum is almost a continuum and there is no mass gap, as well as from that of the Randall–Sundrum model [97] of warped extra dimensions, where the KK resonances are TeV–seperated.

Interest in little string theories and, in particular, in their holographic duals with a linear dilaton background, has been revived in view of the recently proposed "clockwork mechanism" [48, 49, 50], that is a new way of generating an exponential hierarchy without necessitating the existence of small parameters at the fundamental level. In the discrete version of the mechanism, $\widetilde{N}+1$ particles in $4D$ spacetime that are of the same type, that can be scalars, fermions, gauge bosons or gravitons, are accommodated in a one–dimensional lattice, with the distance of the particles from one of the two boundaries of the lattice exhibiting an exponential profile. It can be shown that the particle closest to the other boundary couples to it with an exponentially supressed coupling. In the continuum version, one takes the limit $\widetilde{N} \to \infty$ and treats the lattice as a dimension extra to those of $4D$ spacetime. Interestingly, it turns out that the relevant $5D$ metric is identical to the one corresponding to (V.77) in the linear dilaton background. Note that the factor of proportionality, namely the analogue of $\frac{1}{\sqrt{N} l_S}$ in (F.66) in LST, is $\sqrt{-\Lambda}$ in the clockwork (upon imposing the symmetry $y \to -y$); it may thus be thought of as a measure of the vaccum energy in the bulk. Finally, notice the similarities in the ways LST and the clockwork address the hierarchy problem, as well as the fact that the clockwork KK spectrum containts a mass gap followed by a near–continuum of resonances just as the LST KK graviton spectrum.

V.5 The effective supergravity of LST

The material presented in this section corresponds to: I. Antoniadis, A. Delgado, C. Markou and S. Pokorski, *The effective supergravity of Little String Theory*, Eur. Phys. J. C **78** (2018) no.2, 146.

We would now like to find and study the effective supegravity of the toy model (V.77) which corresponds to a simplified version of the holographic dual of LST that preserves spacetime supersymmetry. To this end, we first write (V.77) in the Einstein frame by means of a conformal transformation

$$e^{-1}\mathcal{L}_{LST} = \frac{1}{2}\mathcal{R} - \frac{1}{2}(\partial_M \Phi)(\partial^M \Phi) - e^{\frac{2}{\sqrt{3}}\Phi}\Lambda \,, \tag{V.78}$$

where we have set $\kappa = 1$, which obviously contains a runaway potential for the dilaton. Naturally, gauged $\mathcal{N} = 2$, $D = 5$ supergravity is the simplest extended supergravity that may accommodate this setup, as it is the $U(1)$ gauging that generates a scalar potential, see (V.58). As an aside, note that a gauging of the $SU(2)$ subgroup of the automorphism group in $\mathcal{N} = 2$, $D = 5$ supergravity also generates a scalar potential [98] that could perhaps accommodate the linear dilaton. We thus have to write $\mathcal{L}+\mathcal{L}'$, namely the sum of (V.20) and (V.58), for the case in which there is a single real physical scalar s, that will correspond to the dilaton d.o.f., and consequently only one vector multiplet. Setting $X^0 \equiv t$, we now need an Ansatz for $\mathcal{N} = \mathcal{N}(s,t)$ of (V.47). Motivated by results regarding the graviton–dilaton system in the case of the compactification of heterotic string thery in $5D$ [99], we choose

$$\mathcal{N} = ts^2 + as^3 \,, \tag{V.79}$$

where a is a constant parameter and $1/t$ can be identified with the $5D$ heterotic string coupling. Note that the first and the second term of (V.79) correspond to the tree–level contribution and the one-loop correction respectively. Comparing (V.47) to (V.79), we find that

$$\begin{gathered} C_{000} = C_{001} = 0 \\ C_{011} = \tfrac{1}{3\beta^3} \quad , \quad C_{111} = \tfrac{a}{\beta^3} \,. \end{gathered} \tag{V.80}$$

The solution of the constraint (V.50) is then

$$t = \frac{1 - a s^3}{s^2} \tag{V.81}$$

and, using (V.40), we find

$$G_{tt} = \frac{1}{2} s^4 \quad , \quad G_{st} = \frac{1}{2} a s^4 \quad , \quad G_{ss} = \frac{1}{s^2} + \frac{1}{2} a^2 s^4 \,. \tag{V.82}$$

Moreover, using (V.41), (V.43) and (V.53), we find that

$$g_{ss} = \frac{3}{s^2} \quad , \quad f_s^a = \frac{\sqrt{3}}{s} \quad , \quad \Gamma^s_{ss} = -\frac{1}{s} \quad , \quad T_{sss} = \frac{3}{\beta}\frac{1}{s^3} \,. \tag{V.83}$$

The system made out of (V.62) and (V.64) thus takes the form

$$\begin{gathered} P_s = -\tfrac{\sqrt{3}}{2} P_0' \\ P_0'' + \tfrac{2}{s} P_0' - \tfrac{2}{s^2} P_0 = 0 \,, \end{gathered} \tag{V.84}$$

whose solution is

$$\begin{gathered} P_0 = A_P s + B_P \tfrac{1}{s^2} \\ P_s = -\tfrac{\sqrt{3}}{2}\Big(A_P - 2 B_P \tfrac{1}{s^3}\Big) \quad , \quad P^a = f_s^a g^{ss} P_s = -\tfrac{A_P}{2} s + B_P \tfrac{1}{s^2} \,. \end{gathered} \tag{V.85}$$

where A_P, B_P are constant parameters. Using the third of equations (V.60), we also find that

$$P = -3 A_P \Big(\frac{A_P}{4} s^2 + B_P \frac{1}{s}\Big) \tag{V.86}$$

so that the kinetic term and the potential for s in $\mathcal{L} + \mathcal{L}'$ take the form

$$e^{-1}\mathcal{L}_{dilaton} = -\frac{1}{2}\frac{3}{s^2}(\partial_M s)(\partial^M s) + 3 g^2 A_P \left(\frac{A_P}{4} s^2 + B_P \frac{1}{s}\right) . \tag{V.87}$$

We now redefine

$$\sqrt{3} \ln s = \Phi \,, \tag{V.88}$$

so that Φ is described by

$$e^{-1}\mathcal{L}_{dilaton} = -\frac{1}{2}(\partial_M \Phi)(\partial^M \Phi) + 3 g^2 A_P \left(\frac{A_P}{4} e^{\frac{2}{\sqrt{3}}\Phi} + B_P e^{-\frac{1}{\sqrt{3}}\Phi}\right) \tag{V.89}$$

and is identified with the canonically normalized dilaton of (V.78) upon making the identification

$$\frac{3}{4}g^2 A_P^2 = -\Lambda \quad , \quad B_P = 0\,. \tag{V.90}$$

Note that, using (V.85), we have that

$$P_0 = A_P e^{\frac{1}{\sqrt{3}}\Phi} \quad , \quad P^a = -\frac{A_P}{2} e^{\frac{1}{\sqrt{3}}\Phi}\,. \tag{V.91}$$

Furthermore, let us study how the linear dilaton background

$$\Phi = Cy\,, \tag{V.92}$$

where C a constant parameter which, using (F.66) is given by

$$C = -\frac{1}{\sqrt{3N}l_S}\,, \tag{V.93}$$

affects $\mathcal{N}=2$ supersymmetry. The background bulk metric is (see the appendix D for the conventions and some calculations)

$$ds^2 = e^{-\frac{2}{\sqrt{3}}Cy}(\eta_{\mu\nu}dx^\mu dx^\nu + dy^2)\,, \tag{V.94}$$

under the fine–tuning condition

$$C = \pm\frac{gA_P}{\sqrt{2}}\,. \tag{V.95}$$

Without loss of generality, we choose

$$C = \frac{gA_P}{\sqrt{2}}\,. \tag{V.96}$$

To have at least one unbroken supersymmetry, the fermion transformations must vanish in the vacuum for at least one linear combination of the supersymmetry parameters. Using equations (V.91), the relevant terms of the fermion transformations given by (V.22) and (V.59) take the following form in $4D$ spacetime (in the vacuum)

$$\begin{aligned}\widetilde{\delta}\psi_{\mu i} &= \tfrac{i}{2\sqrt{3}}\Gamma_\mu\Big(iC\Gamma^5\epsilon_i + \tfrac{gA_P}{\sqrt{2}}\varepsilon_{ji}\delta^{jk}\epsilon_k\Big)\\ \widetilde{\delta}\lambda_i &= -\tfrac{1}{2}e^{\frac{1}{\sqrt{3}}Cy}\Big(iC\Gamma^5\epsilon_i + \tfrac{gA_P}{\sqrt{2}}\varepsilon_{ji}\delta^{jk}\epsilon_k\Big)\,.\end{aligned} \tag{V.97}$$

Consequently, we find that

$$\begin{aligned}\widetilde{\delta}(\lambda_1 - i\Gamma^5\lambda_2) &= 0\\ \widetilde{\delta}(\lambda_1 + i\Gamma^5\lambda_2) &\sim \epsilon_2 - i\Gamma^5\epsilon_1\,.\end{aligned} \tag{V.98}$$

We thus conclude that $\mathcal{N}=2$ supersymmetry is partially broken to $\mathcal{N}=1$ and identify $\lambda_1 - i\Gamma^5\lambda_2$ with the fermion residing in a multiplet of the unbroken $\mathcal{N}=1$ supersymmetry that has direction

$$\epsilon_2 = i\Gamma^5\epsilon_1 \tag{V.99}$$

and $\lambda_1 + i\Gamma^5\lambda_2$ with the Goldstino of the broken $\mathcal{N} = 1$ supersymmetry. To determine the dependence of ϵ_i on y, we impose in the vacuum

$$\widetilde{\delta}\psi_{5i} = \partial_5\epsilon_i + \frac{igA_P}{2\sqrt{6}}\Gamma_5\varepsilon_{ji}\delta^{jk}\epsilon_k \stackrel{!}{=} 0\,, \tag{V.100}$$

which gives

$$\epsilon_1 = e^{\pm\frac{C}{2\sqrt{3}}y}\widetilde{\epsilon} \quad , \quad \epsilon_2 = \mp e^{\pm\frac{C}{2\sqrt{3}}y}\, i\Gamma_5\,\widetilde{\epsilon}, \tag{V.101}$$

where $\widetilde{\epsilon}$ is a constant spinor. We choose

$$\epsilon_1 = e^{-\frac{C}{2\sqrt{3}}y}\widetilde{\epsilon} \quad , \quad \epsilon_2 = e^{-\frac{C}{2\sqrt{3}}y}\, i\Gamma_5\,\widetilde{\epsilon}\,, \tag{V.102}$$

so that (V.102) are compatible with (V.99).

In addition, using the second of equations (V.60), as well as (V.83) and (V.91) we find that

$$P_{aa} = \frac{1}{2}P_0 + 2\sqrt{2}(f_s^a)^{-3}T_{sss}P^a = -\frac{A_P}{2}e^{\frac{1}{\sqrt{3}}\Phi}\,. \tag{V.103}$$

Consequently, the terms (V.58) take the form

$$\begin{aligned} e^{-1}\mathcal{L}' \;&=\; \tfrac{3g^2A_P^2}{4}e^{\frac{2}{\sqrt{3}}\Phi} - \tfrac{i\sqrt{6}}{8}gA_P\,e^{\frac{1}{\sqrt{3}}\Phi}\,\bar{\psi}^i_M\Gamma^{MN}\psi^j_N\delta_{ij} \\ &\quad + \tfrac{gA_P}{2\sqrt{2}}\,e^{\frac{1}{\sqrt{3}}\Phi}\,\bar{\lambda}^i\Gamma^M\psi^j_M\delta_{ij} - \tfrac{igA_P}{4\sqrt{6}}\,e^{\frac{1}{\sqrt{3}}\Phi}\,\bar{\lambda}^i\lambda^j\delta_{ij}\,. \end{aligned} \tag{V.104}$$

Furthermore, using (V.61) and (V.91) we find that

$$h^I = \frac{A_P}{2}\upsilon^I e^{\frac{1}{\sqrt{3}}\Phi} \quad , \quad h^{Ia} = -\frac{A_P}{2\sqrt{2}}\upsilon^I e^{\frac{1}{\sqrt{3}}\Phi}\,, \tag{V.105}$$

where we have assumed that $\upsilon^I\upsilon_I = 1$ for simplicity. Then, using (V.28) as well as the second of equations (V.26), we find respectively that

$$\begin{gathered} h_I = \tfrac{A_P}{2}G_{IJ}\upsilon^J e^{\frac{1}{\sqrt{3}}\Phi} \quad , \quad h^a_I = -\tfrac{A_P}{2\sqrt{2}}G_{IJ}\upsilon^J e^{\frac{1}{\sqrt{3}}\Phi}\,, \\ \Phi_{Iaa} = -\tfrac{A_P}{8}\sqrt{\tfrac{2}{3}}G_{IJ}\upsilon^J e^{\frac{1}{\sqrt{3}}\Phi}\,. \end{gathered} \tag{V.106}$$

Putting everything together, the final Lagrangian $\widetilde{\mathcal{L}} = \mathcal{L} + \mathcal{L}'$ takes the form

$$
\begin{aligned}
e^{-1}\widetilde{\mathcal{L}} \;=\;& \tfrac{1}{2}\mathcal{R}(\omega) - \tfrac{1}{2}(\partial_M\Phi)(\partial^M\Phi) - \tfrac{1}{8}e^{\frac{4}{\sqrt{3}}\Phi}F^0_{MN}F^{MN0} - \tfrac{1}{4}ae^{\frac{4}{\sqrt{3}}\Phi}F^0_{MN}F^{MN1} \\
&-\tfrac{1}{4}(e^{-\frac{2}{\sqrt{3}}\Phi} + \tfrac{1}{2}a^2e^{\frac{4}{\sqrt{3}}\Phi})F^1_{MN}F^{MN1} - \tfrac{1}{2}\bar{\psi}^i_M\Gamma^{MNP}\mathcal{D}_N\psi_{Pi} - \tfrac{1}{2}\bar{\lambda}^i\widetilde{\not{\mathcal{D}}}\lambda_i \\
&-\tfrac{i}{2}(\partial_N\Phi)\,\bar{\lambda}^i\Gamma^M\Gamma^N\psi_{Mi} - \tfrac{A_P\widetilde{\upsilon}}{16\sqrt{2}}\,e^{\frac{5}{\sqrt{3}}\Phi}\,\bar{\lambda}^i\Gamma^M\Gamma^{\Lambda P}\psi_{Mi}\,F^0_{\Lambda P} \\
&-\tfrac{A_P}{8\sqrt{2}}\Big(\tfrac{1}{2}a\widetilde{\upsilon}\,e^{\frac{5}{\sqrt{3}}\Phi} + \upsilon^1e^{-\frac{1}{\sqrt{3}}\Phi}\Big)\,\bar{\lambda}^i\Gamma^M\Gamma^{\Lambda P}\psi_{Mi}\,F^1_{\Lambda P} - \tfrac{iA_P\widetilde{\upsilon}}{64}\sqrt{\tfrac{2}{3}}e^{\frac{5}{\sqrt{3}}\Phi}\,\bar{\lambda}^i\Gamma^{MN}\lambda_i\,F^0_{MN} \\
&-\tfrac{iA_P}{32}\sqrt{\tfrac{2}{3}}\Big(\tfrac{1}{2}a\widetilde{\upsilon}\,e^{\frac{5}{\sqrt{3}}\Phi} + \upsilon^1e^{-\frac{1}{\sqrt{3}}\Phi}\Big)\,\bar{\lambda}^i\Gamma^{MN}\lambda_i\,F^1_{MN} + \tfrac{e^{-1}}{6\sqrt{6}}C_{IJK}\epsilon^{MNP\Sigma\Lambda}F^I_{MN}F^J_{P\Sigma}A^K_\Lambda \\
&-\tfrac{3iA_P\widetilde{\upsilon}}{32\sqrt{6}}e^{\frac{5}{\sqrt{3}}\Phi}\,[\bar{\psi}^i_M\Gamma^{MNP\Sigma}\psi_{Ni}F^0_{P\Sigma} + 2\bar{\psi}^{Mi}\psi^N_iF^0_{MN}] \\
&-\tfrac{3iA_P}{16\sqrt{6}}\Big(\tfrac{1}{2}a\widetilde{\upsilon}\,e^{\frac{5}{\sqrt{3}}\Phi} + \upsilon^1e^{-\frac{1}{\sqrt{3}}\Phi}\Big)\,[\bar{\psi}^i_M\Gamma^{MNP\Sigma}\psi_{Ni}F^1_{P\Sigma} + 2\bar{\psi}^{Mi}\psi^N_iF^1_{MN}] \\
&+\tfrac{3g^2A_P^2}{4}e^{\frac{2}{\sqrt{3}}\Phi} - \tfrac{i\sqrt{6}}{8}gA_P\,e^{\frac{1}{\sqrt{3}}\Phi}\,\bar{\psi}^i_M\Gamma^{MN}\psi^j_N\delta_{ij} \\
&+\tfrac{gA_P}{2\sqrt{2}}\,e^{\frac{1}{\sqrt{3}}\Phi}\,\bar{\lambda}^i\Gamma^M\psi^j_M\delta_{ij} - \tfrac{igA_P}{4\sqrt{6}}\,e^{\frac{1}{\sqrt{3}}\Phi}\,\bar{\lambda}^i\lambda^j\delta_{ij} \\
&+(\text{4–fermion terms})\,.
\end{aligned}
\tag{V.107}
$$

where A^1_M corresponds to the gauge field of the vector multiplet and we have set $\widetilde{\upsilon} = \upsilon^0 + a\upsilon^1$. Since the parameter A_P appears only through the combination gA_P in $\mathcal{L}'$, we may set $A_P = 1$ for simplicity. Moreover, bearing in mind the commentary after equation (V.79), at tree–level we can set $a = 0$. The final Lagrangian then takes the form

$$
\begin{aligned}
e^{-1}\widetilde{\mathcal{L}} \;=\;& \tfrac{1}{2}\mathcal{R}(\omega) - \tfrac{1}{2}(\partial_M\Phi)(\partial^M\Phi) - \tfrac{1}{8}e^{\frac{4}{\sqrt{3}}\Phi}F^0_{MN}F^{MN0} - \tfrac{1}{4}e^{-\frac{2}{\sqrt{3}}\Phi}F^1_{MN}F^{MN1} \\
&-\tfrac{1}{2}\bar{\psi}^i_M\Gamma^{MNP}\mathcal{D}_N\psi_{Pi} - \tfrac{1}{2}\bar{\lambda}^i\widetilde{\not{\mathcal{D}}}\lambda_i - \tfrac{i}{2}(\partial_N\Phi)\,\bar{\lambda}^i\Gamma^M\Gamma^N\psi_{Mi} \\
&-\tfrac{\upsilon^0}{16\sqrt{2}}\,e^{\frac{5}{\sqrt{3}}\Phi}\,\bar{\lambda}^i\Gamma^M\Gamma^{\Lambda P}\psi_{Mi}\,F^0_{\Lambda P} - \tfrac{\upsilon^1}{8\sqrt{2}}e^{-\frac{1}{\sqrt{3}}\Phi}\bar{\lambda}^i\Gamma^M\Gamma^{\Lambda P}\psi_{Mi}\,F^1_{\Lambda P} \\
&-\tfrac{i\upsilon^0}{64}\sqrt{\tfrac{2}{3}}e^{\frac{5}{\sqrt{3}}\Phi}\,\bar{\lambda}^i\Gamma^{MN}\lambda_i\,F^0_{MN} - \tfrac{i\upsilon^1}{32}\sqrt{\tfrac{2}{3}}e^{-\frac{1}{\sqrt{3}}\Phi}\bar{\lambda}^i\Gamma^{MN}\lambda_i\,F^1_{MN} \\
&+\tfrac{e^{-1}}{24}\epsilon^{MNP\Sigma\Lambda}\Big(2F^0_{MN}F^1_{P\Sigma}A^1_\Lambda + F^1_{MN}F^1_{P\Sigma}A^0_\Lambda\Big) \\
&-\tfrac{3i\upsilon^0}{32\sqrt{6}}e^{\frac{5}{\sqrt{3}}\Phi}\,[\bar{\psi}^i_M\Gamma^{MNP\Sigma}\psi_{Ni}F^0_{P\Sigma} + 2\bar{\psi}^{Mi}\psi^N_iF^0_{MN}] \\
&-\tfrac{3i\upsilon^1}{16\sqrt{6}}e^{-\frac{1}{\sqrt{3}}\Phi}[\bar{\psi}^i_M\Gamma^{MNP\Sigma}\psi_{Ni}F^1_{P\Sigma} + 2\bar{\psi}^{Mi}\psi^N_iF^1_{MN}] \\
&+\tfrac{3g^2}{4}e^{\frac{2}{\sqrt{3}}\Phi} - \tfrac{ig\sqrt{6}}{8}\,e^{\frac{1}{\sqrt{3}}\Phi}\,\bar{\psi}^i_M\Gamma^{MN}\psi^j_N\delta_{ij} \\
&+\tfrac{g}{2\sqrt{2}}\,e^{\frac{1}{\sqrt{3}}\Phi}\,\bar{\lambda}^i\Gamma^M\psi^j_M\delta_{ij} - \tfrac{ig}{4\sqrt{6}}\,e^{\frac{1}{\sqrt{3}}\Phi}\,\bar{\lambda}^i\lambda^j\delta_{ij} \\
&+(\text{4–fermion terms})\,.
\end{aligned}
\tag{V.108}
$$

Notice that (V.108) has three free parameters: g, υ^0 and υ^1. The KK spectrum of every $5D$

field of the theory contains a $4D$ zero–mode and a mass gap

$$M_{gap} = \frac{\sqrt{3}}{2} C = \frac{\sqrt{3}}{2\sqrt{2}} g \,, \tag{V.109}$$

where we have used (V.96) with $A_P = 1$, followed by a near–continuum of narrow KK resonances.

Since $\mathcal{N} = 2$ is partially broken, the final spectrum must consist of multiplets of the unbroken $\mathcal{N} = 1$, $D = 4$ supersymmetry (V.99) that is left intact on the $4D$ slices of the $5D$ bulk. Note that, in terms of representations of the $4D$ Lorentz–group, the $5D$ graviton zero–mode, which has five helicity states, is made out of a $4D$ zero–mode graviton, a massless $4D$ vector $\widetilde{A}_\mu$ and a massless scalar, the radion. The totality of zero–modes forms the following $\mathcal{N} = 1$ multiplets:

- A massless $\mathcal{N} = 1$ pure supergravity multiplet, made out of the $4D$ graviton and the linear combination of $4D$ gravitinos $\psi^1_\mu - i\Gamma^5\psi^2_\mu$.

- A massive spin–3/2 $\mathcal{N} = 1$ multiplet, with a mass controlled by g. It consists of the orthogonal linear combination of $4D$ gravitinos $\psi^1_\mu + i\Gamma^5\psi^2_\mu$, that acquires mass by absorbing the Goldstino $\lambda^1 + i\Gamma^5\lambda^2$, as well as two massive spin–1 and one massive spin–1/2 field.

- The remaining degrees of freedom are neatly packaged in a massless spin–1 and a massless spin–1/2 $\mathcal{N} = 1$ multiplet.

Note that, upon breaking the remaining supersymmetry, the two gravitini might recombine to form a Dirac gravitino [100] or not, and the exact freeze–out mechanism will depend on the nature of their mass.

V.6 Introducing branes

In this section, we present part of the yet unpublished piece of work: I. Antoniadis, A. Delgado, C. Markou and S. Pokorski, *in preparation.*

To begin with, we compactify y on a $S^1/\mathbb{Z}_2$ orbifold, at the fixed points of which we introduce branes, namely two branes with tensions V_1 and V_2 (in the string frame) at $y = 0$ and at $y = L$ respectively. We perform the analysis in both the string and the Einstein frame, as the former will be used in order to explore the Planck mass hierarchy and the latter to study the breaking of supersymmetry due to the branes.

- **String frame:**

In the string frame, the dilaton action is (without using matter sources)

$$\begin{aligned} S_{dil} \;=\; & \int d^5x \Big[\sqrt{-g} e^{-\sqrt{3}\Phi} \Big(\tfrac{1}{2}\mathcal{R} + \tfrac{3}{2}(\partial\Phi)^2 - \Lambda \Big) \\ & - \sqrt{-g_1} e^{-\alpha_1\Phi} V_1 \delta(y) - \sqrt{-g_2} e^{-\alpha_2\Phi} V_2 \delta(y - L) \Big] \end{aligned} \tag{V.110}$$

where $g_{1\mu\nu} = g_{\mu\nu}|_{y=0}$, $g_{2\mu\nu} = g_{\mu\nu}|_{y=L}$ and α_1, α_2 are generically arbitrary parameters. Varying with respect to Φ, $g^{\mu\nu}$ and g^{55} we find the following e.o.m. respectively (see appendix E for the derivation)

$$\begin{aligned} \sqrt{-g}e^{-\sqrt{3}\Phi}\Big(\tfrac{\sqrt{3}}{2}\mathcal{R} - \tfrac{3\sqrt{3}}{2}(\partial\Phi)^2 + 3\Box\Phi - \sqrt{3}\Lambda\Big) & \\ -\alpha_1\sqrt{-g_1}e^{-\alpha_1\Phi}V_1\delta(y) - \alpha_2\sqrt{-g_2}e^{-\alpha_2\Phi}V_2\delta(y-L) &= 0\,, \end{aligned} \tag{V.111}$$

$$\begin{aligned} \sqrt{-g}e^{-\sqrt{3}\Phi}\Big(\tfrac{1}{2}G_{\mu\nu} + \tfrac{3}{4}g_{\mu\nu}(\partial\Phi)^2 - \tfrac{\sqrt{3}}{2}g_{\mu\nu}\nabla_5\partial^5\Phi + \tfrac{1}{2}g_{\mu\nu}\Lambda\Big) & \\ +\tfrac{1}{2}g_{\mu\nu}\big(\sqrt{-g_1}e^{-\alpha_1\Phi}V_1\delta(y) + \sqrt{-g_2}e^{-\alpha_2\Phi}V_2\delta(y-L)\big) &= 0\,. \end{aligned} \tag{V.112}$$

and

$$\tfrac{1}{2}G_{55} + \tfrac{\sqrt{3}}{2}\nabla_5\partial_5\Phi + \tfrac{3}{4}g_{55}(\partial\Phi)^2 - \tfrac{\sqrt{3}}{2}g_{55}\nabla_5\partial^5\Phi + \tfrac{1}{2}g_{55}\Lambda = 0\,. \tag{V.113}$$

Assuming 4D Poincaré invariance, we make the following Ansatz for the metric

$$g_{MN} = e^{2A(y)}\eta_{MN} \tag{V.114}$$

where A is a function of y. The corresponding Ricci scalar is then

$$\mathcal{R} = -\Big(12A'^2 + 8A''\Big)e^{-2A} \tag{V.115}$$

and the components of the Einstein tensor are

$$G_{\mu\nu} = 3(A'^2 + A'')\,\eta_{\mu\nu} \quad , \quad G_{55} = 6A'^2\,. \tag{V.116}$$

Setting $\Phi' = \partial_5\Phi$, we have that

$$\Box\Phi = e^{-2A}(3A'\Phi' + \Phi'') \quad , \quad \nabla_5\partial_5\Phi = \Phi'' + \Gamma^5_{55}\Phi' = \Phi'' + A'\Phi' \tag{V.117}$$

so that the e.o.m. of Φ, $g^{\mu\nu}$ and g^{55} take respectively the form

$$\begin{aligned} \sqrt{3}(6A'^2 + 4A'') + \tfrac{3\sqrt{3}}{2}\Phi'^2 - 9A'\Phi' - 3\Phi'' + \sqrt{3}\Lambda \\ \alpha_1 e^{A+(\sqrt{3}-\alpha_1)\Phi}V_1\delta(y) + \alpha_2 e^{A+(\sqrt{3}-\alpha_2)\Phi}V_2\delta(y-L) = 0\,, \end{aligned} \tag{V.118}$$

$$\begin{aligned} 3\sqrt{3}(A'^2 + A'') + \tfrac{3\sqrt{3}}{2}\Phi'^2 - 3A'\Phi' - 3\Phi'' + \sqrt{3}\Lambda e^{2A} \\ +\sqrt{3}e^{A+(\sqrt{3}-\alpha_1)\Phi}V_1\delta(y) + \sqrt{3}e^{A+(\sqrt{3}-\alpha_2)\Phi}V_2\delta(y-L) = 0\,, \end{aligned} \tag{V.119}$$

and

$$3A'^2 + \frac{3}{4}\Phi'^2 + \frac{1}{2}e^{2A}\Lambda = 0\,. \tag{V.120}$$

One can also decouple A'' and Φ'', so that the equations (V.118)–(V.120) are rewritten as

$$\begin{aligned} \sqrt{3}(3A'^2 + A'') - 6A'\Phi' + \sqrt{3}\Lambda(1 - e^{2A}) \\ (\alpha_1 - \sqrt{3})e^{A+(\sqrt{3}-\alpha_1)\Phi}V_1\delta(y) + (\alpha_2 - \sqrt{3})e^{A+(\sqrt{3}-\alpha_2)\Phi}V_2\delta(y-L) = 0\,, \end{aligned} \tag{V.121}$$

$$
\begin{aligned}
2\sqrt{3}A'^2 - \tfrac{\sqrt{3}}{2}\Phi'^2 + \tfrac{15}{3}A'\Phi' + \Phi'' + \tfrac{1}{\sqrt{3}}\Lambda(3-4e^{2A}) \\
\Big(\alpha_1 - \tfrac{4}{\sqrt{3}}\Big)e^{A+(\sqrt{3}-\alpha_1)\Phi}V_1\delta(y) + \Big(\alpha_2 - \tfrac{4}{\sqrt{3}}\Big)e^{A+(\sqrt{3}-\alpha_2)\Phi}V_2\delta(y-L) = 0\,,
\end{aligned}
\tag{V.122}
$$

$$
3A'^2 + \frac{3}{4}\Phi'^2 + \frac{1}{2}e^{2A}\Lambda = 0\,. \tag{V.123}
$$

This is a nonlinear set of equations which is difficult to solve generically. A supersymmetry–inspired method for finding the solution (in the Einstein frame) has been proposed in [101].

Given the above, let us assume that the general solution of (V.121)–(V.123) takes the simple form

$$
\Phi = \begin{cases} a_1 y + b_1\,, & y \leqslant 0 \\ a_2 y + b_2\,, & 0 \leqslant y \leqslant L \\ a_3 y + b_3\,, & y \geqslant L \end{cases} \tag{V.124}
$$

and

$$
A = \begin{cases} c_1 y + d_1\,, & y \leqslant 0 \\ c_2 y + d_2\,, & 0 \leqslant y \leqslant L \\ c_3 y + d_3\,, & y \geqslant L\,. \end{cases} \tag{V.125}
$$

As the solutions for the dilaton and metric should be continuous (but generically not their derivatives also), we have that

$$
\begin{aligned}
b_1 = b_2 \equiv b \quad , \quad a_2 L + b_2 = a_3 L + b_3 \\
d_1 = d_2 \equiv d \quad , \quad c_2 L + d_2 = c_3 L + d_3\,.
\end{aligned}
\tag{V.126}
$$

Moreover, away from the branes, the solutions (V.124) and (V.125) should satisfy (V.121)–(V.123), while at $y=0$ and at $y=L$ they should satisfy the jump conditions

$$
\begin{aligned}
\Phi'|_{r_i-\epsilon}^{r_i+\epsilon} = \Big(\tfrac{4}{\sqrt{3}} - \alpha_i\Big)\, e^{A(r_i)+\big(\sqrt{3}-\alpha_i\big)\Phi(r_i)}V_i \\
A'|_{r_i-\epsilon}^{r_i+\epsilon} = \Big(1 - \tfrac{\alpha_1}{\sqrt{3}}\Big)\, e^{A(r_i)+\big(\sqrt{3}-\alpha_i\big)\Phi(r_i)}V_i\,,
\end{aligned}
\tag{V.127}
$$

where $r_i = 0$ or L.

Substituting (V.124) and (V.125) for $y<0$, $0<y<L$ and $y>L$ in (V.121), we find the following:

$$
\begin{aligned}
c_1 = c_2 = c_3 = 0 \\
e^{2d_1} = e^{2d_2} = e^{2d_3} = 1\,,
\end{aligned}
\tag{V.128}
$$

so, using (V.126), we have that

$$
d_3 = d = \pi n \quad , \quad n = 0, 1, 2, \ldots \tag{V.129}
$$

Moreover, substituting (V.124) and (V.125) for $y<0$, $0<y<L$ and $y>L$ in (V.122), we find the following:

$$
\begin{aligned}
c_1 = c_2 = c_3 = 0 \\
\tfrac{3}{2}a_1^2 = \Lambda(3-4e^{2d_1}) \quad , \quad \tfrac{3}{2}a_2^2 = \Lambda(3-4e^{2d_2}) \quad , \quad \tfrac{3}{2}a_3^2 = \Lambda(3-4e^{2d_3})
\end{aligned}
\tag{V.130}
$$

and, using the second of equations (V.128), we find that

$$\frac{3}{2}a_1^2 = \frac{3}{2}a_2^2 = \frac{3}{2}a_3^2 = -\Lambda\,. \tag{V.131}$$

Consequently,

$$a_1 = \pm a_2 \quad , \quad a_2 = \pm a_3\,. \tag{V.132}$$

Note that by substituting (V.124) and (V.125) for $y < 0$, $0 < y < L$ and $y > L$ in (V.123), we obtain the very same results (V.130).

Finally, substituting (V.124) and (V.125) in (V.127), we find that

- $y = 0$:

$$\begin{aligned} a_2 - a_1 &= \left(\tfrac{4}{\sqrt{3}} - \alpha_1\right) V_1 \exp\left[d + \left(\sqrt{3} - \alpha_1\right) b\right] \\ c_2 - c_1 &= \left(1 - \tfrac{\alpha_1}{\sqrt{3}}\right) V_1 \exp\left[d + \left(\sqrt{3} - \alpha_1\right) b\right] \end{aligned} \tag{V.133}$$

- $y = L$:

$$\begin{aligned} a_3 - a_2 &= \left(\tfrac{4}{\sqrt{3}} - \alpha_2\right) V_2 \exp\left[c_2 L + d + \left(\sqrt{3} - \alpha_2\right)(a_2 L + b)\right] \\ c_3 - c_2 &= \left(1 - \tfrac{\alpha_2}{\sqrt{3}}\right) V_2 \exp\left[c_2 L + d + \left(\sqrt{3} - \alpha_2\right)(a_2 L + b)\right] \end{aligned} \tag{V.134}$$

Using the first equation of (V.130) in the second equations of (V.133) and (V.134) we find that

$$\alpha_1 = \alpha_2 = \sqrt{3}\,. \tag{V.135}$$

Consequently, using (V.128), (V.132) and (V.135) in the first equations of (V.133) and (V.134) we find that

$$-a_1 = a_2 = -a_3 \equiv a\,, \tag{V.136}$$

where a is not to be confused with the parameter a of (V.79), and that

$$V_1 = 2\sqrt{3}a = -V_2\,. \tag{V.137}$$

Using (V.126) and (V.136) we also find that

$$b_3 = b + 2aL\,. \tag{V.138}$$

To conclude, the dilaton solution is

$$\Phi(y) = \begin{cases} -ay + b\,, & y \leqslant 0 \\ ay + b\,, & 0 \leqslant y \leqslant L \\ -ay + b + 2aL\,, & y \geqslant L \end{cases} \tag{V.139}$$

while the solution for the factor A in the metric is

$$A(y) = \pi n \quad , \quad n = 0, 1, 2, \ldots \tag{V.140}$$

with

$$V_1 = 2\sqrt{3}a = -V_2 \quad , \quad \frac{3}{2}a^2 = -\Lambda\,. \tag{V.141}$$

To derive the Planck mass hierarchy, we begin by noting that the constant b that appears in the dilaton background (V.139) can be absorbed by the y–coordinate via a redefinition, so we set $b = 0$. From equation (V.72), which implies that the gauge coupling on the branes is independent of the string coupling and thus of the dilaton, we deduce that the SM should not couple to the dilaton in the model under consideration, which means that a priori we can introduce, for example, the Higgs field H on any of the two (flat) branes as follows:

$$S_{Higgs} = \int d^4x \Big[\eta_{\mu\nu} \partial^\mu H^\dagger \partial^\nu H - \lambda (H^\dagger H - v_0^2)^2 \Big] . \tag{V.142}$$

Consequently, the Higgs vev, and thus all mass parameters, are at the TeV scale on both branes. Moreover, the 4D Planck mass is

$$M_{Pl}^2 = M_5^3 \int_{-L}^{L} dy\, e^{-\sqrt{3}\Phi} = M_5^3 \int_{-L}^{L} dy\, e^{-\sqrt{3}a|y|} = \frac{2M_5^3}{\sqrt{3}a}(1 - e^{-\sqrt{3}aL}) , \tag{V.143}$$

so, assuming that M_5 is at the TeV scale, a should be negative in order to generate a large M_{Pl}. The SM may be introduced on either of the two branes.

- **Einstein frame:**

We now switch to the Einstein frame by performing the conformal transformation

$$\widetilde{g}_{MN} = e^{-\frac{2}{\sqrt{3}}\Phi} g_{MN} , \tag{V.144}$$

which implies that the Ricci scalar in the string frame can be written as

$$R = e^{-\frac{2\Phi}{\sqrt{3}}} \Big(\widetilde{R} - \frac{8}{\sqrt{3}} \widetilde{\Box}^2 \Phi - 4(\widetilde{\partial}\Phi)^2 \Big) , \tag{V.145}$$

where the tilde signifies that the respective quantity is calculated in the Einstein frame. Note that in the previous section, by abuse of notation, we have used the same symbols for the quantities calculated in the string frame and their counterparts in the Einstein frame. The dilaton action then becomes

$$\begin{aligned} \widetilde{S}_{dil} \;\; = \;\; & \int d^5x \Big[\sqrt{-\widetilde{g}} \Big(\tfrac{1}{2}\widetilde{R} - \tfrac{1}{2}(\widetilde{\partial}\Phi)^2 - e^{\frac{2\Phi}{\sqrt{3}}}\Lambda \Big) \\ & - \sqrt{-\widetilde{g}_1} e^{\left(\frac{4}{\sqrt{3}} - \alpha_1\right)\Phi} V_1 \delta(y) - \sqrt{-\widetilde{g}_2} e^{\left(\frac{4}{\sqrt{3}} - \alpha_2\right)\Phi} V_2 \delta(y - L) \Big] , \end{aligned} \tag{V.146}$$

where we have discarded the total derivative $\widetilde{\Box}^2\Phi$. Treating Φ and $\widetilde{g}_{MN}$ as independent fields and using that

$$\sqrt{-\widetilde{g}_{1,2}} = \sqrt{\frac{-\widetilde{g}}{\widetilde{g}_{55}}} , \tag{V.147}$$

we find the e.o.m. of Φ, $g^{\mu\nu}$ and g^{55} to be respectively

$$\begin{aligned} & \sqrt{-\widetilde{g}}\, \widetilde{\Box}^2 \Phi - \tfrac{2}{\sqrt{3}} \sqrt{-\widetilde{g}} e^{\frac{2\Phi}{\sqrt{3}}} \Lambda - \sqrt{-\widetilde{g}_1} \left(\tfrac{4}{\sqrt{3}} - \alpha_1 \right) e^{\left(\frac{4}{\sqrt{3}} - \alpha_1\right)\Phi} V_1 \delta(y) \\ & - \sqrt{-\widetilde{g}_2} \left(\tfrac{4}{\sqrt{3}} - \alpha_2 \right) e^{\left(\frac{4}{\sqrt{3}} - \alpha_2\right)\Phi} V_2 \delta(y - L) = 0 , \end{aligned} \tag{V.148}$$

$$\begin{aligned}&\sqrt{\widetilde{g}_{55}}\Big[\widetilde{G}_{\mu\nu}+\widetilde{g}_{\mu\nu}\Big(\tfrac{1}{2}(\widetilde{\partial}\Phi)^2+e^{\frac{2\Phi}{\sqrt{3}}}\Lambda\Big)\Big]\\&+\widetilde{g}_{\mu\nu}\Big(e^{\left(\frac{4}{\sqrt{3}}-\alpha_1\right)\Phi}V_1\delta(y)+e^{\left(\frac{4}{\sqrt{3}}-\alpha_2\right)\Phi}V_2\delta(y-L)\Big)=0\,,\end{aligned}\tag{V.149}$$

and

$$\widetilde{G}_{55}+\widetilde{g}_{55}\Big(\tfrac{1}{2}(\widetilde{\partial}\Phi)^2+e^{\frac{2\Phi}{\sqrt{3}}}\Lambda\Big)-(\Phi')^2=0\,.\tag{V.150}$$

Similarly to the analysis in the string frame , we make the following Ansatz for the metric

$$\widetilde{g}_{MN}=e^{2\widetilde{A}(y)}\eta_{MN}\tag{V.151}$$

where $\widetilde{A}$ is a function of y. Consequently,

$$\widetilde{G}_{\mu\nu}=3(\widetilde{A}'^2+\widetilde{A}'')\,\eta_{\mu\nu}\quad,\quad\widetilde{G}_{55}=6\widetilde{A}'^2\quad,\quad\widetilde{\Box}\Phi=e^{-2\widetilde{A}}(3\widetilde{A}'\Phi'+\Phi'')\,.\tag{V.152}$$

so that the e.o.m. of Φ, $g^{\mu\nu}$ and g^{55} take respectively the form

$$\begin{aligned}&\Phi''+3\widetilde{A}'\Phi'-\tfrac{2}{\sqrt{3}}\Lambda e^{2\left(\widetilde{A}+\frac{\Phi}{\sqrt{3}}\right)}-(\tfrac{4}{\sqrt{3}}-\alpha_1)\,e^{\widetilde{A}+\left(\frac{4}{\sqrt{3}}-\alpha_1\right)\Phi}V_1\delta(y)\\&-(\tfrac{4}{\sqrt{3}}-\alpha_2)\,e^{\widetilde{A}+\left(\frac{4}{\sqrt{3}}-\alpha_2\right)\Phi}V_2\delta(y-L)=0\,,\end{aligned}\tag{V.153}$$

$$\begin{aligned}&3(\widetilde{A}''+\widetilde{A}'^2)+\tfrac{1}{2}(\Phi')^2+\Lambda e^{2\left(\widetilde{A}+\frac{\Phi}{\sqrt{3}}\right)}+\\&e^{\widetilde{A}+\left(\frac{4}{\sqrt{3}}-\alpha_1\right)\Phi}V_1\delta(y)+e^{\widetilde{A}+\left(\frac{4}{\sqrt{3}}-\alpha_2\right)\Phi}V_2\delta(y-L)=0\,,\end{aligned}\tag{V.154}$$

and

$$6\widetilde{A}'^2-\tfrac{1}{2}(\Phi')^2+\Lambda e^{2\left(\widetilde{A}+\frac{\Phi}{\sqrt{3}}\right)}=0\,.\tag{V.155}$$

Now let us assume that the general solution of (V.153)–(V.155) takes the form

$$\Phi=\begin{cases}A_1y+B_1\,, & y\leqslant 0\\ A_2y+B_2\,, & 0\leqslant y\leqslant L\\ A_3y+B_3\,, & y\geqslant L\end{cases}\tag{V.156}$$

and

$$\widetilde{A}=\begin{cases}\Gamma_1y+\Delta_1\,, & y\leqslant 0\\ \Gamma_2y+\Delta_2\,, & 0\leqslant y\leqslant L\\ \Gamma_3y+\Delta_3\,, & y\geqslant L\,.\end{cases}\tag{V.157}$$

Due to continuity we have that

$$\begin{aligned}B_1=B_2\equiv\widetilde{b}\quad&,\quad A_2L+B_2=A_3L+B_3\\ \Delta_1=\Delta_2\equiv\widetilde{d}\quad&,\quad\Gamma_2L+\Delta_2=\Gamma_3L+\Delta_3\,.\end{aligned}\tag{V.158}$$

Moreover, away from the branes, the solutions (V.156) and (V.157) should satisfy (V.153)–(V.155), while at $y=0$ and at $y=L$ they should satisfy the jump conditions

$$\begin{aligned}\Phi'|_{r_i-\epsilon}^{r_i+\epsilon}&=\Big(\tfrac{4}{\sqrt{3}}-\alpha_i\Big)\,e^{A(r_i)+\left(\frac{4}{\sqrt{3}}-\alpha_i\right)\Phi(r_i)}V_i\\ A'|_{r_i-\epsilon}^{r_i+\epsilon}&=-\tfrac{1}{3}\,e^{A(r_i)+\left(\frac{4}{\sqrt{3}}-\alpha_i\right)\Phi(r_i)}V_i\,,\end{aligned}\tag{V.159}$$

where $r_i = 0$ or L.

Substituting (V.156) and (V.157) in (V.153), we find the following:

- $y < 0$

$$3\Gamma_1 A_1 - \frac{2}{\sqrt{3}}\Lambda \exp\left[2\Big(\Gamma_1 y + \Delta_1 + \frac{1}{\sqrt{3}}(A_1 y + B_1)\Big)\right] = 0\,, \tag{V.160}$$

which gives

$$\begin{aligned} \Gamma_1 &= -\tfrac{1}{\sqrt{3}} A_1 \\ \tfrac{3}{2} A_1^2 &= -\Lambda e^{2\left(\Delta_1 + \frac{1}{\sqrt{3}} B_1\right)}\,. \end{aligned} \tag{V.161}$$

- $0 < y < L$

$$3\Gamma_2 A_2 - \frac{2}{\sqrt{3}}\Lambda \exp\left[2\Big(\Gamma_2 y + \Delta_2 + \frac{1}{\sqrt{3}}(A_2 y + B_2)\Big)\right] = 0\,, \tag{V.162}$$

which gives

$$\begin{aligned} \Gamma_2 &= -\tfrac{1}{\sqrt{3}} A_2 \\ \tfrac{3}{2} A_2^2 &= -\Lambda e^{2\left(\Delta_2 + \frac{1}{\sqrt{3}} B_2\right)}\,. \end{aligned} \tag{V.163}$$

- $y > L$

$$3\Gamma_3 A_3 - \frac{2}{\sqrt{3}}\Lambda \exp\left[2\Big(\Gamma_3 y + \Delta_3 + \frac{1}{\sqrt{3}}(A_3 y + B_3)\Big)\right] = 0\,, \tag{V.164}$$

which gives

$$\begin{aligned} \Gamma_3 &= -\tfrac{1}{\sqrt{3}} A_3 \\ \tfrac{3}{2} A_3^2 &= -\Lambda e^{2\left(\Delta_3 + \frac{1}{\sqrt{3}} B_3\right)}\,. \end{aligned} \tag{V.165}$$

It is straightforward to verify that substituting (V.156) and (V.157) in (V.154) and in (V.155) we obtain the very same results. Let us note that, comparing (V.161) to (V.163) and using (V.158), we find that

$$A_2 = \pm A_1\,, \tag{V.166}$$

while, comparing (V.163) to (V.165) we find that

$$A_3 = \pm A_2\, e^{\Delta_3 - \Delta_2 + \frac{1}{\sqrt{3}}(B_3 - B_2)} = \pm A_2\,, \tag{V.167}$$

where in the last step we have used (V.158).

Moreover, substituting (V.156) and (V.157) in (V.159) we find that

- y=0

$$\begin{aligned} A_2 - A_1 &= \left(\tfrac{4}{\sqrt{3}} - \alpha_1\right) e^{\Delta_1 + \left(\frac{4}{\sqrt{3}} - \alpha_1\right) B_1} V_1 \\ \Gamma_2 - \Gamma_1 &= -\tfrac{1}{3} e^{\Delta_1 + \left(\frac{4}{\sqrt{3}} - \alpha_1\right) B_1} V_1\,, \end{aligned} \tag{V.168}$$

- y=L

$$A_3 - A_2 = \left(\tfrac{4}{\sqrt{3}} - \alpha_2\right) e^{\Gamma_2 L + \Delta_2 + \left(\frac{4}{\sqrt{3}} - \alpha_2\right)(A_2 L + B_2)} V_2$$
$$\Gamma_3 - \Gamma_2 = -\tfrac{1}{3} e^{\Gamma_2 L + \Delta_2 + \left(\frac{4}{\sqrt{3}} - \alpha_2\right)(A_2 L + B_2)} V_2 \qquad \text{(V.169)}$$

where the continuity conditions (V.158) have been assumed.

Comparing the two equations of (V.168) to each other and using (V.161) and (V.163) we find that

$$\alpha_1 = \sqrt{3}\,. \qquad \text{(V.170)}$$

Similarly, comparing the two equations of (V.169) to each other and using (V.163) and (V.165) we find that

$$\alpha_2 = \sqrt{3}\,. \qquad \text{(V.171)}$$

Using (V.170) in (V.168), we find that

$$A_2 = A_1 + \frac{1}{\sqrt{3}} e^{\Delta_1 \frac{1}{\sqrt{3}} B_1} V_1\,, \qquad \text{(V.172)}$$

which, using (V.166), yields

$$A_2 = -A_1 \equiv \widetilde{a} \qquad \text{(V.173)}$$

and

$$V_1 = 2\sqrt{3}\widetilde{a}\, e^{-\widetilde{d} - \frac{1}{\sqrt{3}}\widetilde{b}}\,. \qquad \text{(V.174)}$$

Furthermore, using (V.171) in (V.169), we find that

$$A_3 = A_2 + \frac{1}{\sqrt{3}} e^{\Gamma_2 L + \Delta_2 + \frac{1}{\sqrt{3}}(A_2 L + B_2)} V_2\,, \qquad \text{(V.175)}$$

which, using (V.167), yields

$$A_3 = -A_2 \equiv -\widetilde{a} \qquad \text{(V.176)}$$

and

$$V_2 = -2\sqrt{3}\widetilde{a}\, e^{-\Gamma_2 L - \Delta_2 - \frac{1}{\sqrt{3}}(A_2 L + B_2)} = -2\sqrt{3}\widetilde{a}\, e^{-\widetilde{d} - \frac{1}{\sqrt{3}}\widetilde{b}} = -V_1\,, \qquad \text{(V.177)}$$

where in the penultimate step we have used (V.163).

Finally, using (V.173) and (V.176) in (V.158) we find that

$$B_3 = \widetilde{b} + 2\widetilde{a}L \quad , \quad \Delta_3 = \widetilde{d} - \frac{2}{\sqrt{3}}\widetilde{a}L\,. \qquad \text{(V.178)}$$

Putting everything together, we find the dilaton solution to be

$$\Phi(y) = \begin{cases} -\widetilde{a}y + \widetilde{b}\,, & y \leqslant 0 \\ \widetilde{a}y + \widetilde{b}\,, & 0 \leqslant y \leqslant L \\ -\widetilde{a}y + \widetilde{b} + 2\widetilde{a}L\,, & y \geqslant L \end{cases} \qquad \text{(V.179)}$$

while the solution for the factor $\widetilde{A}$ in the metric is

$$\widetilde{A}(y) = \begin{cases} \frac{1}{\sqrt{3}}\widetilde{a}y + \widetilde{d}\,, & y \leqslant 0 \\ -\frac{1}{\sqrt{3}}\widetilde{a}y + \widetilde{d}\,, & 0 \leqslant y \leqslant L \\ \frac{1}{\sqrt{3}}\widetilde{a}y + \widetilde{d} - \frac{2}{\sqrt{3}}\widetilde{a}L\,, & y \geqslant L\,, \end{cases} \qquad \text{(V.180)}$$

with

$$V_1 = 2\sqrt{3}\widetilde{a}\, e^{-\widetilde{d}-\frac{1}{\sqrt{3}}\widetilde{b}} = -V_2 \quad , \quad \frac{3}{2}\big(\widetilde{a}e^{-\widetilde{d}-\frac{1}{\sqrt{3}}\widetilde{b}}\big)^2 = -\Lambda\,. \tag{V.181}$$

Consequently, comparing (V.141) and (V.181), we have that

$$a = \widetilde{a}\, e^{-\widetilde{d}-\frac{1}{\sqrt{3}}\widetilde{b}}\,. \tag{V.182}$$

Note that, by comparing (V.92) and (V.179), the parameter C of the previous section is in fact identified with $\widetilde{a}$.

V.7 Supersymmetry–preserving branes

As in the previous section, we present here part of the yet unpublished piece of work: I. Antoniadis, A. Delgado, C. Markou and S. Pokorski, *in preparation*.

We now remain in the Einstein frame and set for simplicity

$$\widetilde{d} = \widetilde{b} = 0 \quad \Rightarrow \quad a = \widetilde{a} \quad , \quad \frac{3}{2}a^2 = -\Lambda\,. \tag{V.183}$$

Comparing (V.183) to (V.90), we find that

$$a = \pm\frac{gA_P}{\sqrt{2}}\,, \tag{V.184}$$

as expected by equation (V.95). Note that we have *not* set $A_P = 1$ here. Moreover, we have that

$$e^m_M = e^{\widetilde{A}}\delta^m_M\,, \tag{V.185}$$

so

$$e^{55} = e^5_5 g^{55} = e^{-\widetilde{A}} \quad , \quad e^{a\nu} = e^a_\mu g^{\mu\nu} = e^{-\widetilde{A}}\eta^{\mu\nu}\delta^a_\mu \quad , \quad e_{\mu b} = e^a_\mu \eta_{ab} = e^{\widetilde{A}}\eta_{ab}\delta^a_\mu\,. \tag{V.186}$$

Consequenty,

$$\omega_\mu{}^{a5} = \widetilde{A}'\delta^a_\mu \quad , \quad \omega_5{}^{ab} = \omega_5{}^{a5} = 0 \tag{V.187}$$

Using (V.180), we compute that

$$\omega_\mu{}^{a5} = \delta^a_\mu \begin{cases} \frac{1}{\sqrt{3}}a\,, & y<0\,, y>L \\ 0\,, & y = 0, L \\ -\frac{1}{\sqrt{3}}a\,, & 0<y<L\,, \end{cases} \tag{V.188}$$

so that

$$D_\mu\epsilon_i = \Gamma_\mu\Gamma_5\epsilon_i \begin{cases} \frac{1}{2\sqrt{3}}a\,, & y<0\,, y>L \\ 0\,, & y = 0, L \\ -\frac{1}{2\sqrt{3}}a\,, & 0<y<L\,, \end{cases} \tag{V.189}$$

and

$$D_5\epsilon_i = \partial_5\epsilon_i\,. \tag{V.190}$$

In addition, we have that

$$f^a_x \partial\!\!\!/ \phi^x = e^{-\widetilde{A}}\Gamma_5 \Phi' \tag{V.191}$$

and we also compute that

$$\Phi' = \begin{cases} -a\,, & y<0\,,\, y>L \\ 0\,, & y=0, L \\ a\,, & 0<y<L\,, \end{cases} \tag{V.192}$$

so that

$$f^a_x \partial\!\!\!/ \phi^x = e^{-\widetilde{A}}\Gamma_5 \begin{cases} -a\,, & y<0\,,\, y>L \\ 0\,, & y=0, L \\ a\,, & 0<y<L\,. \end{cases} \tag{V.193}$$

Consequently, using (V.184), as well as the expressions (V.91), the (relevant parts of the) fermion transformations are:

- $y<0\,,\, y>L$

$$\begin{aligned} \tilde{\delta}\psi_{\mu i} &= \tfrac{1}{2\sqrt{3}}\tfrac{gA_P}{\sqrt{2}}\Gamma_\mu\Big(\pm\Gamma_5\epsilon_i + i\varepsilon_{ji}\delta^{jk}\epsilon_k\Big) + \ldots \\ \tilde{\delta}\lambda^a_i &= \tfrac{i}{2}\tfrac{gA_P}{\sqrt{2}}e^{-\widetilde{A}}\Big(\pm\Gamma_5\epsilon_i + i\varepsilon_{ji}\delta^{jk}\epsilon_k\Big) + \ldots \end{aligned} \tag{V.194}$$

- $y=0, L$

$$\begin{aligned} \tilde{\delta}\psi_{\mu i} &= \tfrac{i}{\sqrt{3}}\tfrac{gA_P}{\sqrt{2}}\Gamma_\mu\varepsilon_{ji}\delta^{jk}\epsilon_k + \ldots \\ \tilde{\delta}\lambda^a_i &= -\tfrac{1}{2}\tfrac{gA_P}{\sqrt{2}}e^{-\widetilde{A}}\varepsilon_{ji}\delta^{jk}\epsilon_k + \ldots \end{aligned} \tag{V.195}$$

- $0<y<L$

$$\begin{aligned} \tilde{\delta}\psi_{\mu i} &= \tfrac{1}{2\sqrt{3}}\tfrac{gA_P}{\sqrt{2}}\Gamma_\mu\Big(\mp\Gamma_5\epsilon_i + i\varepsilon_{ji}\delta^{jk}\epsilon_k\Big) + \ldots \\ \tilde{\delta}\lambda^a_i &= \tfrac{i}{2}\tfrac{gA_P}{\sqrt{2}}e^{-\widetilde{A}}\Big(\mp\Gamma_5\epsilon_i + i\varepsilon_{ji}\delta^{jk}\epsilon_k\Big) + \ldots \end{aligned} \tag{V.196}$$

Without loss of generality, we now choose the positive sign in (V.184). Then for $y<0\,,\, y>L$, we have that

$$\begin{aligned} \widetilde{\delta}(\lambda_1 - i\Gamma_5\lambda_2) &\sim \epsilon_2 + i\Gamma_5\epsilon_1 \\ \widetilde{\delta}(\lambda_1 + i\Gamma_5\lambda_2) &= 0\,, \end{aligned} \tag{V.197}$$

while for $0<y<L$, we have that

$$\begin{aligned} \widetilde{\delta}(\lambda_1 - i\Gamma_5\lambda_2) &= 0 \\ \widetilde{\delta}(\lambda_1 + i\Gamma_5\lambda_2) &\sim \epsilon_2 - i\Gamma_5\epsilon_1\,. \end{aligned} \tag{V.198}$$

We thus conclude that the supersymmetry corresponding to the linear combination $\lambda_1 - i\Gamma_5\lambda_2$ is preserved between the branes but broken "outside" of them, while the one corresponding to $\lambda_1 + i\Gamma_5\lambda_2$ is broken between the branes but preserved "outside" of them. The direction of the unbroken supersymmetry is thus

$$\begin{aligned} \epsilon_2 &= i\Gamma_5\epsilon_1\,, \text{ between the branes} \\ \epsilon_2 &= -i\Gamma_5\epsilon_1\,, \text{ outside.} \end{aligned} \tag{V.199}$$

Interestingly, the direction of the preserved supersymmetry between the branes is identical to that of the preserved supersymmetry (V.99) in the infinite line y in the treatment of the previous section, namely before the introduction of the branes.

Finally, we have that everywhere

$$\widetilde{\delta}\psi_{5i} = \partial_5\epsilon_i + \frac{i}{2\sqrt{3}}\frac{gA_P}{\sqrt{2}}\Gamma_5\varepsilon_{ji}\delta^{jk}\epsilon_k \stackrel{!}{=} 0\,, \tag{V.200}$$

which gives

$$\begin{aligned} \epsilon_1 &= \exp\Big(\pm\tfrac{1}{2\sqrt{3}}\tfrac{gA_P}{\sqrt{2}}y\Big)\widetilde{\epsilon} \\ \epsilon_2 &= \mp\exp\Big(\pm\tfrac{1}{2\sqrt{3}}\tfrac{gA_P}{\sqrt{2}}y\Big)i\Gamma_5\widetilde{\epsilon} = \mp i\Gamma_5\epsilon_1\,, \end{aligned} \tag{V.201}$$

where $\widetilde{\epsilon}$ is a constant spinor. Consequently, between the branes we accept the solution

$$\epsilon_2 = \exp\Big(-\frac{1}{2\sqrt{3}}\frac{gA_P}{\sqrt{2}}y\Big)i\Gamma_5\widetilde{\epsilon} = i\Gamma_5\epsilon_1\,, \tag{V.202}$$

while outside the branes

$$\epsilon_2 = -\exp\Big(\frac{1}{2\sqrt{3}}\frac{gA_P}{\sqrt{2}}y\Big)i\Gamma_5\widetilde{\epsilon} = -i\Gamma_5\epsilon_1\,. \tag{V.203}$$

Note that (V.202) is precisely the same as the corresponding equation (V.102) before the introduction of the branes.

We would now like to explore supersymmetry on the branes. To this end, we start by writing the dilaton and metric backgrounds (V.179) and (V.180) for $y < L$ respectively as

$$\Phi(y) = a|y| \quad , \quad \widetilde{A} = -\frac{1}{\sqrt{3}}a|y|\,. \tag{V.204}$$

From (V.22) we find that the dilaton trasforms under supersymmetry as

$$\delta\Phi = \frac{1}{2}i\bar{\epsilon}^i\lambda_i\,, \tag{V.205}$$

so, using (V.170), we then find that the (relavant part of the) transformation of the brane term at $y = 0$

$$\mathcal{L}_1 = -\sqrt{-\widetilde{g}_1}e^{\frac{1}{\sqrt{3}}\Phi}V_1\delta(y) \tag{V.206}$$

is

$$\delta\mathcal{L}_1 = -\frac{1}{2\sqrt{3}}i\,\bar{\epsilon}^i\lambda_i\,e^{-\sqrt{3}a|y|}\,V_1\delta(y)\,. \tag{V.207}$$

Moreover, from (V.22) we compute the (relevant part of the) fermion transformation for $y < L$

$$\delta\lambda_i = -\frac{1}{2}ie_m^M\Gamma^m\partial_M\Phi\epsilon_i = -\frac{1}{2}i\,a\,\operatorname{sgn}(y)\,e^{\frac{1}{\sqrt{3}}a|y|}\,\Gamma^5\epsilon_i\,. \tag{V.208}$$

so that transformation of the bulk fermion kinetic term of (V.20) is

$$\delta\mathcal{L}_{kin} = -\frac{1}{2} ia\,\overline{\lambda}^i \epsilon_i\, e^{-\sqrt{3}a|y|}\,\delta(y)\,. \tag{V.209}$$

We now use the first of equations (V.181) to compare (V.207) to (V.209) and find that

$$\overline{\epsilon}^i\lambda_i \stackrel{!}{=} \frac{1}{2}\overline{\lambda}^i\epsilon_i\,. \tag{V.210}$$

The LHS of (V.210) becomes

$$\overline{\epsilon}^i\lambda_i = \overline{\epsilon}^1\lambda_1 + \overline{\epsilon}^2\lambda_2 = \overline{\epsilon}^1\lambda_1 + (\pm i\Gamma^5\epsilon_1)^\dagger\Gamma_0(\pm i\Gamma^5\lambda_1) = \overline{\epsilon}^1\lambda_1 - \overline{\epsilon}^1\lambda_1 = 0\,, \tag{V.211}$$

where the signs $\pm$ stand for the direction of the unbroken supersymmetry between and outside of the branes respectively, according to (V.199). Similarly, the RHS of (V.210) vanishes for both directions. The equation (V.210) is thus satisfied for both directions of the unbroken supersymmetry. We, therefore, conclude that the brane at $y = 0$ does not break $\mathcal{N} = 1$, $D = 4$ supersymmetry further.

Now let us consider the dilaton and metric backgrounds (V.179) and (V.180) for $y > 0$

$$\Phi(y) = -a|y-L| + aL \quad , \quad \widetilde{A} = \frac{1}{\sqrt{3}}a|y-L| - \frac{1}{\sqrt{3}}aL\,, \tag{V.212}$$

for which

$$\delta\lambda_i = \frac{1}{2} i\,a\;\mathrm{sgn}(y-L)\,e^{-\frac{1}{\sqrt{3}}a|y-L|+\frac{1}{\sqrt{3}}aL}\,\Gamma^5\epsilon_i\,. \tag{V.213}$$

Consequently, the transformation of the fermion bulk kinetic term is

$$\delta\mathcal{L}_{kin} = \frac{1}{2} ia\,\overline{\lambda}^i\epsilon_i\,e^{\sqrt{3}a|y-L|-\sqrt{3}aL}\,\delta(y-L)\,. \tag{V.214}$$

Moreover, the brane term at $y = L$

$$\mathcal{L}_2 = -\sqrt{-\widetilde{g}_2}\,e^{\frac{1}{\sqrt{3}}\Phi}V_2\delta(y-L) \tag{V.215}$$

transforms as

$$\delta\mathcal{L}_2 = -\frac{1}{2\sqrt{3}} i\,\overline{\epsilon}^i\lambda_i\,e^{\sqrt{3}a|y-L|-\sqrt{3}aL}\,V_2\delta(y-L)\,. \tag{V.216}$$

Using the first of equations (V.181) to compare (V.216) to (V.214), we find again that the condition (V.210) is satisfied for both directions of the unbroken supersymmetry, just as in the case of the brane at $y = 0$. We thus conclude that the introduction of the two branes preserves $\mathcal{N} = 1$, $D = 4$ supersymmetry, which is related to the fact that the linear dilaton can be thought of as a flux, as its derivative with respect to the extra dimension is a constant.

A – Some useful superspace identities

$[\ldots]$ denote antisymmetrization with weight one, for example

$$\partial_{[\mu}B_{\nu\rho]} = \frac{1}{6}\,\partial_\mu\, B_{\nu\rho} \pm 5 \text{ permutations}\,.$$

For the conjugates we use the following conventions

$$(\overline{D}_{\dot\alpha})^\dagger = -D_\alpha \quad , \quad \overline{W}_{\dot\alpha} = -(W_\alpha)^*\,. \tag{A.1}$$

Note also that several times throughout the thesis we write

$$\theta^2 = \theta\theta = \theta^\alpha\theta_\alpha \quad , \quad \overline{D}^2 = \overline{DD} = \overline{D}_{\dot\alpha}\overline{D}^{\dot\alpha} \tag{A.2}$$

and

$$\int d^2x \int d^2\theta\, f = -\frac{1}{4}\int d^4x D^2 f \quad , \quad \int d^4x \int d^2\overline{\theta}\, g = -\frac{1}{4}\int d^4x \overline{D}^2 g\,. \tag{A.3}$$

Moreover, the following identites hold

$$\begin{aligned}
[D_\alpha, \overline{DD}] &= -4i(\sigma^\mu\overline{D})_\alpha\partial_\mu\,, & [\overline{D}_{\dot\alpha}, DD] &= 4i(D\sigma^\mu)_{\dot\alpha}\partial_\mu\,.\\
2i\,\partial_\mu\chi\sigma^\mu\overline{\widetilde{\theta}} &= \overline{\widetilde{\theta}D}\,D\chi, & \overline{DD}\,\overline{\widetilde{\theta}\chi} &= -2\,\overline{\widetilde{\theta}D}\,\overline{D\chi},\\
2\,\widetilde{\theta}\sigma^\mu\partial_\mu\overline{\omega} &= i\,\widetilde{\theta}^\alpha\overline{D}_{\dot\alpha}D_\alpha\overline{\omega}^{\dot\alpha}, & \overline{DD}\,\widetilde{\theta}\omega &= \widetilde{\theta}^\alpha\overline{D}_{\dot\alpha}\overline{D}^{\dot\alpha}\omega_\alpha\,,
\end{aligned} \tag{A.4}$$

where χ_α (left–handed) and $\overline{\omega}_{\dot\alpha}$ (right–handed) are $\mathcal{N}=1$ chiral spinor superfields, namely $\overline{D}_{\dot\alpha}\chi_\beta = \overline{D}_{\dot\alpha}\overline{\omega}_{\dot\beta} = 0$. In addition, for a chiral superfield Y, which namely satisfies $\overline{D}_{\dot\alpha}\,Y = 0$, the following identities hold

$$D^\alpha\overline{D}_{\dot\alpha}D_\alpha Y = -\frac{1}{2}\overline{D}_{\dot\alpha}D^2 Y \tag{A.5}$$

and

$$\frac{1}{16}\,\overline{DD}\,DD\,Y = -\Box Y \tag{A.6}$$

while for a complex linear superfield $\mathbb{L}$

$$\overline{\eta}_{\dot\alpha}\,\overline{DD} = -2\,\overline{D}_{\dot\alpha}\,\overline{\eta D} \quad , \quad \overline{DD}\,D_\alpha\mathbb{L} = -2\,\overline{D}_{\dot\alpha}D_\alpha\overline{D}^{\dot\alpha}\mathbb{L} = 4i(\sigma^\mu\overline{D})_\alpha\partial_\mu\mathbb{L}\,.$$

B – Alternative derivation

In the following we give a derivation of the Lagrangian for the partial breaking with a single–tensor multiplet alternative to the one presented in section IV.2. Let us consider the $\mathcal{N}=2$ supersymmetric Lagrangian (III.65) and *assume* that the transformations take the form (IV.18), such that there is a fermion transforming like a Goldstino. The deformation induces a new term in the variation of the Lagrangian under the second supersymmetry:

$$\delta^*_{def}\mathcal{L}_{kin.} = \sqrt{2}\,\widetilde{M}^2 \int d^2\theta d^2\overline{\theta}\ \mathcal{H}_L(\theta\eta + \overline{\theta}\overline{\eta})\,, \tag{B.1}$$

where $\mathcal{H}$ satisfies the Laplace equation in the limit $\widetilde{M}^2 \to 0$. The expression (B.1) selects the $\theta\theta\overline{\theta}$ and $\overline{\theta}\overline{\theta}\theta$ components of $\mathcal{H}_L$. To maintain $\mathcal{N}=2$ invariance, these components must transform as derivatives under the first supersymmetry. This is the case if the highest component of $\mathcal{H}_L$ is zero or a derivative

$$\int d^2\theta d^2\overline{\theta}\,\mathcal{H}_L = \text{tot.deriv.} \tag{B.2}$$

whose solution is

$$\mathcal{H}_L = \widetilde{\mathcal{G}}(\Phi) + \overline{\widetilde{\mathcal{G}}}(\overline{\Phi}) - 2L\Big(\mathcal{G}_{\Phi\Phi}(\Phi) + \overline{\mathcal{G}}_{\overline{\Phi}\overline{\Phi}}(\overline{\Phi})\Big) \tag{B.3}$$

where $\mathcal{G}$, $\widetilde{\mathcal{G}}$ are holomorphic functions of Φ and $\mathcal{G}_\Phi = \frac{d}{d\Phi}\mathcal{G}(\Phi)$ (we use the derivatives merely for convenience). The prefactor -2 of L terms is conventional. Consequently,

$$\mathcal{H} = K(\Phi,\overline{\Phi}) + L\Big(\widetilde{\mathcal{G}}(\Phi) + \overline{\widetilde{\mathcal{G}}}(\overline{\Phi})\Big) - L^2\Big(\mathcal{G}_{\Phi\Phi}(\Phi) + \overline{\mathcal{G}}_{\overline{\Phi}\overline{\Phi}}(\overline{\Phi})\Big)\,, \tag{B.4}$$

where $K(\Phi,\overline{\Phi})$ is a function of Φ, $\overline{\Phi}$ and, using the Laplace equation, we obtain

$$\mathcal{H} = \Big(\overline{\Phi}\mathcal{G}_\Phi(\Phi) + \Phi\overline{\mathcal{G}}_{\overline{\Phi}}(\overline{\Phi})\Big) - L^2\Big(\mathcal{G}_{\Phi\Phi}(\Phi) + \overline{\mathcal{G}}_{\overline{\Phi}\overline{\Phi}}(\overline{\Phi})\Big)\,, \tag{B.5}$$

since terms linear in L do not contribute to the integral $\int d^2\theta d^2\overline{\theta}\,.$

Now let us consider again the derformation (B.1) of the lagrangian. With the use of (B.5), it becomes (since terms proportional to L^0 do not contribute):

$$\begin{aligned}
\delta^*_{def}\mathcal{L}_{kin.} &= -2\sqrt{2}\widetilde{M}^2\int d^2\theta d^2\overline{\theta}\ L\,\Big(\mathcal{G}_{\Phi\Phi}(\Phi) + \overline{\mathcal{G}}_{\overline{\Phi}\overline{\Phi}}(\overline{\Phi})\Big)(\theta\eta + \overline{\theta}\overline{\eta}) \\
&= \frac{\widetilde{M}^2}{\sqrt{2}}\int d^2\theta\ \overline{D}\overline{D}\Big[L\,\mathcal{G}_{\Phi\Phi}(\Phi)\,\overline{\theta}\overline{\eta}\Big] + \text{h.c.} \\
&= -\widetilde{M}^2\sqrt{2}\int d^2\theta\ (\overline{\eta}\overline{D}L)\,\mathcal{G}_{\Phi\Phi}(\Phi) + \text{h.c.} = i\widetilde{M}^2\,\delta^*\int d^2\theta\ \mathcal{G}_\Phi(\Phi) + \text{h.c.}
\end{aligned} \tag{B.6}$$

Consequently, the deformed lagrangian

$$\mathcal{L}_{def,kin.} = \int d^2\theta d^2\overline{\theta}\ \mathcal{H}(L, \Phi, \overline{\Phi}) - i\widetilde{M}^2 \int d^2\theta\ \mathcal{G}_\Phi(\Phi) + \text{h.c.} \tag{B.7}$$

is invariant under the first, linearly realized, supersymmetry as well as under the second, nonlinearly realized, one. It is obvious that the Lagrangians corresponding to (III.67) and (B.5) are equivalent upon identifying $\mathcal{G}_\Phi(\Phi) = iW(\Phi)$.

C – Auxiliary d.o.f. in the short and long representations

In the version of super–Maxwell theory with the use of the short representation $\mathcal{W}$ which contains the auxiliary scalar D, its lagrangian is quadratic in D:

$$\mathcal{L}_D = \frac{1}{2} A\, D^2 + \frac{1}{2}(B+\xi)D, \qquad\qquad A>0, \tag{C.1}$$

where A and B are functions of other scalar fields (with A being the gauge kinetic metric) and the constant ξ is the FI coefficient. To integrate over D, it is legitimate to solve the field equation $2AD + B + \xi = 0$ and substitute the result into $\mathcal{L}_D$ to obtain the scalar potential

$$\mathcal{L}_D = -\frac{(B+\xi)^2}{8A} = -\mathcal{V}. \tag{C.2}$$

This theory does not have any symmetry and the (supersymmetric) ground state is at $\langle B\rangle = -\xi$. The contribution of $\mathcal{L}_D$ to the field equations of the scalars, denoted collectively by z, appearing as variables of A and B is of course given by

$$\partial_z \mathcal{L}_D = -\partial_z \frac{(B+\xi)^2}{8A} = -\partial_z \mathcal{V}. \tag{C.3}$$

To go to the long representation $\widehat{\mathcal{W}}$ of the Maxwell multiplet, one has to make the replacement $D = \partial^\mu V_\mu$, with $V_\mu = -4\,\mathrm{Im}\,\mathbb{V}_\mu$, which yields a quadratic lagrangian for the divergence of a vector field,

$$\mathcal{L} = \frac{1}{2} A(\partial^\mu V_\mu)^2 + \frac{1}{2}(B+\xi)\,\partial^\mu V_\mu, \qquad\qquad A>0, \tag{C.4}$$

instead of expression (C.1). Now, the FI term is a derivative which does not contribute to the dynamical equations and the field equation for V_μ is

$$\partial_\mu[2A\,\partial^\nu V_\nu + B] = 0. \tag{C.5}$$

Its solution

$$\partial^\nu V_\nu = -\frac{B+c}{2A} \tag{C.6}$$

involves an integration constant c which replaces the FI coefficient ξ. The more subtle point is the procedure to obtain the Lagrangian after the integration of $\partial^\mu V_\mu$, since the right–hand side of the solution is not a derivative of off–shell fields.

This situation is not new in the literature. Redefine

$$V_\mu = \frac{1}{6}\,\epsilon_{\mu\nu\rho\sigma}A^{\nu\rho\sigma}, \qquad\qquad F_{\mu\nu\rho\sigma} = 4\,\partial_{[\mu}A_{\nu\rho\sigma]}. \tag{C.7}$$

Since

$$\partial^\mu V_\mu = \frac{1}{24}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}, \qquad\qquad (\partial^\mu V_\mu)^2 = -\frac{1}{24}F^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}, \tag{C.8}$$

the lagrangian (C.4) becomes

$$\mathcal{L}_F = -\frac{1}{48}A\,F^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma} + \frac{1}{48}(B+\xi)\,\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}. \tag{C.9}$$

It is part of $\mathcal{N}=8$ supergravity, with $A=e$, and the introduction of the ξ term has been studied as a potential source for a cosmological constant [102]. Another example is the massive Schwinger model [102, 103], where the Maxwell lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\theta\,\epsilon^{\mu\nu}\partial_\mu A_\nu + A^\mu j_\mu \tag{C.10}$$

(j_μ is a conserved fermion current) does not propagate any field. In the gauge $A_0 = 0$,

$$\mathcal{L} = \frac{1}{2}(\partial_0 A_1)^2 + \theta\,\partial_0 A_1, \tag{C.11}$$

and the field equation $\partial_0^2 A_1 = j_1$ implies the presence of a physically relevant arbitrary integration constant in $F_{01} = \partial_0 A_1$, to be identified with the parameter θ.

Returning to our lagrangian (C.4) and solution (C.6), if we substitute the solution into the lagrangian, $\partial^\mu V_\mu$ becomes a function of the scalar fields z, it is not any longer a derivative and the ξ–term would then become physically relevant and contribute to the field equation of z. We obtain

$$\mathcal{L} = -\frac{(B+\xi)^2}{8A} + \frac{(\xi-c)^2}{8A} = -\mathcal{V} \tag{C.12}$$

and the contribution of $\mathcal{L}$ to the field equations of the scalar fields z is of course $\partial_z\mathcal{L} = -\partial_z\mathcal{V}$. Comparing with expression (C.3), equivalence is obtained if we identify the integration constant with the FI coefficient ξ,

$$c = \xi. \tag{C.13}$$

except if A is constant (the super–Maxwell theory has then canonical kinetic terms), in which case the second constant term in the potential is irrelevant. With this procedure, both versions of the theory depend on a single arbitrary constant $c=\xi$, the FI coefficient of the super-Maxwell theory.

Notice that a derivative term may in general contribute to currents. The canonical energy-momentum tensor for a "Lagrangian" $\mathcal{L}_\xi = \xi\,\partial^\mu V_\mu$ is

$$T_{\mu\nu} = \xi\,[\partial_\nu V_\mu - \eta_{\mu\nu}\,\partial^\rho V_\rho] \tag{C.14}$$

which is not zero, conserved ($\partial^\mu T_{\mu\nu} = 0$) and an improvement term (so that the total energy–momentum is zero, assuming the absence of boundary contributions):

$$T_{00} = \xi\,\vec{\nabla}\cdot\vec{V}, \qquad\qquad T_{0i} = \xi\,\partial_i V_0. \tag{C.15}$$

D – 5D conventions and calculations

Our convention for the five–dimensional Minkowski metric is

$$\eta_{mn} = \text{diag}(-,+,+,+,+)\,, \tag{D.1}$$

where $m, n, \ldots$ are inert indices and $m = 1, \ldots, 5$. For Γ–matrices we write

$$\Gamma_{mn} \equiv \Gamma_{[m}\Gamma_{n]} \equiv \frac{1}{2}(\Gamma_m\Gamma_n - \Gamma_n\Gamma_m)\,. \tag{D.2}$$

We also have that

$$\Gamma^5 = \Gamma_5 = \gamma^5 = \gamma_5\,, \tag{D.3}$$

where γ^5 is the standard γ^5 in four–dimensions, such that in the Dirac representation

$$\Gamma^5 = \gamma^5 = \begin{pmatrix} 0_{2\times2} & 1_{2\times2} \\ 1_{2\times2} & 0_{2\times2} \end{pmatrix}\,. \tag{D.4}$$

The five–dimensional bulk metric of the LST dual is given by

$$g_{MN} = \begin{pmatrix} e^{-\frac{2}{\sqrt{3}}Cy}\eta_{\mu\nu} & 0_{4\times1} \\ 0_{1\times4} & e^{-\frac{2}{\sqrt{3}}Cy} \end{pmatrix} = e^{-\frac{2}{\sqrt{3}}Cy}\eta_{MN}\,. \tag{D.5}$$

In our conventions, the Einstein equation takes the form

$$G_{MN} = T_{MN}\,, \tag{D.6}$$

where G_{MN} and T_{MN} are the Einstein and the energy–momentum tensor respectively. Moreover, we have that

$$G_{MN} = \frac{3}{2}\Big[\frac{1}{2}\partial_M\Xi\partial_N\Xi + \partial_M\partial_N\Xi - \eta_{MN}\Big(\partial_l\partial^l\Xi - \frac{1}{2}\partial_l\Xi\partial^l\Xi\Big)\Big]\,, \tag{D.7}$$

where $\Xi = \Xi(y) = \frac{2}{\sqrt{3}}Cy$ in our case. This gives

$$G_{55} = \frac{3}{2}\Big(\frac{d\Xi}{dy}\Big)^2 = 2C^2\,. \tag{D.8}$$

In addition,

$$T_{MN} = (\partial_M\Phi)(\partial_N\Phi) - g_{MN}\Big(\frac{1}{2}(\partial_K\Phi)(\partial^K\Phi) + e^{\frac{2}{\sqrt{3}}\Phi}\Lambda\Big)\,, \tag{D.9}$$

so $T_{55} = \frac{1}{2}C^2 - \Lambda$. The Einstein equation $G_{55} = T_{55}$ then gives

$$C = \frac{gA_P}{\sqrt{2}}\,, \tag{D.10}$$

where we have used (V.90).

In the linear dilaton background, the only non–vanishing components of the vielbein e^m are

$$e^a_\mu = e^{-\frac{1}{\sqrt{3}}Cy}\delta^a_\mu \quad , \quad e^5_5 = e^{-\frac{1}{\sqrt{3}}Cy}\,, \tag{D.11}$$

where $\mu, \nu, \ldots$ are the coordinate and $a, b, \ldots$ the frame indices on the four–dimensional brane respectively. Moreover,

$$e^{a5} = g^{55}e^a_5 = 0 \quad , \quad e^{55} = g^{55}e^5_5 = e^{\frac{2}{\sqrt{3}}Cy}e^5_5 \tag{D.12}$$

and

$$e^{a\nu} = g^{\nu\kappa}e^a_\kappa = e^{\frac{2}{\sqrt{3}}Cy}\eta^{\nu\kappa}e^a_\kappa \quad , \quad e_{\mu b} = \eta_{ab}e^a_\mu\,. \tag{D.13}$$

Consequently,

$$\partial\!\!\!/\Phi = (\partial_M\Phi)\Gamma^M = (\partial_M\Phi)e^M_m\Gamma^m = (\partial_M\Phi)(e^m_M)^{-1}\Gamma^m = C(e^5_5)^{-1}\Gamma^5 = Ce^{\frac{1}{\sqrt{3}}Cy}\Gamma^5\,. \tag{D.14}$$

Using the second of the equations (V.91), the second of the equations (V.97) then takes the form (in the vacuum)

$$\tilde{\delta}\lambda_i = -\frac{1}{2}e^{\frac{1}{\sqrt{3}}Cy}\left(iC\Gamma^5\epsilon_i + \frac{gA}{\sqrt{2}}\varepsilon_{ji}\delta^{jk}\epsilon_k\right). \tag{D.15}$$

The components of the spacetime spin–connection are given by

$$\omega_M{}^{mn}(e) = 2e^{[mN}e_{[N,M]}{}^{n]} + e^{m\Lambda}e^{nP}e_{[\Lambda,P]}{}^{l}e_{Ml}\,. \tag{D.16}$$

Consequently,

$$\omega_\mu{}^{ab}(e) = \Big(- e^{[a5}e_{\mu,5}{}^{b]} + \frac{1}{2}e^{a\Lambda}e^{b5}e_{\Lambda,5}{}^{l}e_{\mu l} - \frac{1}{2}e^{b\Lambda}e^{a5}e_{\Lambda,5}{}^{l}e_{\mu l}\Big) = 0\,, \tag{D.17}$$

since $e^{a5} = 0$. Moreover,

$$\begin{aligned}
\omega_\mu{}^{a5}(e) &= \Big(- e^{[a5}e_{\mu,5}{}^{5]} + \tfrac{1}{2}e^{a\Lambda}e^{55}e_{\Lambda,5}{}^{l}e_{\mu l}\Big) \\
&= \Big(\tfrac{1}{2}e^{55}e^a_{\mu,5} + \tfrac{1}{2}e^{a\nu}e^{55}e^b_{\nu,5}e_{\mu b}\Big) \\
&= e^{55}\left(\partial_5 e^{-\frac{C}{\sqrt{3}}y}\right)\Big(\tfrac{1}{2}\delta^a_\mu + e^{\frac{1}{\sqrt{3}}Cy}\eta^{\nu\kappa}\delta^a_\kappa\delta^b_\nu\eta_{cb}e^c_\mu\Big) = -\tfrac{C}{\sqrt{3}}\delta^a_\mu\,.
\end{aligned} \tag{D.18}$$

Similarly, we find that

$$\omega_5{}^{ab} = \omega_5{}^{a5} = 0\,. \tag{D.19}$$

Since $\partial_\mu\epsilon_{i1} = 0$, we have that (in the vacuum) on the brane

$$D_\mu\epsilon_i = \frac{1}{4}\omega^{mn}_\mu\Gamma_{mn}\epsilon_i = -\frac{C}{2\sqrt{3}}\Gamma_\mu\Gamma_5\epsilon_i\,. \tag{D.20}$$

Then, using the first of the equations (V.91), the first of the equations (V.97) takes the following form on the brane

$$\tilde{\delta}\psi_{\mu i} = \frac{i}{2\sqrt{3}}\Gamma_\mu\Big(iC\Gamma^5\epsilon_i + \frac{gA}{\sqrt{2}}\varepsilon_{ji}\delta^{jk}\epsilon_k\Big)\,, \tag{D.21}$$

while the 5–th component of the first of the equations (V.97) takes the form

$$\tilde{\delta}\psi_{5i} = \partial_5\epsilon_i + \frac{igA}{2\sqrt{6}}\Gamma_5\varepsilon_{ji}\delta^{jk}\epsilon_k\,. \tag{D.22}$$

E – Derivation of e.o.m.

We use the following definitions

$$(\partial\Phi)^2 \equiv g^{MN}\partial_M\Phi\partial_N\Phi \quad , \quad (\widetilde{\partial}\Phi)^2 \equiv \widetilde{g}^{MN}\partial_M\Phi\partial_N\Phi$$
$$\Box\Phi \equiv \frac{1}{\sqrt{-g}}\partial_M[\sqrt{-g}g^{MN}\partial_N\Phi] \quad , \quad \widetilde{\Box}\Phi \equiv \frac{1}{\sqrt{-\widetilde{g}}}\partial_M[\sqrt{-\widetilde{g}}\,\widetilde{g}^{MN}\partial_N\Phi]\,. \tag{E.1}$$

- **Dilaton**

We set

$$\begin{aligned}\mathcal{L}_{dil} \quad = \quad & \sqrt{-g}e^{-\sqrt{3}\Phi}\left(\tfrac{1}{2}\mathcal{R} + \tfrac{3}{2}(\partial\Phi)^2 - \Lambda\right) \\ & -\sqrt{-g_1}e^{-\alpha_1\Phi}V_1\delta(y) - \sqrt{-g_2}e^{-\alpha_2\Phi}V_2\delta(y-L)\end{aligned} \tag{E.2}$$

so that the dilaton's equation of motion is

$$\frac{\partial\mathcal{L}_{dil}}{\partial\Phi} - \partial_M\frac{\partial\mathcal{L}_{dil}}{\partial(\partial_M\Phi)} = 0\,, \tag{E.3}$$

which yields

$$\begin{aligned}&-\sqrt{3}\sqrt{-g}e^{-\sqrt{3}\Phi}\left(\tfrac{1}{2}\mathcal{R} + \tfrac{3}{2}(\partial\Phi)^2 - \Lambda\right) + \alpha_1\sqrt{-g_1}e^{-\alpha_1\Phi}V_1\delta(y) \\ &+\alpha_2\sqrt{-g_2}e^{-\alpha_2\Phi}V_2\delta(y-L) - 3\partial_M[\sqrt{-g}e^{-\sqrt{3}\Phi}g^{MN}\partial_N\Phi] = 0\,,\end{aligned} \tag{E.4}$$

namely

$$\begin{aligned}&\sqrt{-g}e^{-\sqrt{3}\Phi}\left(\tfrac{\sqrt{3}}{2}\mathcal{R} - \tfrac{3\sqrt{3}}{2}(\partial\Phi)^2 + 3\Box\Phi - \sqrt{3}\Lambda\right) \\ &-\alpha_1\sqrt{-g_1}e^{-\alpha_1\Phi}V_1\delta(y) - \alpha_2\sqrt{-g_2}e^{-\alpha_2\Phi}V_2\delta(y-L) = 0\,,\end{aligned} \tag{E.5}$$

where we have used (E.1) to show that

$$-3\partial_M[\sqrt{-g}e^{-\sqrt{3}\Phi}g^{MN}\partial_N\Phi] = -3\sqrt{-g}e^{-\sqrt{3}\Phi}[\Box\Phi - \sqrt{3}(\partial\Phi)^2]\,. \tag{E.6}$$

- **Metric**

We now vary S_{dil} with respect to g^{MN}

$$\begin{aligned}
\delta S_{dil} &= \int d^5x\Big[\tfrac{1}{2}e^{-\sqrt{3}\Phi}\,\tfrac{\delta(\sqrt{-g}\mathcal{R})}{\delta g^{MN}} + e^{-\sqrt{3}\Phi}\,\tfrac{\delta\sqrt{-g}}{\delta g^{MN}}\Big(\tfrac{3}{2}(\partial\Phi)^2-\Lambda\Big) + \tfrac{3}{2}\sqrt{-g}e^{-\sqrt{3}\Phi}\,\tfrac{\delta[(\partial\Phi)^2]}{\delta g^{MN}} \\
&\quad -\tfrac{\delta\sqrt{-g_1}}{\delta g^{MN}}e^{-\alpha_1\Phi}V_1\delta(y) - \tfrac{\delta\sqrt{-g_2}}{\delta g^{MN}}e^{-\alpha_2\Phi}V_2\delta(y-L)\Big]\delta g^{MN} \\
&= \int d^5x\Big\{\sqrt{-g}e^{-\sqrt{3}\Phi}\,\Big[\tfrac{1}{2}\Big(\tfrac{\delta\mathcal{R}}{\delta g^{MN}} + \tfrac{\mathcal{R}}{\sqrt{-g}}\tfrac{\delta\sqrt{-g}}{\delta g^{MN}}\Big) + \tfrac{1}{\sqrt{-g}}\tfrac{\delta\sqrt{-g}}{\delta g^{MN}}\Big(\tfrac{3}{2}(\partial\Phi)^2-\Lambda\Big) \\
&\quad +\tfrac{3}{2}\tfrac{\delta[(\partial\Phi)^2]}{\delta g^{MN}}\Big] - \tfrac{\delta\sqrt{-g_1}}{\delta g^{MN}}e^{-\alpha_1\Phi}V_1\delta(y) - \tfrac{\delta\sqrt{-g_2}}{\delta g^{MN}}e^{-\alpha_2\Phi}V_2\delta(y-L)\Big\}\delta g^{MN} \\
&= \int d^5x\Big\{\sqrt{-g}e^{-\sqrt{3}\Phi}\,\Big[\tfrac{1}{2}G_{MN} + \tfrac{3}{2}\partial_M\Phi\partial_N\Phi - \tfrac{3}{4}g_{MN}(\partial\Phi)^2 + \tfrac{1}{2}g_{MN}\Lambda\Big]\delta g^{MN} \\
&\quad +\tfrac{1}{2}g_{\mu\nu}\big[\sqrt{-g_1}e^{-\alpha_1\Phi}V_1\delta(y) + \sqrt{-g_2}e^{-\alpha_2\Phi}V_2\delta(y-L)\big]\delta g^{\mu\nu}\Big\} + \delta S_{Chris}\,,
\end{aligned} \tag{E.7}$$

where

$$\delta S_{Chris} = \frac{1}{2}\int d^5x\sqrt{-g}e^{-\sqrt{3}\Phi}\nabla_P(g^{\Sigma N}\delta\Gamma^P_{N\Sigma} - g^{\Sigma P}\delta\Gamma^M_{M\Sigma}) \tag{E.8}$$

and we have used that

$$\delta\mathcal{R} = \mathcal{R}_{MN}\delta g^{MN} + \nabla_P(g^{\Sigma N}\delta\Gamma^P_{N\Sigma} - g^{\Sigma P}\delta\Gamma^M_{M\Sigma})\,, \tag{E.9}$$

$$\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}\,g_{MN}\delta g^{MN} \quad , \quad \delta\sqrt{-g_{1,2}} = -\frac{1}{2}\sqrt{-g_{1,2}}\,g_{\mu\nu}\delta g^{\mu\nu}\,, \tag{E.10}$$

$$G_{MN} = \mathcal{R}_{MN} - \frac{1}{2}g_{MN}\mathcal{R} \tag{E.11}$$

and that

$$\frac{\delta[(\partial\Phi)^2]}{\delta g^{MN}} = \frac{\delta g^{K\Lambda}}{\delta g^{MN}}\partial_K\Phi\partial_\Lambda\Phi = \partial_M\Phi\partial_N\Phi\,. \tag{E.12}$$

We now have that

$$\begin{aligned}
\delta S_{Chris} &= -\tfrac{1}{2}\int d^5x\sqrt{-g}\partial_P(e^{-\sqrt{3}\Phi})(g^{\Sigma N}\delta\Gamma^P_{N\Sigma} - g^{\Sigma P}\delta\Gamma^M_{M\Sigma}) \\
&= -\tfrac{1}{4}\int d^5x\sqrt{-g}\partial_P(e^{-\sqrt{3}\Phi})\big[g^{\Sigma N}g^{P\Lambda}(\nabla_N\delta g_{\Sigma\Lambda} + \nabla_\Sigma\delta g_{N\Lambda} - \nabla_\Lambda\delta g_{N\Sigma}) \\
&\qquad -g^{\Sigma P}g^{M\Lambda}(\nabla_M\delta g_{\Sigma\Lambda} + \nabla_\Sigma\delta g_{M\Lambda} - \nabla_\Lambda\delta g_{M\Sigma})\big] \\
&= -\tfrac{1}{4}\int d^5x\sqrt{-g}\partial_P(e^{-\sqrt{3}\Phi})\big[g^{\Sigma N}g^{P\Lambda}(2\nabla_N\delta g_{\Sigma\Lambda} - \nabla_\Lambda\delta g_{N\Sigma}) \\
&\qquad -g^{\Sigma P}g^{M\Lambda}\nabla_\Sigma\delta g_{M\Lambda}\big] \\
&= \tfrac{\sqrt{3}}{2}\int d^5x\sqrt{-g}\,e^{-\sqrt{3}\Phi}\Big[\nabla_M\partial_N\Phi - \sqrt{3}\partial_M\Phi\partial_N\Phi \\
&\qquad -g_{MN}\nabla^P\partial_P\Phi + \sqrt{3}g_{MN}(\partial\Phi)^2\Big]\delta g^{MN}
\end{aligned} \tag{E.13}$$

where we have performed integration by parts twice and we have used that

$$\delta\Gamma^P_{N\Sigma} = g^{P\Lambda}\Big(\nabla_{(N}\delta g_{\Sigma)\Lambda} - \frac{1}{2}\nabla_\Lambda\delta g_{N\Sigma}\Big)\,, \tag{E.14}$$

$$(\delta g^{MN})g_{N\Sigma} = -g^{MN}\delta g_{N\Sigma} \tag{E.15}$$

and that

$$\nabla_M \partial_N \Phi = \nabla_N \partial_M \Phi \,. \tag{E.16}$$

We then combine (E.7) and (E.13) and find that the equation of motion of $g^{\mu\nu}$ is

$$\begin{gathered}\sqrt{-g}e^{-\sqrt{3}\Phi}\Big(\tfrac{1}{2}G_{\mu\nu} + \tfrac{3}{4}g_{\mu\nu}(\partial\Phi)^2 - \tfrac{\sqrt{3}}{2}g_{\mu\nu}\nabla_y\partial^y\Phi + \tfrac{1}{2}g_{\mu\nu}\Lambda\Big) \\ +\tfrac{1}{2}g_{\mu\nu}\big(\sqrt{-g_1}e^{-\alpha_1\Phi}V_1\delta(y) + \sqrt{-g_2}e^{-\alpha_2\Phi}V_2\delta(y-L)\big) = 0\end{gathered} \tag{E.17}$$

and that that of g^{55} is

$$\tfrac{1}{2}G_{55} + \tfrac{\sqrt{3}}{2}\nabla_5\partial_5\Phi + \tfrac{3}{4}g_{55}(\partial\Phi)^2 - \tfrac{\sqrt{3}}{2}g_{55}\nabla_5\partial^5\Phi + \tfrac{1}{2}g_{55}\Lambda = 0\,, \tag{E.18}$$

where we have assumed that the dilaton depends only on the extra dimension.

F – Nonlinear supersymmetry and $\mathcal{N} = 1$ supergravity

F.1 Two equivalent Lagrangians

The present appendix is with minor modifications: I. Antoniadis and C. Markou, *The coupling of Non-linear Supersymmetry to Supergravity*, Eur. Phys. J. C **75** (2015) no.12, 582.

In the constrained superfield formalism of non-linear supersymmetry, the goldstino is described by the fermionic component of a chiral superfield X, that satisfies the nilpotent constraint $X^2 = 0$ [1, 2, 3, 4]. The scalar component (sgoldstino) is then eliminated by the constraint and is replaced by a goldstino bilinear. The most general low energy (without super-derivatives) Lagrangian, invariant (upon space-time integration) under global supersymmetry, is then given by

$$\mathscr{L}_{VA} = [X\bar{X}]_D + ([fX]_F + \text{h.c.})\,, \tag{F.1}$$

where $f \neq 0$ is a complex parameter. The subscripts D and F denote D and F-term densities, integrated over the full or the chiral superspace, respectively, and correspond to the Kähler potential and superpotential of $N = 1$ supersymmetry. It can be shown [4, 5] that $\mathscr{L}_{VA}$ is equivalent to the Volkov-Akulov Lagrangian [6] on-shell.

The coupling to supergravity in the superconformal context [104], (F.1) takes the form

$$\mathscr{L} = -\left[(1 - X\bar{X})S_0\bar{S}_0\right]_D + \left([(fX + W_0 + \frac{1}{2}TX^2)S_0^3]_F + \text{h.c.}\right), \tag{F.2}$$

where we have used the superconformal tensor calculus [105, 13] with S_0 being the superconformal compensator superfield. We have also used a Lagrange multiplier T in order to impose the constraint $X^2 = 0$ explicitly in $\mathscr{L}$, while the factor $\frac{1}{2}$ is put merely for convenience. W_0 is a complex constant parameter whose importance will appear shortly. The Kähler potential corresponding to (F.2) is given by

$$K(X, \bar{X}) = -3\ln(1 - X\bar{X}) = -3\left[-X\bar{X} - \frac{(-X\bar{X})^2}{2} + \ldots\right] = 3X\bar{X}. \tag{F.3}$$

We would now like to find a geometrical formulation of (F.2), that is, to eliminate X and write an equivalent Lagrangian that contains only superfields describing the geometry of spacetime, such as the superspace chiral curvature $\mathscr{R}$ [14, 106, 107]. For that, we observe that the following Kähler potential K':

$$K' = -3\ln(1+X+\bar{X}) = -3\left(X+\bar{X}-\frac{(X+\bar{X})^2}{2}+\ldots\right) = 3X\bar{X}-3(X+\bar{X}), \quad \text{(F.4)}$$

is related to the Kähler potential K via a Kähler tranformation of the type

$$\begin{aligned} K &\to K' = K - 3(X+\bar{X}) \\ W &\to W' = e^{3X}W. \end{aligned} \quad \text{(F.5)}$$

This tells us that $\mathscr{L}$ is equivalent to $\mathscr{L}'$, where

$$\mathscr{L}' = -\left[(1+X+\bar{X})S_0\bar{S}_0\right]_D + \left([(fX+W_0+\frac{1}{2}TX^2)e^{3X}S_0^3]_F + \text{h.c.}\right). \quad \text{(F.6)}$$

Using the constraint $X^2=0$, we have

$$\begin{aligned} \mathscr{L}' &= -\left[(1+X+\bar{X})S_0\bar{S}_0\right]_D + \left([(fX+W_0(1+3X)+\frac{1}{2}TX^2)S_0^3]_F + \text{h.c.}\right) \\ &= -[S_0\bar{S}_0]_D + \left([(\lambda X + W_0 - X\frac{\mathscr{R}}{S_0}+\frac{1}{2}TX^2)S_0^3]_F + \text{h.c.}\right), \end{aligned} \quad \text{(F.7)}$$

where we have set $\lambda = f + 3W_0$ and we have used the identity [13]

$$[X\cdot\mathscr{R}\cdot S_0^2]_F = \left[S_0\bar{S}_0(X+\bar{X})\right]_D + \text{total derivatives.} \quad \text{(F.8)}$$

In (F.7), X enters only in F-terms without derivatives and can be thus integrated out. Solving the equation of motion for X, we have

$$\lambda - \frac{\mathscr{R}}{S_0} + TX = 0 \quad \Rightarrow \quad X = \frac{\frac{\mathscr{R}}{S_0}-\lambda}{T} \quad \text{(F.9)}$$

and substituting back into (F.7), we get

$$\begin{aligned} \mathscr{L}' &= -[S_0\bar{S}_0]_D + \left([(-\frac{1}{2T}(\frac{\mathscr{R}}{S_0}-\lambda)^2+W_0)S_0^3]_F + \text{h.c.}\right) \\ &= \left[(-\frac{1}{2}\frac{\mathscr{R}}{S_0}+W_0-\frac{1}{2T}(\frac{\mathscr{R}}{S_0}-\lambda)^2)S_0^3\right]_F + \text{h.c.}, \end{aligned} \quad \text{(F.10)}$$

where we have used again the identity (F.8). We can now view $\frac{1}{T}$ as a Lagrange multiplier that imposes the constraint

$$(\frac{\mathscr{R}}{S_0}-\lambda)^2 = 0\,. \quad \text{(F.11)}$$

Consequently, we have established an equivalence between the constrained Lagrangians (F.2) and (F.10); they both describe the coupling of non-linear supersymmetry to supergravity, with $\mathscr{L}'$ providing its geometric formulation with the use of a constraint imposed on $\mathscr{R}$ instead of X. This constraint was proposed in [14] for $\lambda = 0$. In what follows we will confirm the equivalence by writing these Lagrangians in terms of component fields.

- Constraining a chiral superfield X

In the following we use the method and conventions of [108] except from a factor of 1/6 which we omit in the expression of $\mathscr{R}$ but introduce at the Lagrangian level. We also set the gravitational coupling $\kappa^2 = 8\pi G_N$ (given here in natural units) to be equal to one, in accordance with the usual convention. After gauge-fixing the superconformal symmetry by using the convenient gauge $S_0 = 1$, the Lagrangian (F.2) can be written as follows:

$$\begin{gathered}\mathscr{L} = \int \mathrm{d}^2\Theta 2\mathscr{E}\left\{\frac{3}{8}(\bar{\mathscr{D}}\bar{\mathscr{D}} - \frac{8}{6}\mathscr{R})e^{-K/3} + W\right\} + \text{h.c.} \\ \text{with } W(X) = fX + W_0 \text{ and } X^2 = 0\,,\end{gathered} \tag{F.12}$$

where $\mathscr{D}$ is the super-covariant derivative and $\mathscr{E}$ the chiral superfield density that is constructed from the vielbein e_a^m:

$$\mathscr{E} = \frac{1}{2}e\left\{1 + i\Theta\sigma^a\bar{\psi}_a - \Theta\Theta[\bar{M} + \bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b]\right\}\,. \tag{F.13}$$

Here ψ_a is the gravitino, Θ the fermionic coordinates of the curved superspace and $\sigma^a = (-1, \vec{\sigma})$, $\sigma_\alpha^{ab\beta} = \frac{1}{4}(\sigma^a_{\alpha\dot{\alpha}}\bar{\sigma}^{b\dot{\alpha}\beta} - \sigma^b_{\alpha\dot{\alpha}}\bar{\sigma}^{a\dot{\alpha}\beta})$ with $\vec{\sigma}$ the Pauli matrices. Note that the Lagrange multiplier T in (F.2) has been used to impose the constraint $X^2 = 0$, which can be solved, fixing the scalar component (sgoldstino) in terms of the goldstino G and the auxiliary field F of X [4].

We now substitute X, $\mathscr{E}$ and $\mathscr{R}$ with their respective expressions in component fields:

$$\begin{gathered}X = \frac{G^2}{2F} + \sqrt{2}\Theta G + (\Theta\Theta)F \equiv A + \sqrt{2}\Theta G + (\Theta\Theta)F \\ \mathscr{R} \equiv -M - \Theta B - (\Theta\Theta)C \\ \Xi \equiv (\bar{\mathscr{D}}\bar{\mathscr{D}} - \frac{8}{6}\mathscr{R})\bar{X} \equiv -4\bar{F} + \frac{4}{3}M\bar{A} + \Theta D + (\Theta\Theta)E\,.\end{gathered} \tag{F.14}$$

The exact components of $\mathscr{R}$ and Ξ are computed in [108] (our convention for $\mathscr{R}$ differs by 1/6 with respect to [108]). M and b_a are the auxiliary fields of the $N = 1$ supergravity multiplet in the old-minimal formulation. Then

$$\begin{aligned}-\frac{3}{4}\left[\mathscr{E}(\bar{\mathscr{D}}\bar{\mathscr{D}} - \frac{8}{6}\mathscr{R})\bar{X}X\right]_F &= -\frac{3}{8}[2\mathscr{E}\Xi X]_F = -\frac{3}{8}e(EA - 4F\bar{F} + \frac{4}{3}MF\bar{A} - \frac{\sqrt{2}}{2}(DG)) \\ &\quad + \frac{3}{16}\mathrm{i}e(y\sigma^a\bar{\psi}_a) + \frac{3}{8}e[\bar{M} + \bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b][-4A\bar{F} + \frac{4}{3}MA\bar{A}]\,,\end{aligned} \tag{F.15}$$

where

$$y = \sqrt{2}G(-4\bar{F} + \frac{4}{3}M\bar{A}) + DA\,. \tag{F.16}$$

This expression is simplified significantly if we choose to use the unitary gauge, setting $G = 0$ and thus $A = y = 0$:

$$-\frac{3}{4}\left[\mathscr{E}(\bar{\mathscr{D}}\bar{\mathscr{D}} - \frac{8}{6}\mathscr{R})\bar{X}X\right]_F = \frac{3}{2}eF\bar{F}\,. \tag{F.17}$$

Moreover, also in the unitary gauge, one can compute

$$[2\mathscr{E}(fX + W_0)]_F = efF - e\left[\bar{M} + \bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b\right]W_0\,. \tag{F.18}$$

Now, using the property

$$(\bar{\psi}_a \bar{\sigma}^{ab} \bar{\psi}_b)^\dagger = \frac{1}{4}\left[\bar{\psi}_a(\bar{\sigma}^a\sigma^b - \bar{\sigma}^b\sigma^a)\bar{\psi}_b\right]^\dagger = \frac{1}{4}\left[\psi_b(\sigma^b\bar{\sigma}^a - \sigma^a\bar{\sigma}^b)\psi_a\right] = \psi_a\sigma^{ab}\psi_b\,, \tag{F.19}$$

the Lagrangian (F.12) in terms of component fields becomes

$$\begin{aligned}\mathscr{L} = &-\frac{1}{2}eR - \frac{1}{3}eM\bar{M} + \frac{1}{3}eb^a b_a + \frac{1}{2}e\epsilon^{abcd}(\bar{\psi}_a\bar{\sigma}_b\widetilde{\mathscr{D}}_c\psi_d - \psi_a\sigma_b\widetilde{\mathscr{D}}_c\bar{\psi}_d)\\ &+efF - eW_0[\bar{M} + \bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b] + e\bar{f}\bar{F} - e\bar{W}_0[M + \psi_a\sigma^{ab}\psi_b] + 3eF\bar{F}\,,\end{aligned} \tag{F.20}$$

where R is the Ricci scalar. The equations of motion for the auxiliary fields b^a, M, F are then

$$\begin{aligned} b^a &= 0\\ M = -3W_0&, \bar{M} = -3\bar{W}_0\\ F = -\frac{\bar{f}}{3}&, \bar{F} = -\frac{f}{3}. \end{aligned} \tag{F.21}$$

Substituting back into (F.20) we get

$$\begin{aligned}\mathscr{L} \;=\; &-\frac{1}{2}eR + \frac{1}{2}e\epsilon^{abcd}(\bar{\psi}_a\bar{\sigma}_b\widetilde{\mathscr{D}}_c\psi_d - \psi_a\sigma_b\widetilde{\mathscr{D}}_c\bar{\psi}_d)\\ &-eW_0\bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b - e\bar{W}_0\psi_a\sigma^{ab}\psi_b + 3e|W_0|^2 - \frac{1}{3}e|f|^2.\end{aligned} \tag{F.22}$$

In this form, it is obvious that the Lagrangian reduces to the usual $N = 1$ supergravity, together with a gravitino mass term:

$$\boxed{m_{3/2} = |W_0|}. \tag{F.23}$$

Imposing that the cosmological constant (i.e. the vacuum expectation value of the scalar potential) vanishes, one finds

$$3|W_0|^2 - \frac{1}{3}|f|^2 = 0 \Rightarrow \boxed{|f|^2 = 9|W_0|^2}. \tag{F.24}$$

This means that $W_0 \neq 0$, which justifies the use of the constant piece W_0 in the superpotential in $\mathscr{L}$. Then, the final form of $\mathscr{L}$ is

$$\mathscr{L} = -\frac{1}{2}eR + \frac{1}{2}e\epsilon^{abcd}(\bar{\psi}_a\bar{\sigma}_b\widetilde{\mathscr{D}}_c\psi_d - \psi_a\sigma_b\widetilde{\mathscr{D}}_c\bar{\psi}_d) - eW_0\bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b - e\bar{W}_0\psi_a\sigma^{ab}\psi_b\,. \tag{F.25}$$

It is important to notice that the use of the constrained superfield X is what has generated the gravitino mass term: the final form of the Lagrangian in flat space is just the pure $N = 1$ supergravity, but with a massive gravitino. The use of the unitary gauge $G = 0$ results in the gravitino absorbing the goldstino and becoming massive, in analogy with the well-known Brout-Englert-Higgs mechanism.

- Constraining the superspace curvature superfield $\mathscr{R}$

After gauge-fixing the superconformal symmetry by imposing $S_0 = 1$, the Lagrangian (F.10) can be written as follows:

$$\begin{aligned} \mathscr{L}' &= -\int d^2\Theta \mathscr{E}(\mathscr{R} - 2W_0) + \text{h.c.}, \\ &(\mathscr{R} - \lambda)^2 = 0. \end{aligned} \tag{F.26}$$

$\mathscr{L}'$ then yields

$$\begin{aligned} \mathscr{L}' = -\frac{1}{2}eR - \frac{1}{3}eM\bar{M} + \frac{1}{3}eb^a b_a + \frac{1}{2}e\epsilon^{abcd}(\bar{\psi}_a\bar{\sigma}_b\widetilde{\mathscr{D}}_c\psi_d - \psi_a\sigma_b\widetilde{\mathscr{D}}_c\bar{\psi}_d) \\ -eW_0[\bar{M} + \bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b] - e\bar{W}_0[M + \psi_a\sigma^{ab}\psi_b]. \end{aligned} \tag{F.27}$$

Now let us solve the constraint which is the second of the equations (F.26). For that, we substitute the second of the equations (F.14) into the constraint and find the set of the following equations:

$$\begin{aligned} (M+\lambda)^2 &= 0 \\ (M+\lambda)B_\alpha &= 0 \\ 4(M+\lambda)C &= (BB), \end{aligned} \tag{F.28}$$

where

$$\begin{aligned} B_\alpha = \sigma^a_{\alpha\dot{\alpha}}\bar{\sigma}^{b\dot{\alpha}\beta}\psi_{ab\beta} - \mathrm{i}\sigma^a_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}_a M + \mathrm{i}\psi_{a\alpha}b^a \quad \text{with} \quad \psi_{ab} \equiv \widetilde{\mathscr{D}}_a\psi_b - \widetilde{\mathscr{D}}_b\psi_a \\ C = -\frac{1}{2}R + \mathcal{O}\{M, b_a, \psi_a\} \neq 0\,. \end{aligned} \tag{F.29}$$

Equations (F.28) yield:

$$M = -\lambda \quad \text{and} \quad b^a = 0\,. \tag{F.30}$$

Indeed, B in this case depends only on the gamma-trace or the divergence of the gravitino, $\bar{\sigma}^a\psi_a$ and $\widetilde{\mathscr{D}}^a\psi_a$ (using the Clifford algebra property of sigma-matrices $(\sigma^a\bar{\sigma}^b + \sigma^b\bar{\sigma}^a)_\alpha^{\ \beta} = -2\eta^{ab}\delta_\alpha^\beta$) , that can be put to zero by an appropriate gauge choice. Alternatively, one can show that B vanishes on-shell as will be demonstrated below.

Using (F.30), eq. (F.27) becomes:

$$\begin{aligned} \mathscr{L}' = -\frac{1}{2}eR - \frac{1}{3}e|\lambda|^2 + \frac{1}{2}e\epsilon^{abcd}(\bar{\psi}_a\bar{\sigma}_b\widetilde{\mathscr{D}}_c\psi_d - \psi_a\sigma_b\widetilde{\mathscr{D}}_c\bar{\psi}_d) \\ +eW_0\bar{\lambda} + e\bar{W}_0\lambda - eW_0\bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b - e\bar{W}_0\psi_a\sigma^{ab}\psi_b\,. \end{aligned} \tag{F.31}$$

Substituting now $\lambda = f + 3W_0$, one finds that the cosmological constant is given by $3e|W_0|^2 - \frac{1}{3}e|f|^2$ and the Lagrangian (F.31) is identical to (F.22). Note that the vanishing of the cosmological constant

$$-\frac{1}{3}e|\lambda|^2 + eW_0\bar{\lambda} + e\bar{W}_0\lambda = 0 \tag{F.32}$$

gives two possible solutions for λ:

$$\boxed{\lambda = 6W_0} \quad \text{and} \quad \boxed{\lambda = 0}\,, \tag{F.33}$$

corresponding to $f = \pm 3W_0$ that solve the condition (F.24).

Now let us derive the equation of motion for the gravitino from (F.31):

$$\frac{1}{2}\epsilon^{abcd}\sigma_b\widetilde{\mathscr{D}}_c\bar{\psi}_d = -\bar{W}_0\sigma^{ab}\psi_b\,. \tag{F.34}$$

Contracting (F.34) with $\widetilde{\mathscr{D}}_a$, we obtain the following equation:

$$\sigma^{ab}\widetilde{\mathscr{D}}_a\psi_b = 0\,. \tag{F.35}$$

Moreover, contracting the hermitian conjugate

$$\frac{1}{2}\epsilon^{abcd}\bar{\sigma}_b\widetilde{\mathscr{D}}_c\psi_d = W_0\bar{\sigma}^{ab}\bar{\psi}_b\,. \tag{F.36}$$

of (F.34) with σ_a, we have that

$$\epsilon^{abcd}\sigma_a\bar{\sigma}_b\widetilde{\mathscr{D}}_c\psi_d \sim \epsilon^{abcd}\sigma_{ab}\widetilde{\mathscr{D}}_c\psi_d \sim \sigma^{cd}\widetilde{\mathscr{D}}_c\psi_d = 0\,, \tag{F.37}$$

where we have used (F.35) and

$$\epsilon^{abcd}\sigma_{ab} = -2\mathrm{i}\sigma^{cd}\,. \tag{F.38}$$

Consequently,

$$\sigma_a\bar{\sigma}^{ab}\bar{\psi}_b = 0 \Rightarrow \sigma^a\bar{\psi}_a = 0\,, \tag{F.39}$$

where we have used the identity

$$\sigma^a\bar{\sigma}^b\sigma^c - \sigma^c\bar{\sigma}^b\sigma^a = 2\mathrm{i}\epsilon^{abcd}\sigma_d\,. \tag{F.40}$$

Now let us consider B_α of eq. (F.29). Its last term $\mathrm{i}\psi_a b^a$ vanishes due to the equation of motion for b^a, while its second term vanishes due to equation (F.39). B_α's first term is:

$$\sigma^a\bar{\sigma}^b\psi_{ab} = \sigma^a\bar{\sigma}^b(\widetilde{\mathscr{D}}_a\psi_b - \widetilde{\mathscr{D}}_b\psi_a) = (\sigma^a\bar{\sigma}^b - \sigma^b\bar{\sigma}^a)\widetilde{\mathscr{D}}_a\psi_b = 4\sigma^{ab}\widetilde{\mathscr{D}}_a\psi_b = 0\,, \tag{F.41}$$

where we have used the definition

$$\sigma^{ab} \equiv \frac{1}{4}(\sigma^a\bar{\sigma}^b - \sigma^b\bar{\sigma}^a) \tag{F.42}$$

and the relation (F.35). Consequently $B_\alpha = 0$ on-shell, which justifies the solution $M = -\lambda$ and $b^a = 0$ we chose previously.

F.2 Without imposing direct constraints

In this section, we would like to start with a regular $\mathscr{R}^2$ supergravity and recover the constraint in an appropriate limit where the additional (complex) scalar arising from $\mathscr{R}^2$ becomes superheavy and decouples from the low energy spectrum. Indeed, by analogy with ordinary General Relativity in the presence of an R^2-term (with R the scalar curvature), an $\mathscr{R}^2$ supergravity can be re-written as an ordinary Einstein $N = 1$ supergravity coupled to an extra chiral multiplet.[1] Let us then consider the Lagrangian

$$\mathscr{L} = \left[\left(-\frac{1}{2}\frac{\mathscr{R}}{S_0} + W_0 + \frac{1}{2}\rho(\frac{\mathscr{R}}{S_0} - \lambda)^2\right)S_0^3\right]_F + \text{h.c.}\,, \tag{F.43}$$

[1] Note that $\mathscr{R}^2$ supergravity is not the supersymmetrization of R^2 gravity which is described by a D-term $\mathscr{R}\bar{\mathscr{R}}$, bringing two chiral multiplets to be linearized [105, 13].

where ρ is a real parameter. In the limit $|\rho| \to \infty$, one would naively expect to recover the constraint $(\mathscr{R} - \lambda)^2 \to 0$, and thus (F.43) should be reduced to (F.10). In principle, one could linearize (F.43) with the use of a chiral superfield S and then demonstrate that in the limit $|\rho| \to \infty$, $\mathscr{L}$, $\mathscr{L}'$ and $\bar{\mathscr{L}}$ are all equivalent. If this were true, one would expect that S corresponds to the goldstino superfield and that supersymmetry is non-linearly realized (in the limit $|\rho| \to \infty$), as is the case for the chiral nilpotent superfield X. In other words, the mass of the scalar component of S would approach infinity as $|\rho| \to \infty$ and would, therefore, decouple from the spectrum. However, upon computing the scalar potential and the scalar mass matrix corresponding to (F.43), we found that this is not the case. This means that the parameter space (λ, W_0, ρ) does not allow for a supersymmetry breaking minimum that realizes the sgoldstino decoupling and the equivalence between $\bar{\mathscr{L}}$ with $\mathscr{L}$ and $\mathscr{L}'$. The detailed analysis can be found in the end of this section.

To solve this problem, we start with a more general class of $f(\mathscr{R})$ supergravity actions. More precisely, we modify $\bar{\mathscr{L}}$ with the addition of a suitable term that is supressed by ρ in the limit $|\rho| \to \infty$:[2]

$$\mathscr{L}'' = \left[\left(-\frac{1}{2}\frac{\mathscr{R}}{S_0} + W_0 + \frac{1}{2}\rho(\frac{\mathscr{R}}{S_0} - \lambda)^2 + \frac{1}{\rho}\left(S\frac{\mathscr{R}}{S_0} - F(S)\right)\right) S_0^3\right]_F + \text{h.c.}\,, \tag{F.44}$$

where S is a chiral superfield coupled to gravity and $F(S)$ is a holomorphic function of the superfield S. This extra term has already been studied in the literature and is known as $f(\mathscr{R})$ supergravity [109, 110]. Indeed, S can be integrated out by its equation of motion at finite ρ:

$$\mathscr{R} = F' S_0, \tag{F.45}$$

where $F' = \frac{\partial F}{\partial S}$. This equation can be in principle solved to give S as a function of $\mathscr{R}$ and replacing it back in (F.44) one finds an $f(\mathscr{R})$ theory.

We will now study the physical implications of $\mathscr{L}''$ in the limit $\rho \to \infty$ so as to confirm the equivalence between $\mathscr{L}$, $\mathscr{L}'$ and $\mathscr{L}''$ (without loss of generality, we take ρ positive). We first use eq. (F.45) to replace $\mathscr{R}$ in terms of S in the third term of (F.44), instead of doing the reverse as described above. Using then the identity (F.8), we get

$$\mathscr{L}'' = -\left[(1 - \frac{1}{\rho}(S + \bar{S}))S_0\bar{S}_0\right]_D + \left\{[(W_0 + \frac{1}{2}\rho(F' - \lambda)^2 - \frac{1}{\rho}F)S_0^3]_F + \text{h.c.}\right\}. \tag{F.46}$$

We now fix the gauge according to $S_0 = 1$ and set ϕ to be the lowest component of S. Then the Kähler potential and the superpotential corresponding to $\mathscr{L}''$ are given by (we use the same symbols K and W as in section (F.1) as there is no confusion)

$$\begin{aligned} K &= -3\ln\left(1 - \frac{1}{\rho}(\phi + \bar{\phi})\right) \\ W &= W_0 + \frac{1}{2}\rho(F' - \lambda)^2 - \frac{1}{\rho}F, \end{aligned} \tag{F.47}$$

where now $F' = \frac{\partial F}{\partial \phi}$.

[2] In principle, we may replace $1/\rho$ by $1/\hat{\rho}(\rho)$ with $|\hat{\rho}(\rho)| \to \infty$ when $\rho \to \infty$. One can show however that our results do not change and thus we make the simple choice $\hat{\rho} = \rho$.

It follows that

$$\exp(K) = \frac{\rho^3}{(\rho - \phi - \bar{\phi})^3} \tag{F.48}$$

and

$$g_{\phi\bar{\phi}} = \frac{\partial}{\partial\phi}\frac{\partial}{\partial\bar{\phi}}K = \frac{3}{(\rho - \phi - \bar{\phi})^2} \quad , \quad g^{\phi\bar{\phi}} = \frac{(\rho - \phi - \bar{\phi})^2}{3}. \tag{F.49}$$

Also

$$D_\phi W = \partial_\phi W + K_\phi W = \rho F''(F' - \lambda) - \frac{1}{\rho}F' + \frac{3}{\rho - \phi - \bar{\phi}}\left(W_0 + \frac{1}{2}\rho(F' - \lambda)^2 - \frac{1}{\rho}F\right). \tag{F.50}$$

Putting everything together, we get that the scalar potential V is given by:

$$V = \exp(K)\left[g^{\phi\bar{\phi}}(D_\phi W)(\bar{D}_{\bar{\phi}}\bar{W}) - 3\bar{W}W\right] = \frac{\rho^2}{3(\rho - \phi - \bar{\phi})^2}\tilde{V}, \tag{F.51}$$

where

$$\begin{aligned} \tilde{V} &= \rho^4|F''(F' - \lambda)|^2 + \rho^3\left[-(\phi + \bar{\phi})|F''(F' - \lambda)|^2 + \frac{3}{2}|F' - \lambda|^2\left(F''(\bar{F}' - \bar{\lambda}) + \text{h.c.}\right)\right] \\ &+ \rho^2\left[-\bar{F}'F''(F' - \lambda) + 3\bar{W}_0F''\,(F' - \lambda) + \text{h.c.})\right] + \rho\left[(\phi + \bar{\phi})\bar{F}'F''(F' - \lambda) - 3\bar{F}F''(F' - \lambda)\right. \\ &\left. - \frac{3}{2}F'(\bar{F}' - \bar{\lambda})^2 + \text{h.c.}\right] + \rho^0\left[|F'|^2 - 3F'\bar{W}_0 - 3\bar{F}'W_0\right] + \rho^{-1}\left[-(\phi + \bar{\phi})|F'|^2 + 3F'\bar{F} + 3\bar{F}'F\right]. \end{aligned} \tag{F.52}$$

For $\rho \to \infty$, the leading behaviour of V is given by

$$V = \frac{\rho^4}{3}|F''(F' - \lambda)|^2. \tag{F.53}$$

It is positive definite with a minimum at zero when $F' = \lambda$ or $F'' = 0$. In the following, we will analyze the case $F' = \lambda$; its curvature defines the (canonically normalized) scalar mass given by

$$m_\phi = \frac{\rho^3}{3}(F'')^2 \tag{F.54}$$

which goes to infinity at large ρ and ϕ decouples. At the minimum $F' = \lambda$, the potential at large ρ becomes constant, proportional to $|\lambda|^2 - 3\lambda\bar{W}_0 - 3\bar{\lambda}W_0$. This term vanishes precisely if equation (F.32), or equivalently (F.24), holds. We conclude that in the model (F.44) the cosmological constant can be tuned to zero (in the limit $\rho \to \infty$) by using the same condition as for the model (F.10). As shown in Section 2.2, this is the case for two possible values of λ:

$$\boxed{\lambda = 6W_0} \quad \text{or} \quad \boxed{\lambda = 0}. \tag{F.55}$$

Now let us investigate the minimum of the potential at finite but large ρ. We shall construct the solution as a power series in $1/\rho$ around the asymptotic field value of the minimum $\phi=\phi_0$ that solves $F'=\lambda$. A simple inspection of the potential (F.52) shows that it is sufficient to consider only even powers in $1/\rho$:

$$\begin{aligned} \phi &= \phi_0+\frac{\phi_1}{\rho^2} \\ F'(\phi) &= F'(\phi_0)+(\phi-\phi_0)F''(\phi_0)+\frac{1}{2}(\phi-\phi_0)^2F'''(\phi_0)+\ldots \end{aligned} \tag{F.56}$$

or equivalently,

$$F'(\phi)=\lambda+\frac{c}{\rho^2}+\frac{d}{\rho^4}+... \tag{F.57}$$

where

$$c=\phi_1F_0'' \quad , \quad d=\frac{1}{2}\phi_1^2F_0'''\,. \tag{F.58}$$

We then compute the derivative of $\tilde{V}$ with respect to ϕ and keep only the terms that do not vanish in the limit $\rho\to\infty$:

$$\begin{aligned} \tilde{V}_\phi=\frac{\partial\tilde{V}}{\partial\phi} &= \rho^4(\bar{F}''F'''|F'-\lambda|^2+|F''|^2F''(\bar{F}'-\bar{\lambda}))-\rho^3(\phi+\bar{\phi})|F''|^2F''(\bar{F}'-\bar{\lambda}) \\ &+\rho^2[F''^2(3\bar{W}_0-\bar{F}')-|F''|^2(\bar{F}'-\bar{\lambda})+F'''(F'-\lambda)(3\bar{W}_0-\bar{F}')] \\ &+\rho F''^2[\bar{F}'(\phi+\bar{\phi})-3\bar{F}]+\rho^0F''[\bar{F}'-3\bar{W}_0]. \end{aligned} \tag{F.59}$$

This expression vanishes if every coefficient at each order vanishes.

We now substitute the expansion (F.56), (F.57) into $\tilde{V}_\phi$ (ignoring orders that vanish as ρ^{-2} and higher) and impose each coefficient to be set to zero so as to have an extremum. Assuming for simplicity that W_0,λ,ϕ_0,c,d are real, we find the following constraints on the function F:

$$\begin{aligned} cF_0'' &= \lambda-3W_0 \\ F_0 &= 2W_0\phi_0 \\ c^2F_0''' &= \frac{2}{3}(\lambda-3W_0)\,, \end{aligned} \tag{F.60}$$

which yield

$$\begin{aligned} F(\phi) &= 2\phi_0W_0+\lambda(\phi-\phi_0)+\frac{\lambda-3W_0}{2c}(\phi-\phi_0)^2+\frac{1}{3!}\frac{2(\lambda-3W_0)}{3c^2}(\phi-\phi_0)^3+\ldots \\ &= 2\phi_0W_0+\lambda(\phi-\phi_0)\pm\frac{3W_0}{2c}(\phi-\phi_0)^2\pm\frac{1}{3!}\frac{2W_0}{c^2}(\phi-\phi_0)^3+\ldots\,, \end{aligned} \tag{F.61}$$

where in the second line above, we used the two possible values of λ (F.55), $\lambda=6W_0$ for the $+$ sign and $\lambda=0$ for the $-$ sign, for which the potential vanishes at the minimum.

At the minimum, the F-auxiliary term of S, $\mathcal{F}_\phi$, is given by:

$$\begin{aligned} \langle|\mathcal{F}^\phi|\rangle=\langle\left|e^{K/2}\sqrt{g^{\phi\bar{\phi}}}\bar{D}_{\bar{\phi}}\bar{W}\right|\rangle &\overset{\rho\to\infty}{\longrightarrow} \frac{\rho^2}{\sqrt{3}}\langle|F''(F'-\lambda)|\rangle+(\text{subleading terms}) \\ &= \frac{1}{\sqrt{3}}\langle|F_0''c|\rangle+\mathcal{O}(1/\rho^2)=\frac{1}{\sqrt{3}}|\lambda-3W_0| \\ &= \sqrt{3}|W_0|\neq 0\,, \end{aligned} \tag{F.62}$$

where in the third line we used $\lambda = 0$ or $\lambda = 6W_0$. We conclude that supersymmetry is spontaneously broken in this limit along the direction of ϕ, which can be identified with the scalar superpartner of the goldstino that becomes superheavy and decouples. The supersymmetry breaking scale remains finite and is given by $f = 3|W_0|$. Therefore, we identify the fermionic component of S with the goldstino and ϕ with its superpartner, the sgoldstino. According to (F.54), the latter decouples from the spectrum in the limit $\rho \to \infty$, which is equivalent to imposing the nilpotent constraint for the goldstino superfield $X^2 = 0$ on $\mathscr{L}$. Finally, the gravitino mass is given by

$$\boxed{m_{3/2} = \langle |W|e^{K/2}\rangle \to |W_0|} \text{ as } \rho \to \infty, \tag{F.63}$$

which completes the proof of equivalence between $\mathscr{L}$, $\mathscr{L}'$ and $\mathscr{L}''$.

We will now demonstrate why the Lagrangian

$$\bar{\mathscr{L}} = \left[\left(-\frac{1}{2}\frac{\mathscr{R}}{S_0} + W_0 + \frac{1}{2}\rho(\frac{\mathscr{R}}{S_0} - \lambda)^2\right) S_0^3\right]_F + \text{h.c.} \tag{F.64}$$

does not reproduce (F.10) with the constraint (F.11) in the limit $\rho \to \infty$. We first set

$$\begin{aligned} a &= W_0 + \frac{1}{2}\rho\lambda^2 \\ b &= 1 + 2\rho\lambda\,, \end{aligned} \tag{F.65}$$

assuming again reality of all parameters for simplicity. We then introduce a chiral superfield

$$S = A + \sqrt{2}\Theta\chi + (\Theta\Theta)F$$

(A and F are not the same as in the previous sections) such that

$$\begin{aligned} \bar{\mathscr{L}} &= \left[\left(a - \frac{1}{2}b\frac{\mathscr{R}}{S_0} + \frac{1}{2}\rho\frac{\mathscr{R}^2}{S_0^2}\right) S_0^3\right]_F + \text{h.c.} \\ &= \left[\left(a - \frac{1}{2}b\frac{\mathscr{R}}{S_0} + \frac{S}{S_0}\frac{\mathscr{R}}{S_0} - \frac{1}{2\rho}\frac{S^2}{S_0^2}\right) S_0^3\right]_F + \text{h.c.} \end{aligned} \tag{F.66}$$

It follows that $b > 0$ in order to have canonical gravity for a metric tensor with signature $(-+++)$. It is obvious from (F.66) that we have linearized our initial theory (F.64), which now describes the coupling of supergravity to a chiral superfield S that satisfies the equation of motion

$$S = \rho\mathscr{R}. \tag{F.67}$$

Next, using the identity (F.8) and fixing the gauge at $S_0 = 1$, we have

$$\bar{\mathscr{L}} = -[b - S - \bar{S}]_D + \left([a - \frac{1}{2\rho}S^2]_F + \text{h.c.}\right) \tag{F.68}$$

and the corresponding Kähler potential and superpotential are

$$K = -3\ln(b - A - \bar{A}) \quad , \quad W = a - \frac{1}{2\rho}A^2\,. \tag{F.69}$$

$\langle A_R \rangle$	$-b \leqslant A_R < \frac{b}{2}, b > 0$
$-\frac{b}{2}$	true always
$b+\sqrt{b^2-6a\rho}$, $b^2-6a\rho \geq 0$	never true
$b-\sqrt{b^2-6a\rho}$, $b^2-6a\rho \geq 0$	true if $b^2 > 8a\rho$ and $b^2 \geq -2a\rho$

Table F.1: Possible values of $\langle A_R \rangle$ for $\langle A_I \rangle = 0$.

The scalar potential V is given by

$$V = e^K \left[g^{A\bar{A}} (D_A W)(\bar{D}_{\bar{A}} \bar{W}) - 3\bar{W} W \right] . \tag{F.70}$$

Note that positivity of the kinetic terms implies that $b - 2A_R > 0$, where we have set $A = A_R + \mathrm{i} A_I$. We now compute

$$e^K = \frac{1}{(b-A-\bar{A})^3} \ , \ g_{A\bar{A}} = \frac{\partial}{\partial A}\frac{\partial}{\partial \bar{A}} K = \frac{3}{(b-A-\bar{A})^2} \ , \ g^{A\bar{A}} = \frac{(b-A-\bar{A})^2}{3} \ , \tag{F.71}$$

and

$$D_A W = \partial_A W + K_A W = -\frac{A}{\rho} + \frac{3}{b-A-\bar{A}} (a - \frac{1}{2\rho} A^2). \tag{F.72}$$

Putting everything together, we get that

$$\begin{aligned} V &= \frac{A\bar{A}}{3\rho^2(b-A-\bar{A})} - \frac{A}{\rho(b-A-\bar{A})^2}(a - \frac{1}{2\rho}\bar{A}^2) - \frac{\bar{A}}{\rho(b-A-\bar{A})^2}(a - \frac{1}{2\rho}A^2) \\ &= \frac{1}{\rho^2(b-2A_R)^2} \left\{ \frac{1}{3}(A_R^2 + A_I^2)(b + A_R) - 2a\rho A_R \right\} . \end{aligned} \tag{F.73}$$

The range of A_R is given by

$$-b \leqslant A_R < \frac{b}{2} \quad , \quad b > 0 \, , \tag{F.74}$$

so that the scalar potential is bounded from below.

To find the minimum of the potential, we demand that

$$\langle \frac{\partial V}{\partial A_R} \rangle = \langle \frac{\partial V}{\partial A_I} \rangle = 0. \tag{F.75}$$

The second of the equations above gives

$$\langle A_I (b + A_R) \rangle = 0. \tag{F.76}$$

If $\langle A_R \rangle = -b$, then

$$\langle \frac{\partial V}{\partial A_R} \rangle = 0 \Rightarrow \langle A_I^2 \rangle = -2a\rho - b^2 \overset{\rho\to\infty}{\longrightarrow} -\infty, \tag{F.77}$$

so this case is rejected. Consequently $\langle A_I \rangle = 0$. Then

$$\langle \frac{\partial V}{\partial A_R} \rangle = 0 \Rightarrow \langle (b + 2A_R)(A_R^2 - 2bA_R + 6a\rho) \rangle = 0, \tag{F.78}$$

$\langle A_R\rangle$	$-b \leqslant A_R < \frac{b}{2}, b>0$
0	true always
$-\frac{b}{2}+\frac{\sqrt{b^2+24a\rho}}{2}$, $b^2+24a\rho \geq 0$	true if $b^2 > 8a\rho$
$-\frac{b}{2}-\frac{\sqrt{b^2+24a\rho}}{2}$, $b^2+24a\rho \geq 0$	true if $a\rho \leq 0$

Table F.2: Possible values of $\langle A_R\rangle$ for $\langle V\rangle = 0$ and $\langle A_I\rangle = 0$.

$g^{A\bar{A}}\frac{\partial^2 V}{\partial\phi_i\partial\phi_j}$	A_R	A_I
A_R	$-\frac{1}{\rho^2}\frac{b}{9}$	0
A_I	0	$\frac{1}{\rho^2}\frac{b}{9}$

Table F.3: The (canonically normalized) scalar squared-mass matrix for $\langle A_R\rangle = -\frac{b}{2}$, $\langle A_I\rangle = 0$ and $\langle V\rangle = 0$.

which yields three solutions whose compatibility with the condition (F.74) is given in Table F.1. Only the solutions $\langle A_R\rangle = -\frac{b}{2}$ and $\langle A_R\rangle = b-\sqrt{b^2-6a\rho}$ are compatible with the range of A_R. Now we would like to check whether one of them is compatible with the condition

$$\langle V\rangle = 0. \tag{F.79}$$

Equation (F.79) has two solutions whose compatibility with the condition (F.74) is given in Table F.2.

It is straightforward to see that the solution $\langle A_R\rangle = b-\sqrt{b^2-6a\rho}$ is compatible with (F.79) only if $b^2 = 8a\rho$ (for $\langle A_R\rangle \neq 0$) or if $a=0$ (for $\langle A_R\rangle = 0$). The first case is rejected, since then $\langle A_R\rangle = b/2$ and the metric $g_{A\bar{A}}$ diverges. The second case is rejected, because then $\langle D_A W\rangle = 0$ and there is thus no spontaneous supersymmetry breaking. On the other hand, the solution $\langle A_R\rangle = -\frac{b}{2}$ is compatible with (F.79) for $b^2+24a\rho = 0$. It can also lead to spontaneous supersymmetry breaking, as

$$\langle e^{K/2}\sqrt{g^{A\bar{A}}}\bar{D}_{\bar{A}}\bar{W}\rangle \sim ab^{-3/2} \neq 0 \text{ for finite } \rho\,. \tag{F.80}$$

However, it is easy to see that the scalar squared-masses corresponding to A_R and A_I have opposite signs and thus the point $(\langle A_R\rangle = -\frac{b}{2}, \langle A_I\rangle = 0)$ is a saddle point of the potential and not a minimum, see Table F.3. Moreover, all the eigenvalues of the scalar mass matrix approach 0 as $|\rho| \to \infty$ and thus the extra scalar (sgoldstino) does not decouple. We conclude that neither of the two solutions for $\langle A_R\rangle$ can be used to tune the cosmological constant to zero for every value of ρ, consistently with the decoupling of the extra scalar.

Instead, we can investigate what happens if the condition (F.79) holds for the potential only in the limit $\rho \to \infty$. For both possible solutions

$$\begin{aligned} \langle A_I\rangle = 0 \quad , \quad \langle A_R\rangle = b-\sqrt{b^2-6a\rho} \approx \rho\lambda - 1 + 3\frac{W_0}{\lambda}\,, \lambda \neq 0 \\ \langle A_I\rangle = 0 \quad , \quad \langle A_R\rangle = -\frac{b}{2} = -\frac{1}{2} - \rho\lambda \end{aligned} \tag{F.81}$$

we find that $V \to 0$ for $\rho \to \infty$; however, none of the eigenvalues of the scalar mass matrix approaches ∞ at $\rho \to \infty$ (they approach 0 instead), which is again incompatible with the sgoldstino decoupling. We conclude that the parameter space of the model (F.64) does not allow for the realization of the non-linear supersymmetry coupled to gravity. Thus, (F.64) has to be modified suitably which is what we have proposed using $f(\mathcal{R})$ supergravity.

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Chrysoula MARKOU **2018**

Sujet : Supersymétrie $\mathcal{N} = 2$: réalisations non–linéaires, brisure spontanée et dimensions supplémentaires

Résumé : Le sujet de cette thèse est la brisure partielle de la supersymétrie N=2 en quatres et cinq dimensions de l'espace–temps. Dans le premiere cas, nous étudions hors de la couche de masse la réalisation non–linéaire de la supersymétrie N=1 brisée en utilisant de superchamps N=2 nilpotents qui contiennent les degrés de liberté de Goldstone du multiplet massif de N=1 de spin–3/2. Ces superchamps du Goldstino peuvent être de Maxwell ou de simple–tenseur. La brisure partielle est généré par une combinaison des termes électriques et magnétiques de Fayet–Iliopoulos, dont les coefficients peuvent être considérés comme de paramètres des déformations des transformations de l'algèbre de la supersymétrie brisée, ou, en plus, comme de déformations des superchamps eux–mêmes. D'interactions entre les multiplets du Goldstino deformés avec des multiplets pas deformés donne lieu à un mécanisme de super–Brout–Englert–Higgs mais sans la gravité, en raison duquel un multiplet N=1 vectoriel absorbe un multiplet N=1 linéaire et devient massif. Dans le deuxième cas, nous étudions sur la couche de masse la brisure partielle qui est généré par la valeur moyenne du dilaton qui est linéaire de la dimension supplémentaire et ce dernier est un modèle du dual holographique de little string theory. Un jaugement particulier de la supergravité N=2, D=5 peut incorporer cet modèle, et la supersymétrie N=1 reste intacte en quatres dimensions. Après compactification de la dimension supplémentaire, nous trouvons que l'introduction de branes est compatible avec la direction de la supersymétrie pas brisée.

Mots clés : Brisure partielle, supersymétrie non–linéaire, mécanisme de super–Brout–Englert–Higgs, supergravité effective, modèles du dilaton linéaire, little string theory

Subject : $\mathcal{N} = 2$ Supersymmetry: nonlinear realisations, spontaneous breaking and extra dimensions

Abstract: The subject matter of the present thesis is the partial breaking of N=2 supersymmetry in four and in five spacetime dimensions. In the first case, we study the off–shell nonlinear realization of the broken N=1 superymmetry with the use of nilpotent N=2 superfields that contain the Goldstone degrees of freedom of the massive spin–3/2 multiplet of N=1 supersymmetry. These Goldstino superfields can either be the Maxwell or the single–tensor multiplet. The partial breaking is induced by a combination of magnetic and electric Fayet–Iliopoulos terms, the coefficients of which can be seen as deformation parameters of the transformations of the broken supersymmetry algebra or, futhermore, as deformations of the superfields themselves. Interactions of deformed Goldstino multiplets with undeformed multiplets generate a super–Brout–Englert–Higgs effect but without gravity, in which an N=1 vector multiplet absorbs an N=1 linear multiplet and becomes massive. In the second case, we study on–shell the partial breaking that is induced by the background value of the dilaton that is linear in the extra dimension, with the latter being a toy model of the holographic dual of little string theory. A particular gauging of N=2, D=5 supergravity can accommodate this model, with N=1 supersymmetry remaining intact in four dimensions. Upon compactification of the extra dimension, we find that the introduction of branes is compatible with the direction of the unbroken supersymmetry.

Keywords : Partial breaking, nonlinear supersymmetry, super–Brout–Englert–Higgs mechanism, effective supergravity, linear dilaton, little string theory