THE PRESENT KNOWLEDGE OF WEAK QUARK MIXING ANGLES IN THE SIX-QUARK SCHEME

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ABSTRACT

The mixing of quarks through the weak interaction is described by a unitary matrix which can be parametrized in terms of three angles and one phase. Experimental results on weak decays of hyperons and B mesons, on neutrino production of charm quarks, and on the B meson lifetime are used to obtain, in a combined fit, values for the three mixing angles in the Kobayashi-Maskawa scheme: $\sin\theta_1 = 0.231 \pm 0.003$, $0.025 < \sin\theta_2 < 0.06$ and $\sin\theta_3 < 0.02$. In the Maiani parametrization, the angles obtained are: $\sin\theta = 0.231 \pm 0.003$, $|\sin\gamma| = 0.044 {}^{+0.007}_{-0.005}$ and $\sin\beta < 0.004$.

1. INTRODUCTION

If the mixing of quarks by the weak interaction is described phenomenologically, the six-quark mixing scheme proposed by Kobayashi and Maskawa [1] serves as a useful parametrization of the connection between generations of quarks. The elements U_{ik} of the quark mixing matrix (i = u,c,t; k = d,s,b) are parametrized in terms of three angles θ_1, θ_2 and θ_3, and one phase δ, possibly related to CP violation (Table 1a). If CP violation is due to quark mixing, then this phase δ is related to the parameter ε, describing the admixture of wrong CP parity in the long- and short-lived neutral K-meson states, measured to be $\varepsilon = (2.28 \pm 0.05) \times 10^{-3} \times \exp(i\pi/4)$ [2]. An approximate relation derived by Pakvasa and Sugawara [3] is $|\varepsilon| \approx |m_t - m_c)/m_c| \sin 2\theta_2 \tan\theta_3 \sin\delta/(2\sqrt{2}\cos\theta_1)$ where m_t and m_c are the top- and charm-quark masses. An alternative parametrization of the matrix U has been given by Maiani [4] in terms of angles θ, γ and β and a phase δ' (see Table 1b).

A more detailed calculation of the CP parameter in the K meson system gives [5-8]

$$\sqrt{2}|\varepsilon| = B\frac{\sin\beta\ \sin\gamma\ \sin\delta'}{\sin\theta}\{-1+\frac{\eta_3}{\eta_1}\ln\frac{m_t^2}{m_c^2}+\frac{\eta_2}{\eta_1}\frac{m_t^2}{m_c^2}[\sin^2\gamma-\frac{\sin\beta\ \sin\gamma}{\sin\theta}\cos\delta']\} \qquad (1)$$

where B is the K^o-$\bar{K}^o$ transition matrix element, normalized to its value for a specific model ("vacuum insertaion value"), m_t and m_c are the top and charm quark masses and η_1, η_2, η_3 represent QCD corrections. CP violating amplitudes, in this model, are proportional to the product of three, presumably small, angles.

Experimentally, information on the weak mixing angles comes from measurements of weak decays of light and heavy quarks and from neutrino production of charm quarks as observed in dimuon events, as summarized in previous papers [9-13]. New results on the B-meson lifetime [14,15] and on hyperon semileptonic decays [16] give new stringent constraints. In a recent paper [17], the impact of all these constraints on the weak mixing angles has been analyzed. I give here an updated version of this analysis using the most recently available data on the B lifetime [15,18] and on B meson semileptonic inclusive decays [19,20]. I first go through constraints on the coupling parameters U_{ik}, and then proceed to derive bounds on the mixing angles.

2. CONSTRAINTS ON MATRIX ELEMENTS

2.1 Light-quark couplings

2.1.1 Coupling U_{ud}

This coupling parameter has been determined from a comparison of measured rates of nuclear beta decays with that of muon decay. Two different evaluations of this quantity have been made and their results are $U_{ud} = 0.9730 \pm 0.0024$ [10] and $U_{ud} = 0.9739 \pm 0.0025$ [21]. Combining these two, one obtains

$$U_{ud} = 0.9733 \pm 0.0024. \tag{2}$$

2.1.2 Coupling U_{us}

In a series of experiments, the WA2 Collaboration has studied five different hyperon semileptonic decays, i.e. the leptonic weak decays $\Sigma^- \to n e^- \bar{\nu}$, $\Sigma^- \to \Lambda e^- \bar{\nu}$, $\Xi^- \to \Lambda e^- \bar{\nu}$, $\Xi^- \to \Sigma^0 e^- \bar{\nu}$, and $\Lambda \to p e^- \bar{\nu}$. Including radiative corrections and using in addition the neutron lifetime [22], this experiment gives [16]

$$U_{us} = 0.231 \pm 0.003. \tag{3}$$

This represents a substantial improvement over former analyses [10,21], although the value for U_{us} is exactly the same as the one obtained ten years ago [23]. It is still debated at which level corrections for SU(3) breaking effects have to be applied.

2.2 Charm-quark couplings

2.2.1 Coupling U_{cd}

This coupling can be determined from measurements of single charm production in neutrino and antineutrino reactions.

The differential cross-sections for neutrino charm production on isoscalar targets are:

$$\frac{d\sigma^{\nu}}{dxdy} = \frac{G^2 M E_{\nu} x}{\pi} \left[U_{cd}^2 (u(x) + d(x)) + |U_{cs}|^2 \, 2s(x)\right] \tag{4}$$

$$\frac{d\sigma^{\bar{\nu}}}{dxdy} = \frac{G^2 M E_{\bar{\nu}} x}{\pi} \left[U_{cd}^2 (\bar{u}(x) + \bar{d}(x)) + |U_{cs}|^2 \, 2\bar{s}(x)\right] \tag{5}$$

where u(x), d(x) and s(x) are the quark density distributions in the proton, G is the Fermi coupling constant, M the nucleon mass, E_{ν} the neutrino laboratory energy, and x and y the Bjorken scaling variables.

In order to obtain the coupling parameter U_{cd}, the contribution of charm production from the strange sea s and $\bar{s}$ quarks has to be eliminated. According to the cross-section given in (4) and (5), this can be done [24] by using the weighted difference of neutrino and antineutrino cross-sections:

$$BU_{cd}^2 = \frac{(\sigma^{\nu}_{\mu^-\mu^+}/\sigma^{\nu}_{\mu^-}) - (R\sigma^{\bar{\nu}}_{\mu^+\mu^-}/\sigma^{\bar{\nu}}_{\mu^+})}{1 - R} \; \frac{2}{3} \tag{6}$$

where R is the ratio of antineutrino to neutrino total cross-sections, $R = \sigma^{\bar{\nu}}_{\mu^+}/\sigma^{\nu}_{\mu^-} = 0.48 \pm 0.02$ [25], $R^{\nu} = \sigma^{\nu}_{\mu^-\mu^+}/\sigma^{\nu}_{\mu^-}$ is the dimuon to singlemuon cross-section ratio in neutrino induced reactions corrected for the threshold effects due to the charm mass (slow rescaling) [26], and B denotes the semileptonic branching ratio of that mixture of charmed particles which is produced in the neutrino reactions.

In an analysis of their large sample of neutrino- and antineutrino-induced dimuon events, the CDHS Collaboration [24] obtains $BU_{cd}^2 = (0.41 \pm 0.07)10^{-2}$. With a value of $B = (7.1 \pm 1.3)$ % based on e^+e^- results and on the composition of charmed particles in neutrino reactions from emulsion experiments, a value of

$$|U_{cd}| = 0.24 \pm 0.03 \tag{7}$$

is obtained.

2.2.2 Coupling U_{cs}

In charged current reactions this coupling appears always together with the strange-sea structure function $xs(x)$ or its integral $S = \int xs(x)dx$. The quantity measured is $|U_{cs}|^2 \cdot 2S$. In the absence of an independent determination of S, only the upper limit for 2S given by SU(3) symmetry, $2S \leq \bar{U} + \bar{D}$, and a corresponding lower limit on $|U_{cs}|$ can be obtained. The product $|U_{cs}|^2 \cdot 2S$ can be extracted in three ways from the neutrino and antineutrino dimuon production data [12]. We use here the results from the x distribution of neutrino dimuons [12,24]

$$\frac{|U_{cs}|^2}{U_{cd}^2} = (6.26 \pm 0.73)\,\frac{1 + \alpha^*}{\alpha} \tag{8}$$

and the one from the cross-sections of neutrino and antineutrino-induced dimuon production using the semileptonic branching ratio of D mesons:

$$|U_{cs}|^2 = (0.41 \pm 0.09)\,\frac{1 + \alpha^*}{\alpha}, \tag{9}$$

where $\alpha = 2S(\bar{U} + \bar{D})$ is the ratio of momentum fractions carried by strange and non-strange sea quarks in the nucleon, and $\alpha^* = 2S(U_{us}^2 + U_{cs}^2/r_s)/(\bar{U} + \bar{D})$ is the same ratio modified by the threshold suppression factor r_s for the charm-quark mass, which is $r_s = 1.5$ for the experiment considered [24,25].

2.3 Bottom-quark couplings

2.3.1 Ratio $|U_{ub}|/|U_{cb}|$

At electron-positron storage rings, the reaction $e^+e^- \to \Upsilon(4S) \to B\bar{B}$ can be used as a B meson source. The most sensitive search for decays of b quarks into u quarks can be done by measuring the inclusive lepton momentum spectrum of B meson decays. Data corresponding to an integrated luminosity of 50 pb^{-1} have been collected by the CLEO[20] and CUSB [19] collaborations at the Cornell Electron Storage Ring. The CLEO group measured the momentum spectra of 3750 electron events and 2115 muon events, while the CUSB group reports about the momentum spectrum of 900 electron events. In principle, the two possible semileptonic decays $B \rightarrow \ell\nu X_u$ and $B \rightarrow \ell\nu X_c$ can be distinguished by measuring the end point of the lepton momentum spectrum. In practice, the analysis is model-dependent because of the theoretical uncertainty about which X_u state with which mass is populated in the decay. Altarelli et al. [27] have calculated the expected lepton momentum spectrum using as an input the observed electron momentum spectrum in decays of charmed D mesons. Based on this model, the limits on the $b \rightarrow u$ decay width are

$\Gamma(b \rightarrow u)/\Gamma(b \rightarrow c) < 0.04$ at 90 % C.L. (CLEO [20])

$\Gamma(b \rightarrow u)/ (b \rightarrow c) < 0.045$ at 90 % C.L. (CUSB [19])

By taking into account the ratio of phase space available, one obtains

$$|U_{ub}|/|U_{cb}| < 0.12 \text{ at } 90\ \% \text{ C.L.} \qquad (10)$$

2.3.2 B lifetime

The lifetime τ_B of b flavoured hadrons, apart from phase-space factors, depends on the magnitude of the couplings U_{cb} and U_{ub}. In fact [28]

$$\tau_B = 10^{-14}\ s/(3.68|U_{cb}|^2 + 7.8\ |U_{ub}|^2).$$

The previous upper limit obtained by the JADE Collaboration [29], $\tau_B < 1.4\text{x}10^{-12}$s at 95 % C.L., is significantly improved by the recent measurements of the MARK II and MAC Collaborations [15,18]. In these experiments, semileptonic decays of b-flavoured hadrons are tagged by identifying an electron in an electromagnetic calorimeter or a muon from its penetration through a layer of iron and by requiring that this lepton has a high transverse momentum p_T relative to the thrust axis of a jet. The method is extensively discussed in the talk of J.Yelton at this meeting [18]. The results are:

$$\tau_B = (12.0 \begin{smallmatrix} +4.5 \\ -3.6 \end{smallmatrix} \pm 3.0) \times 10^{-13}\ s \qquad \text{Mark II Collab. [15]}$$
$$\tau_B = (16 \pm 4 \pm 3) \times 10^{-13}\ s \qquad \text{MAC Collab. [18]} \qquad (11)$$

2.4 Combined fit

Using the constraints of eqs. (2,3) and (7) to (11), we obtain for a minimum $\chi^2 = 1.1/3$ D.F. the values $\sin\theta_1 = 0.231 \pm 0.003$, $\alpha = 2S/(\bar{U} + \bar{D}) = 0.49 \pm 0.07$, and values of $\sin\theta_2$ and $\sin\theta_3$ with the error contours in the $(\sin\theta_2, \sin\theta_3)$plane given in fig.1. These contours vary slightly with the value of the phase δ. From the resulting error contours, we obtain a finite value of $0.025 < \sin\theta_2 < 0.06$, and an upper limit for $\sin\theta_3 < 0.02$.

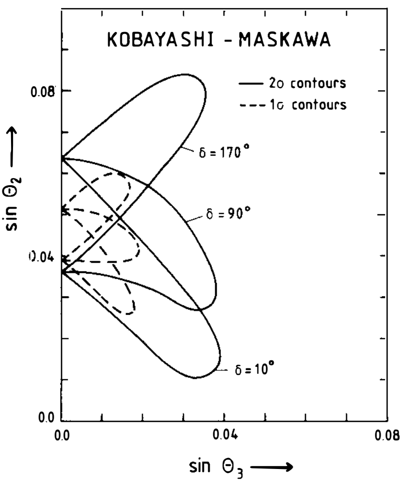

Fig.1:

Error contours in the ($\sin\theta_2$, $\sin\theta_3$) plane for three values of the phase δ (10^o, 90^o and 170^o). One standard deviation contours (dashed line) and two standard deviation contours (solid lines) are shown.

We conclude from this analysis that the second mixing angle θ_2 is smaller than the first one, θ_1, i.e. $\sin\theta_2/\sin\theta_1 < 0.26$. The third angle, θ_3, is still compatible with zero, with the upper limit $\sin\theta_3 < 0.02$ at the 67 % C.L. This pattern of decreasing mixing angles means that weak transitions between members of different quark families are suppressed more for heavy quarks than for light ones.

Analogously for the Maiani parametrization [4], the error contours in the plane of the parameters $\sin\gamma$ (corresponding approximately to $\sin\theta_2$) and $\sin\beta/\tan\theta$ (corresponding to $\sin\theta_3$) are given in fig.2. Here error contours at the one (1σ)

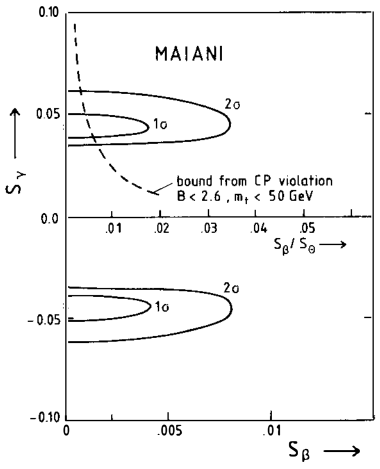

Fig.2:

Error contours in the plane of mixing parameters $\sin\gamma$ and $\sin\beta/\sin\theta$ in the Maiani parametrization. One standard deviation (1σ) and two standard deviation (2σ) contours are given. Also shown is a limit on the range of angles from the measured value of the CP parameter $|\varepsilon|$.

and two (2σ) standard deviation level are given. These contours are nearly independent of the phase angle δ' in the Maiani parametrization. The values for the angles are $\sin\theta = 0.231 \pm 0.003$, $|\sin\gamma| = 0.044 {}^{+0.007}_{-0.005}$ and $\sin\beta < 0.004$.

From the range of values of the mixing angles in either parametrization, the values of the Kobayashi-Maskawa matrix elements can be ontained. These are given in table 2. It is evident that the error on these matrix elements from the common fit is, for most of them, much smaller than the one obtained from individual experimental bounds on one matrix element.

It also appears from the values in table 2 that, apart from the diagonal elements, and the three off-diagonal elements observed directly up to now, (U_{us}, U_{cs} and U_{cb}), there is only one other non-diagonal element (U_{ts}) whose magnitude is such as to allow direct observation, possibly in the reaction $\nu + s \to \mu^- + t$ or in the decay $t \to s + X$. The other two elements, connecting the first and third families, are of a magnitude which makes their detection very difficult, and therefore represent a challenge to future experimentation.

This analysis is done in the framework of a six-quark model. If the number of quark flavours is larger than six, the unitarity condition on the elements of the 3 x 3 matrix U_{ik} is replaced by $\Sigma_{i=1}^{3} |U_{ik}|^2 \leq 1$ and $\Sigma_{k=1}^{3} |U_{ik}|^2 \leq 1$. In this generalized case, the ranges of values for U_{ik} are given in table 3.

The pattern of decreasing mixing angles can be parametrized in still another way as suggested by Wolfenstein [30]. Realizing that experimentally $|\sin\gamma| \sim (\sin\theta)^2$, the parametrization of table 4 is proposed. The experimental limits on these parameters are then: $\lambda = 0.231 \pm 0.003$, $A = 0.82 {}^{+0.13}_{-0.10}$ and $\rho^2 + \eta^2 < 0.2$.

If one goes beyond a purely experimental determination of the mixing angles, a description of CP violation in terms of quark mixing requires all three angles to be finite, as shown in eq.(1) and as discussed here by A.Buras [31]. This means that $\sin\theta_3$ (or $\sin\beta$), although experimentally compatible with zero, has a lower limit depending on the value of the K^o-$\bar{K}^o$ transition matrix element B. If an upper limit of $B < 2.6$ from theoretical arguments of Guberina et al. [32] is used, the allowed values for the angles $\sin\beta$ and $\sin\gamma$ in fig.2 have to lie above the hyperbolic line drawn from eq.(1). If the vacuum insertion value $B = 1$ [5] is assumed, the allowed region becomes smaller, and even more so if the PCAC-value $B = 0.33$ is taken [7].

The question whether CP violation is indeed due to a phaseshift δ in the quark-mixing matrix can be studied further by searching for the second kind of CP violation not due to the mass matrix but to CP violation in the weak transition from the eigenstate $K_2 = (K^o - K^o)/\sqrt{2}$ with negative CP eigenvalue to a 2π final state. While the first kind of CP violation is described by the parameter ε, the corresponding amplitude for this second kind is called ε'. From present experimental knowledge the amplitude ε' is still compatible with zero, and an upper limit on the amplitude ratio is $|\varepsilon'/\varepsilon| < 0.02$ at 90 % C.L. [2], consistent with superweak models of CP violation [33]. If, however, CP violation is due to the quark mixing matrix, a finite value of ε' is expected. From the range of mixing angles allowed a lower limit on the ratio of these amplitudes can be derived: $|\varepsilon'/\varepsilon| > 2 \times 10^{-3}$ [34]. Future experiments on the ratio of decay rates of $K_L \to \pi^0\pi^0$ and $K_L \to \pi^+\pi^-$ undertaken now at Fermilab, Brookhaven and CERN will show whether this model will emerge as the true picture of CP violation.

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Table 1

Parametrizations of the quark mixing matrix

a) Kobayashi-Maskawa parametrization [1]

$$U = \begin{pmatrix} c_1 & s_1c_3 & s_1s_3 \\ -s_1c_2 & c_1c_2c_3 - e^{i\delta}s_2s_3 & c_1c_2s_3 + e^{i\delta}s_2c_3 \\ s_1s_2 & -c_1s_2c_3 - e^{i\delta}c_2s_3 & -c_1s_2s_3 + e^{i\delta}c_2c_3 \end{pmatrix}$$

b) Maiani parametrization [4]

$$U \equiv \begin{pmatrix} c_\beta c_\theta & c_\beta s_\theta & s_\beta \\ -s_\gamma c_\theta s_\beta e^{i\delta'} - s_\theta c_\gamma & c_\gamma c_\theta - s_\gamma s_\beta s_\theta e^{i\delta'} & s_\gamma c_\beta e^{i\delta'} \\ -s_\beta c_\gamma c_\theta + s_\gamma s_\theta e^{-i\delta'} & -c_\gamma s_\beta s_\theta - s_\gamma c_\theta e^{-i\delta'} & c_\gamma c_\beta \end{pmatrix}$$

Table 2

Elements of quark mixing matrix $|U_{ik}|$ from fit of experimental constraints (1 standard deviation range)

	d	s	b
u	0.9723 - 0.9737	0.228 - 0.234	0.000 - 0.004
c	0.228 - 0.234	0.9711 - 0.9727	0.039 - 0.051
t	0.005 - 0.015	0.038 - 0.050	0.9987 - 0.9993

Table 3

Elements of quark mixing matrix $|U_{ik}|$ from experimental constraints if number of quark flavours is larger than 6

	d	s	b
u	0.9709 - 0.9737	0.228 - 0.234	0.000 - 0.006
c	0.21 - 0.27	0.78 - 0.98	0.039 - 0.051
t	0.00 - 0.12	0.00 - 0.58	0.000 - 0.999

Table 4

Parametrization of the quark mixing matrix according to Wolfenstein [30]

	d	s	b
u	$1 - \lambda^2/2$	λ	$\lambda^3 A(\rho - i\eta)$
c	$-\lambda$	$1 - \lambda^2/2$	$\lambda^2 A$
t	$\lambda^3 A(1 - \rho - i\eta)$	$-\lambda^2 A$	1

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