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Zeeman splitting in structure and composition of ultramagnetized spherical nuclei

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ABSTRACT

Properties and mass distribution of ultramagnetized atomic nuclei that arise in heavy-ion collisions, magnetar crusts, during Type II supernova explosions and neutron star mergers are analyzed. For magnetic field strength range 0.1–10 *teratesla* the Zeeman effect leads to linear nuclear magnetic response with susceptibility parameter exceeding significantly nuclear g-factor. Respectively, binding energies increase for open shell and decrease for closed shell nuclei. Noticeable enhancement in a yield of corresponding explosive nucleosynthesis products with antimagic numbers corroborate with observational results.

1. Introduction

Ultrastrong magnetization with a field strength exceeding *teratesla* (TT) arise in heavy ion collisions [1], magnetar crusts [2], at core-collapse supernovae (SNe) [3] and neutron star mergers [4]. Time duration for such magnetic amplification matches fairly wide range from typical nuclear time in a case of collisions to subseconds for SNe and neutron star mergers and to hundred years for magnetars. Nuclides produced in such processes contain an information on matter structure and explosion mechanisms.

Ultramagnetized astrophysical objects and/or magnetar concept were introduced to interpret activities of soft gamma repeaters (SGRs) and anomalous X-ray pulsars (AXPs). Large values, up to tenths TT, for strength of dipole-surface-field components are revealed from observed periods and period derivatives of these pulsars when assuming a magnetic-braking spin-down mechanism. Many properties of SGRs and AXPs indicate [2,5] significant multipole and toroidal magnetic fields being substantially stronger than corresponding dipole components. Such extremely strong magnetization of intensities up to tens TT can develop due to the violent convection bringing magneto-rotational instabilities (MRI) and/or dynamo-action process and contribute to shock wave formation in conjunction with numerical simulations of SN explosions, e.g., [6–8] and neutron star mergers [4]. Consequently, nuclides in ejected matter behind bifurcation point are plausibly formed at conditions of strong magnetic fields of several TT.

In this work we analyze effect of relatively *weak* magnetic field in nuclear structure and composition as well as discuss possibilities for using radionuclides to probe internal regions of respective sites. Interaction of nucleon magnetic moment m_{N} with a field H leads to energy level shift $\Delta = m_{\mathrm{N}} H$ associated with the Zeeman effect at relatively *small* values H. Dramatic change in nuclear structure corresponds to conditions of level crossing. The nuclear level spacing $\Delta\epsilon \sim 1$ MeV gives respective field strength scale $\Delta H_{\mathrm{cross}} \sim \Delta\epsilon/\mu_{\mathrm{N}} \sim 10$ TT. Here μ_{N} denotes the nuclear magneton. In a case of smaller magnetic inductions $H < 10$ TT one can use a linear approximation, cf., [9]. In next section we demonstrate that magnetic susceptibility at a field strength of *teratesla* exceeds significantly respective ground state g-factor corresponding to vanishing magnetic induction. Effect of magnetic field in nuclear composition is considered in sect. 3. Conclusions are in sect. 4.

2. Zeeman splitting in structure of atomic nuclei

Total Hamiltonian $\hat{\mathrm{H}}$ for nuclei within non-relativistic approximation and a linear limit in a weak magnetic field $\mathbf{H}$ reads

$$\hat{\mathrm{H}} = \hat{\mathrm{H}}^0 - \hat{\mathrm{H}}_{\mathrm{M}}, \;\; \hat{\mathrm{H}}_{\mathrm{M}} = \sum_{i=1}^{A}((\hat{\tau}_3^i + 1/2)\hat{\mathbf{l}}_i + g(\hat{\tau}_3^i)\hat{\mathbf{s}}_i)\boldsymbol{\omega}_{\mathrm{L}}\,, \quad (1)$$

where the sum runs over nucleons. $\hat{\mathrm{H}}^0$ represents the Hamiltonian for isolated nuclei, $\hat{\mathbf{l}}$ and $\hat{\mathbf{s}}$ denote operators of orbital momentum

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and spin. An interaction of nucleon dipole magnetic moments with magnetic field is represented by the Hamiltonian $\hat{H}_M$ with terms containing the vector $\boldsymbol{\omega}_L = \mu_N \mathbf{H}$, and $\hat{\tau}_3$ denotes the isospin projection operator. Respectively, $g(\tau_3)$ give well known g-factors for protons $g(1/2) \approx 5.586$ and neutrons $g(-1/2) \approx -3.826$.

The nuclear binding energy B is given as an energy difference between non-interacting *free* nucleons E_N and a nucleus E_A consisting from them: $E_N - E_A$. Under conditions of thermodynamic equilibrium at a temperature T corresponding energy is expressed through the free energy $F = -kTln(\Sigma)$ as

$$E = Mc^2 + E_m; \; E_m = F - T\frac{\partial F}{\partial T}. \quad (2)$$

Here, the partition function $\Sigma = \sum_i \exp\{-e_i/kT\}$ with energies e_i of nuclear particles in ith state, k denotes the Boltzmann constant, and Mc^2 is a rest mass energy. Using Eq. (2) for *free* nucleons the energy component due to an interaction with magnetic field is written as $E_m(\tau_3) = -\frac{g(\tau_3)}{2}\omega_L \, \mathrm{th}(g(\tau_3)\omega_L/2kT)$, where th$(x)$ is a hyperbolic tangent. For values of temperature considered here ($T \sim 10^{10}$ K) and magnetic inductions ($H \sim \;\; 1$ TT) we get $E_m \sim -10^{0.5}$ keV. Thus, the binding energy B decreases for magic nuclei with a closed shell, zero magnetic moment and, consequently, zero energy of interaction with a field. In cases of anti-magic nuclei with open shells significant (see below) magnetic moment leads to an additional increase of binding energy B in a field. The leading component of such a magnetic contribution is represented as a sum over single particle energy levels ϵ_i, $B_m = \sum_{i-occ} \epsilon_i$, see [3] and refs. therein.

Self-consistent mean field approach constitutes useful framework for reliable description and analysis of properties of atomic nuclei. By making use of angular momentum representation for spherical nuclei, single particle energies ϵ_{nljm_j} and wave functions $|nljm_j\rangle$ are characterized by following quantum numbers (see [10]): n is the radial quantum number, angular momentum l, total spin j and spin projection m_j on magnetic field direction. Then one can write the change of binding energy in magnetic field as

$$\Delta B_m = B_m(H) - B_m(0) = \sum_{\mathrm{occ}} \langle nljm_j|\hat{H}_M|nljm_j\rangle. \quad (3)$$

For further consideration we notice that spin-orbit coupling strength $\delta_{so} \approx 5/A^{1/3}$ MeV (see [10]). Consequently, total spins j and spin projections m_j remain well conserved quantum numbers up to magnetic induction $H \leq \delta_{so}/\mu_N$. For atomic nuclei of average mass numbers $A \sim 50$ at corresponding field strengths $H \leq 10^{1.5}$ TT (see also sect. 1) from Eq. (3) one obtains

$$\Delta B_m = \kappa \omega_L, \quad (4)$$

$$\kappa = \sum_{i-\mathrm{v}; m,s} |\langle l^i m, \frac{1}{2} s | j^i m^i_j \rangle|^2 ((\tau^i_3 + 1/2)m + g(\tau^i_3)s) =$$

$$= |g_{lj}(1/2)| Z_v (1 + \frac{1 - Z_v}{2j_p}) + |g_{lj}(-1/2)| N_v (1 + \frac{1 - N_v}{2j_n})$$

$$g_{lj}(\tau_3) = \begin{cases} ((\tau_3 + 1/2)l + g(\tau_3)/2), \\ \quad \text{for } j = l + 1/2, \\ ((\tau_3 + 1/2)(l+1) - g(\tau_3)/2)j/(j+1), \\ \quad \text{for } j = l - 1/2, \end{cases}$$

where $\langle lm, \frac{1}{2}s|jm_j\rangle$ represents the Clebsch–Gordan coefficient. Here the sum runs over valent ($i - \mathrm{v}$) protons $\tau_3 = 1/2$ and neutrons $\tau_3 = -1/2$ on open shells with respective numbers given by values $Z_v = Z - Z_{CSh}$ and $N_v = N - N_{CSh}$, where Z_{CSh} and N_{CSh} indicate numbers of protons and neutrons on closed shells. It is worthy to point out here that magnetic susceptibility parameters κ represent the combined effect of independent valent nucleons confined spatially to a finite volume due to a self-consistent mean field. Thus, values κ are significantly different from nuclear g-factors corresponding to an interaction of nuclear magnetic moment in a ground state with magnetic field. Within the shell model nuclear magnetic moment is determined by a valent unpaired nucleon [10]. The associated nuclear g-factor is given by values $g_{lj}(\tau_3)$ of Eq. (4) and, therefore, determined by the single particle state with maximum spin projection m_j of positive and negative signs for protons and neutrons, see Table 1.

Table 1
Susceptibility parameter κ for atomic nuclei of iron shell closure region. Contributions of $Z_v = Z - 20$ protons and $N_v = N - 20$ neutrons are indicated by $\kappa(1/2)$ and $\kappa(-1/2)$. Here m_j is the spin projection on field direction of a nucleon with the smallest separation energy.

m_j	Z_v	$\kappa(1/2)$	N_v	$\kappa(-1/2)$
7/2	1	5.79	8	0.0
5/2	2	9.93	7	1.91
3/2	3	12.40	6	3.28
1/2	4	13.23	5	4.10
−1/2	5	12.40	4	4.37
−3/2	6	9.93	3	4.10
−5/2	7	5.79	2	3.28
−7/2	8	0.0	1	1.91

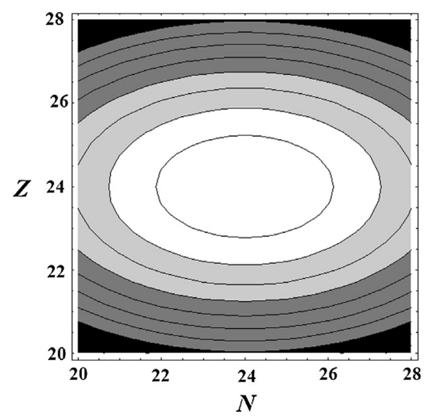

Fig. 1. Contour plot for nuclear magnetic susceptibility κ versus number of protons Z and neutrons N in a case of $1f_{7/2}$ shell, i.e., iron shell closure. Dark regions indicate small values κ (in corners $\kappa_{magic} = 0$), brighter pixels give larger values with maximum $\kappa_{max} \approx 17.51$ at $Z = N = 24$.

As is evident from Eq. (4) parameters $g_{lj}(\tau_3)$ for protons (i.e., $\tau_3 = 1/2$) increase at growing angular momentum l while for neutrons the value $g_{lj}(-1/2)$ is almost l independent Therefore, noticeable magnetic enhancement of nuclear binding energy can be expected in a case of proton shells with large values l. As a matter of fact such a case corresponds to pronounced nuclear magic numbers $Z = N = 28$, 50, 82 etc., with considerable nucleon numbers on closing shells [10]. In particular, the magic number $Z = N = 28$ brings the most tightly bound iron group nuclides at the Earth based environment (i.e., for sufficiently small field). In following we concentrate on an analysis of magnetic reactivity of atomic nuclei with open $1f_{7/2}$ shell. Such a choice of symmetric nuclei with large binding energy corresponding to iron, i.e., $1f_{7/2}$ shell closure gives a clear idea of an influence of magnetism on the structure and properties of atomic nuclei in superstrong magnetic fields.

As is evident from Table 1 and Fig. 1 magnetic susceptibility parameter κ vanishes for double magic nuclei. As one can see the largest effect of such additional magnetic coupling is displayed for nuclei with half-filled open shells. For protons effect is about 3 times larger as compared to neutrons. For titanium isotopes from Table 1 we obtain $\kappa_{Ti} \approx 14$. From Eqs. (2) and (4) it is easy to see that the magnetic component of titanium energy $E_{\rm Ti} \sim -10^{2.5}$ keV exceeds considerably analogous values for nucleon components. Thus, the magnetic field leads to an increase in binding energies of nuclei with open shells.

Consequently, composition of stable nuclei in magnetic field can be modified. For instance, for often considered isobars ^{44}Z, e.g., ^{44}Sc and ^{44}Ti, from Table 1 we have $\kappa_{\rm Ti} - \kappa_{\rm Sc} \approx 3.32$ and at a field strength $H \approx 2.5$ TT the nucleus ^{44}Ti is stronger bound than ^{44}Sc. It is worthy to recall here that ^{44}Ti becomes the most tightly bound even-even symmetric nucleus at $H \approx 20$ TT, see [3,9]. For a *weak* field limit we notice similar results obtained from consideration within the covariant density functional theory [11] and shell correction approach [9] which exclude an interaction of total classical magnetic moment component of a nucleus with magnetic field.

3. Nuclear composition in strong magnetic field

Magnetic modification of nuclear structure considered above can affect production of atomic nuclei at respective explosion. Nuclear statistical equilibrium (NSE) approach gives effective tool used very successfully for description and analysis of abundances of iron group and nearby chemical elements for over half a century. At such conditions nuclide yields Y is determined mainly by binding energies of corresponding atomic nuclei $Y \sim \exp\{B/kT\}$. Magnetic effects in NSE were considered [3] and refs. therein. Recall that at temperatures ($T \leq 10^{10}$ K) and field strengths ($H \geq$ 0.1 TT) the magnetic field dependence of relative output value $y = Y(H)/Y(0)$ is determined by a change in the binding energy of nuclei in a field H and can be written in the following form

$$y \approx \exp\{\Delta B/kT\} \approx \exp\{(E_{\rm N} + \kappa\ \omega_{\rm L})/kT\}\ . \qquad (5)$$

We used here Eqs. (2), (3) and (4).

In cases of magic numbers magnetic susceptibility vanishes, $\kappa_{\rm magic} = 0$, see Table 1, and magnetic field dependence in the synthesis of nuclei is due to a change in the interaction energy $E_{\rm N}$ of *free* nucleons with a field. The magnetization of nondegenerate nucleon gas and respective magnetic pressure component leads to an effective decrease in binding energy of magic nuclei and, as a result, to a suppression of a yield of corresponding chemical elements. We notice, however, that the suppression factor is less significant in a case of realistic geometry of magnetic field, see [9]. Significant values of magnetic moment and parameter κ contribute to an increase in coupling of nucleons for ultramagnetized antimagic nuclei in a field. An increase of nucleosynthesis products caused by such amplification is slightly sensitive to a structure of magnetic field [9].

We consider the normalized yield ratio of antimagic even-even symmetric nuclei $1f_{7/3}$ shell and double magic ^{56}Ni, i.e., $[i/{\rm Ni}] \equiv y_i/y_{\rm Ni}$. As is seen in Fig. 2 the production volume of ^{44}Ti and ^{48}Cr grows sharply with increasing magnetic induction while the yield of ^{52}Fe changes relatively slightly. We recall in this regards a puzzle of large abundance of titanium isotope ^{44}Ti obtained in direct observations of type II SN remnants [3,9,12]. Observational data suggest that the yield of ^{44}Ti nuclei for some SNe Type II exceeds significantly model predictions and data for SNe Type I. As is seen from Eqs. (2), (4), (5), Table 1 and Fig. 2 an excess on order of magnitude corresponds to field strength of 1.5 TT. As is evident from Eqs. (4) and Fig. 2 such conditions imply even stronger enrichment by ^{48}Cr isotope since maximum of magnetic susceptibility κ corresponds to half filled shells. In case of iron shell closure, i.e., $1f_{7/2}$ shell, such a condition is met at $Z = N = 24$ (see sect. 2). Then significant value of parameter $\kappa_{\rm Cr} = 17.51$ results in strong magnetic enhancement in production of ^{48}Cr nuclide. The radioactive decay chain $^{48}{\rm Cr} \rightarrow ^{48}{\rm V} \rightarrow\ ^{48}{\rm Ti}$ gives rise to an excess of major titanium isotope. At the same time enhancement in production ^{52}Fe is less pronounced.

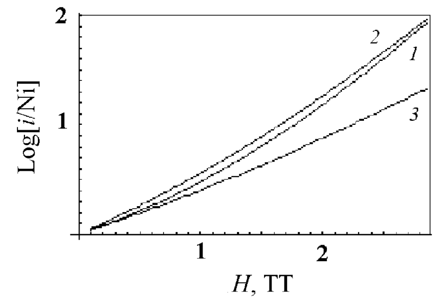

Fig. 2. Magnetic field dependence of yield ratio $[i/{\rm Ni}]$ (see text) of ^{56}Ni and ^{44}Ti − 1, ^{48}Cr − 2, and ^{52}Fe − 3 at $kT = 500$ keV.

4. Conclusion

We considered structure and composition of atomic nuclei in ultrastrong magnetic fields relevant for heavy-ion collisions, supernovae, neutron star mergers and magnetar crusts. The Zeeman effect is shown to dominate for field strengths 0.1–10 *teratesla*. Respective linear magnetic response is given as a combined reactivity of valent nucleons and enhances binding energy for open shell nuclei. For magic nuclei with closed shells the binding energy effectively decreases because of additional pressure induced by a field in free nucleon gas. As a result, composition of atomic nuclei created in ultramagnetized matter depends on a magnetic field strength. For $1f_{7/2}$ shell nuclei (iron group) the magnetic modification of nuclear structure enhances a yield of nucleosynthesis products of smaller mass numbers. In particular, growing of titanium portion at field strength of several TT is favorably compared to direct observational data of ^{44}Ti volume in SN remnants [3,9,12]. Such magnetization intensity is consistent with SN explosion energetics [3,9]. As is shown these conditions imply significant enhancement in portion of major titanium isotope ^{48}Ti in galactic chemical composition.

We notice finally that magnetic increase of binding energy considered above gives rise to a suppression of neutron capture reactions [13] important for synthesis of heavy chemical elements.

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