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Review

Advanced Suspension Techniques in Interferometric Gravitational Wave Detectors: An Overview

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Abstract: Interferometric gravitational wave (GW) detectors are sophisticated instruments that require suspended mirrors to be effectively isolated from all forms of vibrations and noise. This isolation is crucial for enabling the detectors to function efficiently at low frequencies, which directly impacts their capacity to detect distant events from the universe's past. To address this challenge, various suspension systems have been developed, utilizing passive, active, or hybrid control mechanisms. The effectiveness of these systems in suppressing noise determines the lowest detectable frequencies. Designing and managing mirror suspensions present significant challenges across all interferometric GW detectors. Detectors such as LIGO, VIRGO, TAMA300, KAGRA, and GEO600 implement unique suspension designs and techniques to enhance their performance. A comprehensive comparison of these systems would offer valuable insights. This paper provides an overview of the different suspension systems employed in major global interferometric GW detectors, alongside a brief examination of proposed future detectors. It discusses the rationale behind each design, the materials utilized, and other relevant details, serving as a useful resource for the gravitational wave detector community.

Keywords: gravitational wave detectors; mirror suspension systems; vibration isolation; interferometry; low-frequency noise suppression

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1. Introduction

Gravitational waves (GWs) are disturbances in spacetime produced by the acceleration of massive objects, propagating at the speed of light. These waves were a fundamental prediction of Einstein's General Relativity (GR) [1]. The strength of a gravitational wave is characterized by its dimensionless strain, denoted as h, which decreases with increasing distance from the source. In September 2015, the Advanced Laser Interferometer Gravitational-Wave Observatory (aLIGO) achieved the first direct detection of a gravitational wave signal, marking the first observation of black hole mergers via spacetime distortions and further confirming GR's predictions [2,3]. Additionally, gravitational waves from binary neutron star collisions have provided critical insights into astrophysical processes [4]. Beyond confirming GR, gravitational wave astronomy has emerged as a powerful tool for testing alternative theories of gravity. With increasing detector sensitivity, deviations from GR may be identified through gravitational wave polarization analysis, offering potential signatures of extended gravitational theories. This key aspect has been extensively discussed in the literature, including [5,6], which highlight how interferometric response functions can distinguish different gravitational theories.

This paper reviews the suspension systems employed in current and future interferometric GW detectors. Early GW detection efforts were pioneered by Joseph Weber and others [5,7–10]. Since initial detectors lacked the necessary sensitivity, low-temperature detectors were introduced [11–14], though these also struggled to detect weak GWs. A detailed historical overview can be found in [15–18]. Currently, several ground-based GW detectors are operational, including LIGO in the United States [19,20], VIRGO in Italy [21,22], GEO600 in Germany [23,24], and TAMA300 and KAGRA in Japan [25–27]. Space-based observatories, such as LISA and other proposed missions, aim to extend GW detection capabilities [28–35]. Interferometric GW detectors function by suspending mirrors, or test masses, so their motion approximates free fall in a local gravitational field [2]. Effective isolation from environmental noise is essential for optimal performance, achieved through seismic isolation platforms and pendulum suspensions [2]. The pendulum mechanism provides high-quality horizontal isolation above its resonance frequency while minimizing mechanical losses via dissipation dilution [36,37]. Thermal noise from fibers, joints, mirrors, and coatings—especially Brownian and thermo-elastic noise—dominates within the 10–100 Hz frequency range, impacting detector performance [38]. Since thermal noise significantly influences low-frequency sensitivity [39,40], advanced techniques such as monolithic suspensions and cryogenically cooled mirrors have been adopted. The following sections provide an in-depth discussion of interferometric GW detectors, their suspension systems, and the noise sources affecting their performance. The remainder of this paper is organized as follows: Section 2 provides an overview of detection of gravitational waves. In Section 3, the various noise sources that impact detector performance is discussed. Section 4 covers the theory behind mirror suspension systems, while Section 5 offers a brief overview of past, present, and future detectors. Finally, Section 6 concludes the paper.

2. Detection of Gravitational Waves

Gravitational waves (GWs) are distortions in spacetime predicted by Albert Einstein in his General Theory of Relativity. These waves become detectable when massive objects undergo acceleration in strong gravitational fields [41]. The frequency range of GWs spans from approximately 10^{-17} Hz to 10^3 Hz, depending on the specific astrophysical event responsible for their generation. Detailed discussions on various sources of GWs can be found in [42,43]. Among the current methods for GW detection, interferometric GW detectors are the most promising. These detectors operate on the principle that as GWs pass through Earth, they induce tidal effects that cause slight variations in the separation between suspended mirrors (acting as test masses). By measuring these variations, information about the passing GWs can be extracted.

The mathematical relationship for the strain h, which quantifies the GW-induced change in separation between the mirrors, is given by

$$h = \frac{2\Delta L}{L} \tag{1}$$

where L is the original distance between the test masses and ΔL is the change in this distance due to the GW. For astrophysical events occurring near Earth, the strain h can be on the order of 10^{-21} or smaller. Therefore, to detect such weak signals, the amplitude spectral density of the detector's noise must be less than 10^{-23} $\mathrm{Hz}^{-1/2}$ across the relevant frequency range of the GW signal.

Interferometric Gravitational Wave Detectors

Gravitational waves exhibit a quadrupole nature, making interferometric detectors—especially those based on Michelson interferometers—highly suitable for detecting them. These detectors function by suspending mirrors (test masses) over large distances, such that the mirrors' motion closely resembles free fall in the local gravitational environment [2].

The primary optical components of an interferometric GW detector include the following [44]:

1. Fabry–Perot arm cavity: This mechanism causes light to bounce multiple times between mirrors, amplifying the phase shift introduced by passing gravitational waves through each reflection.
2. Power recycling: Light that would otherwise be reflected back toward the laser is redirected coherently back into the interferometer, enhancing the overall laser power.
3. Mode cleaner: This system reduces laser beam fluctuations by ensuring that only the desired optical modes reach the interferometer.

Achieving the desired sensitivity requires isolating the test masses from external noise sources, such as seismic and thermal disturbances. Effective noise isolation is essential to ensure the accurate detection of the minuscule displacements produced by gravitational waves.

3. Noise in Gravitational Wave Detectors

In this section, some of the major sources of noise affecting the performance of GW detectors are presented.

3.1. Seismic Noise

Seismic noise originates from vibrations transmitted through the Earth's surface due to various factors, such as human activities, earthquakes, and oceanic movements. This type of noise impacts both the horizontal and vertical components of the detector, thereby influencing its overall performance. To mitigate seismic noise, it is crucial to provide effective isolation in both directions.

Horizontal isolation can be achieved using inverted pendulum systems, which are effective at reducing noise above their resonance frequency. For vertical isolation, suspending the test masses on springs is a common approach [45–51]. Additionally, incorporating low-frequency active or hybrid damping techniques with various design strategies is essential to achieve the desired level of isolation [52–54].

3.2. Newtonian Noise

Newtonian noise is the result of variations in mass density due to variations in the local gravitational field affecting the suspended mirrors. It is also known as gravity gradient noise. Research has shown that this noise is primarily caused by seismic surface waves, which induce gravitational fluctuations near the test masses of interferometric gravitational wave detectors [55–60].

The root mean square (rms) motion of the test masses caused by Newtonian noise can be expressed as

$$\tilde{x} = \frac{4\pi G\rho}{\omega^2}\beta(\omega)\tilde{W}(\omega) \tag{2}$$

where the following definitions are used:

- ρ is the Earth's density near the test mass;
- G is Newton's gravitational constant;
- ω is the angular frequency of the seismic spectrum;
- $\beta(\omega)$ is a dimensionless reduced transfer function;
- $\tilde{W}(\omega)$ represents the rms displacement averaged over three-dimensional directions.

Several strategies have been proposed to mitigate or eliminate Newtonian noise, including the use of space-based detectors, advanced subtraction techniques, and underground detector installations.

3.3. Thermal Noise

Thermal noise is more profound at low frequencies [61–65]. The spectral density of thermal motion for a suspended mass can be described by the following expression:

$$x^2(\omega) = \frac{4k_B T\omega_0^2\phi(\omega)}{\omega m\left[(\omega_0^2 - \omega^2)^2 + \omega_0^4\phi^2(\omega)\right]} \tag{3}$$

where the following definitions are used:

- k_B is Boltzmann's constant;
- T is the temperature;
- m is the mass of the test mass;
- ω_0 is the angular resonant frequency of the oscillator;
- $\phi(\omega)$ is the loss angle or loss factor of the oscillator.

The dielectric coatings applied on the mirrors to improve their reflectivity in turn contributes to thermal noise to a great extent. Researchers are actively investigating the mechanical losses associated with these coatings and developing new materials to reduce such losses [66].

3.4. Quantum Noise

Quantum noise affects gravitational wave detectors in two ways: photoelectron shot noise or radiation pressure noise. Shot noise, or photoelectron shot noise, limits the sensitivity of the optical readout. The detectable sensitivity, which is influenced by the laser power (P), wavelength (λ), and the arm length of the interferometer (L), can be expressed as

$$\text{Detectable Sensitivity} = \frac{1}{L}\left(\frac{\lambda hc}{2\pi^2 P}\right)^{1/2} \tag{4}$$

where the following definitions are used:

- h is Planck's constant;
- c is the speed of light;
- π is the mathematical constant pi.

Radiation pressure noise arises from the interaction between the laser and the test mass. This type of noise can be mitigated by using heavier mirrors or by employing lower-power lasers [41].

4. Mirror Suspension Systems in Interferometric Gravitational Wave Detectors

Mirror suspension systems play a critical role in the operation of interferometric gravitational wave detectors, such as LIGO, VIRGO, and the future Einstein Telescope. These detectors use mirrors suspended by complex systems to minimize external disturbances, ensuring that the extremely delicate measurements of gravitational waves can be made. The primary function of the suspension system is to isolate the mirrors from seismic vibrations, thermal noise, and other environmental interferences that could mask the faint signals of gravitational waves. The suspension system typically comprises multiple stages, where each stage is designed to isolate specific frequencies of noise. The mirrors, often made of fused silica, are suspended by thin fibers of the same material, which help to reduce thermal noise by lowering mechanical losses. These fibers are designed to be extremely low-loss to avoid adding excess noise to the interferometer's measurement.

One of the most important aspects of the suspension system is the ability to dampen vibrations and allow the mirrors to behave as free test masses over a range of frequencies. This is achieved through passive and active isolation techniques. Passive isolation uses mechanical components such as pendulums and blades, which naturally oscillate at certain frequencies and block others. Active isolation systems involve sensors and actuators that dynamically adjust the position and orientation of the mirrors, countering external disturbances in real-time. In advanced gravitational wave detectors, the suspension system must also account for control and alignment of the mirrors to maintain the high precision needed for interferometry. Small deviations in mirror position or angle can cause loss of signal or false readings, so feedback control systems are used to keep the mirrors in their desired positions.

4.1. Basic Principles of Pendulum Suspension

Pendulum suspension systems rely on the fundamental principles of pendulum motion to achieve stable oscillatory behavior and effective isolation from external forces. These systems involve a mass or body suspended from a fixed point that is free to swing under the influence of gravity. The natural dynamics of pendulum motion, governed by a restoring force that brings the mass back to its equilibrium position, offer key advantages in creating stability and minimizing vibrations. By leveraging these characteristics, pendulum suspensions are widely used in applications ranging from vibration isolation in sensitive equipment to seismic protection systems in large structures.

4.1.1. Pendulum Systems and Their Natural Isolation Properties

Pendulum systems are widely used for their natural ability to isolate vibrations and disturbances, particularly in low-frequency ranges. These systems take advantage of the oscillatory behavior of pendulums to filter out unwanted vibrations, making them highly effective in applications where isolation from external disturbances is critical.

The equation of motion for a simple pendulum with angular displacement θ can be derived from Newton's second law or the Lagrangian approach. For small angular displacements (i.e., when θ is small), the equation of motion is given by

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0, \tag{5}$$

where g is the acceleration due to gravity and L is the length of the pendulum. This results in a natural frequency of oscillation:

$$\omega_n = \sqrt{\frac{g}{L}}. \tag{6}$$

The natural period of oscillation T is given by

$$T = 2\pi\sqrt{\frac{L}{g}}. \tag{7}$$

The key property of the simple pendulum is its ability to act as a low-pass filter for external vibrations. Because the pendulum naturally oscillates at a frequency determined by its length and the force of gravity, any external disturbances with a frequency much higher than its natural frequency will be effectively filtered out. This makes pendulums highly effective for isolating systems from high-frequency vibrations. For example, in seismic isolation systems, pendulums are used to isolate sensitive equipment from ground vibrations caused by earthquakes. By designing pendulum systems with a natural frequency lower than that of seismic waves, the high-frequency ground motion can be filtered out, protecting the equipment [67,68].

A compound or physical pendulum is more complex than a simple pendulum, as it consists of a rigid body of mass M that swings about a fixed pivot point. The motion of the compound pendulum involves the distribution of mass along its length, and its equation of motion is

$$\frac{d^2\theta}{dt^2} + \frac{Mgd}{I}\theta = 0, \tag{8}$$

where d is the distance from the pivot to the center of mass, I is the moment of inertia about the pivot point, and M is the mass of the pendulum. The natural frequency of oscillation for the compound pendulum is

$$\omega_n = \sqrt{\frac{Mgd}{I}}. \tag{9}$$

Due to its distributed mass and moment of inertia, the compound pendulum offers a broader range of natural frequencies compared to the simple pendulum. This allows for more precise control and tuning of isolation characteristics. Compound pendulums are often used in tuned mass dampers for tall buildings, where they reduce the amplitude of oscillations caused by wind or seismic activity. These systems are tuned to the natural frequency of the structure, and the pendulum swings in opposition to external forces, effectively damping vibrations [69,70].

Pendulum systems are often equipped with damping mechanisms to enhance their isolation properties. Damping reduces the amplitude of oscillations over time and ensures that the pendulum returns to its equilibrium position more quickly after a disturbance. The equation of motion for a damped pendulum is

$$\frac{d^2\theta}{dt^2} + 2\zeta\omega_n\frac{d\theta}{dt} + \omega_n^2\theta = 0, \tag{10}$$

where ζ is the damping ratio, and ω_n is the natural frequency. Damped pendulums are particularly useful in situations where rapid stabilization is necessary after a displacement.

In advanced vibration isolation systems, such as those used in precision instruments (e.g., gravitational wave detectors), multiple pendulums are connected in series to create a multi-stage isolation system. Each stage filters out vibrations at progressively lower frequencies, resulting in highly effective isolation from environmental noise. The combined effect of multiple pendulums increases the overall isolation performance and bandwidth. Pendulum-based isolation systems play a critical role in detectors like the Laser Interferometer Gravitational-Wave Observatory (LIGO), where multiple pendulums isolate sensitive equipment from tiny vibrations that could interfere with the detection of gravitational waves [71].

In summary, pendulum systems, whether simple, compound, or damped, provide effective natural isolation properties. By leveraging the dynamics of pendulum motion, these systems filter out high-frequency disturbances and protect sensitive structures or instruments from external vibrations. The tunability and cost-effectiveness of pendulum-based isolation systems make them highly attractive in fields ranging from engineering and construction to scientific research.

4.1.2. Multiple Pendulum Stages to Enhance Isolation

In gravitational wave detection, isolation at very low frequencies (below 1 Hz) is essential to prevent seismic noise from masking the signal. To achieve this, a multi-stage pendulum system, consisting of several pendulums connected in series, is used. Each pendulum stage provides additional attenuation of vibrations, allowing for isolation across a broader frequency range. In such a system, the first stage filters out higher-frequency vibrations, while subsequent stages address progressively lower frequencies. The equation of motion for a multi-stage pendulum system can be derived by considering the dynamics of each stage. For an n-stage system, the effective transfer function $T(\omega)$ that describes the isolation performance is given by

$$T(\omega) = \prod_{i=1}^{n} \left(\frac{\omega_{n_i}^2}{\omega^2 + 2\zeta_i \omega_{n_i} \omega + \omega_{n_i}^2} \right), \tag{11}$$

where ω_{n_i} is the natural frequency of the i-th pendulum stage, ζ_i is the damping ratio for each stage, and ω is the frequency of the external disturbance. By cascading the stages, the system can achieve superior isolation performance, significantly reducing the transmission of low-frequency noise to the sensitive components of the detector [71].

In the case of LIGO, a quadruple pendulum suspension system is employed. Each pendulum in the series is carefully designed to have progressively lower natural frequencies, ensuring that seismic noise at frequencies as low as 0.1 Hz is effectively attenuated. The primary mirror in LIGO is suspended by this multi-stage pendulum system, ensuring that the mirror is isolated from vibrations that could otherwise interfere with the detection of gravitational waves [72]. The success of this approach is evident in the detection of gravitational waves in 2015, where the extreme sensitivity of LIGO's interferometers was made possible in part by these sophisticated isolation systems [3].

In addition to LIGO, the upcoming Einstein Telescope, a proposed third-generation gravitational wave detector, plans to further refine multi-stage pendulum systems. The Einstein Telescope will utilize an even more complex isolation system, potentially incorporating cryogenic cooling to minimize thermal noise and enhancing the performance of the pendulum stages at ultra-low frequencies [71]. The goal is to achieve vibration isolation across an even broader frequency range, enabling the detection of gravitational waves from sources at greater distances and lower frequencies than current detectors.

4.1.3. The Quadruple Pendulum System

In LIGO, each test mass (which serves as the mirror in the interferometer) is suspended by a quadruple-pendulum system, consisting of four pendulum stages connected in series (Figure 1). The purpose of this multi-stage system is to progressively attenuate vibrations at various frequencies. The pendulums act as a series of filters, with each successive stage providing isolation from vibrations over a broader range of frequencies, especially at the lower frequencies that are difficult to isolate.

- **First Stage: Upper Mass Suspension** The first stage of the quadruple-pendulum system in LIGO consists of a heavy upper mass suspended from the structure of the detector by steel wires or cables. This stage primarily addresses higher-frequency noise, including seismic vibrations and acoustic disturbances. The upper mass is made heavier to help lower its natural frequency, making it less responsive to high-frequency disturbances.
- **Second Stage: Intermediate Mass Suspension** The second stage is an intermediate mass, which is suspended from the first mass by another set of steel wires. This mass further reduces the transmission of vibrations from the first stage to the lower pendulum stages. The intermediate mass is also equipped with eddy current damping to reduce oscillations over time, ensuring that the pendulum system remains stable after disturbances.

- **Third Stage: Penultimate Mass** The third stage, known as the penultimate mass, is where significant isolation from low-frequency vibrations begins. This mass is smaller and lighter than the upper stages, allowing it to swing more freely, thereby filtering out lower-frequency vibrations. The penultimate mass is suspended using fused silica fibers, a material chosen for its low mechanical loss properties, which minimizes thermal noise at low frequencies. The use of silica fibers also reduces the coupling of vibrations from the upper stages to the final mass.
- **Fourth Stage: Test Mass (Mirror)** The final stage is the mirror in LIGO's interferometer. The test mass is suspended from the penultimate mass by fused silica fibers to provide isolation from thermal and seismic noise. The natural frequency of this suspension is very low, on the order of millihertz, which is crucial for isolating the mirror from vibrations at frequencies relevant to gravitational wave detection. The test mass is designed to be as still as possible, responding only to gravitational waves while filtering out environmental noise. The effectiveness of this multi-stage system lies in the fact that each pendulum stage provides a degree of isolation, with the isolation factor increasing exponentially for lower frequencies. The combination of four stages allows the system to provide effective isolation from vibrations at frequencies as low as 0.1 Hz, which is essential for the successful detection of gravitational waves [72].

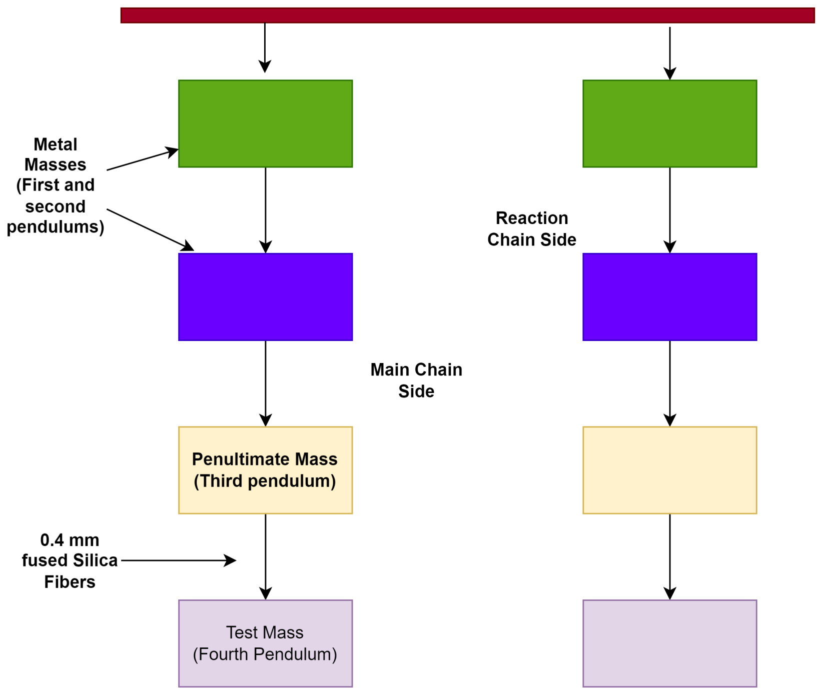

Figure 1. Schematic diagram of the quadruple-pendulum suspension system used in LIGO. The main chain side and reaction chain side are also illustrated.

The quadruple-pendulum system in LIGO provides isolation over a broad frequency range. For high-frequency vibrations (above 10 Hz), the system can achieve isolation factors greater than 10^9, meaning that the vibrations transmitted to the test mass are reduced by nine orders of magnitude compared to the input noise. At lower frequencies, especially in the critical range from 0.1 Hz to 1 Hz, the system still provides substantial isolation, which is crucial for reducing seismic noise. This level of isolation allows the interferometers to detect gravitational waves with frequencies between 10 Hz and several kHz, the range in which astrophysical sources such as binary black hole mergers emit detectable gravitational radiation [3].

The VIRGO detector, located in Italy, also employs a multi-stage pendulum system similar to LIGO's, although VIRGO uses a three-stage system. This system provides isolation to its mirrors by cascading pendulums that progressively filter out vibrations, with each stage lowering the natural frequency of the system. The mirrors in VIRGO are suspended by a combination of steel wires and glass fibers, providing thermal noise reduction and effective isolation from low-frequency seismic vibrations [73].

The proposed Einstein Telescope, a third-generation gravitational wave detector, plans to further refine the multi-stage pendulum system by introducing cryogenic cooling in addition to the mechanical isolation. Cryogenic cooling reduces the thermal noise that arises from the pendulum suspensions and the mirrors themselves. This technique, combined with multi-stage pendulum systems, is expected to achieve even greater sensitivity at lower frequencies, extending the detection range of gravitational waves from astrophysical sources to frequencies as low as 1 Hz [74].

4.1.4. Materials and Their Properties

Fused silica is selected for test mass suspensions in advanced detectors is due to its extremely low mechanical loss, which reduces the coupling of thermal noise into the system. The thermal noise power spectral density $S_{th}(\omega)$ of the suspension can be expressed as

$$S_{th}(\omega) = \frac{4k_B T\phi(\omega)}{m\omega^2}, \tag{12}$$

where k_B is the Boltzmann constant, T is the temperature, $\phi(\omega)$ is the frequency-dependent mechanical loss factor, m is the mass of the suspended test mass, and ω is the angular frequency of the system. Fused silica fibers, with their low ϕ, help minimize the thermal noise contribution at the low frequencies most relevant for gravitational wave detection.

In addition to its low mechanical loss, fused silica possesses excellent tensile strength, which is crucial for supporting the weight of the test masses while maintaining stability. The fibers are thin and long, chosen to maximize their flexibility and to lower the system's resonant frequencies. The long length of the fibers helps reduce the transmission of vibrations from the upper stages to the test mass, while their tensile strength ensures they can withstand the significant weight of the test masses, which in LIGO are 40 kg each [72].

The tensile stress σ on a fiber under the load of the test mass can be expressed as

$$\sigma = \frac{F}{A}, \tag{13}$$

where F is the force exerted by the test mass (equal to the gravitational force mg), and A is the cross-sectional area of the fiber. Fused silica fibers are engineered to withstand these stresses without deforming or breaking, maintaining the stability and alignment of the test mass.

One of the primary roles of the fibers is to minimize the coupling of seismic noise from the ground and upper pendulum stages to the test mass. The fibers themselves act as mechanical filters, attenuating the transfer of vibrations from one stage to the next. Since each stage is connected to the next by these fibers, the isolation performance of the system is critically dependent on the length, diameter, and material properties of the fibers. By carefully designing these parameters, the resonant frequencies of each stage are progressively lowered, improving the overall isolation. The total isolation provided by the multi-stage system, considering the properties of the fibers, can be described by the cumulative transfer function $T(\omega)$, which includes the natural frequencies and damping factors of each stage. The fibers contribute to ensuring that the system's natural frequencies are kept low, allowing isolation from seismic noise at frequencies as low as 0.1 Hz, which is essential for gravitational wave detectors like LIGO [71,72].

In next-generation detectors such as the Einstein Telescope, cryogenic suspensions are being explored to further reduce thermal noise. At cryogenic temperatures, materials like fused silica exhibit even lower mechanical loss, making them ideal for advanced suspension systems. Additionally, research is being conducted into other materials, such as silicon or sapphire, which also exhibit favorable properties at cryogenic temperatures [71].

Material selection in multi-stage pendulum systems is critical for achieving the desired isolation performance in gravitational wave detectors such as LIGO, VIRGO, and future observatories like the Einstein Telescope. The materials used in the suspension system must meet stringent requirements for low mechanical loss, high tensile strength, and minimal thermal noise. Different materials are used at various stages of the pendulum system based on their mechanical and thermal properties, with a key distinction between the materials used for the final stage, where thermal noise is most critical, and the higher stages, where strength and seismic noise suppression are prioritized. The use of fused silica fibers is particularly important in the final stage, where the isolation system is most sensitive. The fibers are long and thin to further reduce the suspension's resonant frequency, providing additional filtering of low-frequency seismic noise. The combination of low mechanical loss and the geometric design of the fibers ensures that thermal noise is kept as low as possible. Thermal noise in the suspension arises from both the internal friction in the fibers and from their interaction with the surrounding environment. Fused silica's low internal friction helps keep the overall thermal noise contribution from the suspension to a minimum, allowing the system to remain sensitive to gravitational waves. Furthermore, fused silica fibers also have a high quality factor Q, which means that the energy stored in oscillations dissipates slowly, further enhancing the isolation properties [72].

In addition to having low mechanical loss, materials in the suspension system must also possess high tensile strength to ensure that the test masses are stably supported. This is particularly important in the upper stages of the suspension, where heavier masses and greater forces are involved. The tensile strength of a material is its ability to withstand stress without breaking, and materials used in the suspension system must be able to endure the forces exerted by the weight of the test masses while maintaining stability over long periods of operation. Fused silica fibers in the final stage have adequate tensile strength to support the 40 kg test masses used in LIGO. However, because fused silica fibers are relatively brittle, other materials with higher tensile strength are used in the upper stages where mechanical strength is more critical.

In the upper stages of the suspension system, metal wires, typically made from materials such as maraging steel, are used due to their high tensile strength and durability. Maraging steel is known for its excellent mechanical properties, including high yield strength and resistance to deformation, which makes it suitable for supporting the heavier masses in the upper stages of the suspension system. The primary purpose of the upper stages is to attenuate high-frequency seismic noise, and the mechanical properties of maraging steel provide the necessary strength and durability to support the system without introducing significant mechanical loss. Since thermal noise is less of a concern in the higher stages, the focus is on using materials that can withstand the stresses of suspension without degrading the performance of the system over time. The use of metal wires in the upper stages and fused silica fibers in the lower stages creates a balance between strength and noise minimization. The upper stages isolate high-frequency seismic noise, while the lower stages, especially the fused silica fibers in the final stage, are designed to minimize thermal noise at low frequencies. Temperature also plays a significant role in material selection for the suspension system. Fused silica has a low coefficient of thermal expansion, making it highly stable in response to temperature fluctuations, which is crucial for maintaining the precise positioning of the mirrors in interferometric detectors.

4.2. Active and Passive Isolation

Gravitational wave detectors such as LIGO, VIRGO, and the planned Einstein Telescope achieve extraordinary sensitivity through a combination of passive isolation provided by multi-stage pendulum systems and active feedback control mechanisms. These complementary techniques are crucial for reducing the impact of noise sources such as seismic, thermal, and technical noise across a wide frequency spectrum. The integration of passive and active isolation allows the detectors to isolate their test masses (mirrors) from environmental disturbances, enabling the precise detection of gravitational waves in the frequency range of interest, which is typically between 10 Hz and 10 kHz.

Passive isolation in these detectors is primarily achieved through multi-stage pendulum suspension systems. The pendulum design attenuates vibrations at high frequencies by filtering out ground motion and environmental disturbances. Each pendulum stage progressively isolates the test mass, enhancing the overall attenuation. In LIGO, a quadruple-pendulum system is used, offering progressive isolation that reduces ground motion by up to eight orders of magnitude at frequencies above 10 Hz [72]. Despite the effectiveness of passive isolation at higher frequencies, it is insufficient to address low-frequency seismic noise, which dominates below 10 Hz. To mitigate low-frequency disturbances, active feedback control systems are employed alongside passive isolation. These systems use sensors to measure the position, velocity, and acceleration of the suspension and test mass, allowing actuators to apply corrective forces in real-time. Inertial sensors such as accelerometers and seismometers detect ground motion, and actuators (e.g., voice coils or electrostatic actuators) apply counteracting forces to the suspension system. This real-time feedback control significantly reduces the displacement caused by seismic noise, especially in the 0.1 Hz to 10 Hz frequency band [75].

Active feedback systems also play a crucial role in damping the resonances of the pendulum stages. The multi-stage suspension system has multiple resonant frequencies, which can amplify noise if left uncontrolled. Sensors such as optical levers and interferometric displacement sensors monitor the motion of each pendulum stage, while feedback actuators dampen the resonances, preventing the amplification of suspension noise [71]. This active damping ensures stable operation of the suspension system. By combining passive and active isolation techniques, gravitational wave detectors achieve superior noise reduction across a wide frequency range. The passive isolation provided by the pendulum system effectively attenuates high-frequency noise, while active feedback control dominates in the low-frequency regime. The total transfer function of the combined system can be described as the product of the passive isolation transfer function $T_{\text{passive}}(\omega)$ and the active feedback transfer function $T_{\text{active}}(\omega)$. This combined approach enables detectors like LIGO to isolate test masses from ground vibrations and detect extremely weak gravitational waves [75]. The passive isolation is effective at high frequencies, while active feedback systems significantly improve performance at low frequencies, particularly by reducing seismic noise and stabilizing the pendulum's resonant behavior.

In detectors such as LIGO, the quadruple-pendulum system offers significant passive isolation, while advanced active feedback control systems apply corrections to reduce low-frequency disturbances and enhance overall noise suppression. Future detectors, including the Einstein Telescope, aim to further enhance these isolation techniques through the use of cryogenic systems and more sophisticated control algorithms.

4.2.1. Passive Isolation via Pendulum

Passive isolation is a fundamental mechanism employed in gravitational wave detectors to mitigate the impact of seismic noise and other environmental disturbances on the sensitive test masses (mirrors). One of the most effective methods of passive isolation is the use of multi-stage pendulum suspension systems. These systems are designed to take advantage of the natural dynamics of pendulums, which inherently provide significant isolation at high frequencies.

In a multi-stage pendulum system, several pendulum stages are stacked vertically, each designed to filter out different frequency ranges of seismic noise. The first stage of the pendulum system experiences the highest amplitude of motion due to ground vibrations, while each subsequent stage sees a reduced amplitude of motion. This is due to the mass and length of each pendulum, which act to attenuate the vibrations transmitted through the system. As a result, the test mass, suspended at the lowest stage, remains relatively undisturbed by high-frequency ground motion.

The effectiveness of the multi-stage pendulum system can be quantitatively analyzed using its transfer function. The transfer function $T(\omega)$ describes how input disturbances (like seismic noise) are transmitted through the system. For a simple pendulum, the transfer function can be approximated as

$$T(\omega) = \frac{1}{1 + \left(\frac{\omega_0}{\omega}\right)^2}, \tag{14}$$

where ω_0 is the natural frequency of the pendulum and ω is the frequency of the external disturbance. This equation shows that at frequencies much higher than ω_0, the transfer function approaches zero, indicating that vibrations at those frequencies are effectively blocked. For gravitational wave detectors like LIGO, the multi-stage pendulum system typically consists of four or more stages, with each stage progressively reducing the amplitude of seismic vibrations. The overall isolation performance can be significant with attenuation levels exceeding 100 dB in the frequency range above 10 Hz [72].

4.2.2. Active Control and Damping

Active control systems are essential in gravitational wave detectors for further reducing residual noise that may not be sufficiently mitigated by passive isolation methods. While passive systems like pendulum suspensions excel at filtering out high-frequency vibrations, they may struggle to handle low-frequency noise and residual disturbances that can affect the sensitivity of the detector. Active control strategies utilize a combination of sensors, actuators, and feedback loops to monitor and counteract these unwanted vibrations in real-time. The heart of an active control system lies in its ability to detect disturbances and respond accordingly. Sensors such as laser interferometers, accelerometers, and seismometers are strategically placed to continuously monitor the position, velocity, and acceleration of the suspended test masses and pendulum stages. The data collected by these sensors are fed into a control system that processes the information to determine the necessary corrective actions.

Actuators play a critical role in implementing these corrective measures. Commonly employed actuators in gravitational wave detectors include electromagnetic devices, piezoelectric actuators, and electrostatic actuators. For example, electrostatic actuators can apply forces directly to the test mass, adjusting its position to counteract detected disturbances. The effectiveness of the actuator depends on its responsiveness and the speed at which it can act to mitigate vibrations.

This process can be mathematically described by the following equation:

Active damping techniques such as electrostatic and magnetic damping are particularly effective for counteracting residual vibrations. Electrostatic damping employs electric fields generated by electrodes placed near the test mass. By applying controlled voltages to these electrodes, forces can be exerted on the test mass, allowing for fine-tuned adjustments that counteract specific vibrational modes [75]. Similarly, magnetic damping utilizes magnetic fields generated by coils or permanent magnets to apply damping forces, offering another layer of noise suppression.

The combination of active control and damping techniques enhances the overall performance of gravitational wave detectors. By effectively addressing low-frequency noise and residual vibrations, these systems improve the sensitivity and stability of the detector, allowing for the precise measurement of gravitational waves. The interplay between passive isolation and active control creates a robust environment for the sensitive test masses, enabling scientists to detect cosmic events with unprecedented accuracy.

4.3. Seismic Isolation Platforms

Seismic isolation platforms are critical components in the design of gravitational wave detectors, such as LIGO and VIRGO, as they significantly reduce the transmission of ground motion to the sensitive test masses suspended above. Ground vibrations caused by seismic activity, traffic, and other environmental factors can introduce noise that interferes with the detector's ability to measure the minute perturbations caused by passing gravitational waves. At the core of the seismic isolation platform design are multi-layer systems that employ various materials and techniques to absorb and dissipate seismic energy. These platforms typically consist of a series of mechanical elements, such as springs and dampers, that are engineered to decouple the sensitive components from the ground vibrations. The basic principle behind seismic isolation is to create a mechanical barrier that reduces the coupling between ground motion and the suspended elements above.

One of the most common types of seismic isolation used in gravitational wave detectors is the use of passive isolation systems, such as rubber and spring systems. These systems are designed to have low stiffness in the horizontal direction, which allows them to deform and absorb energy from incoming seismic waves. The effectiveness of these systems can be quantified using the equation for the natural frequency of a mass–spring system:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}, \tag{15}$$

where f_0 is the natural frequency, k is the stiffness of the spring, and m is the mass of the system. By carefully selecting the values of k and m, engineers can tune the natural frequency of the isolation system to be below the frequencies of significant seismic activity, thereby minimizing the transfer of vibrations to the test masses. In addition to passive systems, active seismic isolation platforms are also employed in modern gravitational wave detectors. These platforms utilize sensors to detect ground motion and actuators to apply counteracting forces, effectively neutralizing incoming vibrations. The feedback control systems continuously adjust the position of the platform in real-time to maintain stability and minimize noise. The combination of these techniques allows for improved performance over a broader frequency range, addressing both high- and low-frequency seismic disturbances.

A notable example of a seismic isolation platform is the "superattenuator" used in LIGO. This system consists of multiple stages of passive and active isolation, incorporating a combination of springs, dampers, and mass blocks. The design effectively attenuates seismic motion through a hierarchical approach, with each stage providing a level of isolation that contributes to the overall performance. The superattenuator's multi-layer structure ensures that seismic waves are significantly damped before reaching the test masses, allowing them to remain undisturbed [76]. Moreover, the placement of seismic isolation platforms is crucial. These platforms are typically located in specially designed, isolated buildings or underground facilities, which further shield the sensitive instruments from environmental vibrations. The building's design often includes features such as thick walls, vibration-damping materials, and decoupled foundations to minimize ground motion interference.

4.3.1. Seismic Isolation Stages

The design of multi-stage seismic isolation platforms is a critical aspect of gravitational wave detectors, as it plays a vital role in reducing the transmission of ground motion to sensitive components such as the test masses. These platforms are engineered to operate in a hierarchical manner, employing a combination of passive and active isolation elements to achieve high levels of noise reduction across a broad frequency spectrum. Multi-stage seismic isolation systems typically consist of several interconnected layers, with each layer designed to mitigate vibrations at specific frequency ranges. The first stage often employs passive elements, such as rubber mounts and springs, which are specifically chosen for their low stiffness in the horizontal direction. This design allows the platform to deform under seismic forces, effectively absorbing energy and reducing vibrations that can propagate to the upper stages.

In addition to passive components, active isolation elements are integrated into the design to further enhance performance. These active systems rely on feedback control mechanisms, where sensors detect residual vibrations and actuators apply corrective forces to counteract them. The combination of passive and active elements allows for effective isolation across a wider frequency range, addressing both high- and low-frequency disturbances. One prominent example of a seismic isolation stage is the inverted pendulum stage (IPS). The IPS is designed to utilize the principles of pendulum motion for isolation. By suspending the mass from a flexible spring or suspension system, the pendulum can move freely, allowing it to respond to ground motion while maintaining a stable equilibrium position. This design effectively isolates the test mass from vibrations, particularly at low frequencies, due to the pendulum's inherent natural frequency characteristics.

Another crucial component of modern seismic isolation platforms is the geophone-based feedback system. Geophones are highly sensitive sensors that measure ground motion and vibrations. In a geophone-based feedback system, these sensors continuously monitor the vibrations at the isolation platform and provide real-time data to the control system. The control system processes these data and adjusts the actuators accordingly to minimize residual motion. The use of geophones enhances the overall performance of the isolation system, enabling rapid response to seismic events and improving the stability of the suspended components [76]. The integration of these advanced technologies into multi-stage seismic isolation platforms allows for gravitational wave detectors to achieve remarkable sensitivity and stability. By effectively combining passive and active elements, along with specialized systems like the inverted pendulum stage and geophone-based feedback systems, these platforms can significantly reduce the influence of ground motion, paving the way for precise measurements of gravitational waves.

4.3.2. Active Seismic Isolation

Active seismic isolation is a critical component in the design of gravitational wave detectors, enabling the precise measurement of gravitational waves by minimizing the impact of ground vibrations on the suspended test masses. These systems utilize adaptive control techniques that rely on real-time feedback from ground motion sensors, such as seismometers and geophones, to maintain the stability of the test masses against environmental disturbances. The core principle behind active seismic isolation is the use of sensors to continuously monitor ground motion and vibrations that could potentially affect the performance of the detector. Seismometers and geophones are highly sensitive devices that detect even the slightest ground movements. The data collected from these sensors are processed by a control system, which interprets the information to determine the nature and magnitude of the disturbances. This feedback loop allows for real-time adjustments to the isolation system, ensuring that the test masses remain stable and undisturbed.

The real-time adjustments made by the active control system are crucial for maintaining the stability of the test masses. When ground motion is detected, the control system quickly calculates the required response and activates the appropriate actuators, which can include electromagnetic or piezoelectric devices. These actuators apply forces to the suspended components, counteracting the effects of the ground vibrations and ensuring that the test mass remains in its intended position. One of the significant advantages of active seismic isolation is its ability to adapt to changing environmental conditions. As seismic activity varies, the control system can dynamically adjust its response, providing robust isolation across a wide range of frequencies. This adaptability is vital, especially in scenarios where low-frequency seismic noise is prevalent, which can significantly impact the sensitivity of gravitational wave detectors.

4.4. Damping Mechanisms

Damping mechanisms are essential components of suspension systems in gravitational wave detectors, such as LIGO and VIRGO, as they serve to mitigate residual vibrations that could compromise the sensitivity of these highly sophisticated instruments. These mechanisms work by converting vibrational energy into heat, thereby reducing the amplitude of oscillations and enhancing the overall stability of the test masses suspended within the detection system. One of the most commonly used damping mechanisms in suspension systems is viscous damping, which is characterized by the resistance encountered by an object moving through a fluid. In gravitational wave detectors, viscous damping is typically achieved using materials such as silicone oils or other viscous fluids, which are integrated into the suspension system. Another important damping mechanism employed in these systems is hysteretic or structural damping, which occurs due to internal friction within the materials of the suspension elements themselves. This type of damping is particularly beneficial at low frequencies, where many disturbances occur. The effectiveness of structural damping can be characterized by the loss factor, η, which is defined as follows:

$$\eta = \frac{W_d}{W_e}, \tag{16}$$

where W_d is the energy dissipated per cycle and W_e is the energy stored per cycle. Materials with higher loss factors are preferred for their ability to dissipate vibrational energy more effectively. In addition to these passive damping methods, active damping techniques are also integrated into suspension systems to enhance vibration suppression. These systems typically utilize sensors to detect residual vibrations and actuators to apply corrective forces in real-time, functioning similarly to the active seismic isolation systems. Combining both passive and active damping mechanisms within suspension systems allows for comprehensive vibration mitigation across a broad frequency range. This multi-faceted approach is crucial for the operation of gravitational wave detectors, which require unprecedented levels of sensitivity to detect minute disturbances caused by gravitational waves.

4.4.1. Mechanical Damping

Mechanical damping is accomplished through the integration of various mechanical components such as springs, dampers, and other elements designed to dissipate vibrational energy and minimize the transfer of disturbances between pendulum stages. Springs are fundamental components of the mechanical damping system, providing the necessary restoring force that allows the pendulum to return to its equilibrium position after being displaced by external forces. The choice of spring materials and their configurations can significantly influence the dynamic behavior of the suspension system. By carefully selecting the spring constants of the suspension elements, engineers can optimize the response of the pendulum stages to minimize vibrations. Dampers, on the other hand, are designed to dissipate energy and reduce oscillations caused by seismic or thermal disturbances. Mechanical dampers are often incorporated into the suspension system to provide viscous damping, which can effectively reduce the amplitude of oscillations.

By tuning the damping coefficients, it is possible to achieve a desirable balance between damping effectiveness and system responsiveness. In addition to springs and dampers, various mechanical components can be employed to minimize energy transfer between pendulum stages. One effective technique involves the use of mass dampers, which are additional masses strategically placed between pendulum stages to absorb and dissipate vibrational energy. These mass dampers can be designed to have specific frequencies that target the predominant vibrations in the system, thus improving the overall isolation effectiveness.

Another technique for reducing energy transfer is the implementation of mechanical filters or tuned mass dampers (TMDs). A TMD is a secondary mass–spring–damper system attached to the primary system (the pendulum) in such a way that it oscillates out of phase with the unwanted vibrations. The TMD's resonant frequency can be tuned to match the frequency of the dominant vibrations, thereby counteracting them and significantly reducing their impact on the test mass. The effectiveness of a TMD can be quantified by its dynamic magnification factor, defined as

$$\mathrm{DMF} = \frac{F_{\mathrm{input}}}{F_{\mathrm{output}}}, \tag{17}$$

where F_{input} is the input force (or disturbance) and F_{output} is the force experienced by the test mass. By minimizing the dynamic magnification factor, the energy transfer between pendulum stages can be significantly reduced.

4.4.2. Electromagnetic Damping

Electromagnetic damping is an advanced technique employed in the suspension systems of gravitational wave detectors, such as LIGO and VIRGO, to achieve precise control over the position of mirrors (test masses) while effectively minimizing residual vibrations. This method utilizes electromagnetic forces generated by coil–magnet pairs to provide a high degree of damping and stability, which is essential for the accurate detection of gravitational waves. The fundamental principle behind electromagnetic damping involves the interaction between an electric current flowing through a coil and a magnetic field generated by a permanent magnet or an electromagnet.

In the context of gravitational wave detectors, electromagnetic damping is implemented through coil–magnet systems attached to the mirrors or their suspension components. As the test mass experiences vibrations or displacements due to external forces, the coil's motion through the magnetic field induces an EMF that produces a damping force. This damping force can be expressed as

$$F_d = -k \cdot v, \tag{18}$$

where F_d is the damping force, k is the electromagnetic damping coefficient, and v is the velocity of the mirror. The damping force acts in the opposite direction of the motion, effectively reducing oscillations and stabilizing the mirror's position. One significant advantage of electromagnetic damping is its ability to provide a tunable and responsive damping force. By adjusting the current flowing through the coil, engineers can control the strength of the damping force, allowing for fine-tuned control over the mirror position. This tunability is particularly advantageous in gravitational wave detection, where precise alignment of the mirrors is crucial for maintaining sensitivity. Additionally, electromagnetic damping can effectively work in conjunction with passive and active damping methods within the suspension system. While passive methods like springs and mechanical dampers provide a baseline level of vibration reduction, electromagnetic damping can address high-frequency residual vibrations that may still affect the test mass. The combination of these techniques allows for a comprehensive approach to vibration control, enhancing the overall stability and performance of the gravitational wave detector.

4.4.3. Thermal Noise Damping

Thermal noise can significantly limit the sensitivity of detectors, making it essential to implement effective thermal noise damping techniques to minimize its impact. Various strategies can be employed to mitigate thermal noise, enhancing the overall performance of gravitational wave observatories. One common approach to reduce thermal noise involves the selection of materials with low mechanical loss for the mirrors and suspension components. Mechanical loss, often quantified by the loss tangent ($\tan\delta$), represents the ratio of energy dissipated to the energy stored in a material during oscillation. Materials with low mechanical loss are preferred as they generate less thermal noise. For instance, fused silica is widely used for mirror fabrication due to its favorable mechanical properties and low loss tangent, typically in the range of 10^{-6} to 10^{-5} at room temperature. The relationship between thermal noise power (S_x) and mechanical loss can be expressed as

$$S_x \propto \frac{k_B T}{Q}, \tag{19}$$

where k_B is the Boltzmann constant, T is the temperature, and Q is the quality factor of the mechanical system. By selecting materials with high Q values, the thermal noise contribution can be significantly reduced.

Another effective technique for thermal noise damping involves the implementation of cryogenic cooling. Cooling the suspension system and test masses to cryogenic temperatures substantially decreases thermal fluctuations, thereby reducing thermal noise. Detectors such as KAGRA employ cryogenic cooling techniques, achieving temperatures around 20 K, which leads to a significant reduction in thermal noise. At cryogenic temperatures, the thermal energy of the system is lowered, resulting in reduced motion of the mirror and suspension components, as indicated by the following relationship:

$$S_x(T) \propto T^2, \tag{20}$$

where $S_x(T)$ represents the thermal noise at temperature T. This relationship highlights that as the temperature decreases, the thermal noise generated by the system decreases quadratically, enhancing the overall sensitivity of the detector.

Additionally, the design of the suspension system itself can be optimized to minimize thermal noise. For instance, employing a multi-stage pendulum suspension can help isolate the test mass from seismic and thermal disturbances, allowing for a more stable measurement environment. This isolation, combined with the use of low-loss materials and cryogenic techniques, creates an effective damping strategy that minimizes thermal noise contributions.

5. Cryogenic Suspension Systems

Cryogenic suspension systems are a critical component in the design of gravitational wave detectors such as KAGRA, where reducing thermal noise is essential for achieving high sensitivity. At cryogenic temperatures, typically around 20 K, the thermal fluctuations that contribute to noise are significantly diminished, enabling the detection of faint gravitational wave signals. This section explores the unique design challenges and benefits associated with cryogenic suspension systems. By cooling the system, KAGRA achieves a significant decrease in thermal noise, improving the overall sensitivity of the detector. However, the implementation of cryogenic suspension systems presents several unique design challenges. One major challenge is the need for materials that maintain their mechanical integrity and low-loss characteristics at cryogenic temperatures. Traditional materials may exhibit changes in mechanical properties when cooled, potentially leading to increased noise or failure of the suspension components. To address this issue, KAGRA employs materials such as sapphire and low-loss metal alloys that retain their desirable properties even at cryogenic temperatures. Another challenge involves the thermal contraction of materials used in the suspension system. As temperatures drop, materials shrink, which can introduce stresses in the suspension structure. This requires careful design considerations to ensure that the components are adequately accommodated and do not compromise the alignment or stability of the mirrors. The use of flexible connections and adjustable components can help mitigate these issues and maintain the desired performance.

The integration of cryogenic systems with the overall detector infrastructure presents logistical challenges. Cooling systems must be designed to effectively manage heat transfer and maintain the necessary low temperatures without introducing additional vibrations or disturbances that could affect the detector's sensitivity. Advanced cryogenic systems utilizing liquid nitrogen or helium are typically employed, necessitating careful engineering to minimize thermal gradients and ensure uniform cooling across the suspension elements. In addition to reducing thermal noise, cryogenic suspension systems offer the benefit of increased stability and reduced susceptibility to environmental disturbances. By isolating the test masses at lower temperatures, KAGRA can achieve a more stable measurement environment, allowing for improved sensitivity in detecting gravitational waves. The multi-stage pendulum suspension design employed in KAGRA further enhances isolation by decoupling the test mass from seismic and thermal disturbances. Thermal noise is a fundamental limitation in the sensitivity of gravitational wave detectors, arising from the random thermal motion of atoms within the materials used in mirrors and suspension systems. As temperature increases, the thermal noise contribution increases, which can limit the ability of detectors to discern gravitational wave signals. The magnitude of thermal noise is primarily determined by the temperature of the system, the material properties, and the resonant frequency of the suspension components.

Cryogenic cooling has proven to be an effective method for reducing thermal noise in gravitational wave detectors. By lowering the operating temperature of the suspension system and test masses, the thermal motion of the constituent atoms is significantly reduced. As a result, the overall thermal noise contribution to the detector's sensitivity is diminished. The quadratic dependence indicates that even small reductions in temperature can lead to substantial decreases in thermal noise. For instance, cooling the system to cryogenic temperatures, typically in the range of 20 K to 40 K, can reduce thermal noise by several orders of magnitude compared to room temperature. In addition to reducing thermal noise, cryogenic cooling enhances the stability of the system. At lower temperatures, materials exhibit reduced thermal expansion and contraction, leading to more consistent and stable geometrical configurations. This stability is crucial for maintaining the precise alignment of optical components within gravitational wave detectors, which is essential for accurate measurements. Moreover, cryogenic systems can also take advantage of specific materials that exhibit lower mechanical loss characteristics at cryogenic temperatures. For example, fused silica, which is commonly used for mirrors, has a lower loss tangent at cryogenic temperatures, leading to further reductions in thermal noise. The design of cryogenic suspension systems in gravitational wave detectors is crucial for achieving optimal sensitivity and minimizing thermal noise. One of the key aspects of these systems is the selection of materials that perform well at low temperatures. At cryogenic temperatures, materials can exhibit different mechanical properties, which can significantly affect the performance of the suspension system. For instance, sapphire has emerged as a popular choice for mirrors due to its excellent mechanical properties, low thermal expansion coefficient, and high optical quality. Sapphire mirrors not only provide improved stability at low temperatures but also possess a lower mechanical loss compared to traditional materials like fused silica, contributing to a reduction in thermal noise. In addition to sapphire mirrors, the use of specialized fibers, such as fused silica fibers, is essential in the suspension system. Fused silica fibers exhibit low mechanical loss and high tensile strength, making them suitable for suspending mirrors while minimizing energy dissipation. These fibers can maintain their properties even at cryogenic temperatures, providing the necessary structural support without compromising the detector's sensitivity. The choice of materials is vital, as the thermal characteristics of the components directly influence the performance of the entire system.

However, designing cryogenic suspension systems presents several challenges, particularly in maintaining mirror alignment. The extreme temperatures can lead to differential thermal expansion and contraction among the components, which may result in misalignment of the mirrors. This misalignment can adversely affect the interferometric measurements that gravitational wave detectors rely on. To mitigate this issue, advanced engineering techniques, such as flexible suspension designs and active alignment systems, are employed. These techniques enable real-time adjustments to the mirror positions, ensuring optimal alignment despite temperature variations. Another significant challenge is minimizing heat transfer to the suspension system. The introduction of heat can increase thermal noise, which counteracts the benefits of cryogenic cooling. Effective thermal insulation techniques are essential in this context. Superinsulation, typically comprising multiple layers of reflective materials, is often utilized to reduce heat transfer from the environment to the cryogenic components. Additionally, heat exchangers and thermal filters can be integrated into the design to manage the thermal loads effectively. Furthermore, the integration of cryogenic systems with the detector infrastructure necessitates careful consideration of heat flow pathways. Strategies to enhance thermal isolation, such as employing intermediate stages that further decouple the test mass from thermal influences, play a crucial role in maintaining low temperatures at the mirror level. These design elements contribute to the overall effectiveness of the cryogenic suspension system in minimizing thermal noise and maximizing sensitivity.

6. Mirror Suspensions in Past, Present, and Future Detectors

As discussed in the preceding sections, ground-based gravitational wave (GW) detectors require isolation from all forms of noise to ensure accurate detection. The primary goal is to keep the test masses stationary, despite the presence of various vibrational sources. To achieve this, a range of active, passive, and hybrid isolation techniques have been developed and implemented in ground-based interferometric GW detectors. Additionally, new designs are continuously being proposed for future detectors to further improve performance. This section provides an overview of the various isolation systems used or proposed for interferometric GW detectors. It covers key aspects such as the design of these systems, the rationale behind choosing specific designs, the operational frequency ranges they cater to, and the materials employed in their construction.

6.1. TAMA300

TAMA300 employs a Fabry–Pérot Michelson interferometer (FPMI) with power recycling, situated at the Mitaka campus of the National Astronomical Observatory of Japan. The primary goal of TAMA300 is to achieve a target sensitivity of 5×10^{-20} m/$\sqrt{\text{Hz}}$ in horizontal displacement [77]. To meet this sensitivity requirement, it is necessary to isolate the mirrors by a factor of approximately 10^8 at 300 Hz. The optical arms of the interferometer extend to 300 m in length and achieve a sensitivity of 10^{-21} Hz$^{-1/2}$ at 1 kHz [2]. The laser used in TAMA300 is a 10 W injection-locked Nd:YAG laser with a wavelength of 1064 nm. The two main objectives of the TAMA300 project are to develop technologies for future generation detectors and to detect gravitational waves originating from nearby galaxies.

Construction of TAMA300 began in 1995, with data collection occurring between 1999 and 2004. The interferometer faces various noise challenges: seismic noise at lower frequencies, shot noise at higher frequencies, and thermal noise at intermediate frequencies [44]. To achieve the required isolation factor of 10^8 at 300 Hz, TAMA300 employs an advanced isolation system comprising a double pendulum and stacks. The double pendulum system provides an isolation factor of 10^5, while the remaining 10^3 is achieved using a stack composed of rubber and metal blocks. This stack system is designed to offer isolation across all degrees of freedom [77]. The mirrors are suspended using a double pendulum system mounted on an optical table supported by a three-layer stack [78] and an active isolation system with pneumatic actuators [79]. Metal wires are utilized to suspend the mirror, which is mounted on a vibration-isolated breadboard supported by three legs, each comprising a three-layer stack [80]. The mirror suspension employs a double pendulum with flexible eddy current damping [81]. The entire system within the vacuum chamber is controlled by pneumatic actuators attached to the legs of the base plate via double balanced bellows. The stack system incorporates fluorocarbon rubber encased in welded bellows to prevent contamination of the optics, ensuring good vacuum compatibility [77]. The bellows are essential in this system, as they cover the rubber to prevent outgassing from the rubber in the ultra-high vacuum environment. The stack system provides effective vibration isolation for frequencies above 10 Hz. For further details on the stack construction, refer to [77].

TAMA Seismic Attenuation System (SAS)

An improvement over the previous isolation system is the TAMA Seismic Attenuation System (SAS) (Figure 2). It has a three-layer vertical isolation system and a five-layer pendulum. June 2007 saw the completion of the SAS installation [82,83]. Targeting −180 dB at 10 Hz, the main objective of the TAMA SAS design is to produce strong low-frequency seismic attenuation. The advanced Mirror Suspension System (SUS) and active inertial damping stage of the SAS provide steady interferometer operation. In order to reduce internal resonances and preserve thermal noise performance, the SUS uses magnetic dampers. Using a combination of passive filters and active controls to reduce resonances at low frequencies, the SAS is engineered to offer isolation down to 100 mHz [82,84].Two different control schemes are used for isolation: IP Control, which uses three coil–magnet actuators [85] at the top of the SAS to control horizontal directions, three linear variable differential transformers [86] as position sensors, and three accelerometers [87]; and Platform and Test Mass Control, in which signals from the test mass rotation (pitch and yaw) are detected by a local optical lever and then fed back to the test mass actuators.

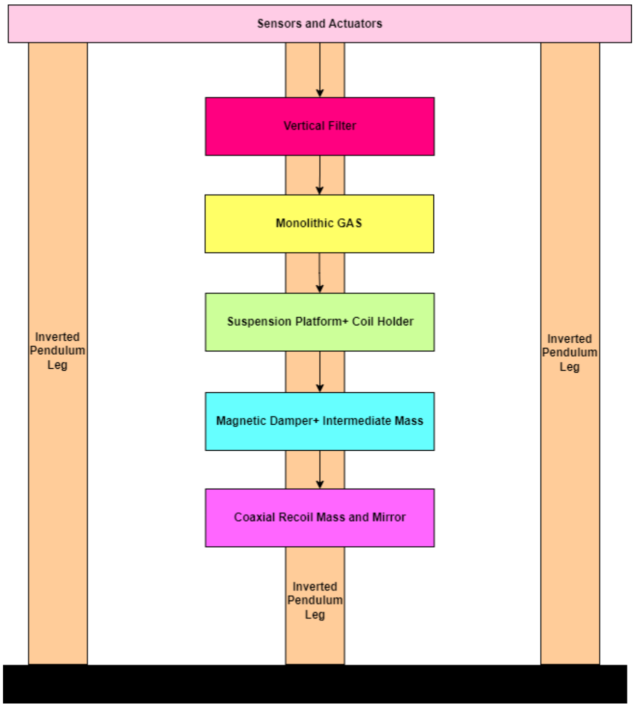

Figure 2. A schematic representation of the TAMA SAS/SUS system shows an inverted pendulum structure supporting two stages of vertical isolators along with the mirror suspension subsystem. The SUS component is further elaborated in [82].

6.2. GEO600

GEO600 is a gravitational wave detector located near Sarstedt, south of Hanover, Germany [88]. GEO600 has been designed to detect gravitational waves within a frequency range of 50 Hz to 1.5 kHz, which makes it highly sensitive to both astrophysical signals and local noise sources. The detector uses a Michelson interferometer configuration, with two 600-meter-long arms [89]. This layout enables precise measurements of spacetime distortions caused by gravitational waves.

A critical part of GEO600's sensitivity arises from its suspension system, which is responsible for isolating the test masses from seismic and environmental noise. The last stage of the suspension system is composed of fused silica fibers, a material chosen for its low mechanical loss and excellent thermal properties at room temperature. Fused silica helps to minimize thermal noise, which is one of the dominant noise sources in gravitational wave detection. The suspension system employs a triple-pendulum structure for horizontal isolation, where each stage progressively reduces the transmission of ground vibrations to the test mass (Figure 3). For vertical isolation, GEO600 uses cantilever springs, which act as mechanical filters to prevent vertical ground motion from reaching the sensitive mirrors. The combination of the triple pendulum for horizontal isolation and the cantilever springs for vertical isolation is highly effective in filtering out seismic noise in the frequency range of interest. In addition to passive isolation mechanisms, GEO600 incorporates active damping systems. The suspension system includes six coil–magnet actuators, which apply precise forces to counteract residual vibrations. These actuators are paired with sensors that measure the position of the suspension system and provide feedback to the control system. Piezoelectric elements are also employed to fine-tune the alignment of the mirrors and dampen high-frequency vibrations that cannot be addressed by the passive isolation stages alone. This combination of passive and active isolation allows GEO600 to achieve a peak sensitivity of $2 \times 10^{-22}\,\mathrm{Hz}^{-1/2}$ at 600 Hz.

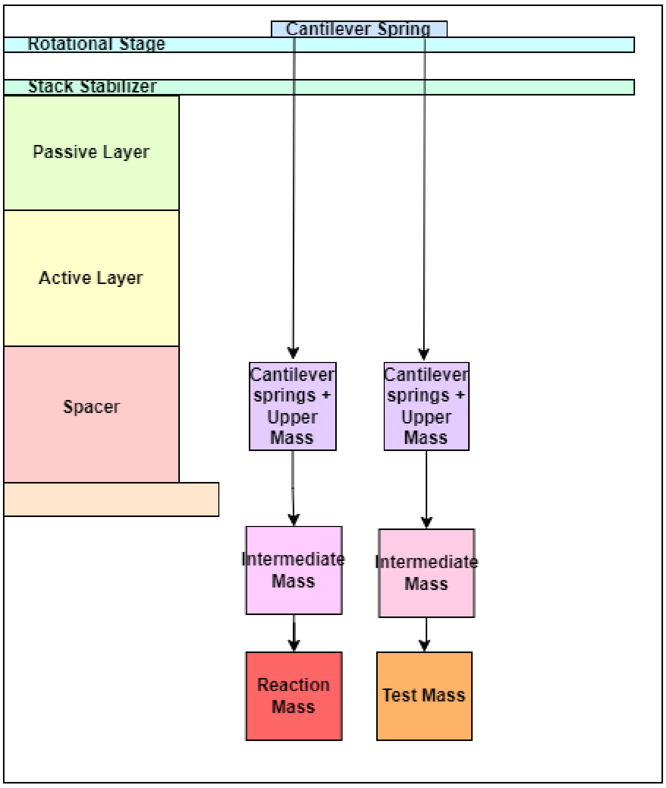

Figure 3. The schematic of GEO600 main suspension system. The lower stage utilizes a monolithic fused silica design to reduce thermal noise.

One of the advanced technologies used in GEO600 to improve its sensitivity is signal recycling. This technique allows the detector to enhance its sensitivity at specific frequency bands by reinjecting the gravitational wave signal back into the interferometer. Additionally, the use of low-noise electrostatic actuators at the test masses enables precise control of mirror positions with minimal added noise, further improving the detector's performance in its operating frequency range. The suspension system of GEO600 is highly complex, consisting of eleven different types of multiple pendulum suspensions. These suspensions are enclosed in ultra-high vacuum (UHV) chambers to eliminate noise from air molecules and other environmental contaminants. The multiple pendulum design provides several layers of isolation from seismic noise and other sources of environmental interference, ensuring that the test masses remain as still as possible despite ground motion. To further isolate the system from seismic noise, GEO600 includes a pre-isolation stage, which consists of stack isolators. These isolators form a hybrid system that combines passive isolation elements, such as mechanical springs and masses, with active components like feedback-controlled actuators. This hybrid isolation stage ensures that the bulk of seismic noise is filtered out before it reaches the sensitive components of the detector. GEO600 employs a monolithic last stage in its suspension system to reduce thermal noise. Monolithic designs, where the mirror and suspension system are made from a single piece of fused silica, help to minimize internal friction and energy dissipation, which are major contributors to thermal noise. The use of fused silica in the last stage is particularly effective in reducing mechanical loss at the suspension points, thereby lowering the overall thermal noise in the system.

The mode cleaner suspension is specifically designed to isolate the mode cleaner optics from external vibrations and noise. It consists of a top plate supported by three legs and a double-pendulum suspension. To prevent excitation of the pendulum modes, four local control loops with a bandwidth of 3 Hz are implemented. These control loops use shadow sensors and feedback coils to monitor and adjust the position of the suspension, thereby ensuring stable performance. The mode cleaner mirrors are hung using two wire slings that help control mirror motions at the intermediate mass level. The coil–magnets act as actuators within the local control loops to provide automatic alignment of the optics, contributing to precise mirror positioning. Additionally, fused silica prisms are employed to visually identify the break-off points of the suspension wires from the mirror's circumference, which aids in maintaining proper alignment. A reaction pendulum system provides rapid feedback to the mode cleaner's length-control system, ensuring optimal isolation and stability. A schematic diagram of this suspension system is shown in Figure 3. In order to achieve the high sensitivity required for detecting gravitational waves, the main suspension of the interferometer utilizes Quasi-Monolithic Triple Pendulum Suspensions (QMTPSs). This system comprises a triple-pendulum structure that includes two vertical cantilever-spring stages for additional seismic isolation. The term "quasi-monolithic" refers to the fact that the lower two stages are constructed from individual parts that are joined together using advanced techniques such as flame welding and hydroxy-catalysis bonding. These techniques significantly enhance the thermal noise properties of the suspension by minimizing internal friction.

For active damping, geophones and piezo actuators are included in the upper stage, where they help counteract residual motion. The fused silica mirror, which serves as the test mass, is suspended by four fused silica fibers from the intermediate mass. This monolithic stage forms the third and lowest stage of the triple-pendulum structure. Initially, test mirrors were suspended in steel-wire loops during the commissioning phase of the power-recycled Michelson interferometer; however, to reduce thermal noise, the final stage uses fused silica fibers. Both the upper mass and the intermediate mass are suspended by steel wires. To manage the various modes of vibration, magnet–coil actuators are employed at the upper mass, damping motion in all six degrees of freedom. This ensures that the suspension remains stable and isolates the mirrors from seismic disturbances. The main suspension also utilizes three legs to support the stack stabilizer and the rotational stage. Each leg is equipped with both passive and active seismic isolation to reduce ground motion transmission to the test mass. Rotational isolation is provided by a flex-pivot mounted on the top plate. Additionally, six collocated feedback systems are used to dampen all degrees of freedom of the upper mass, ensuring that the entire suspension system remains stable under varying environmental conditions. The fused silica stage, as the lowest pendulum stage, plays a critical role in isolating the system from thermal noise, which is essential for maintaining the detector's sensitivity. A reaction pendulum system, consisting of triple pendulums suspended just 3 mm behind the mirror, provides rapid feedback for the Michelson interferometer's length-control system.

6.3. Laser Interferometric Gravitational Wave Observatory (LIGO)

LIGO was initially funded by the National Science Foundation (NSF) and is operated by Caltech and MIT. The LIGO interferometers are capable of measuring distances on the order of 10^{-19} m, demonstrating an extraordinary sensitivity to minute distortions in spacetime caused by gravitational waves. To achieve such sensitivity, LIGO employs both active and passive isolation techniques to minimize vibrations and external noise [90]. The first layer of isolation is provided by an active stage, the Internal Seismic Isolation (ISI) system, which actively dampens ground vibrations. This system adjusts in real-time to compensate for seismic noise and disturbances. Beyond the active stage, passive isolation is achieved through a four-stage pendulum system, also known as a "quad" suspension, where the mirrors (test masses) are suspended from fused silica fibers. This quadruple-pendulum structure allows for successive stages of mechanical isolation, where each stage reduces the transmission of vibrations from the ground to the mirrors. There are two sides to the suspension mass: the main side and the reaction mass side (Figure 1). The reaction mass is used to isolate the main test mass (the mirror) from unnecessary vibrations by providing a reference point against which vibrations can be countered using active damping mechanisms.

LIGO's main suspension design incorporates several key features [91]. The fused silica mirrors, each weighing 10.7 kg, are suspended as single pendulums using a loop of steel piano wire, a design that minimizes mechanical noise while offering stable suspension. Damping of low-frequency pendulum modes is achieved directly at the mirrors, where magnets affixed to the back and sides via metal standoffs interact with coil–magnet pairs for precise position control. Additionally, global control, necessary to maintain the interferometer in its correct operating position, is also implemented through these magnets, ensuring that the mirrors remain aligned and properly positioned despite environmental disturbances. The combination of active isolation provided by the ISI system and passive isolation from the quadruple-pendulum design ensures that LIGO can effectively filter out a wide range of noise sources, allowing it to detect extremely faint gravitational wave signals with high precision [92].

6.4. Advanced LIGO

Advanced LIGO is an updated version of the original LIGO, which began its upgrade process in 2008 and became operational in 2015, leading to the first direct detection of gravitational waves in 2015 (GW150914, GW151226, etc.) The system consists of two interferometers, each with 4 km long arms, located at the Hanford and Livingston sites. Compared to LIGO, Advanced LIGO offers significantly better sensitivity—a factor of 10 improvement—which translates into a thousandfold increase in the observable volume of space and, consequently, an increased event detection rate. The improvements in Advanced LIGO include better seismic isolation and enhanced handling of thermal noise.

In terms of suspension systems, the original LIGO employed a four-layer passive isolation stack with coarse and fine actuators, where the mirrors were suspended from a single pendulum stage using steel wires. In contrast, Advanced LIGO uses a more advanced suspension system. The Hydraulic External Pre-Isolator (HEPI) provides the first stage of isolation, followed by a two-stage active isolation platform [93]. The suspension itself consists of a quadruple-pendulum design, offering four stages of isolation. The final stage employs monolithic fused silica suspensions to hang the mirrors. These advancements were partly inspired by the suspension design of GEO600, where the triple-pendulum system was modified to a quadruple system to meet the more stringent isolation requirements for low-frequency noise damping.

Key features of Advanced LIGO's suspension system include the following:

- Thermal and Seismic Noise Reduction: Advanced LIGO targets a seismic and thermal noise level of $10^{-10} \mathrm{m}/\sqrt{\mathrm{Hz}}$ at 10 Hz. To achieve this, sapphire mirrors weighing 40 kg are suspended in the final stage using monolithic fused silica fibers. The fibers are welded to fused silica 'ears' or prisms that are silicate-bonded to the flat sides of the penultimate mass and the mirror below.
- Seismic Isolation: The system utilizes a quadruple-pendulum suspension with maraging steel blades for seismic isolation. Horizontal isolation is provided by the pendulum's natural motion, while vertical isolation is achieved through soft blade springs.

- Damping and Control Noise Reduction: To minimize control noise, damping is applied at the top mass. In addition, a quiet reaction pendulum is employed for global control actuation.

Each Advanced LIGO detector features 11 vacuum chambers that house both the core optics, located in the Beam Splitter Chambers (BSCs), and the auxiliary optics, which are situated in the Horizontal Access Module (HAM) chambers. In the HAM chambers, the first stage of isolation is achieved through HEPI, which is a system that uses quiet hydraulic actuators, inductive position sensors, geophones, and ground seismometers to deliver active isolation at very low frequencies. At the final stage of the quadruple-pendulum suspension, the test masses are hung. The suspension system is divided into two chains: the main chain and the reaction chain. Each chain consists of four masses, which together weigh approximately 120 kg, with the lower two masses accounting for a total of 80 kg. The mass distribution in the main chain is around 20 kg, 20 kg, 40 kg, and 40 kg from top to bottom. Control of the interferometer is classified into local and global control. Local control mitigates the rigid-body modes of the suspension by using displacement sensors (BOSEM sensors) positioned around the top mass in both the main and reaction chains, applying control individually to each top mass with six BOSEMs. Global control, on the other hand, ensures proper alignment and positioning of the suspended components. It is applied between the main and reaction chains at the three lower stages, utilizing BOSEMs at the upper intermediate mass, AOSEMs at the penultimate mass, and an electrostatic drive at the test mass.

6.5. VIRGO

VIRGO is a Michelson interferometer with two arms, each spanning three kilometers, located in Santo Stefano a Macerata, near Pisa, Italy. It is a collaborative effort involving Italy, France, the Netherlands, Poland, Hungary, and Spain. The project was approved in 1993–94 and became operational in 2000. VIRGO's sensitivity ranges between 10 Hz and 10,000 Hz. The VIRGO suspension system, known as the "superattenuator", suspends the mirrors using four thin silica fibers. This structure is approximately 10 m tall and is housed in a vacuum. The superattenuator's principle is based on the fact that a device suspending a mirror through a simple pendulum provides effective isolation from horizontal seismic noise at a frequency of f/f_o, where f is the frequency of motion of the pendulum's suspension point, and f_o is the pendulum's natural frequency [94]. Adding more stages to the suspension chain increases the attenuation. Since vertical ground motion can also affect the laser beam path length, reducing vertical seismic noise to a level comparable with horizontal noise is essential. In the superattenuator, this is achieved by connecting the suspension wires to cantilever springs mounted beneath each pendulum mass, creating a cascade of vertical oscillators. The top stage of the superattenuator consists of a three-legged inverted pendulum with a top platform. The upper mass, called the marionette, supports the optical components, including the mirrors. Like LIGO, VIRGO uses a reference mass along with the test mass (mirror), which serves as a reaction mass for controlling the mirror's position. The first filter of the suspension chain is connected to a ring above it, which is suspended by the three legs of the inverted pendulum (IP). This assembly forms the top stage, from which the entire pendulum system hangs. The mirror stage includes the marionette, mirror, and reference mass. The marionette is suspended from its center of mass using maraging steel wire to minimize the torque required for controlling the angular position [94]. Thermal noise is a critical factor in suspension design, as explored in detail in [95]. Although a full discussion of the control system is beyond the scope of this paper, the system is generally responsible for aligning the suspension, locally controlling and damping normal modes of the payload, and assisting in lock acquisition. VIRGO was decommissioned in 2011, and the upgraded Advanced VIRGO was subsequently installed.

6.6. Advanced VIRGO

The 2011 decommissioning of the original VIRGO detector led to the installation of an upgraded model called Advanced VIRGO, which boasts a tenfold improvement in sensitivity and 1000 times greater visibility of the universe. Advanced VIRGO operates in the frequency range of 10 Hz to 104 Hz. It is also an interferometer with suspended mirrors and 3 km of perpendicular arms. Together, advanced VIRGO and advanced LIGO began accepting readings in August 2017 (O2Phase). Currently, Advanced VIRGO is a member of a network of GW detectors that also includes GEO HF in Germany and the two Advanced LIGO detectors in the US. When comparing Advanced VIRGO to the original version, there have been a number of significant improvements. Dual recycling is now available in the interferometer, including a signal recycling (SR) cavity in addition to power recycling [73]. The detector can be optimized for different astrophysical sources by adjusting the sensitivity curve through the tuning of the signal recycling parameter. The beam spot size on the test masses has been enlarged, placing the beam waist close to the center of the 3 km Fabry–Perot (FP) cavities in order to reduce the thermal noise in the mirror coatings at mid-frequencies. Larger vacuum links must be installed in the center of the beam, and new mode-matching telescopes must be installed at the interferometer's input and output. There are difficulties in locking all the cavities at once, so a system of backup green lasers is being designed to help lock the entire interferometer. A tenfold increase in optical power must be handled by the input optics, necessitating the creation of high-power-tolerant Faraday isolators and specifically engineered electro-optic modulators. The output mode cleaner is being redesigned in response to the introduction of a DC readout system.

The AdV test masses will be 42 kg heavier than the VIRGO test masses in order to compensate for the greater radiation pressure changes. For the most crucial optics, fused silica with ultra-low absorbance and great homogeneity has been chosen, and sophisticated polishing methods guarantee a flatness of better than 0.5 nm rms in the central regions of the test masses. To reduce thermal noise and optical losses in the cavities—which are essential for attaining high-frequency sensitivity—low-loss, low-absorption coatings are used. New diaphragm bases, either hanging around the mirrors or ground-connected inside the vacuum connections, will be placed to decrease phase noise from back-scattered light, as this can also affect detector sensitivity. Every photodiode utilized in the science mode will be kept in a vacuum and isolated from earthquakes. In order to facilitate this, five of these minitowers—a miniature vibration isolation system—have been erected inside vacuum chambers. The requirement to suspend larger mirrors, baffles, and compensation plates while enhancing controllability and lowering mechanical losses led to an improvement in the payload design as well. Last but not least, a factor of almost 100 has been removed from residual pressure thanks to the improvement of the vacuum system, which includes enormous cryotraps at the ends of the 3 km pipes.

Four cables, driven by a series of coil–magnet actuators, are used to suspend the mirrors from the marionette [73]. This configuration is suspended from Filter 7, the final filter in the superattenuator chain (Figure 4). This uses the previously discussed monolithic suspension method. The ends of each fiber are welded onto two T-shaped fused silica blocks, called anchors, and attached to the mirror and the marionette on opposite sides. The anchor is fastened to the mirror's little fused silica extension, known as the ear, on the mirror side using a silica bonding process. The ear is silica-bonded to the mirror before the fiber assembly since it is difficult to machine the ear straight from the mirror. The fiber, anchor, ear, and mirror form a continuous structure and create a monolithic suspension when the silica bonding is correctly performed. The marionette is connected to the other end of the fiber, which has been welded to a different anchor, by an interface made of stainless steel called the upper clamp assembly. Minimal modifications are made to the superattenuator (SA) mechanism utilized in VIRGO to create Advanced VIRGO. With frequencies exceeding a few Hz, this hybrid attenuation system, which combines passive and active components, may reduce seismic noise by more than 10 orders of magnitude in each of the six degrees of freedom (DoFs). The inverted pendulum (IP), the seismic filter chain, and the mirror suspension are the three primary parts of the SA. Acernese et al. [96] provides a thorough explanation of the SA and its functionality.

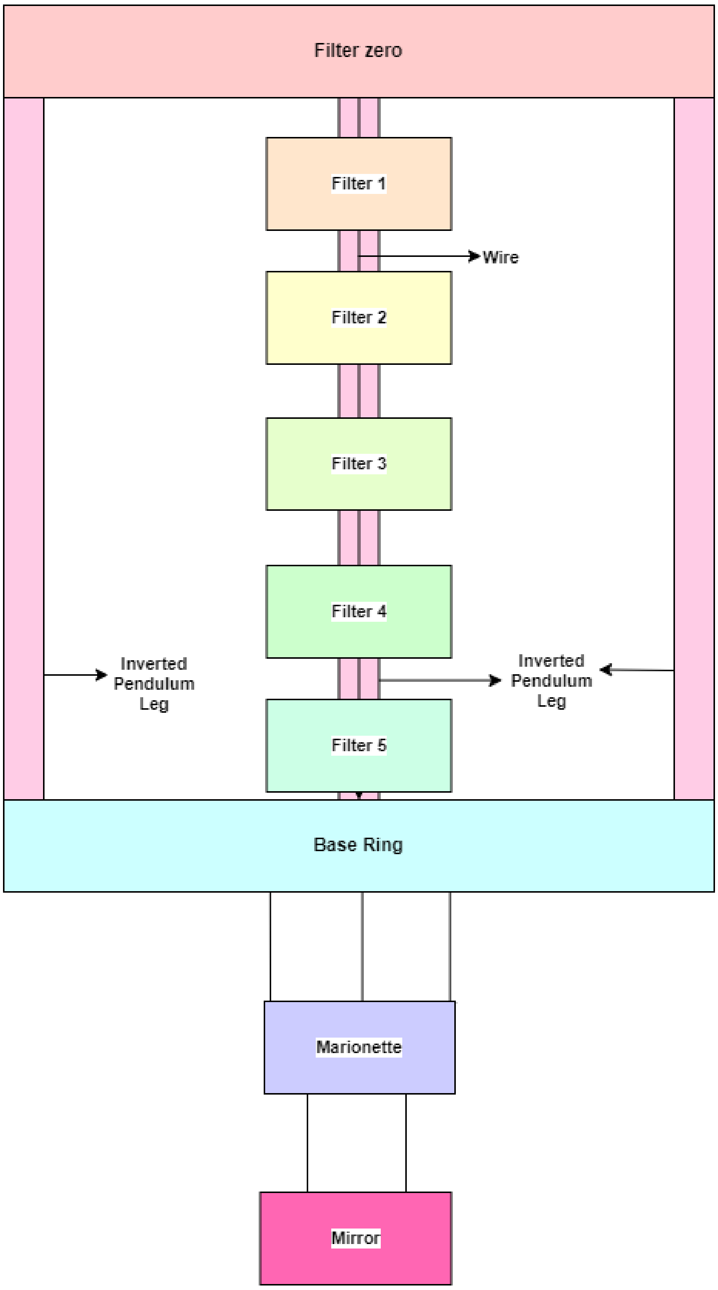

Figure 4. Schematic diagram of the superattenuator chain used in Advanced VIRGO, showing the multi-stage pendulum system for seismic isolation. The mirrors are suspended from a marionette using four wires and moved by coil–magnet actuators ([73]). This setup is attached to the last filter in the chain, Filter 7. Monolithic suspension is achieved by silica-bonding fused silica fibers to T-shaped anchors, which connect the marionette to the mirror, forming a continuous structure for enhanced stability.

The last step has been revamped for Advanced VIRGO, especially with modifications to Filter 7. Now, the ring encircling Filter 7 is equipped with a pinwheel arrangement of six LVDTs and six coil–magnet actuators, providing complete control over the filter's motion in all directions. At the top stage of the inverted pendulum (IP) along the Z-, X-, and θ_y-axes, actuation takes place in the ultra-low frequency region (frequencies $f < 10$ mHz), where the Z-axis coincides with the optical axis of the suspended mirror. This is achieved by use of three coils arranged on the upper ring in a pinwheel configuration [97]. Both the Filter 7 stage and the marionette are controlled at intermediate frequencies (10 mHz $< f <$ 1 Hz). While the marionette is controlled along four DoFs (Z, θ_x, θ_y, and θ_z) using eight coils, the Filter 7 stage is actuated along six DoFs using six actuators [98]. Four coils installed on the actuation cage are used to directly provide actuation to the mirror along the Z-axis at higher frequencies (over a few Hz). In addition, these coils allow for accurate mirror alignment by facilitating tiny adjustments in the θ_x and θ_y axes. To lessen the impacts of ground tilt, experimental tests have been conducted using a tilt control system integrated with piezoelectric actuators. These actuators, with a dynamic range of 40 µm and force capacity up to 4500 N, were paired with three linear variable differential transformers (LVDTs) at the base of the IP to monitor vertical displacement. However, these systems are not currently used for active tilt control. Instead, the IP employs a blending strategy combining accelerometer and LVDT data from the top ring, ensuring precise and stable control over an extensive frequency range. The readers are advised to refer to [99,100] for further details.

6.7. KAGRA

The Kamioka Gravitational Wave Detector, or KAGRA for short, was formerly known as the Large Scale Cryogenic Gravitational Wave Telescope (LCGT). It was developed by the gravitational wave studies group at the University of Tokyo's Institute for Cosmic Ray Research (ICRR) and began operations in 2020 [101]. The original purpose of TAMA300 and Cryogenic Laser Interferometer Observatory (CLIO) was to investigate the viability of KAGRA. KAGRA is a suspension point interferometer with two arms, each measuring three kilometers. Unlike all GW detectors to date, it is subterranean and employs cryogenic mirrors to eliminate a significant amount of thermal and seismic noise. Sapphire is used by KAGRA for their mirrors and suspension fibers because it has a high mechanical Q-factor that is nearly 10^8 at cryogenic temperatures [102]. This helps to reduce thermal noise. The test masses in KAGRA are sapphire mirrors that are suspended by a Type-A suspension, which is a nine-stage vibration isolation system (Seismic Attenuation System (SAS)) that is 13.5 m high (Figure 5). The lower four stages—collectively referred to as the cryogenic payload—are cooled to 20K, while the upper five stages—known as the Type-A tower—run at normal temperature. From top to bottom, the platform (PF), marionette (MN), intermediate mass (IM), and test mass (TM) are the four cryogenic phases. The displacement sensors and actuators are housed within equivalent recoil masses (RMs) that encircle the TM chain, which consists of the TM, IM, and MN. To keep unwanted magnetic forces from damaging the suspension, the RM is made of low-magnetism stainless steel, while the TM is made of sapphire. SUS316L stainless steel is used to make the other suspension parts [102]. Schematics of the KAGRA cryogenic suspension system and cryogenic payload ([103]) are given in Figure 5.

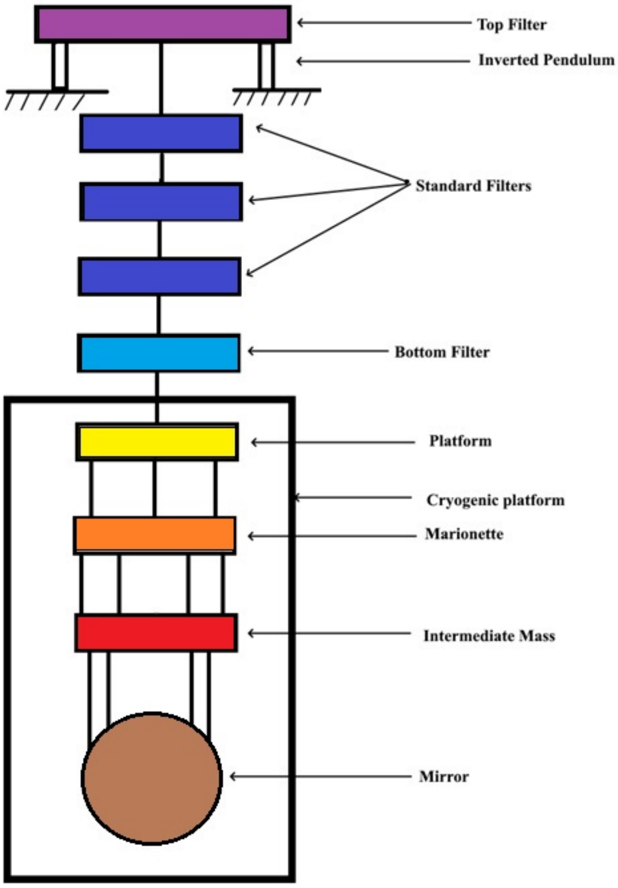

Figure 5. Schematic diagram of the KAGRA suspension system, showcasing the nine-stage Seismic Attenuation System (13.5 m tall). The upper five stages operate at room temperature, while the lower four stages are cooled to 20K.

A 3.3-meter-long maraging steel wire suspends the platform (PF) from the Type-A tower's bottom stage (bottom filter, BF). The test mass (TM) chain and the associated recoil masses (RM chain) are suspended independently from the PF. Three copper beryllium (CuBe) blade springs hold a single maraging steel rod that connects the TM chain to the PF. Four CuBe wires are used to hang the intermediate mass (IM) from the marionette (MN), and four sapphire fibers, which are fastened to four sapphire blade springs on the IM, are used to suspend the TM from the IM. Similar suspension arrangements are used by the RM chain, which makes use of two looped CuBe wires, four CuBe wires, and three CuBe wires. Since a geometric anti-spring (GAS) filter is used to isolate vertical vibration in suspensions at ambient temperature, the cryogenic payload's overall weight must be optimized to match the 200 kg design load of the GAS filter. Because of this, the cryogenic payload's design weight of 198 kg includes a safety buffer that permits the installation of extra weights for fine-tuning the GAS load. Table 1 provides an overview of the key sensors and actuators used, their placement on different suspension platforms, and their specific functions. The sensors are primarily responsible for detecting various degrees of motion and misalignment, while the actuators provide the necessary corrections to maintain optimal alignment and control. The sensors include angular- and length-sensing optical levers, which monitor the angular and linear displacements of the test masses (TMs) and marionette (MN) components. Additionally, photo-reflective displacement sensors measure relative displacements within the suspension system, helping maintain stability. The actuators are designed to exert controlled forces on different suspension stages.

The coarse alignment control actuator, located on the pre-isolation stage (PF) and marionette (MN), provides initial alignment by adjusting the suspension's configuration. Meanwhile, the coil–magnet actuators offer precise positioning and active control of the test mass (TM), intermediate mass (IM), and marionette (MN), ensuring minimal disturbances and accurate alignment of the optical components. This integration of sensing and actuation mechanisms plays a crucial role in maintaining the interferometer's sensitivity, effectively suppressing seismic and environmental noise while ensuring the stable operation of the detector.

Table 1. Sensors and actuators, their platforms, and functions.

Component	Platform	Function
	Sensors	
Angular-sensing optical levers	On Marionette (MN) and Test Mass (TM)	Sensing angular motion
Length-sensing optical lever	On Test Mass (TM)	Sensing its motion along the optical axis of the main interferometer
Photo-reflective displacement sensors	On Marionette Recoil Mass (MNR) and Intermediate Recoil Mass (IRM)	Sensing relative displacement between a TM chain and a RM chain
	Actuators	
Coarse alignment control actuator (Refer to [102] for construction details)	On PF and MN	For alignment control
Coil–magnet actuator	MN, IM, and TM have magnets, and their RMs have corresponding coils	Precise control of the suspension

6.8. Third-Generation Detectors

Several proposed next-generation gravitational wave (GW) detectors, both terrestrial and spaceborne, aim to enhance our understanding of the universe by significantly improving sensitivity and capabilities compared to existing facilities. One of the most ambitious projects is the Einstein Telescope (ET), a proposed third-generation underground GW detector to be located in Europe. With an arm length of 10 km, the ET aims to incorporate advanced technologies to enhance sensitivity further. Notably, it plans to utilize a cryogenic system for cooling some of the main optics, leveraging new quantum technologies to minimize light fluctuations, and implementing comprehensive infrastructure alongside active noise-mitigation measures to address environmental disturbances. The Einstein Telescope is expected to be operational by 2035, providing a powerful tool for exploring the cosmos in greater detail.

In the United States, Cosmic Explorer is another proposed ground-based detector, designed with a remarkable arm length of 40 km. This facility aims to achieve sensitivity levels surpassing those of Advanced LIGO through the implementation of cryogenic mirror suspensions. By focusing on reducing thermal noise and improving isolation techniques, Cosmic Explorer is expected to play a crucial role in the next phase of gravitational wave astronomy [104]. On the international front, the Laser Interferometer Space Antenna (LISA) represents the first proposed spaceborne gravitational wave detector. It consists of a constellation of three spacecraft arranged in an equilateral triangle, with sides measuring 2.5 million km. These spacecraft will operate along an Earth-like heliocentric orbit, enabling them to detect gravitational waves in the low-frequency band, particularly those produced by massive astrophysical events such as merging supermassive black holes [105]. LISA is the first proposed spaceborne gravitational wave detector, developed through a collaboration between the European Space Agency (ESA) and NASA. It consists of three spacecraft arranged in an equilateral triangle, each separated by 2.5 million km. These spacecraft will maintain their formation by utilizing precise laser interferometry to detect gravitational waves. Unlike ground-based detectors, LISA is designed to observe low-frequency gravitational waves in the millihertz range, which are primarily emitted by astrophysical events such as supermassive black hole mergers, compact binary systems, and extreme mass-ratio inspirals. By operating in a heliocentric orbit, LISA will avoid many of the noise sources that affect terrestrial detectors, thereby enabling unprecedented sensitivity to these long-wavelength signals. Expected to launch in the 2030s, LISA will play a crucial role in multi-messenger astronomy, complementing observations from electromagnetic and neutrino-based telescopes. In addition, TianQin is a proposed spaceborne gravitational wave detector consisting of three spacecraft situated 100,000 km apart in a triangular formation, designed to operate in geocentric orbits. Planned for deployment in the 2030s, TianQin aims to provide insights into various cosmic phenomena, including those related to dark matter and dark energy [106]. Another notable initiative is the Taiji Program in Space, also known as Taiji, a proposed Chinese satellite-based gravitational wave observatory set for launch in 2033. It will feature a triangle of three spacecraft orbiting the Sun, interconnected by laser interferometers to detect gravitational waves with unprecedented sensitivity [107]. Lastly, Deci-hertz Interferometer Gravitational Wave Observatory (DECIGO) is a proposed Japanese space-based gravitational wave observatory comprising three satellites positioned 1000 km apart in heliocentric orbits. DECIGO aims to detect low-frequency gravitational waves in the decihertz range, which will enable studies of various astrophysical sources, including early universe phenomena [108]. Together, these next-generation detectors represent a significant advancement in our ability to observe gravitational waves, promising to deepen our understanding of the universe from its very origins to the complex dynamics of cosmic events.

7. Conclusions

This paper presents a comprehensive review of the suspension systems and technologies utilized in interferometric gravitational wave (GW) detectors. The performance of GW detectors is primarily influenced by their vibration isolation capabilities, particularly in the low-frequency range. Various GW detectors employ distinct designs and techniques, making it essential to compile this information for the benefit of the community. This review summarizes the different suspension systems implemented in interferometric GW detectors globally, including those proposed for future use. The authors believe that this paper will serve as a valuable reference document for researchers and practitioners in the gravitational wave detector community, fostering knowledge sharing and encouraging further advancements in this critical field.

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