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The strong decay of higher vector $\Upsilon(11020)$ state in the framework of non-relativistic quark model

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Abstract An analytical study of a source of Beauty mesons (B mesons) by the Strong decay of $\Upsilon(11020)$ is performed to obtain various modes of B mesons. The non-relativistic quark model framework is used to predict the masses of the bottomonium mesons. The strong decay of the $\Upsilon(11020)$ state is calculated utilizing Quark Pair Creation model. In addition, the branching ratio of the strong decay has been calculated for each decay width and using them to estimate the number of B-meson pairs for each strong decay width. The results show reasonable agreement with recent Particle Data Group data, in particular with the total strong decay width of $\Upsilon(11020)$. The $\Upsilon(11020)$ particle is expected to be a good candidate for the conventional $\Upsilon(6S)$ state and provides both types of strange and non-strange B mesons. We found new conclusions for the strong decay widths of the $\Upsilon(11020)$ in the relativistic phase space. The strong width of the B^*B^* channel is predicted to be the highest width by the ratio ($\cong 51\%$) concerning higher vector $\Upsilon(11020)$ State. In addition, the specific $B_sB_s^*$ channel is expected to be the highest width relative to higher Υ resonances.

1 Introduction

(B mesons) [1]. The measurements have exhibited three discrete Υ resonance states: Υ(4S), $\Upsilon(10860)$, and $\Upsilon(11020)$, also recently have found the $\Upsilon(10750)$ [2, 3] state which is possibly considered the Υ(3D) state. [4]. The first two resonances are the most important sources to produce Beauty mesons in the B factories. In addition, the $\Upsilon(11020)$ resonance is expected to be an important source of B mesons. Among these bound states of bottomonium mesons, Υ(4S) is particularly interesting because its resonance mass is especially favorable for the production and study of B mesons [5]. It is barely above the $B\bar{B}$ energy threshold, but is too tiny to produce B^*-exciting mesons or any extra particle. The B factories were developed to take advantage of this unique situation. In the literature, the $B^0 - \bar{B}^0$ mixing is noted for the first time midst the running time of the ARGUS detector (1982:1992) [6–9], where the $e^+e^- \longrightarrow \Upsilon(4S) \longrightarrow B\bar{B}$ is caused to obtain pure $B\bar{B}$ pairs. And $\Upsilon(10860)$ is the first resonance that can produce two types of beauty mesons; B and B_s [4]. The collision in B factories can create several modes of B_s pairs in addition to B pairs, which supplies a great opportunity to increase the knowledge of the properties of bottomonium states, B mesons, and B_s mesons. $\Upsilon(11020)$, the strong candidate of Υ(6S), with leptonic width $\Gamma_{ee}^{\Upsilon(11020)} = 0.130 \pm 0.030$ keV and the quantum numbers $J^{PC} = 1^{-}-$ [4] was detected in positron-electron annihilation in 1985 [10, 11]. Its measured mass and total width are (11000±4 MeV) and (24_6^{+8} MeV), respectively. The last measure of BaBar collaboration [12] of the cross section for the $\left(e^+e^- \longrightarrow b\bar{b}\right)$ by the scan of energy during the range of (10540:11200) MeV to obtain the mass and total width of $\Upsilon(11020)$ as (10996±2 MeV), and (37±3 MeV), respectively. There is no measured data about the strong decay partial widths of the $\Upsilon(11020)$ state and their branching ratio in the Particle Data Group [4]. Practically just the total decay width is available, but a recent study by (Husken et al.) [3] presented a new estimated branching ratio of this state. The overall unanimity is that $\Upsilon(11020)$ assigns as the S-wave vector bottomonium state Υ(6S) [13–21].

Undoubtedly, B-meson physics has a grand role in probing the deeps of new physics (NP) known as the Beyond Standard Method (BSM). That originates through the rare B mesons decay, which involves: the inclusive rare decay as $B \rightarrow X_s\mu^+\mu^-$ and the exclusive rare decay as $B \rightarrow K^*\mu^+\mu^-$ [22] in addition to very rare decay as $B_s \rightarrow \mu^+\mu^-$ and with the ratio of $(\mathfrak{B}(B_s \rightarrow \mu^+\mu^-)_{CMS+LHCb2017} \cong 3 \times 10^-9)$ [22, 23]. At the same time, it has a significant effect on solving some puzzles in the Standard Model (SM), such as CP violation and specifying some CKM matrix elements, also increasing the knowledge of the properties of

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bottomonium states and B mesons. Thereupon, searching for $B\bar{B}$ mesons sources is one of the major tasks of the particles colliders in recent and future years. Without a doubt, higher $b\bar{b}$ bottomonium mesons achieve this aim through the strong decays to B mesons.

Unfortunately, there are few studies on the strong decay of various higher bottomonium mesons as sources of beauty mesons. So we need more studies in these sectors, especially with advanced and upcoming generations of particle colliders [24, 25]. In this paper, the strong decay of $\Upsilon(11020)$ bottomonium meson is studied as a source for B mesons. This paper could be considered a step towards this exciting and challenging target. The outline of this work is defined as follows: Sect. 2 is devoted to the explanation of the theoretical framework, including the mass spectrum and the mesonic states of the bottomonium mesons in the non-relativistic quark model and the main lines of the QPC ($^3P_0$) model. Section 3 represents the numerical results and discussion. Finally, we present our conclusion in Sect. 4.

2 Theoretical framework

The quark model (QM) is a powerful model for studying the properties of hadrons [26–30]. And the various types of decays provide a better understanding of hadrons. We are concerned with the strong decays of $\Upsilon(11020)$ the initial state that separates into two B mesons. There are many models of strong decay, for example, the Cornell model [31], fluxtube model [32], microscopic models [33], $^3S_1$ model [34], and the $^3P_0$ model [35], but the last is the widespread and simplest model; it describes the strong decay phenomenon well [36, 37]. Here, applying the non-relativistic potential quark model to obtain the predicted mesonic masses and utilizing the $^3P_0$ model to get the widths of strong decay for the $\Upsilon(11020)$ meson as Υ(6S) state.

2.1 The predicted masses

In this section, we calculate the spectra predictions in the frame of the non-relativistic quark model for the predicted masses of bottomonium mesonic states including the mass of $\Upsilon(11020)$ as displayed in Table 2. We apply the conventional potential (color Coulomb potential in addition to a scalar linear potential) [38]. We consider the spin-dependent effects, so we incorporate corrections [39, 40] arising from vector gluon exchange and the efficient scalar confinement interaction. Additionally, we include the centrifugal term (orbital-angular momentum term) [41], and because we have separated the angular part of the Schr$\ddot{o}$dinger equation, we obtain the radial Schr$\ddot{o}$dinger equation, which we used in our calculations. The used generic formula of the potential is defined as:

$$V(r) = \left(\frac{-}{4}3\frac{\alpha_s}{r} + br\right) + \frac{\ell(\ell+1)}{2\mu r^2} + \frac{32\pi\alpha_s\delta_\sigma(r)\mathbf{S}_b\mathbf{S}_{\bar{b}}}{9m_b^2} + \frac{1}{m_b^2}\left(\frac{2\alpha_s}{r^3} - \frac{b}{2r}\right)\mathbf{L}\cdot\mathbf{S} + \frac{1}{m_b^2}\frac{4\alpha_s}{r^3}\mathrm{T} \tag{2.1}$$

Where, the reduced mass is $\mu = \frac{m_b m_{\bar{b}}}{m_b + m_{\bar{b}}}$,

And,

$$\delta_\sigma(r) = \left(\frac{\sigma}{\sqrt{\pi}}\right)^3 e^{-\sigma^2 r^2}, \tag{2.2}$$

$$\langle SM_s|\mathbf{S}_b\mathbf{S}_{\bar{b}}|SM_s\rangle = \frac{S(S-1)}{2} - \frac{3}{4}. \tag{2.3}$$

Where, S is the total spin of the meson [42], and T is the tensor operator [40]:

$$\mathrm{T} = \mathbf{S}_q\cdot\hat{r}\mathbf{S}_{\bar{q}}\cdot\hat{r} - \frac{1}{3}\mathbf{S}_q\cdot\mathbf{S}_{\bar{q}} \tag{2.4}$$

The matrix element of the spin-orbit operator $\mathbf{L}.S$ is given by,

$$\langle\mathbf{L}.S\rangle = \frac{J(J+1)}{2} - \frac{L(L+1)}{2} - \frac{S(S+1)}{2} \tag{2.5}$$

This operator considers diagonalization in a base $|J, L, S\rangle$. The parameters used in this potential for bottomonium mesons ($b\bar{b}$) are determined by fitting. The fitting is for sixteen experimental bottomonia states, tabulated in Table 2. All parameters are tabulated in Table 1. These parameters result in a good description of the masses of these mesons as shown in Table 2.

Table 1 The used potential parameters of the bottomonium mesons

Parameters	The values
α_s	0.4037
b (GeV^2)	0.1623
σ (GeV)	2.3937
m_b (GeV)	4.8089

Table 2 The expected masses of bottomoniu ($b\bar{b}$) mesons in (MeV) using the non-relativistic model

Meson States	$n^{2S+1}L_J$	$\mathbf{M}_{Theor.}$	$\mathbf{M}_{Exp.}$ [4]	Ref. [47]	Ref. [18]	Ref. [17]	Ref. [20]	$\lvert\Delta m\rvert$
$\Upsilon(1S)$	1^3S_1	9453	9460.30 ± 0.264	–	9463	9465	9502	7.30
$\Upsilon(2S)$	2^3S_1	10013	10023.26 ± 0.31	–	10017	10003	10015	10.26
$\Upsilon(3S)$	3^3S_1	10340	10355.2 ± 0.5	–	10356	10354	10349	15.20
$\Upsilon(4S)$	4^3S_1	10602	10579.4 ± 1.2	10597	10612	10635	10607	22.60
$\Upsilon(5S)$	5^3S_1	10831	$10885.2^{+2.6}_{-}1.6$	10794	10822	10878	10818	54.20
$\Upsilon(6S)$	6^3S_1	11039	11000±4 [2]	10997	11001	11102	10995	39.00
$\Upsilon(7S)$	7^3S_1	11232	–	–	11157	–	–	–
$\Upsilon_1(5D)$	5^3D_1	11083	–	11029	11041	–	11023	–
$\eta(1S)$	1^1S_0	9389	9398.7 ± 2.0	–	9398	9402	9455	9.70
$\chi_{b2}(1P)$	1^3P_2	9933	9912.21 ±0.26	–	9910	9897	9886	20.79
$\chi_{b1}(1P)$	1^3P_1	9910	9892.78 ±0.26	–	9889	9876	9874	17.22
$\chi_{b0}(1P)$	1^3P_0	9877	9859.44 ±0.24	–	10259	9847	9855	17.56
$h_b(1P)$	1^1P_1	9924	9899.3 ±0.8	–	9894	9882	9879	24.70
$\chi_{b2}(2P)$	2^3P_2	10268	10268.65 ±0.22	–	10269	10261	10246	0.65
$\chi_{b1}(2P)$	2^3P_1	10249	10255.46 ±0.22	–	10255	10246	10236	6.46
$\chi_{b0}(2P)$	2^3P_0	10226	10232.5 ±0.4	–	10235	10226	10221	6.50
$\chi_{b2}(3P)$	3^3P_2	10536	10524.0±0.8	–	10539	10550	10521	12.00
$\chi_{b1}(3P)$	3^3P_1	10518	10513.4 ±0.7	–	10527	10538	10513	4.60

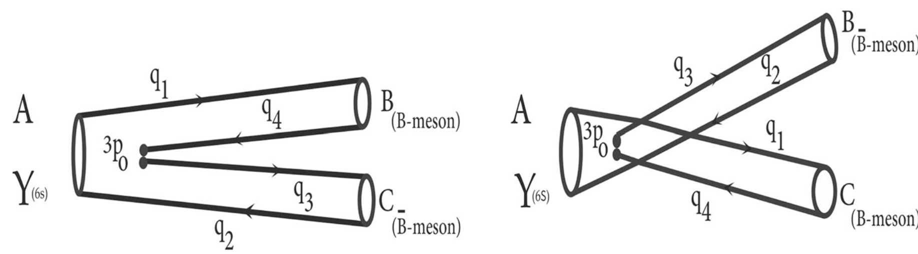

Fig. 1 The two possible diagrams take part to $A \to B + C(\Upsilon(6S) \to B\bar{B})$ strong decay process under the (OZI) allowed rule

2.2 The QPC model

The Quark-Pair-Creation QPC model, known as the $^3P_0$ model, was presented by Micu [35]. Additionally, the Orsay group is credited with enhancing the $^3P_0$ model [36, 43]. The $^3P_0$ model is considered the simplest and the most qualified model for describing and calculating the strong decay of extensive spectra of hadrons [40, 44, 45] into two particles under the Okubo-Zweig-Iizuka(OZI) allowed rule for the hadronic strong decay, as shown in Fig.1. In this part, the $^3P_0$ model is introduced in its basic mathematical formula. The model is designed in a perfectly phenomenological method to describe how the quark pair ($q\bar{q}$) production from the vacuum.

Generally, the hadron strong decay can be expressed A $\to$ $B + C$ process utilizing the helicity amplitude:

$$\langle BC|T_r|A\rangle = \delta^3\left(\vec{k}_i - \vec{k}_f\right)\mathcal{A}_m^{M_{J_A}M_{J_B}M_{J_C}}\left(\vec{K}\right) \tag{2.6}$$

Here, the T_r transition operator gives us the significant imaginary of the quark pair production amplitude from the vacuum in the $^3P_0$ state with the quantum number 0^{++}. The three-momenta of mesons in the initial and final states are represented by $\vec{k}_i$, $\vec{k}_f$, respectively. M_{J_A}, M_{J_B}, and M_{J_C} represent the orbital magnetic momenta, respectively. The transition operator T_r is the most significant in this model and reads:

$$T_r = -3\sum_m \gamma\langle 1m; m-1\rangle \int b_3^\dagger(k_3)b_4^\dagger(k_4)\delta^3\left(\vec{k}_3+\vec{k}_4\right) \times \mathcal{Y}_{1m}\left(\frac{\vec{k}_3-\vec{k}_4}{2}\right)\chi_{1-m}^{34}\phi_0^{34}\omega_0^{34}d^3k_3d^3k_4 \tag{2.7}$$

Where γ is the overall parameter, which is the free undetermined dimensionless parameter of the model. $\mathcal{Y}_{1m}$, χ_{1-m}^{34}, ϕ_0^{34} and ω_0^{34} represent the solid harmonic oscillator, spin, flavor, and color wave functions of quark pair production from the vacuum, and 3 and 4 symbolize the quark pair ($q\bar{q}$), respectively. The helicity amplitude $\mathcal{A}_m^{M_{J_A}M_{J_B}M_{J_C}}$, utilizing Eq. (2.6) and Eq. (2.7), is written as:

$$\mathcal{A}_m^{M_{J_A}M_{J_B}M_{J_C}} = \sum\nolimits_{M_{L_A},M_{S_A},M_{L_B},M_{S_B},M_{L_C},M_{S_C},m} \langle L_BM_{L_B}; S_BM_{S_B}|J_BM_{J_B}\rangle\langle L_CM_{L_C}; S_CM_{S_C}|J_CM_{J_C}\rangle$$

$$\times \langle L_A M_{L_A}; S_A M_{S_A} | J_A M_{J_A} \rangle \langle 1m; 1-m|00\rangle \left\langle \chi^{q_1q_4}_{S_B M_{S_B}} \chi^{q_3q_2}_{S_C M_{S_C}} | \chi^{q_1q_2}_{S_A M_{S_A}} \chi^{q_3q_4}_{1-m} \right\rangle$$
$$\times \left[\langle \phi^{q_1q_4}_B \phi^{q_3q_2}_C | \phi^{q_1q_2}_A \phi^{q_3q_4}_0 \rangle I\left(\vec{K}, m_1, m_2, m_3\right) + (-1)^{1+S_A+S_B+S_C} \langle \phi^{q_3q_2}_B \phi^{q_1q_4}_C | \phi^{q_1q_2}_A \phi^{q_3q_4}_0 \rangle I\left(-\vec{K}, m_1, m_2, m_3\right) \right] \tag{2.8}$$

where, (L_i, S_i and J_i) represent the orbital angular momentum, total spin, and total angular momentum, respectively, and i refers to A, or B, or C hadron. In Fig. 1 right-panel, it is easy to replace $B \longleftrightarrow C$, $m_1 \longleftrightarrow m_2$, and $\vec{K} \longleftrightarrow -\vec{K}$. In a meson center-of-mass frame, we take into consideration $\vec{K} \equiv K_B = -K_C$. The Clebsch-Gordan coefficients of orbital angular momentum, total spin, and total angular momentum for initial meson, the quark pair-creation, and final mesons (B and C), respectively, are as the following:

- $\langle L_A M_{L_A}; S_A M_{S_A} | J_A M_{J_A} \rangle$.
- $\langle 1m; 1-m|00\rangle$,
- $\langle L_B M_{L_B}; S_B M_{S_B} | J_B M_{J_B} \rangle$.
- $\langle L_C M_{L_C}; S_C M_{S_C} | J_C M_{J_C} \rangle$

Fig. 1 left side, is expressed by momentum-space integral:

$$I\left(\vec{K}, m_1, m_2, m_3\right) = \int d^3k \mathcal{Y}_{1m}\left(\vec{k}\right) \psi_{n_A, L_A, m_{L_A}}\left(\vec{k} + \vec{K}\right) \psi^*_{n_B, L_B, m_{L_B}}\left(\vec{k} + \frac{m_3}{m_1 + m_3}\vec{K}\right) \times \psi^*_{n_C, L_C, m_{L_C}}\left(\vec{k} + \frac{m_3}{m_2 + m_3}\vec{K}\right) \tag{2.9}$$

where, m_1 and m_2 are the quark masses of initial meson, and m_3, m_4; $m_3 = m_4$ are representing the quark pair production masses from the vacuum. We utilize the momentum-space simple harmonic oscillator (SHO) wavefunctions $\psi^{SHO}_{nLM_L}(\vec{k})$. The partial decay amplitude is formulated utilizing the formula of Jacob-Wick [46] for the helicity amplitude like the following:

$$\mathcal{A}^{LS}_m = \frac{\sqrt{4\pi(2L+1)}}{2J_A+1} \sum_{M_{J_B}, M_{J_C}} \langle L0SM_{J_A} | J_A M_{J_A} \rangle \langle J_B M_{J_B} J_C M_{J_C} | S M_{J_A} \rangle \mathcal{A}^{M_{J_A} M_{J_B} M_{J_C}}_m (K_{\hat{z}}) \tag{2.10}$$

With taking into consideration:

$$M_{J_A} = M_{J_B} + M_{J_C}, \quad \vec{L} = \vec{J}_A - \vec{S}, \vec{S} = \vec{J}_B + \vec{J}_C. \tag{2.11}$$

The partial decay width is formulated in the relativistic phase space [16],

$$\Gamma^{LS}_{A \to B+C}(K) = 2\pi K \left(\frac{E_B(K) E_C(K)}{M_A}\right) |\mathcal{A}^{LS}_m(K)|^2 \tag{2.12}$$

Where M_A is the mass of particle A. The total energy of B, and C mesons are $E_B = \sqrt{M_B^2 + K^2}$, and $E_C = \sqrt{M_C^2 + K^2}$, respectively. The total decay width for A meson is represented by the sum over all orbital angular momentum and spin values.

$$\Gamma^{\text{total}} = \sum_{L,S} \Gamma^{LS}_{A \to B+C}(K) \tag{2.13}$$

3 Numerical results and discussion

The matrix method is used to solve the Schrödinger equation numerically. The obtained phenomenological parameters are from Quantum Chromodynamics theory and the non-relativistic quark model, see Table 1, are used to find the masses of bottomonium mesons. Accordingly, the calculated masses are compared to recent data [4]. Table 2 displays the calculated masses for the bottomonium mesons with the notation of state $\left(n^{2S+1}L_J\right)$. The accuracy of the used method has been noted by comparison of our results with other works [17, 18, 20, 47] and the available experimental data [2, 4]. This leads to estimate the difference between our calculations and the experimental one by taking the absolute of the difference in masses $|\Delta m| = |M_{\text{exp.}} - M_{\text{Theor.}}|$ as shown in the last column in Table 2. For Υ(6S), the other works used various models, as (S. Godfrey and K. Moats) [17] employed the RQM model and expect 11102 MeV, (J. Segovia et al.) used the constituent quark model including the spontaneous chiral symmetry breaking and foretell 10995 MeV [20], the Modified GI model including a screening effect was applied by (J.Z. Wang et al.) [18] foresee 11001 MeV, and finally (Qi Li et al.) applied the screened potential and the predicting result 10997 MeV [47]. Our predicted mass is 11039 MeV by the non-relativistic quark model with the most updated PDG (2022) data. Although all these models utilize different approaches, they conclude that Υ(11020) resonance is enabled to consider as a candidate to be the Υ(6S) state.

The varied QPC models were presented in the literature, and they usually differ in the adoption of the pair-creation strength γ, phase space conventions, and used hadronic space wave functions. In this article, we choose the simple pair creation strength, which is a constant vertex, as presented by Micu [35]. We adopt the momentum-space simple harmonic oscillator (SHO) wavefunctions

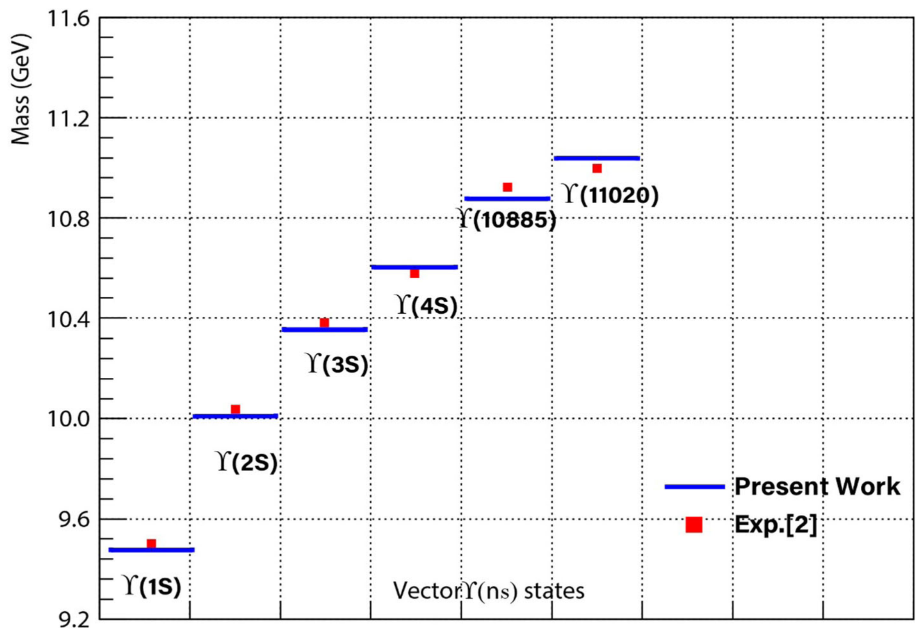

Fig. 2 Comparison of the mass spectra of S-wave Upsilon states including the mass of ϒ(6S) in GeV with the recent experimental

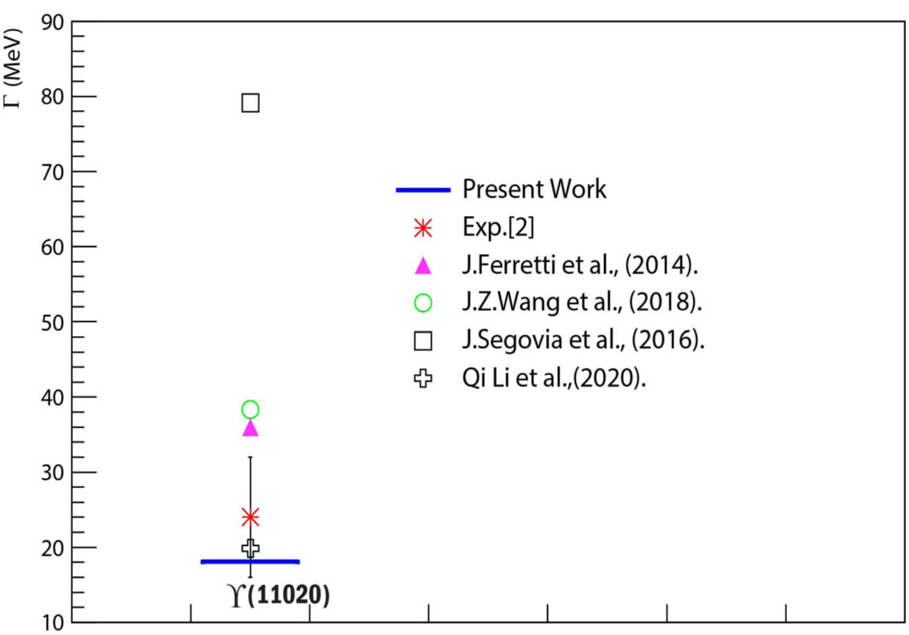

Fig. 3 Comparison of the total strong decay width of ϒ(11020) as ϒ(6S) with the recent experimental and other theoretical works

as wavefunctions which proved its success in studying the strong decay process of the hadrons in literature; in addition, we employ the technique of Barnes and Swanson to compute the decay amplitude of the QPC model [39, 48–50]. These techniques assume that the radial parts of the hadronic space wave functions must be describable in specific functional forms, which are included in (SHO) simple harmonic oscillator wave functions [51]. In addition, we utilized the same techniques as many references [16, 39, 44, 50, 52]. Also, the meson wave functions possess good flavor symmetry and can rely on a fixed simple harmonic oscillator parameter β based on the literature [16, 39, 53, 54] where the fixed value of β gives an agreement with experimental data. So, we adopt the fixed β oscillator parameter.

We adopt utilizing the SHO wavefunction with $\gamma = 0.89$ as well as (J. Ferretti and E. Santopinto) [16] used the SHO wavefonction too, but with $\gamma = 0.732$. And (J. Segovia et al.), (J.Z. Wang et al.), and (Qi Li et al.) [18, 20, 47] employed a realistic wave function with different values of the strength pair creation γ equals 7.09, 0.205, and 0.232, respectively. We used our $^3P_0$ model with the most recent PDG (2022) data [4] which we enabled to predict a total decay width of ϒ(11020) to be 18.652 MeV to get the most updated estimation of the total width. Although the fact that these models take partially different is explicit in the previous, they all arrive at the same conclusion: the ϒ(11020) resonance might be considered a possible candidate for the ϒ(6S) state.

Table 3 The experimental input masses of B and Bs mesons included in the existing calculation of $\Upsilon(11020)$ strong decay widths

Meson States	$n^{2S+1}L_J$	$\mathbf{M}_{Exp.}$ [4]
B	1^1S_0	5279.34
B^*	1^3S_1	5324.71
B_s	1^1S_0	5366.92
B_s^*	1^3S_1	5415.40

Table 4 The predicted values for the strong decay widths of Υ(6S) in (MeV) compared with [16–18, 20, 47] in addition to [3] and the recent PDG [4]

Initial meson	State	Decay mode	Our work Γ (MeV)	BR %	[3] (MeV)	BR [3] %	[16] (MeV)	[17] (MeV)	[20] (MeV)	[18] (MeV)	[47]SP (MeV)
$\Upsilon(6S)$	6^3S_1	BB	0.307	1.28	0.19–2.06	(0.8–8.6)	0	1.32	4.18	7.81	3.8
		BB^*	4.258	17.74	0.46–2.88	(1.9–12)	8	7.59	15.49	16.5	9.02
		$BB(1P_1)$	0	0	–	–	0	3.91	40.08	8.27	0
		$BB(1P_1^{'})$	0	0	–	–	0	4.82	3.95	0	0
		B^*B^*	12.242	51.00	0.04–1.49	(0.2–6.2)	26	5.89	11.87	4.43	3.13
		B_sB_s	0.053	0.22	–	–	0	0.00131	0.07	0.101	0.28
		$B_sB_s^*$	0.631	2.63	–	–	0	0.136	1.5	0.78	2.38
		$B_s^*B_s^*$	1.161	4.84	0.001–2.33	(0.04–9.7)	2	0.310	2.02	0.448	1.5
		$\Gamma^{Total Theor.}_{\Upsilon(6s)}$	$\cong$ 18.652		0.691–8.76		36	25.30	79.16	38.3	20.11
		$\Gamma^{Total Exp.}_{\Upsilon(6s)}$ [4]	$\cong 24^{+8}_{-6}$		$\cong 24^{+8}_{-6}$		$\cong 24^{+8}_{-6}$	$\cong 24^{+8}_{-6}$	$\cong 24^{+8}_{-6}$	$\cong 24^{+8}_{-6}$	$\cong 24^{+8}_{-6}$

Figure 2 represents the calculated masses of Upsilon states including the mass of $\Upsilon(6S)$ in GeV, in the frame of the non-relativistic quark model, listed in Table 2 in comparison with the experimental data [4]. The solid horizontal lines represent the theoretical calculations and the symbols are the experimental data. We observed little distinction between the expected theoretical masses and the practical values.

Figure 3 illustrates the calculated total decay width of $\Upsilon(11020)$ represented by the horizontal line compared with different theoretical results [16, 18, 20], and with experimental data [4]. In this research, we have performed an analytical study of the strong decay of $\Upsilon(11020)$ meson to give a clear vision of the source of the Beauty mesons pairs. Here, we inspect the techniques of the QPC model that have been structured to give us the strong decay widths of the mesons to confirm if the theoretical calculations concur significantly with the recent measured published data in (PDG) [4]. The free dimensionless parameter γ represents the $q\bar{q}$ quark pair creation strength is found by fitting; γ =0.89 with the recent measured data [4].

The quark pair creation model has an overall parameter γ, which may have a great influence on the absolute decay widths. So, we calculate the uncertainty by using the chi-square method. The χ^2 value obtained from our prediction is 0.45. Hence, the calculated value is within the experimental error σ. Then, it is consistent with the experimental data $\cong 24^{+8}_{-}6$ MeV. So, we can expect the $\Upsilon(11020)$ meson a good candidate for the Υ(6S) state.

For the rest of parameters, we used the fixed β oscillator parameter of 0.5 GeV which is determined from the literature [16, 39]. The b quark mass is shown in Table 1, the $m_{u=d} = 0.33GeV$ and $m_s = 0.55GeV$ are acquired from the literature [16, 39], the mass of $\Upsilon(11020)$ bottomonium meson state is taken from experimental data as shown in Table 2; column four, and B-meson states are taken from experimental data as in Table 3 [4] to calculate the total strong decay widths.

3.1 $\Upsilon(11020)$ state

Many standard calculations have characterized the $\Upsilon(11020)$ state as the 6S state as in Refs. [13–21]. But Ref. [55] assigned it as the Upsilon 7S state. However, in Ref. [47], the authors investigated S - D mixing for the $\Upsilon(11020)$ state, and they inferred that the $\Upsilon(11020)$ state can be assigned as a pure Υ(6S) state. The extracted practical mass of the $\Upsilon(11020)$ state is 11000 ± 4 MeV in PDG [4]. In the non-relativistic quark model, the expected mass for Υ(7S) is about 11232 MeV, and for Υ_1(5D) state is 11083 MeV while for Υ(6S) state is 11039 MeV as in Table 2. So, we found that Υ(6S) state is the nearest to the practical mass of $\Upsilon(11020)$. Thus, the $\Upsilon(11020)$ is a possible candidate for the Υ(6S) state. With this scenario, its total strong decay width as a 6S state is determined to be 18.652 MeV, which concurs with a world average measured data of $24^{+8}_{-}6$ MeV for the observed $\Upsilon(11020)$ state [4], and also it agrees with the result of Ref. [47] that equals 20.11 for total strong decay width. Based on the previous discussion, we expected the $\Upsilon(11020)$ to be a good candidate for the conventional Υ(6S) state.

Unfortunately, there are no partial widths of strong decay in PDG [4] of Υ(6S) state, but there is just an experimental total decay width value. Lately Ref.[3] provides us with a new estimated branching ratio of the $\Upsilon(11020)$ which is in agreement with us in the ratio of the BB channel, and the $B_s^* B_s^*$ channel. That makes us need more measured data and calculated strong widths and their branching ratio. So, this study is vital to fill this lack of measured data. The predicted values for strong decay widths of Υ(6S) are tabulated in Table 4. From this table, it is remarkable that the current result of the total decay width shows a reasonable agreement in comparison with the observation. The threshold of Υ(6S) resonance is greater than that of $B_s B_s$ threshold, which allows both B_s mesons production and B mesons production.

The strong decay widths for the Υ(11020) bottomonium state, as the widths of the Υ(6S) state, are listed in Table 4. Figure 3 shows the comparison of the total predicted strong decay width by our QPC model with practically current PDG value [4] and theoretical predictions from other works. Υ(11020) state or so-called Υ(6S) state has J^{PC} quantum numbers state equal to 1^{--}. The notation of this state is $n^{2S+1}L_J = 6^3S_1$, and its threshold is above mesons pair $B\bar{B}$. The strong decay channels of Υ(6S) are BB , BB^*, B^*B^*, B_sB_s, $B_sB_s^*$, and $B_s^*B_s^*$ as in Table 4. Υ(6S) provides us with various partial decay widths, which is essential in the searching for the sources of Beauty mesons, we offered our predictions for each partial strong decay width utilizing our QPC model, and then we have done finding branching ratio BR to each one as in Table 4. These ratios are very important to estimate the numbers of B-meson pairs for every mode.

3.2 Non-strange B-meson type

Υ(6S) provides us with three modes of strong decay widths: BB, BB^*, B^*B^*. Table 4 lists these modes and their strong decay widths. The dominant decay of the non-strange type and also of the Υ(11020) meson is $\Upsilon(6S) \rightarrow B^*B^*$, and the partial decay width of $\Upsilon(6S) \rightarrow BB^*$ provides us with a sizable value for the strong partial decay width of the non-strange B-meson type. In the framework of the QPC ($^3P_0$) model, we predict the following ratio relative to the dominant partial width:

$$\frac{\Gamma(BB^*)}{\Gamma(B^*B^*)} = \frac{\Gamma^{\text{QPC model}}_{\Upsilon(6S)\longrightarrow BB^*}}{\Gamma^{\text{QPC model}}_{\Upsilon(6S)\longrightarrow B^*B^*}} \approxeq 0.35 \tag{3.1}$$

Also, our prediction of

$$\frac{\Gamma(BB)}{\Gamma(B^*B^*)} = \frac{\Gamma^{\text{QPC model}}_{\Upsilon(6S)\longrightarrow BB}}{\Gamma^{\text{QPC model}}_{\Upsilon(6S)\longrightarrow B^*B^*}} \approxeq 0.02 \tag{3.2}$$

The partial decay widths of other Refs. [16, 18, 20] are different from each other and about our new predictions. But Ref. [47] has good agreement with present work for total strong decay width, although differs from us in the partial widths values. We note in Table 4 that the partial widths of $BB(1P_1)$ and $BB(1P_1^{'})$ equal zero. That returns to simply kinematics because the $B + B(1P_1)$ or $B + B(1P_1^{'})$ masses are large relative to Υ(11020) mass. If we want to estimate the number of events for $\Upsilon(6S)$, we need to know the total cross section σ that we can obtain from Ref. [56]; Fig. 10, we can approximately determine that the total cross section in the $\Upsilon(6S)$ peak is about 400 pb, with around 300 pb of that being non-resonant. Also, we can find from Ref.[2]; (Table 1) that the Belle has four datasets very close to the $\Upsilon(6S)$ at 11.0039, 11.0148 GeV, 11.0185, and 11.0208 GeV, with luminosities of 976, 771, 859, and 982 $pb^{-}1$. From that, 100 pb cross section times (976+771+859+982) $pb^{-}1$ integrated luminosity = 358.8 $\times 10^3$ events for $\Upsilon(6S)$ (where the appreciation of the cross section for $\Upsilon(6S)$ is around 100 pb for all datasets). Now we can utilize our calculations for the BR to estimate the number of B-meson pairs for each kind to be 4.593 $\times 10^3$, 63.651 $\times 10^3$, and 182.988 $\times 10^3$ events for BB, BB^*, and B^*B^*, respectively. Our predictions provide new conclusions to be tested by the upcoming particle colliders' experiments.

3.3 Strange B-meson type

It provides us with B_sB_s, $B_sB_s^*$, and $B_s^*B_s^*$. Where our QPC model predictions of OZI-allowed two-body widths of strange B-meson type appear in Table 4. Υ(11020) meson dominantly decays into the $B_s^*B_s^*$ channel relative to this type. The calculated ratios of strong partial widths (3.3) based on the QPC ($^3P_0$) model are

$$\Gamma(B_sB_s) : \Gamma(B_sB_s^*) : \Gamma(B_s^*B_s^*) \approxeq 1 : 12 : 22 \tag{3.3}$$

which is distinct from the strong widths values of other works [16, 18, 20]. For example, Ref. [16] provides only $B_s^*B_s^*$ channel while the ratios of strange B-meson channel widths for Ref. [20] are

$$\Gamma(B_sB_s) : \Gamma(B_sB_s^*) : \Gamma(B_s^*B_s^*) \approxeq 1 : 21 : 29 \tag{3.4}$$

Also, the strong decay widths ratios from Ref. [18] show that

$$\Gamma(B_sB_s) : \Gamma(B_sB_s^*) : \Gamma(B_s^*B_s^*) \approxeq 1 : 8 : 4 \tag{3.5}$$

The strong widths ratios for Ref. [47] are as the following:

$$\Gamma(B_sB_s) : \Gamma(B_sB_s^*) : \Gamma(B_s^*B_s^*) \approxeq 1 : 8 : 5 \tag{3.6}$$

Finally, the strong widths ratios for Ref. [17] are as follows:

$$\Gamma(B_s B_s) : \Gamma(B_s B_s^*) : \Gamma(B_s^* B_s^*) \approxeq 1 : 136 : 310 \tag{3.7}$$

In fact, the differences in predictions for these varied QPC models return to many factors as the adoption of pair-creation strength γ value, the used space meson wave functions, the masses of mesons, and the masses of quarks. So the influences of these differences reflect the various values for the strong partial decay widths $\Gamma(BB, ..., B_s B_s, ...)$ from version to other for the QPC model. Our prediction decay width value of the $B_s^* B_s^*$ channel is the highest, and that expectation concurs with Ref.[17]. Our total decay width concurs with the measured value of total decay width. So the partial width value of the $B_s^* B_s^*$ channel is expected to be the highest relative to $\Upsilon(6S)$ state in the strange type. The first Upsilon resonance above the strange B-meson pairs energy threshold is considered the $\Upsilon(10860)$, which provides the $B_s B_s^*$ channel with a width value = 0.50 MeV(PDG) [4]. While our prediction of the $B_s B_s^*$ channel width from the $\Upsilon(6S)$ state is 0.631 MeV, that means the $\Upsilon(6S)$ will provide us the highest width value of the $B_s B_s^*$ channel during all higher Upsilon resonances. Like we estimated the non-strange B-meson type events, we can estimate the strange B-meson type events as: 0.789×10^3, 9.463×10^3, and 17.366×10^3 events for $B_s B_s$, $B_s B_s^*$, and $B_s^* B_s^*$ the strong decay widths modes. They are significant predictions that provide new conclusions to be tested by incoming particle colliders' experiments in the future.

4 Conclusion

In this research, we represent an analytic study for the strong decays of $\Upsilon(11020)$ as the source of B-mesons. The non-relativistic quark model is used to get the expected spectrum of the bottomonium mesons inclusive of the mass of $\Upsilon(11020)$. Moreover, the QPC model was used to obtain strong decay partial widths of $\Upsilon(6S)$. The ratios of strong decay widths were computed for each of the partial widths, and these ratios are significant for the experimenters as they use them to estimate the number of pairs of B mesons produced. In addition, we presented to the experimenters our estimations of the number of B meson pairs for each a strong width decay mode. The predictions of the present work showed a reasonable agreement with the observation [4]. According to the expected total strong decay width and mass for $\Upsilon(11020)$ state, it has expected to be the $\Upsilon(6S)$ state. We found that $\Upsilon(6S)$ state is a source of non-strange B-meson type: BB, BB^*, B^*B^* which supplies us with very significant source of B^*B^* also provides a sizible source of BB^*, and supplies with a small source of BB pair mesons. In addition, $\Upsilon(6S)$ is a source for the strange B-meson type: the $B_s B_s$, $B_s B_s^*$, and $B_s^* B_s^*$ mesonic pairs, and concerning $B_s B_s$ pair has a very tiny strong decay partial width, and relative to $B_s^* B_s^*$ pair is considered the highest width for this strange type. Although $\Upsilon(6S)$ offers not an enormous source of $B_s B_s^*$ pair mesons, we anticipate it to be the most significant source concerning the Υ family. We need more theoretical and experimental studies around the strong decay of $\Upsilon(11020)$, where its decay widths (B mesons) consider a powerful tool to study numerous issues of SM and BSM. In addition, they provide us with the knowledge of the nature of the $\Upsilon(11020)$ and Beauty mesons.

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Declarations

Conflict of interest The authors declare that no conflicts of interest or personal relationships have influenced this work.

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