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Introduction

The study of radioactive decay has long been central to nuclear physics, providing critical insights into the forces that govern nuclear stability and structure. While alpha decay and cluster radioactivity are well-established modes, charged-particle emissions such as one- and two-proton radioactivity represent more exotic phenomena, accessible only in nuclei beyond the proton drip line. These processes probe the interplay of nuclear binding and Coulomb repulsion, offering a direct means to explore the limits of nuclear existence [1-4]. Proton radioactivity, first observed in the 1970s, is understood as a quantum tunneling process through a combined Coulomb and centrifugal barrier. The decay half-life is highly sensitive to the released energy, orbital angular momentum, and nuclear deformation, making it an excellent probe of single-particle configurations [5]. Empirical and semi-empirical models, particularly those incorporating angular momentum and deformation corrections have significantly improved agreement with experiment. Building on these approaches, recent formulations extend predictive accuracy across a broad range of spherical and deformed nuclei [6,7]. In this work, we extend these developments by analyzing proton decay systematics within an improved empirical framework optimized against experimental data.

Methodology

The half-life of proton radioactivity depends on the tunnelling of the emitted proton through the Coulomb, nuclear, and centrifugal barriers. To capture this behavior, we applied empirical relations that incorporate angular momentum and nuclear deformation, originally proposed in earlier studies [5], and refined them for systematic application to the selected dataset.

The first empirical expression is given by:

$$\log(T_1(s)) = (aZ + b)Q^{-\frac{1}{2}} + c + c_0 \frac{l(l+1)}{\sqrt{(A-1)(Z-1)A^{-2/3}}} \quad (1)$$

where Z and A are the charge and mass numbers of the parent nucleus, Q is the decay energy, and l is the orbital angular momentum. For spherical emitters, the fitted parameters were $a = 0.344$, $b = 4.963$, $c = -31.125$, and $c_0 = 2.595$, with an RMSD of $\sigma = 0.153$. For deformed emitters ($Z = 53-67$), the optimized values were $a = 0.364$, $b = 4.647$, $c = -30.930$, and $c_0 = 2.624$, with $\sigma = 0.323$.

A second formulation introduces mass-dependent corrections:

$$\log(T_1(s)) = a + bA^{1/6}Z^{1/2} + cZQ^{-1/2} + c_0 \frac{l(l+1)}{\sqrt{(A-1)(Z-1)A^{-2/3}}} \quad (2)$$

For spherical emitters, the parameters were $a = -23.063$, $b = -0.422$, $c = 0.417$ and $c_0 = 2.599$, yielding $\sigma = 0.183$. For deformed emitters, the corresponding set was $a = -23.934$, $b = -0.394$, $c = 0.438$ and $c_0 = 2.617$, with $\sigma = 0.316$.

Results and Discussion

Proton decay half-lives were computed for a representative set of isotopes using the modified empirical relations described earlier. Experimental half-life values obtained from experimental databases and existing literature [8-10] were employed to validate the framework, and the root-mean-square deviation (RMSD) was used as a measure of predictive reliability. Both formulations, Eqs. (1) and (2), reproduced experimental data with strong consistency, with the mass-dependent correction in Eq. (2) yielding improved performance for spherical emitters. Nuclear deformation was found to significantly influence decay dynamics, particularly in the region $Z = 53-67$, where the predicted half-lives

Exhibited systematic shifts relative to spherical cases. Similarly, angular momentum dependence was clearly observed, with $l=0, 2,$ and 5 transitions confirming the expected increase in half-lives due to the centrifugal barrier. These results collectively demonstrate the robustness of the refined empirical approach in describing proton emission across diverse nuclear regimes.

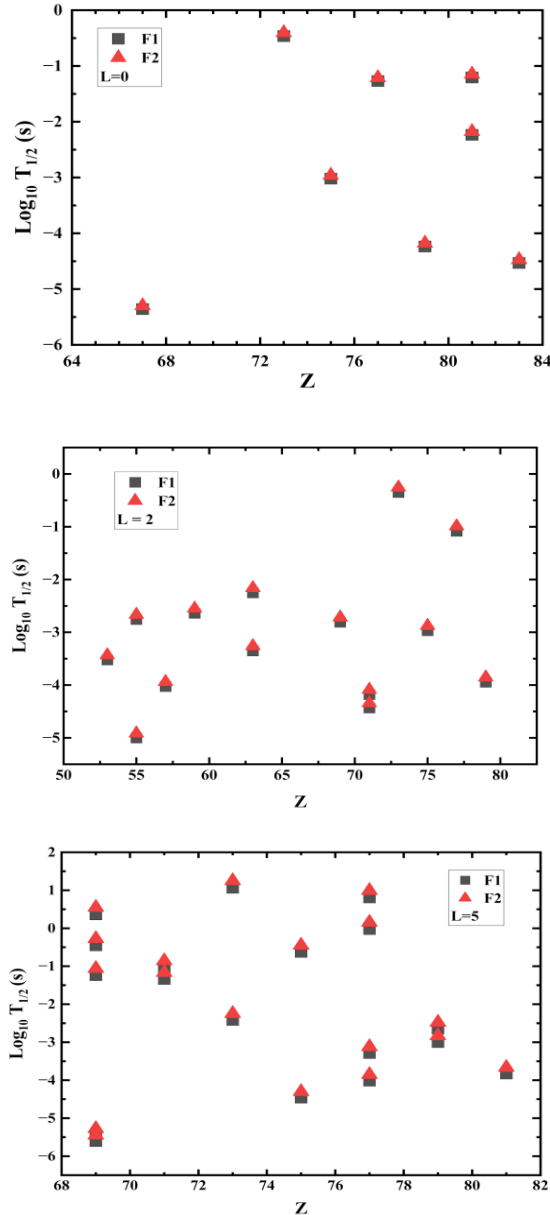


Fig. 1: The calculated half –lives results based on empirical formulas as a function of the atomic number for $L=0, 2, 5$.

The computational findings are illustrated in the accompanying graphs, which display the variation of half-lives as a function of atomic number for different angular momenta. The plots highlight the systematic improvement achieved by the refined relations and the distinct

trends between spherical and deformed nuclei. When benchmarked against other semi-empirical models, the present framework consistently provided closer agreement with experiment, not only for ground-state emitters but also for isomeric states. This enhanced predictive accuracy emphasizes the importance of deformation and angular momentum corrections in empirical models and establishes a reliable tool for investigating proton-rich nuclei near the drip line.

Conclusion

The refined empirical formulations, embedding deformation and angular momentum dependencies, exhibit marked improvement in reproducing experimental half-lives of proton emitters. Their success in describing both ground- and isomeric-state transitions underscores the robustness of the framework and its applicability across spherical and deformed systems. Extending these relations to forthcoming measurements of exotic nuclei near the proton drip line will provide stringent tests of nuclear structure models and advance our understanding of quantum tunnelling phenomena in unstable nuclear matter.

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