

# Black Holes with Ringed Accretion Disks

**A. M. Al Zahrani**

Physics Department, King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia

E-mail: amz@kfupm.edu.sa

**Abstract.** We show that certain electromagnetic field configurations around black holes may lead to the formation of ringed accretion disks. These are structures made of disjoint rings of orbiting matter. The mechanism behind this effect lies in how the electromagnetic fields influence charged particle dynamics in such a way that they create zones where stable circular orbits are forbidden. These forbidden regions may lead to breaking up the regular continuous disk into multiple distinct rings.

## 1. Introduction

Accretion disks provide a primary observational window on black holes, but their physics is highly complex. Yet in low density disks, charged particles have relatively long mean free paths and can be treated as collisionless. Their orbits are then essentially circular. This appears to be the case for Sgr A\* and likely for M87, as well as many other systems [1].

The innermost stable circular orbit (ISCO) is a central concept in the study of black hole accretion disks. The ISCO marks the inner edge of the accretion disk when its luminosity is low compared to the Eddington luminosity [2]. For a Schwarzschild black hole, a neutral particle has its ISCO at  $6M$  (in geometrized units where  $G = c = 1$ ). Real astrophysical environments are rarely so simple and other forces can shift the ISCO inward or outward from this textbook value.

Magnetic fields are one of the most important influences. Observations show that black holes, both stellar-mass and supermassive, are often threaded by magnetic fields produced by currents in their surrounding accretion disks [3, 4, 5, 6, 7]. These fields can be extremely strong:  $\sim 10^8$  G near stellar-mass black holes and  $\sim 10^4$  G for supermassive ones. Furthermore, there are several profound reasons for the existence of weakly charged black holes [8, 9, 10]. The charge of Sgr A\* is estimated to be in the range  $10^8$ – $10^{15}$  coulombs [8, 9]. Even modest electromagnetic fields can dramatically change the dynamics of a charged particle, since the Lorentz force can compete with or even dominate over the purely gravitational pull.

The influence of electromagnetic fields on charged particle orbits has been intensively studied in several settings [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19]. A widely used model is the Wald solution for a uniform, axially symmetric magnetic field. However, this uniform field is only an approximation in certain regions. A more realistic picture comes from modeling the field generated by a current loop in the equatorial plane. To a good approximation, this field is uniform inside the loop but decreases as a dipole outside [20, 21]. Several studies have investigated the motion of charged particles near black holes with dipole fields [22, 23, 24].

In this paper, we demonstrate how the presence of a trace charge on a Schwarzschild black hole immersed in an axisymmetric magnetic field can create a region in which circular orbits are forbidden. We also show that for a Schwarzschild black hole, when the magnetic field is a dipole



field, there is a gap between the ISCO of the dipole field and the current loop generating the field. These configurations may lead to a ringed accretion disk. The existence of ringed accretion disks was suggested in other studies, but due to completely different mechanisms [25, 26, 27].

## 2. Charged particles near a charged Schwarzschild black hole with a uniform magnetic field

The radial equation of motion for a charged particle of mass  $m$  and charge  $e$  reads

$$r^2 r'^2 = (\mathcal{E}r - q)^2 - [r^2 + (\mathcal{L} - br^2)^2]f. \quad (1)$$

where  $f = 1 - 2M/r$ ,  $q = eQ/m$ ,  $b = eB/2m$ . The constants of motion  $\mathcal{E}$  and  $\mathcal{L}$  are the specific energy and azimuthal angular momentum of the particle [19]. It can be rewritten as

$$r'^2 = (\mathcal{E} - V_+)(\mathcal{E} - V_-), \quad (2)$$

where

$$V_{\pm} = \frac{q}{r} \pm \sqrt{\left[1 + \left(\frac{\mathcal{L}}{r} - br\right)^2\right]f}. \quad (3)$$

It is  $V_+$  that corresponds to future-directed motion. The two conditions for a circular orbit ( $V_+ = \mathcal{E}$  and  $dV_+/dr = 0$ ) specify the energy  $\mathcal{E}_o$  and azimuthal angular momentum  $\mathcal{L}_o$  of an orbiting particle. A circular orbit is an ISCO if  $d^2V_+/dr^2 = 0$ . We can use the last two conditions to plot  $r_{ISCO}$  vs.  $b$  for any value of  $q$  (see Ref. [19] for details). When  $q$  is positive and large enough, a very interesting phenomenon appears. As Fig. 1 shows, there are two distinct ISCOs when  $-0.95 \gtrsim b \gtrsim -1.18$ . To better understand the meaning of this, we plot  $d^2V_+/dr^2$  for the specific value of  $b = -1.1M^{-1}$  as shown in Fig. 2. Stable circular orbits ( $d^2V_+/dr^2 > 0$ ) exist in two disjoint bands. The first band is between the smaller ISCO radius and the circular orbit where  $\mathcal{L}_o = 0$ . The second band starts at the larger ISCO radius and extends without a limit. Between the two bands, there is a gap where no stable circular orbits can exist.

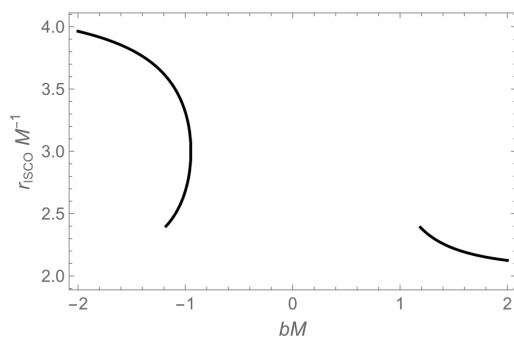


Figure 1: The dependence of the innermost stable circular orbit radius  $r_{ISCO}$  on the magnetic parameter  $b$  when the electric parameter  $q = 10M$ .

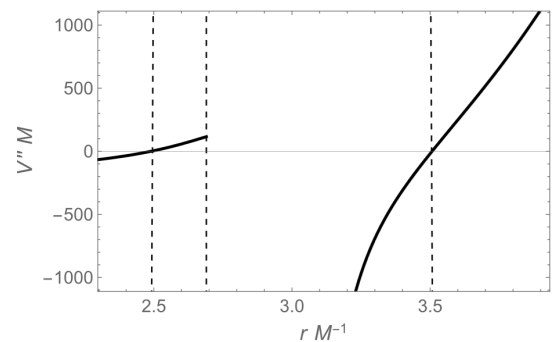


Figure 2:  $d^2V_+/dr^2$  vs.  $r$ . The leftmost (rightmost) dashed line marks the smaller (larger) ISCO. The dashed line in the middle marks the circular orbit at which  $\mathcal{L}_o = 0$ .

## 3. Charged particles near a Schwarzschild black hole with a dipole magnetic field

Now we use the magnetic field due to a current loop of radius  $R$ , up to the dipole term. The effective potential in this case is obtained by setting  $q = 0$ , replacing  $b$  with  $bw(r)$  and squaring

the right-hand side of Eq. 2. We then have

$$U = \left\{ 1 + \left[ \frac{\mathcal{L}}{r} - brw(r) \right]^2 \right\} f. \quad (4)$$

where

$$w(r) = \begin{cases} 1, & 2M < r < R, \\ \frac{\ln f(r) + g(r)}{\ln f(R) + g(R)}, & r \geq R. \end{cases} \quad (5)$$

and

$$g(r) = \frac{2M}{r} \left( 1 + \frac{M}{r} \right). \quad (6)$$

This magnetic field is just Wald's field inside the current loop and has a dipole form outside it. As discussed in [23], the choice of  $R$  has to be physical. For a neutral current loop, we must have  $R \geq 6M$ , the ISCO of neutral particles. If we follow the same procedure as that in the previous section, we can see how the ISCO of the external field  $r_{ISCO}^{out}$  depends on  $b$ . The result is shown in Fig. 3, where the current loop radius  $R$  was taken to be at  $6M$ .

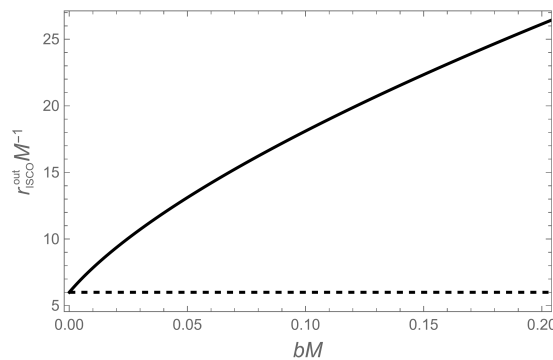


Figure 3: The dependence of the innermost stable circular orbit radius  $r_{ISCO}^{out}$  of the dipole field on the magnetic parameter  $b$  (solid). The dashed line is the current loop radius  $R$

The figure shows that for  $b > 0$  there is always a gap between the current loop radius  $R$  and the ISCO of the outer dipole field. This effect may lead to the formation of a gap in an accretion disk as well.

#### 4. Summary

We demonstrated two electromagnetic configurations that may lead to gaps in low density black hole accretion disks. The first was a weakly charged Schwarzschild black hole immersed in a uniform, axisymmetric magnetic field. This mechanism requires fine tuning of the electric charge and magnetic field and, therefore, is not expected to be astrophysically common.

The second configuration was a Schwarzschild black hole equipped with a current loop. We showed that if the magnetic field parameter and azimuthal angular momentum of charged particles have the same sign then there is always a gap between the current loop and the outer band of circular orbits. This effect is unlikely to be a mathematical artifact of the dipole approximation, since the gap arises entirely within the smooth external field. Astrophysically, we think that this mechanism is far more likely than the first.

It should be mentioned that the gap in both scenarios is for a particular species of particles with a certain charge per mass ratio. Therefore, in real accretion disks, we expect the gap, if exists, to manifest itself as a region of relatively low density.

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