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SU(2|1) chiral superfields and spinning models

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We consider the coupling of dynamical and semi-dynamical (spin) multiplets described by superfields living on the generalized SU(2|1), $d = 1$ chiral superspace. The interaction term of both multiplets is constructed as a superpotential term, where the dynamical multiplet is defined as a chiral multiplet $(\mathbf{2}, \mathbf{4}, \mathbf{2})$, while the semi-dynamical multiplet is associated with a mirror multiplet $(\mathbf{4}, \mathbf{4}, \mathbf{0})$.

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1. Introduction

A new class of systems of $\mathcal{N}=4$, $d=1$ supersymmetric quantum mechanics called "Kähler oscillator" was introduced by S. Bellucci and A. Nersessian [1, 2]. They studied supersymmetric oscillator models on Kähler manifolds with the term of the first-order in time derivatives responsible for the presence of a constant magnetic field. The bosonic Lagrangian of such system can be written as

$$\mathcal{L}_{\text{bos.}} = g\dot{z}\dot{\bar{z}} + \frac{i}{2}\,\mathbf{B}\left(\dot{z}\,\partial_z K - \dot{\bar{z}}\,\partial_{\bar{z}} K\right) - \omega^2 g^{-1}\,\partial_z K\,\partial_{\bar{z}} K, \qquad g = \partial_z\partial_{\bar{z}} K\,(z,\bar{z})\,. \tag{1}$$

It turned out, the presence of oscillator term and the interaction with a magnetic field deforms the standard $\mathcal{N}=4$, $d=1$ Poincaré supersymmetry to the so-called "Weak supersymmetry" [3]. We showed that the deformed superalgebra of the weak supersymmetry corresponds to the worldline supersymmetry SU(2|1) [4, 5]. We initiated a study of deformed supersymmetric quantum mechanics by employing superfield approach based on the worldline supersymmetry SU(2|1) with a mass dimension deformation parameter m.

Multiplets of $\mathcal{N}=4$, $d=1$ supersymmetry are denoted as $(\mathbf{k},\mathbf{4},\mathbf{4}-\mathbf{k})$ with $\mathbf{k}=\mathbf{0},\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4}$. These numbers correspond to the numbers of bosonic physical fields, fermionic physical fields and bosonic auxiliary fields. Wess-Zumino (WZ) type Lagrangians (sometimes referred as Chern-Simons) for $(\mathbf{3},\mathbf{4},\mathbf{1})$ and $(\mathbf{4},\mathbf{4},\mathbf{0})$ were presented in the framework of the $\mathcal{N}=4$, $d=1$ harmonic superspace [6]. For example, the simplest WZ Lagrangian for $(\mathbf{4},\mathbf{4},\mathbf{0})$ reads

$$\mathcal{L}_{\text{WZ}} = \frac{i}{2}\left(z^i\dot{\bar{z}}_i - \dot{z}^i\bar{z}_i\right) + \psi^a\bar{\psi}_a\,, \qquad i=1,2, \qquad a=1,2. \tag{2}$$

Without kinetic Lagrangian containing bosonic terms second-order in time derivatives, this Lagrangian describes a semi-dynamical multiplet. Fermionic fields become auxiliary, while bosonic fields satisfy the following Dirac brackets:

$$\left\{z^i,\bar{z}_j\right\} = i\,\delta^i_j\,. \tag{3}$$

Such bosonic fields in supersymmetric quantum mechanics are usually called spin (isospin) variables.

Coupling of dynamical and semi-dynamical multiplets was proposed by [7]. This idea provided harmonic superfield construction of $\mathcal{N}=4$ extension of Calogero system with the additional spin degrees of freedom z^i, z_j . This work was followed by a further study of "spinning" models considering couplings of dynamical and semi-dynamical multiplets of various types (see, for example, [8] or [9]).

In this paper we briefly consider SU(2|1) superfield approach to spinning models[1] based on the coupling of chiral superfields instead of harmonic ones [11].

2. Superspace and superfields

In this section we follow the notations introduced in [5].

[1]Undeformed version of these models was studied in [10].

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The superalgebra $su(2|1)$ is defined as a deformation of the standard $\mathcal{N}=4$, $d=1$ Poincaré superalgebra:

$$
\begin{aligned}
&\{Q^i, \bar{Q}_j\} = 2\delta^i_j \mathcal{H} - 2m I^i_j,\\
&\left[\mathcal{H}, Q^k\right] = -\frac{m}{2}\, Q^k, \qquad \left[\mathcal{H}, \bar{Q}_l\right] = \frac{m}{2}\, \bar{Q}_l\,,\\
&\left[I^i_j, \bar{Q}_l\right] = \frac{1}{2}\, \delta^i_j \bar{Q}_l - \delta^i_l \bar{Q}_j\,, \qquad \left[I^i_j, Q^k\right] = \delta^k_j Q^i - \frac{1}{2}\, \delta^i_j Q^k,\\
&\left[I^i_j, I^k_l\right] = \delta^k_j I^i_l - \delta^i_l I^k_j\,.
\end{aligned} \tag{4}
$$

The indices i, j $(i=1,2)$ are SU(2) indices. The U(1) generator $\mathcal{H}$ is associated with the Hamiltonian. The generators I^i_j $(I^k_k=0)$ form SU(2) symmetry. In the limit $m=0$, models of the standard $\mathcal{N}=4$ supersymmetric mechanics are restored with $\mathcal{H}$ being a central charge generator.

2.1 Generalized chiral superspace

The SU(2|1) superspace is defined as a supercoset:

$$
\frac{\mathrm{SU}(2|1)}{\mathrm{SU}(2)} \sim \frac{\left\{\mathcal{H}, Q^i, \bar{Q}_j\,, I^i_j\right\}}{\left\{I^i_j\right\}} = \left\{t, \theta_i, \bar{\theta}^j\right\}. \tag{5}
$$

It has a chiral subspace identified with the coset

$$
\frac{\left\{\mathcal{H}, Q^i, \bar{Q}_j\,, I^i_j\right\}}{\left\{\bar{Q}_j\,, I^i_j\right\}} = \{t_{\mathrm{L}}, \theta_i\}\,. \tag{6}
$$

The chiral condition reads

$$
\bar{\mathcal{D}}_j \Phi\,(t_{\mathrm{L}}, \theta_i) = 0, \tag{7}
$$

where $\bar{\mathcal{D}}_j$ and $\mathcal{D}^i$ are SU(2|1) covariant derivatives. A generalization of the chiral subspace is identified with the coset

$$
\frac{\left\{\mathcal{H}, \hat{Q}^i, \bar{\hat{Q}}_j\,, I^i_j\right\}}{\left\{\bar{\hat{Q}}_j\,, I^i_j\right\}} = \left\{\hat{t}_{\mathrm{L}}, \hat{\theta}_i\right\}, \tag{8}
$$

where

$$
\hat{Q}^i = \cos\lambda\; Q^i - \sin\lambda\; \bar{Q}^i, \qquad \bar{\hat{Q}}_j = \cos\lambda\; \bar{Q}_j + \sin\lambda\; Q_j\,. \tag{9}
$$

The generalized chiral condition reads

$$
\left(\cos\lambda\; \bar{\mathcal{D}}_j - \sin\lambda\; \mathcal{D}_j\right) \Phi\left(\hat{t}_{\mathrm{L}}, \hat{\theta}_i\right) = 0. \tag{10}
$$

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2.2 Generalized chiral multiplet

The generalized chiral condition describes a new type of the chiral multiplet $(\mathbf{2},\mathbf{4},\mathbf{2})$ defined on the generalized chiral superspace and depending on two deformation parameters: m, λ. Exactly this multiplet is a basis for the construction of supersymmetric Kähler oscillator models with the frequency of oscillator $\omega = m\sin 2\lambda\,/2$ and the strength of an external magnetic field $\mathbf{B} = m\cos 2\lambda$. Both parameters disappear in the limit $m = 0$ and the rotation parameter λ becomes just an external automorphism parameter of the standard $\mathcal{N} = 4$ Poincaré supersymmetry.

The chiral superfield is solved by

$$\Phi\left(\hat{t}_{\rm L},\hat{\theta}_j\right) = z + \sqrt{2}\,\hat{\theta}_k\xi^k + \hat{\theta}_k\hat{\theta}^k B, \tag{11}$$

where

$$\hat{t}_{\rm L} = t + i\,\bar{\hat{\theta}}^k\hat{\theta}_k\,, \qquad \hat{\theta}_i = \left(\cos\lambda\,\theta_i\,e^{\frac{i}{2}mt} + \sin\lambda\,\bar{\theta}_i\,e^{-\frac{i}{2}mt}\right)\left(1 - \frac{m}{2}\,\bar{\theta}^k\theta_k\right). \tag{12}$$

Superfield invariant action for the chiral superfield Φ is given by

$$S_{\rm kin.} = \int dt\mathcal{L}_{\rm kin.} = \frac{1}{4}\int dt\,d^2\theta\,d^2\bar{\theta}\left(1 + 2m\,\bar{\theta}^k\theta_k\right)K\left(\Phi,\bar{\Phi}\right). \tag{13}$$

where $K\left(\Phi,\bar{\Phi}\right)$ is a Kähler potential. Superpotential is given by the standard superfield action

$$S_{\rm pot.} = \int d\hat{t}_{\rm L}\,d^2\hat{\theta}\,f\left(\Phi\right) + \int d\hat{t}_{\rm R}\,d^2\bar{\hat{\theta}}\,\bar{f}\left(\bar{\Phi}\right). \tag{14}$$

2.3 Mirror multiplet (4,4,0)

The standard $\mathcal{N} = 4$ multiplets have their mirror counterparts characterized by the interchange of two SU(2) groups which form $\mathrm{SU}(2)\times\mathrm{SU}'(2) \rightarrow \mathrm{SO}(4)$ automorphism group of the standard $\mathcal{N} = 4$ Poincaré supersymmetry [12]. Since this interchange $(i, j \longleftrightarrow i', j')$ has no essential impact on Poincaré supersymmetry, $\mathcal{N} = 4$ multiplets and their mirror counterparts are mutually equivalent when dealing with only one multiplet from such a pair. Deformation to SU(2|1) supersymmetry breaks the equivalence, because the first SU(2) group becomes subgroup of SU(2|1) and the second group SU′(2) is broken. It means that SU(2|1) multiplets differ from their mirror counterparts.

The mirror $(\mathbf{4},\mathbf{4},\mathbf{0})$ satisfies the generalized constraints

$$\bar{\tilde{\mathcal{D}}}_i Y^A = 0, \qquad \tilde{\mathcal{D}}^i\bar{Y}^A = 0, \qquad \tilde{\mathcal{D}}_i Y^A = \bar{\tilde{\mathcal{D}}}_i\bar{Y}^A, \qquad \overline{\left(Y^A\right)} = \bar{Y}_A\,, \tag{15}$$

where

$$\bar{\tilde{\mathcal{D}}}_i = \cos\lambda\;\bar{\mathcal{D}}_i - \sin\lambda\;\mathcal{D}_i\,, \qquad \tilde{\mathcal{D}}^i = \cos\lambda\;\mathcal{D}^i + \sin\lambda\;\bar{\mathcal{D}}^i. \tag{16}$$

The mirror multiplet is described by a pair of chiral superfields defined on (8) and satisfying the third condition. It gives the following simple solution:

$$Y^A\left(\hat{t}_{\rm L},\hat{\theta}_i\right) = y^A + \sqrt{2}\,\hat{\theta}_i\psi^{iA} + i\,\hat{\theta}_k\hat{\theta}^k\,\dot{\bar{y}}^A, \qquad \overline{\left(y^A\right)} = \bar{y}_A\,, \qquad \overline{\left(\psi^{iA}\right)} = \psi_{iA}\,. \tag{17}$$

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One can see that the last term is given by a time derivative of bosonic fields instead of auxiliary fields. So, when we write the superpotential action

$$S_{\text{pot.}} = \frac{\mu}{2}\int d\hat{t}_{\rm L}\, d^2\hat{\theta}\, h\left(Y^A\right) + \frac{\mu}{2}\int d\hat{t}_{\rm R}\, d^2\bar{\hat{\theta}}\, \bar{h}\left(\bar{Y}_A\right), \tag{18}$$

we obtain the WZ Lagrangian:

$$S_{\text{pot.}} = \int dt\, \mathcal{L}_{\rm WZ}\,, \qquad \mathcal{L}_{\rm WZ} = \mu\left[i\,\dot{\bar{y}}^A\,\partial_A h\left(y^A\right) + \frac{1}{2}\psi^{iA}\psi^B_i\,\partial_A\partial_B h\left(y^A\right) + \text{c.c.}\right]. \tag{19}$$

2.4 Gauged mirror multiplet (4,4,0)

We can also assume that the chiral superfields Y^A are subjected to the local U(1) transformations

$$\left(Y^1\right)' = e^{\frac{1}{2}(\Lambda-\bar{\Lambda})}\,Y^1, \qquad \left(Y^2\right)' = e^{-\frac{1}{2}(\Lambda-\bar{\Lambda})}\,Y^2, \tag{20}$$

where $\Lambda := \Lambda\left(\hat{t}_{\rm L},\hat{\theta}_i\right)$, $\bar{\Lambda} := \bar{\Lambda}\left(\hat{t}_{\rm R},\bar{\hat{\theta}}^j\right)$. The superfields then satisfy the new gauge invariant constraints

$$\begin{aligned}
&\left(\bar{\tilde{\mathcal{D}}}^i + \frac{1}{2}\left[\bar{\tilde{\mathcal{D}}}^i, X\right]\right)Y^1 = 0, \quad \left(\bar{\tilde{\mathcal{D}}}^i - \frac{1}{2}\left[\bar{\tilde{\mathcal{D}}}^i, X\right]\right)Y^2 = 0, \qquad \text{c.c.},\\
&\left(\tilde{\mathcal{D}}^i - \frac{1}{2}\left[\tilde{\mathcal{D}}^i, X\right]\right)Y^1 = \left(\bar{\tilde{\mathcal{D}}}^i + \frac{1}{2}\left[\bar{\tilde{\mathcal{D}}}^i, X\right]\right)\bar{Y}^1,\\
&\left(\tilde{\mathcal{D}}^i + \frac{1}{2}\left[\tilde{\mathcal{D}}^i, X\right]\right)Y^2 = \left(\bar{\tilde{\mathcal{D}}}^i - \frac{1}{2}\left[\bar{\tilde{\mathcal{D}}}^i, X\right]\right)\bar{Y}^2,
\end{aligned} \tag{21}$$

where the real superfield X is a gauge superfield transforming as

$$X' = X + \Lambda + \bar{\Lambda}\,. \tag{22}$$

According to the gauge invariant constraints it satisfies the additional constraint

$$\tilde{\mathcal{D}}_{(i}\bar{\tilde{\mathcal{D}}}_{j)}X = 0. \tag{23}$$

It is solved by

$$\begin{aligned}
X\left(t,\hat{\theta}_i,\bar{\hat{\theta}}^j\right) \;=\;& x + \sqrt{2}\left(\hat{\theta}_k\bar{\chi}^k + \bar{\hat{\theta}}^k\chi_k\right) + 2\,\bar{\hat{\theta}}_k\hat{\theta}^k\mathcal{A} + \hat{\theta}_k\hat{\theta}^k\,D + \bar{\hat{\theta}}^k\bar{\hat{\theta}}_k\,\bar{D}\\
&+ \sqrt{2}\,i\,\bar{\hat{\theta}}^k\hat{\theta}_k\left(\hat{\theta}_i\dot{\bar{\chi}}^i - \bar{\hat{\theta}}^i\dot{\chi}_i\right) - \frac{1}{4}\,\hat{\theta}_i\hat{\theta}^i\,\bar{\hat{\theta}}^j\bar{\hat{\theta}}_j\,\ddot{x},\\
&\overline{(x)} = x, \qquad \overline{(\mathcal{A})} = \mathcal{A}\,, \qquad \overline{(D)} = \bar{D}, \qquad \overline{(\chi^i)} = \bar{\chi}_i\,.
\end{aligned} \tag{24}$$

This superfield describes the mirror multiplet $(\mathbf{1},\mathbf{4},\mathbf{3})$ that differs from the ordinary one because of the deformation. Using the U(1) gauge freedom $X' = X + \Lambda + \bar{\Lambda}$, we can choose the WZ gauge:

$$X_{\rm WZ} = 2\,\bar{\hat{\theta}}^k\hat{\theta}_k\,\mathcal{A}\,, \qquad \mathcal{A}'(t) = \mathcal{A}(t) - \dot{\alpha}(t)\,. \tag{25}$$

Thus, it can be interpreted as a mirror counterpart of the "topological" gauge multiplet described by the harmonic superfield V^{++} in the WZ gauge [13]. One can introduce accompanying chiral superfields

$$\mathcal{V}_{\rm WZ}\left(\hat{t}_{\rm L},\hat{\theta}_i\right) = \hat{\theta}_k\hat{\theta}^k\mathcal{A}\,, \qquad \bar{\mathcal{V}}_{\rm WZ}\left(\hat{t}_{\rm R},\bar{\hat{\theta}}^j\right) = \bar{\hat{\theta}}^k\bar{\hat{\theta}}_k\,\mathcal{A}\,, \tag{26}$$

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satisfying

$$\tilde{\mathcal{D}}_i\, X_{\rm WZ} = \bar{\tilde{\mathcal{D}}}_i \bar{\mathcal{V}}_{\rm WZ}\,, \qquad \bar{\tilde{\mathcal{D}}}_i\, X_{\rm WZ} = -\,\tilde{\mathcal{D}}_i \mathcal{V}_{\rm WZ}\,. \tag{27}$$

These superfields can be combined in the form of a triplet superfield $\mathcal{V}^{(i'j')}_{\rm WZ} = \hat{\theta}^{k\,(i'}\hat{\theta}^{j')}_{k}\mathcal{A}$ satisfying

$$\tilde{\mathcal{D}}^{i\,(i'}\mathcal{V}^{j'k')}_{\rm WZ} = 0. \tag{28}$$

Discarding the gauge fixing, this constraint describes the multiplet $(\mathbf{3},\mathbf{4},\mathbf{1})$.

According to the (anti)chiral conditions, the superfield solution is modified as

$$\begin{aligned} Y^1\left(t,\hat{\theta}_i,\bar{\hat{\theta}}^j\right) &= e^{-\frac{X}{2}}\, Y^1_{\rm L}\left(\hat{t}_{\rm L},\hat{\theta}_i\right), \qquad \left(Y^1_{\rm L}\right)' = e^{\Lambda}\, Y^1_{\rm L}\,, \\ Y^2\left(t,\hat{\theta}_i,\bar{\hat{\theta}}^j\right) &= e^{\frac{X}{2}}\, Y^2_{\rm L}\left(\hat{t}_{\rm L},\hat{\theta}_i\right), \qquad \left(Y^2_{\rm L}\right)' = e^{-\Lambda}\, Y^2_{\rm L}\,. \end{aligned} \tag{29}$$

Solving the additional constraint the left chiral superfield $Y^A_{\rm L}$ has the following θ-expansion:

$$Y^A_{\rm L}\left(\hat{t}_{\rm L},\hat{\theta}_i\right) = y^A + \sqrt{2}\,\hat{\theta}_i\psi^{iA} + i\,\hat{\theta}_k\hat{\theta}^k\,\nabla_t\bar{y}^A, \qquad \nabla_t\bar{y}^1 = \left(\partial_t + i\mathcal{A}\right)\bar{y}^1, \quad \nabla_t\bar{y}^2 = \left(\partial_t - i\mathcal{A}\right)\bar{y}^2. \tag{30}$$

Finally, the superpotential action must be written as a function of the only possible invariant $Y^1_{\rm L}Y^2_{\rm L}$, since it is the only gauge invariant object defined on the left chiral subspace.

3. Coupling of dynamical and semi-dynamical multiplets

We consider the model of the dynamical multiplet $(\mathbf{2},\mathbf{4},\mathbf{2})$ interacting with the semi-dynamical multiplet $(\mathbf{4},\mathbf{4},\mathbf{0})$. To couple these two multiplets we consider the simplest gauge invariant superpotential for them as

$$S_{\rm int.} = \frac{\mu}{2}\int d\hat{t}_{\rm L}\, d^2\hat{\theta}\; Y^1_{\rm L}Y^2_{\rm L}\, f\left(\Phi\right) - \frac{\mu}{2}\int d\hat{t}_{\rm R}\, d^2\bar{\hat{\theta}}\; \bar{Y}^1_{\rm R}\bar{Y}^2_{\rm R}\, \bar{f}\left(\bar{\Phi}\right), \tag{31}$$

where f is an arbitrary holomorphic function of Φ. The component Lagrangian is then given by

$$\begin{aligned} \mathcal{L}_{\rm int.} \;=\; \mu\Big[& i\left(y^1\,\nabla_t\bar{y}_1 - y^2\,\nabla_t\bar{y}_2\right) f + \psi^{i1}\psi^2_i\, f + B\, y^1 y^2\, \partial_z f \\ & + \xi^i\left(\psi_{i1}\, y^1 - \psi_{i2}\, y^2\right)\partial_z f - \frac{\xi_i\xi^i}{2}\, y^1 y^2\, \partial_z\partial_z f + \text{c.c.}\Big]. \end{aligned} \tag{32}$$

We also construct the Fayet-Iliopoulos Lagrangian in chiral superspace

$$S_{\rm FI} = -\frac{c}{4}\left[\int d\hat{t}_{\rm L}\, d^2\hat{\theta}\,\mathcal{V}_{\rm WZ} + \int d\hat{t}_{\rm R}\, d^2\bar{\hat{\theta}}\,\bar{\mathcal{V}}_{\rm WZ}\right] \quad\Rightarrow\quad \mathcal{L}_{\rm FI} = -\,c\,\mathcal{A}\,, \quad c = \text{const.} \tag{33}$$

The total Lagrangian is a sum of these two terms and the kinetic Lagrangian (13) for the dynamical multiplet on a Kähler manifold.

The total Lagrangian is written as

$$\mathcal{L} = \mathcal{L}_{\rm kin.} + \mathcal{L}_{\rm int.} + \mathcal{L}_{\rm FI}\,. \tag{34}$$

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Eliminating the auxiliary fields B and ψ^{iA} by their equations of motion and performing the following redefinition

$$
\begin{aligned}
&y^1 = v\,(f+\bar f)^{-\frac12}, \qquad y^2 = \bar w\,(f+\bar f)^{-\frac12}, \qquad \xi^i = g^{-\frac12}\,\eta^i,\\
&\bar y_1 = \bar v\,(f+\bar f)^{-\frac12}, \qquad \bar y_2 = w\,(f+\bar f)^{-\frac12}, \qquad \bar\xi_j = g^{-\frac12}\,\bar\eta_j\,,
\end{aligned} \tag{35}
$$

we obtain the total on-shell Lagrangian (up to full time derivatives) as

$$
\begin{aligned}
\mathcal{L} \;=\;& g\,\dot{\bar z}\dot z + \frac{i}{2}\left(\eta^i\dot{\bar\eta}_i - \dot\eta^i\bar\eta_i\right) + \frac{i}{2}\,\mu\left(v\dot{\bar v} + w\dot{\bar w} - \dot v\bar v - \dot w\bar w\right)\\
&- \frac{i}{2}\,m\cos 2\lambda\left(\dot{\bar z}\,\partial_{\bar z}K - \dot z\,\partial_z K\right) + \frac{i\mu\left(\dot{\bar z}\,\partial_{\bar z}\bar f - \dot z\,\partial_z f\right)}{2(f+\bar f)}\left(v\bar v - w\bar w\right)\\
&+ \frac{i}{2}\left(\dot{\bar z}\,\partial_{\bar z}g - \dot z\,\partial_z g\right)g^{-1}\,\eta^k\bar\eta_k - \frac{m\cos 2\lambda}{2}\,\eta^k\bar\eta_k - \frac{\mu\,\partial_z f\,\partial_{\bar z}\bar f}{(f+\bar f)^2}\left(v\bar v - w\bar w\right)g^{-1}\,\eta^k\bar\eta_k\\
&- \frac{\mu\,v\bar w}{(f+\bar f)}\left[\frac{\partial_z\partial_z f}{2} - \frac{\partial_z f\,\partial_z f}{(f+\bar f)} - \frac{\partial_z f}{2}\,g^{-1}\,\partial_z g\right]g^{-1}\,\eta_i\eta^i\\
&- \frac{\mu\,w\bar v}{(f+\bar f)}\left[\frac{\partial_{\bar z}\partial_{\bar z}\bar f}{2} - \frac{\partial_{\bar z}\bar f\,\partial_{\bar z}\bar f}{(f+\bar f)} - \frac{\partial_{\bar z}\bar f}{2}\,g^{-1}\,\partial_{\bar z}g\right]g^{-1}\,\bar\eta^j\bar\eta_j\\
&+ \frac{m\sin 2\lambda}{4}\left[\left(\partial_z\partial_z K - g^{-1}\,\partial_z K\,\partial_z g\right)g^{-1}\,\eta_i\eta^i + \left(\partial_{\bar z}\partial_{\bar z}K - g^{-1}\,\partial_{\bar z}K\,\partial_{\bar z}g\right)g^{-1}\,\bar\eta^j\bar\eta_j\right]\\
&- g^{-1}\left(\frac{\mu\,w\bar v\,\partial_z f}{(f+\bar f)} - \frac{m\sin 2\lambda}{2}\,\partial_z K\right)\left(\frac{\mu\,v\bar w\,\partial_{\bar z}\bar f}{(f+\bar f)} - \frac{m\sin 2\lambda}{2}\,\partial_{\bar z}K\right)\\
&+ \frac14\left(\partial_z\partial_{\bar z}g - g^{-1}\,\partial_z g\,\partial_{\bar z}g\right)g^{-2}\,\eta_i\eta^i\,\bar\eta^j\bar\eta_j + \left[\mu\left(v\bar v + w\bar w\right) - c\right]\mathcal{A}\,.
\end{aligned} \tag{36}
$$

Looking at the last term, one concludes that the U(1) gauge field $\mathcal{A}$ plays the role of a Lagrange multiplier enforcing the constraint

$$
\mu\left(v\bar v + w\bar w\right) - c = 0. \tag{37}
$$

Classical Hamiltonian reads

$$
\begin{aligned}
\mathcal{H} \;=\;& g^{-1}P_z P_{\bar z} - \frac14\left(\partial_z\partial_{\bar z}g - g^{-1}\,\partial_z g\,\partial_{\bar z}g\right)g^{-2}\,\eta_i\eta^i\,\bar\eta^j\bar\eta_j + \frac{2\,\partial_z f\,\partial_{\bar z}\bar f\,S_3}{g\,(f+\bar f)^2}\,\eta^k\bar\eta_k + \frac{m\cos 2\lambda}{2}\,\eta^k\bar\eta_k\\
&+ \frac{S_+}{g\,(f+\bar f)}\left[\frac{\partial_z\partial_z f}{2} - \frac{\partial_z f\,\partial_z f}{(f+\bar f)} - \frac{\partial_z f}{2}\,g^{-1}\,\partial_z g\right]\eta_k\eta^k\\
&+ \frac{S_-}{g\,(f+\bar f)}\left[\frac{\partial_{\bar z}\partial_{\bar z}\bar f}{2} - \frac{\partial_{\bar z}\bar f\,\partial_{\bar z}\bar f}{(f+\bar f)} - \frac{\partial_{\bar z}\bar f}{2}\,g^{-1}\,\partial_{\bar z}g\right]\bar\eta^k\bar\eta_k\\
&- \frac{m\sin 2\lambda}{4}\left[\left(\partial_z\partial_z K - g^{-1}\,\partial_z K\,\partial_z g\right)g^{-1}\,\eta_i\eta^i + \left(\partial_{\bar z}\partial_{\bar z}K - g^{-1}\,\partial_{\bar z}K\,\partial_{\bar z}g\right)g^{-1}\,\bar\eta^j\bar\eta_j\right]\\
&+ g^{-1}\left[\frac{\partial_z f\,S_+}{(f+\bar f)} - \frac{m\sin 2\lambda}{2}\,\partial_z K\right]\left[\frac{\partial_{\bar z}\bar f\,S_-}{(f+\bar f)} - \frac{m\sin 2\lambda}{2}\,\partial_{\bar z}K\right],
\end{aligned} \tag{38}
$$

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where

$$
\begin{aligned}
S_3 &= \frac{\mu}{2}\,(v\bar{v} - w\bar{w})\,, \qquad S_+ = \mu\, v\bar{w}, \qquad S_- = \mu\, w\bar{v}, \\
P_z &= p_z - \frac{i}{2}\, m\cos 2\lambda\; \partial_z K + \frac{i\,\partial_z f\, S_3}{(f+\bar{f}\,)} + \frac{i}{2}\, g^{-1}\,\partial_z g\, \eta^i \bar{\eta}_i\,, \\
P_{\bar{z}} &= p_{\bar{z}} + \frac{i}{2}\, m\cos 2\lambda\; \partial_{\bar{z}} K - \frac{i\,\partial_{\bar{z}} \bar{f}\, S_3}{(f+\bar{f}\,)} - \frac{i}{2}\, g^{-1}\,\partial_{\bar{z}} g\, \eta^j \bar{\eta}_j\,.
\end{aligned} \tag{39}
$$

Poisson (Dirac) brackets are imposed as

$$
\begin{aligned}
&\{p_z\,, z\} = -1, \qquad \{p_{\bar{z}}\,, \bar{z}\} = -1, \qquad \left\{\eta^i, \bar{\eta}_j\right\} = -i\delta^i_j\,, \\
&\{v, \bar{v}\} = i\mu^{-1}, \qquad \{w, \bar{w}\} = i\mu^{-1}.
\end{aligned} \tag{40}
$$

3.1 Spin variables

The generators S_3 and $S_\pm$, written trough spin variables, form the $su(2)$ algebra:

$$
\{S_3, S_\pm\} = \mp\, iS_\pm\,, \qquad \{S_+, S_-\} = -\,2iS_3\,. \tag{41}
$$

The Hamiltonian commutes with the Casimir operator

$$
C_{\mathrm{SU}(2)} = S_+S_- + (S_3)^2\,. \tag{42}
$$

According to the constraint (37) the Casimir operator is determined by the constant

$$
C_{\mathrm{SU}(2)} = \frac{\mu^2\,(v\bar{v} + w\bar{w})^2}{4} = \frac{c^2}{4}\,. \tag{43}
$$

Its quantum counterpart (up to the ordering ambiguity) is given by ($c \approx 2s$)

$$
C_{\mathrm{SU}(2)} = s\,(s+1)\,, \tag{44}
$$

where s is a spin of the quantum system. Since the Hamiltonian commutes with the Casimir operator the spin of the system is preserved.

4. Conclusions

We proposed new models of SU(2|1) supersymmetric mechanics with the use of dynamical and gauged semi-dynamical multiplets. As an alternative of harmonic superspace, we exploited the generalized chiral superspace. We tried to obtain superconformal symmetry for considered models, but it is impossible at least for a single-particle model. Superconformal models can possibly be obtained for two or more chiral dynamical multiplets. It would be interesting to study mirror counterparts of the multiplets $(\mathbf{1},\mathbf{4},\mathbf{3})$ and $(\mathbf{3},\mathbf{4},\mathbf{1})$. The latter one is described by a triplet consisting of real and chiral superfields, *i.e.* X, $\mathcal{V}$ and $\bar{\mathcal{V}}$. For example we can couple the mirror multiplets $(\mathbf{3},\mathbf{4},\mathbf{1})$ and $(\mathbf{4},\mathbf{4},\mathbf{0})$ in SU(2|1) chiral superspace.

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