Journal of Physics: Conference Series **3177** (2026) 012035 doi:10.1088/1742-6596/3177/1/012035

Impact of $f(R)$ gravity on gravitational waves propagation

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Abstract. We explore the propagation of gravitational waves (GWs) in context of metric $f(R)$ gravity under weak field approximation. We observe that the scalar degree of freedom arising from $f(R)$ gravity modifies the wave equation for GWs leading to additional scalar modes of polarization. The background matter distribution influences GWs propagation affecting their speed. Our study also focuses on the observational constraints which limit the detection of these deviations in GWs signals from General Relativity (GR), thus providing insights into the challenges of testing extended gravity theories.

1 Introduction

Gravitational Waves (GWs) provide a new window to look into the universe and allow us to probe regions of spacetime which are otherwise inaccessible through electromagnetic observations. In the standard framework of GR, GWs possess only two transverse-traceless tensor polarization modes. Different extended theories of gravity predict additional degrees of freedom that give rise to extra polarization states of GWs [1]. This gives us the motivation to explore dynamics of GWs in modified gravity theories and look for characteristics which show deviation from GR [2].

In this work, we investigate the dynamics of GWs in metric $f(R)$ gravity under the weak-field approximation.We begin by deriving the gravitational wave (GW) equations in metric $f(R)$ gravity. In the next section we explore the propagation of GWs in the presence of background perturbed scalar field for early and late-time universe. We further examine the distribution of energy among different scalar polarization modes. Finally, we discuss the observational challenges and constraints associated with detecting scalar polarizations.

2 Gravitational waves in $f(R)$ gravity

In $f(R)$ gravity, action is defined as

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\, f(R) + S_m, \tag{1}$$

where $\kappa^2 = 8\pi\mathrm{G}$, $f(R)$ is a function of Ricci scalar R, and S_m is the matter contribution to total action [3]. The corresponding field equations are obtained by varying the action with respect to metric $g_{\mu\nu}$ which in vacuum become,

$$F(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu\nabla_\nu F(R) + \Box F(R)g_{\mu\nu} = 0, \tag{2}$$

where $F(R) \equiv \frac{\partial f}{\partial R}$. In vacuum, the term $\Box F(R)$ does not vanish and represents propagating scalar degree of freedom, $F(R) \equiv \varphi$.

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With a motivation to study GWs, we use linearized gravity condition given as,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{3}$$

$$\varphi = \varphi_0(1 + h_s), \tag{4}$$

where $h_s = \frac{\delta\varphi}{\varphi_0}$. Applying this approximation to $f(R)$ gravity field equations gives linearized field equation in $f(R)$ gravity as

$$R^{(1)}_{\mu\nu} - \frac{1}{2}R^{(1)}\eta_{\mu\nu} = \frac{1}{2}[\partial_\mu\partial^\alpha h_{\nu\alpha} + \partial_\nu\partial^\alpha h_{\mu\alpha} - \partial_\mu\partial_\nu h - \Box h_{\mu\nu}] - \frac{1}{2}\eta_{\mu\nu}(\partial_\mu\partial_\alpha h^{\alpha\mu} - \Box h). \tag{5}$$

We impose the Lorenz gauge condition

$$\nabla^\mu h_{\mu\nu} = 0, \tag{6}$$

and under this condition, equation (5) yields the GW equation as

$$\Box\bar{h}_{\mu\nu} = 0. \tag{7}$$

We define here, $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}h\eta_{\mu\nu} - \eta_{\mu\nu}h_s$. The quantity $\bar{h}_{\mu\nu}$ is not traceless thus there exists additional contribution in GWs from scalar field perturbations.

3 Propagation of scalar perturbations in $f(R)$ gravity

The scalar perturbation h_s in metric $f(R)$ gravity for flat Friedmann-Robertson-Walker (FRW) metric satisfies the modified Klein-Gordon equation

$$\ddot{h}_s + 3H\dot{h}_s + \left(\frac{k^2}{a^2} + m^2_{\text{eff}}\right)h_s = 0, \tag{8}$$

where H is Hubble parameter, a is scale factor, k is wave number, $m^2_{\text{eff}} = \frac{d^2V}{d\phi^2}|_{\phi_0}$, $\phi_0 = \phi_{min}$. In the early universe (Starobinsky: $f(R) = R + \alpha R^2$), $H \approx 10^{14}$ GeV [4, 5, 6].

- Sub-horizon ($k \gg aH$, $m_{\text{eff}} \geq H$) solutions are $h_s \propto a^{-1}e^{\pm ik\tau}$.
- Super-horizon ($k \ll aH$, $m_{\text{eff}} \leq H$) solutions are $h_s \approx A + Be^{-3Ht}$.

In late-time universe, defining Hu-Sawicki $f(R)$ gravity model [7]

$$f(R) = -m^2 c_1 \frac{c_1\left(\frac{R}{m^2}\right)^n}{c_2\left(\frac{R}{m^2}\right)^n + 1}, \tag{9}$$

we obtain $m_{\text{eff}} = 2.58 \times 10^{-39}$GeV, for $m^2 = 0.675 \times 10^{-84}$ GeV^2, n = 1, $R_0 = 12\ H_0^2$, $H_0 \approx 1.5 \times 10^{-42}$ GeV and $h_s \approx e^{-3H_0 t}$ (C cosωt + D sinωt), with $\omega = \frac{\sqrt{4M^2 - 9H_0^2}}{2}$ [8, 9] .

Epoch	H (GeV)	m_{eff} (GeV)	h_s
Early (Sub-horizon)	$\approx 10^{14}$	$\geq H$	Oscillatory solution
Early (Super-horizon)	$\approx 10^{14}$	$\ll H$	Frozen amplitude
Late-time	$\approx 1.5 \times 10^{-42}$	$\approx 2.58 \times 10^{-39}$	Decaying oscillatory amplitude

Table 1: Scalar perturbation amplitudes in early and late universe.

In the early universe, the scalar mode is long-ranged, having a significant effect on GWs and inflationary dynamics. In contrast, in the late-time universe, the scalar mode becomes short-ranged and effectively decouples, recovering GR locally.

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4 Energy distribution in scalar modes

In the Einstein frame, the $f(R)$ gravity action originally expressed in the Jordan frame as a function of the Ricci scalar is transformed through a conformal rescaling of the metric defined as [10]

$$S_E = \int \sqrt{-\tilde{g}}\left[\frac{1}{2\kappa^2}\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\partial_\nu\phi - V(\phi)\right]d^4x + \int \sqrt{-\tilde{g}}\mathcal{L}_{\mathcal{M}}(F^{-1}(\phi)\tilde{g}_{\mu\nu}\psi_M)d^4x, \tag{10}$$

where $\tilde{g}_{\mu\nu} = \Omega^2\ g_{\mu\nu}$, $\Omega^2 = F$ is conformal factor and $\phi = \sqrt{\frac{3}{2\kappa^2}}\mathrm{lnF}$. The scalar field can have kinetic degree of freedom for propagation and potential degree of freedom for mass term as defined in the previous section. Now, scalar breathing mode is massless, and thus no contribution from potential degree of freedom arises. Therefore, in case both longitudinal and breathing scalar modes accquire same amount of energy, the longitudinal mode will have less kinetic contribution. This implies that the longitudinal mode would propagate at slower speed. When GWs propagate in the universe they pass through regions of varying mass densities. In regions of high mass density, $m_{\rm eff}$ increases and thus longitudinal mode dominates over breathing mode. In regions of low mass density, scalaron becomes light and the scalar perturbation exhibit breathing polarization mode. Thus, in this manner, energy carried by scalar perturbation gets redistributed between the longitudinal and breathing scalar modes [11, 12].

5 Observational constraints

Scalar polarization modes have not been observed yet. Current GW detectors are designed to be sensitive only to distortions produced along mutually perpendicular directions in transverse plane (the tensorial plus and cross modes) and not sensitive to transverse breathing mode and longitudinal mode. Thus, scalar distortions either isotropic deformations in the transverse plane (breathing mode) or displacements along the direction of propagation (longitudinal mode) cannot be observed in present detectors[13].

6 Conclusion

We studied GWs in $f(R)$ gravity under weak field approximation. The background perturbations in scalar field distort the wave equation of GWs exciting additional scalar modes (transverse breathing and longitudinal modes). We have discussed how the scalar perturbations propagate with the evolution of the universe. These scalar modes are currently undetected because of limitations due to interferometer design. Detection of scalar polarization modes or any deviation from the predictions of GR in future observations will provide a powerful test for modified gravity theories.

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