The Oblate Sun and Relativity

Our knowledge of the Sun and stars is, like beauty, skin-deep. Most of the light reaching us comes from the photosphere which (for the Sun) has a thickness (or, rather, scale height) of the order of 100 km (compared with the solar radius $R_\odot = 7.0 \times 10^5$ km), a density of order 10^{-7} g/cm^3 (compared with an average density of order 1 g/cm^3), and containing a fraction less than 10^{-10} of the Sun's mass $M_\odot$.

A detailed study of the shape and width of spectral lines from various elements can give the rotational velocity of a star's photosphere (if the velocity is large enough). For the Sun we can observe surface features and thus measure the photosphere's rotational period directly—a 27-day period corresponding to a surface velocity of 2 km/sec. For main-sequence stars there is a fairly unique relation between rotational surface velocity and mass. This relation[1] is shown (somewhat schematically) in Fig. 1 and its most important feature is the drastic drop near 3.5 $M_\odot$: An extrapolation of the high-mass velocities down to the solar mass would lead to rotation periods about 25 times shorter than observed for the solar surface.

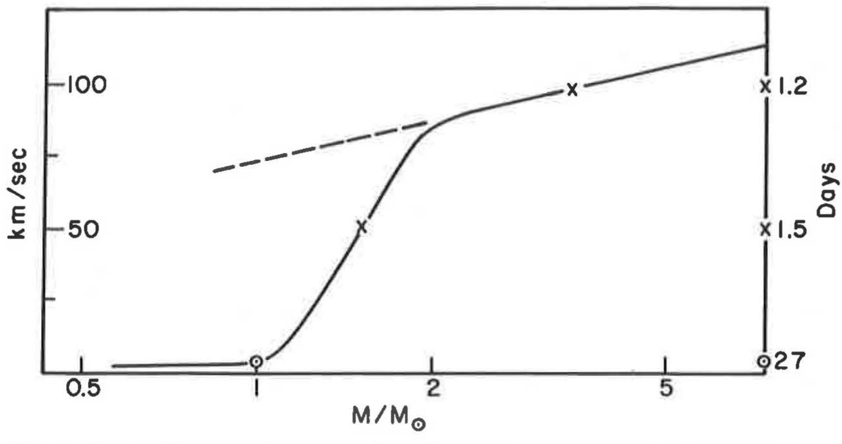

FIG. 1. The surface rotational velocity of a main-sequence star as a function of its mass. The rotational period is given (on the right) for the Sun and two other points.

From a theoretical point of view the high rotational speeds of the massive stars are reasonable and the low speeds for Sun-like stars are puzzling, if one assumes "solid body rotation" (i.e., the same rotational period in the star's interior where the bulk of the mass resides as at the surface). Stars are thought to possess a considerable amount of angular momentum at the early stages of formation. It is gratifying that the Sun's planetary system stores much angular momentum and it is plausible to hope for mechanisms for a protostar to shed sufficient angular momentum so that the ratio of the star's rotational kinetic energy to its thermal energy content is less than unity but not negligibly small. This is the case for the massive main-sequence stars but not for the Sun-like ones; the Sun's typical rotational velocity would have to be about 25 times the surface velocity to be about 10% of typical interior thermal velocities.

By chance, a number of properties of main-sequence stars change over rapidly with mass near 1.5 $M_{\odot}$. One of these properties is the presence of an outer convective zone in the less massive (and cooler) stars; convection in the solar surface is partly responsible for the solar wind and the solar wind ejected from the Sun can provide some magnetic braking of rotation. Rates of transfer of angular momentum are difficult to calculate reliably but Dicke's[2] estimates are such that the solar interior could be rotating rapidly (with a period of the order of a day) providing a drag on the outer convective zone (and photosphere) which balances the solar wind drag on this zone at the observed period of 27 days. Only a small fraction of the Sun's angular momentum would have leaked out through the solar wind in 5×10^9 years (and the convective zone contains only a small fraction of the Sun's mass). Dicke's conjecture of a rapidly rotating interior cannot be proved on purely theoretical grounds, but it has some observable consequences.

A rapidly rotating solar interior will cause some oblateness of the interior mass distribution, since the effective potential $\Phi(r, \theta)$ in a rotating coordinate system is angle-dependent:

$$\Phi(r, \theta) = \phi(r, \theta) - \tfrac{1}{2}r^2\omega^2(r) \sin^2 \theta, \tag{1}$$

where ϕ is the actual gravitational potential, θ the angle which the radius vector $\mathbf{r}$ makes with the rotation axis, and $\omega(r)$ the angular velocity at r. For an interior rotation period of one day, say, the interior oblateness is of order 10^{-4} (or slightly larger), which produces a small quadrupole moment Q. Since the mass in the Sun's outer layers is quite negligible, the gravitational potential near the surface and everywhere outside can be approximated by

$$\phi(r, \theta) = \frac{GM}{r}\left[1 + \frac{Q}{r^2}(3 \cos^2 \theta - 1)\right]. \tag{2}$$

The exact relation between interior rotation and Q depends on the physical conditions in the interior, but the relation between Q and the *surface* oblateness is simple and unique if the material near the photosphere is a perfect gas (no matter how peculiar the material in the deep interior might be): the density and temperature are constant along the photosphere, which is a surface of constant Φ. With the surface oblateness defined as $\Delta \equiv (r_{\text{equator}} - r_{\text{pole}})/r$, the centrifugal term (with *surface* values for r and ω) in Eq. (1) contributes only 1.0×10^{-5} to Δ. A measurement of any excess of Δ above this value then gives directly a value for the quadrupole moment Q in Eq. (2) ($\Delta = 5 \times 10^{-5}$, for instance, gives Q such as could be produced by an interior rotation period of about a day).

A solar surface oblateness of $\Delta = 5 \times 10^{-5}$ corresponds to a height difference of about 35 km (about a third of a photospheric scale height), which subtends an angle difference at the Earth of only $0.05''^{\text{arc}}$. Turbulence in the Earth's atmosphere introduces angular spreads of about $5''^{\text{arc}}$ under sunlight seeing conditions. To determine an isophote line to better than 1% of the spread would be very difficult and Dicke and Goldenberg[3] measured instead variations about the solar limb of the light intensity *outside* of an accurately circular occulting disc: a rotating scanning disc with two diametrically opposed apertures of slightly different size was employed. Any error in the centering of the occulting disc relative to the center of the Sun gives a 1st harmonic intensity variation and was eliminated by a servo system. The 2nd harmonic intensity variations (measured to better than 10^{-3} of the mean intensity) then give the change in intensity from the equator to the pole. The experiment was repeated with different amounts of the solar limb (varying from 6 to $20''^{\text{arc}}$ inside the photosphere) exposed by the occulting disc (as well as about $20''^{\text{arc}}$ outside the photosphere).

These experiments give separately (1) the variation of the brightness "temperature" around an accurately circular thin annulus completely *inside* the solar surface and (2) the intensity variation *outside* of a circle which leads to the oblateness. No measurable brightness variation was found (less than 3°K "temperature" variation from equator to pole). After eliminating various sources of systematic errors, Dicke and Goldenberg[3] found for the solar surface oblateness

$$\Delta = (5 \pm 0.7) \times 10^{-5}. \qquad (3)$$

If one assumes the absence of any mechanism which could give shear-strength to the photospheric gas, the measured value of Δ refers to a surface of constant Φ and Eqs. (1) and (2) give a unique value for the quadrupole moment Q. For a planetary orbit in the plane perpendicular to the Sun's symmetry axis ($\theta = \pi/2$) the angular variation in Eq. (2) does not matter, but the inverse cube dependence on radial distance r contributes to the perihelion advance of an elliptical orbit. The second-order terms of General Relativity modify the Newtonian potential to give[4] (for a spherical Sun)

$$\phi(r) \approx \frac{GM}{r}\left[1 + \frac{r_0\langle r\rangle}{r^2}\right], \tag{4}$$

where $\langle r\rangle$ is the average radius of the orbit and r_0 is a constant of the order of the Sun's "gravitational radius" (~2 km). For a given orbit the effects of an oblate Sun and of General Relativity are similar but the perihelion advance per orbital period is proportional to $\langle r\rangle^{-1}$ from Relativity and to $\langle r\rangle^{-2}$ from oblateness.

For the planet Mercury, General Relativity predicts a perihelion advance of $43.0''^{\text{arc}}$/century and the quadrupole moment Q implied by Eq. (3) contributes another $3.4''^{\text{arc}}$/century. The observations on Mercury are quite accurate but a number of complicated classical corrections have to be allowed for, including (in arc-second/century) about a 5000″ geometric correction (which presumably is well understood) and 575″ dynamic corrections (of which 280″ and 153″ are due to Venus and Jupiter perturbations). I personally cannot judge the probability of some unknown systematic error in this analysis of the observational data but, if there are none and if the solar oblateness contribution is correct, the observations imply a relativistic effect of only $(39.7 \pm 0.5)''^{\text{arc}}$/century, about 10% less than predicted by the orthodox theory of General Relativity. For Venus and Earth the predicted relativistic advance rate is only $8.6''^{\text{arc}}$/century and $3.8''^{\text{arc}}$/century, respectively, and the observations plus analysis are not yet accurate enough to detect discrepancies of the order of 10%.

Some time ago Brans and Dicke[5] proposed a modification to the theory of General Relativity in which a weak scalar field is introduced whose effective coupling strength s is a disposable parameter. With s defined in suitable units this modification multiplies the relativistic predictions for the planetary perihelion advance rate and for the deflection of light by the Sun by factors of $(1 - \frac{4}{3}s)$ and $(1 - s)$, respectively, if $s \ll 1$ (the gravitational red shift is unaffected). The relativistic predictions for proposed[6,7] experiments on time delays of a radar signal passing close to the Sun would also be multiplied by $(1 - s)$. It is hoped that such an experiment or a remeasurement of the deflection of light will give an accurate value (or upper limit) for s in the foreseeable future.

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References

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