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New charged anisotropic solution in $f(Q)$-gravity and effect of non-metricity and electric charge parameters on constraining maximum mass of self-gravitating objects

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Abstract In the present article, A new class of singularity-free charged anisotropic stars is derived in $f(Q)$-gravity regime. To solve the field equations, we assume a particular form of anisotropy along with an electric field and obtain a new exact solution in $f(Q)$-gravity. The explicit mathematical expression for the model parameters is derived by the smooth joining of the obtained solutions with the exterior Reissner–Nordstrom de-Sitter solution across the bounding surface of a compact star along with the requirement that the radial pressure vanishes at the boundary. We have modeled four self-gravitating pulsar objects such as LMC X-4, PSR J1903+327, PSR J1614-2230, and GW190814 in our current study and predict the radii of these objects that fall between 8 and 10 km. Furthermore, the physical validity of the solution is performed for self-gravitating object PSR J1614-2230 with mass $1.97 \pm 0.04\ M_\odot$ with radius 10 km. The solution successfully fulfills all the physical requirements along with the stability and hydrostatic equilibrium conditions for a well-behaved model. The non-metricity $f(Q)$-parameter χ_1 and electric charge parameter η play an important role in the maximum mass of the objects. The maximum mass increases when χ_1 and η increase but a non-collapsing stable object can be obtained when $\chi_1 \leq 0.0205$ and $\eta \leq 0.0006$.

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1 Introduction

A lot of undiscovered cosmic events and activities got revealed theoretically with its marvelous enigmas and astonishing features after the development of general relativity. Although the general theory of relativity has been very effective in describing different cosmological and astrophysical phenomena, it has some constraints in theoretical and observational studies regarding the Universe and various astrophysical events. For example, singularities are often unavoidable in the framework of general relativity. Moreover, general relativity is unable to give an adequate explanation for the dynamics of galaxies, extra-galactic systems, and the cosmos as a whole without considering the existence of dark matter and dark energy. Recent astrophysical observations related to GW170817 and GW190814 events have been obtained with engaging outcomes by the LIGO-VIRGO collaboration. This motivates the researchers to reassess their techniques to model the compact objects participating in binary mergers which act as sources of gravitational wave radiation. Then again, the $GW170817$ event [1] is attributed to the coalescence of two neutron stars of masses falling within the range $0.86 - 2.26\ M_{\dot{o}}$. Now, attaining stellar masses beyond $2\ M_{\dot{o}}$ in the framework of standard general relativity without restoring to exotic matter or rotation proved to be a daunting endeavor for the theorists, but their passion and dedication plunged them deeper into the challenges.

In this situation, it would be compelling to delve into the celestial world to explore the structural properties of astrophysical objects or the important features of spacetime with the help of observational data constraints studied and analyzed in the form of subtle refinement of conventional gravity theory. Under these circumstances, questing for proper expla-

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nations to the mysteries of the Universe, various researchers [2–5] have proposed different modified theories of gravity mainly from a geometric perspective. In literature, one can find some noteworthy modified theories of gravity such as $f(R)$ gravity [6–9], $f(T)$ gravity [10–12], $f(R, \mathcal{T})$ gravity [13–16] which include a modified geometrical component in the appropriate action. In a similar way, the nonmetricity scalar $\mathcal{Q}$ can be chosen as the geometric basis to represent "symmetric teleparallel general relativity" (STGR) [17–20] which is a different but analogous description of gravity. Considering the gravitational Lagrangian to be the function of nonmetricity $\mathcal{Q}$, STGR can be extended to $f(\mathcal{Q})$ gravity [20,21]. In the present astrophysical study of compact stars, we are motivated to utilize $f(\mathcal{Q})$ gravity not only for its an important historical background proposed in a research work by Einstein [22] related to "Teleparallelism" or "teleparallel gravity" but also for its huge application in cosmological [23–27] and astrophysical [28–50] studies.

In the regime of $f(\mathcal{Q})$-gravity, there are various physically plausible compact star models available in the literature [33–36,46,47]. In a recent work [51], a stellar model representing dark energy star in $f(\mathcal{Q})$-gravity proposed that the secondary component of the GW190814 event could be a possible candidate for dark energy star. The speculation came from the fact that the linear form of $f(\mathcal{Q})$ can generate the dark energy stars lie within the mass gap range i.e., 2.5-5 $M_{\odot}$. Some further researches [52,53] considering the non linear form of $f(\mathcal{Q})$ explored the possible existence of dark energy stars with astrophysical implications related to the mass gap range. In this case modified Chaplygin gas is considered under the Krori–Barua (KB) spacetime being equated with the exterior Reissner–Nordström space-time at the boundary to get the solutions to the Einstein–Maxwell field equations in $f(\mathcal{Q})$-gravity. In different aspect, some configurations of different black holes developed in the context of $f(\mathcal{Q})$-gravity could be of particular interest [32,48–50].

In general, stellar matter is considered to be an isotropic fluid to construct stellar configurations of compact stars like neutron star. Nevertheless the presence of various physical phenomena such as superfluidity and superconductivity may rise to the difference in pressure along radial and tangential directions. This is termed as the local anisotropy in pressure which can occur at least in some specific ranges of density [54,55] in the dense matter subject to complex nature of strong interactions in the matter. Additionally, there are some findings which can shed light on the origins of local anisotroy. These findings include interactions with high magnetic field [56–63], nuclear interactions in relativistic regime [64], condensate of pions [65], viscosity [66–69], superfluid core [70–72] which may explain the mechanism of local anisotropy in pressure in the stellar fluid. Undoubtedly, it would be fascinating to study how stellar models will change due to incorporation of anisotropy in the fluid. In one such study considering the effect of anisotropy Bowers and Liang [73] first developed solutions to anisotropic spheres in the framework of general relativity. They have assumed necessary conditions for instance quadratic vanishing of the central anisotropy and nonlinear dependence of anisotropy on radial pressure. Afterwards, the consideration of local anisotropy becomes a realistic one in many astrophysical studies [74–78] of compact objects. In fact, there are publications that unveil the astounding effects of anisotropy on the structural and observable properties of neutron star which include the maximum mass [79,80], moment of inertia [81], redshift [82], tidal deformability [83–85] and non-radial oscillation [86]. Moreover, some unstable configurations can be stabialized by anisotropy [87–89].

There is a new family of solutions [90,91] which represents compact stars with anisotropy in pressure and quintessence dark energy component to sustain its stability in framework of $f(\mathcal{Q})$-gravity. Moreover, a physically anisotropic compact star configurations with inclusion of quintessence can be constructed even in other modified theories of gravity such as in $f(T)$ gravity [92]. Again, in $f(R, T)$ gravity theory one can find mathematically well behaved and physically valid set of solutions [93–95] to the field equations to explore features of a compact star in embedding class one spacetime. Furthermore, several studies on anisotropic compact objects by Nashed and his collaborators in different gravity theories can be found in literature [96–99].

Another important feature to the stellar fluids is the presence of electric field which was first highlighted by Rosseland [100] and subsequently by various researchers [101–103]. Despite the fact that the astrophysical configurations are in general electrically neutral, recent researches [104–106] advocates in behalf of the possibilities that include the existence of massive and charged astrophysical system. The physical mechanism for gaining a net amount of charge to the astrophysical configurations is associated mainly with the accretion process related to the surrounding medium. The immediate effect of the presence of charge is that it can resist the gravitational collapse of the stellar charged sphere by means of strong electric force along with hydrostatic force and anisotropic force. It is argued by Stettner [107] that in comparison to neutral fluid, charged fluid in various situations could offer stable configuration satisfying required physical conditions. Moreover, it is speculated in some works [108,109] that the presence of charge hinders the growth of curvature in spacetime and plays a vital role to overcome singularity problem in the stellar models. Further, it is important to note that the role [110] of charge in fluids can be related to the origin of anisotropy in pressure in the charged stellar fluid.

In the present paper, considering all the fact mentioned above, we concentrate on the mechanism of developing an analytical model for anisotropic and charged fluid spheres in the context of $f(\mathcal{Q})$-gravity theory. We have assumed the anisotropy function and charge function in such a way that these functions have a direct dependence on the $f(\mathcal{Q})$-gravity parameter. Hence, the direct effect of non-metricity scalar ($\mathcal{Q}$) in constructing stellar models can be realized by physical analysis of the solutions to the $f(\mathcal{Q})$-gravity field equations in the present investigation. An ansatz related to the radial component of spherically symmetric metric coefficients tensor is assumed to be non-linear but well behaved so that all the physical quantities in connection to stellar configuration can be determined exactly.

The plan of the paper is structured as follows: in Sect. 2, a brief account of $f(\mathcal{Q})$-gravity theory is given. Then, the corresponding field equations for an anisotropic and charged system are expressed in Sect. 3. Assuming anisotropy and charge function for the fluid and considering the radial component of the metric tensor, solutions to the field equations are obtained and expressed in Sect. 4. In the subsequent Sects. 5 and 6, the physical analysis of the solutions and stability analysis of the stellar system have been presented with graphical illustrations. Final remarks are reported in Sect. 7.

2 Review of the field equations for $f(Q)$-gravity

For the $f(\mathcal{Q})$ gravity, the extended version of gravitational action is expressed as:

$$\mathcal{S} = \underbrace{\int \frac{1}{2}\, f(\mathcal{Q})\sqrt{-g}\, d^4x + \int \mathcal{L}_m \sqrt{-g}\, d^4x}, \tag{1}$$

where $\mathcal{L}_m$ denotes the Lagrangian density, which provides the matter fields in the $f(\mathcal{Q})$ gravity. The expression g represents the determinant of the involved metric tensor $g_{\epsilon\beta}$. In order to understand nonmetricity term $\mathcal{Q}$, the nonmetricity tensor $\mathcal{Q}_{\lambda\epsilon\beta}$ in terms of affine connection is defined as

$$\mathcal{Q}_{\lambda\epsilon\beta} = \nabla_\lambda g_{\epsilon\beta} = \partial_\lambda g_{\epsilon\beta} - \Gamma^{\delta}{}_{\lambda\epsilon} g_{\delta\beta} - \Gamma^{\delta}{}_{\lambda\beta} g_{\epsilon\delta}, \tag{2}$$

where $\Gamma^{\delta}{}_{\epsilon\beta}$ is defining the affine connection, which is further described as:

$$\Gamma^{\delta}{}_{\epsilon\beta} = \{^{\delta}{}_{\epsilon\beta}\} + K^{\delta}{}_{\epsilon\beta} + L^{\delta}{}_{\epsilon\beta}, \tag{3}$$

where $L^{\delta}{}_{\epsilon\beta}$, $\{^{\delta}{}_{\epsilon\beta}\}$ and $K^{\delta}{}_{\epsilon\beta}$ are the disformation, Levi–Civita connection, and contortion tensors respectively. All the above-mentioned important components are expressed as:

$$\begin{aligned}
\{^{\delta}{}_{\epsilon\beta}\} &= \frac{1}{2} g^{\delta\chi}\left(\partial_\epsilon g_{\chi\beta} + \partial_\beta g_{\chi\epsilon} - \partial_\chi g_{\epsilon\beta}\right),\\
L^{\delta}{}_{\epsilon\beta} &= \frac{1}{2}\mathcal{Q}^{\delta}{}_{\epsilon\beta} - \mathcal{Q}_{(\epsilon}{}^{\delta}{}_{\beta)},\\
K^{\delta}{}_{\epsilon\beta} &= \frac{1}{2}T^{\delta}{}_{\epsilon\beta} + T_{(\epsilon}{}^{\delta}{}_{\beta)}.
\end{aligned} \tag{4}$$

In the above relations $T^{\delta}{}_{\epsilon\beta}$ defines the torsion tensor, which is a necessary component to define the anti-symmetric portion of the affine connection, i.e., $T^{\delta}{}_{\epsilon\beta} = 2\Gamma^{\lambda}{}_{[\epsilon\beta]}$. The superpotential in the background of the nonmetricity tensor is calculated as follows:

$$P^{\zeta}{}_{\epsilon\beta} = \frac{1}{4}\left[-\mathcal{Q}^{\zeta}{}_{\epsilon\beta} + 2\mathcal{Q}^{\zeta}_{(\epsilon\ \beta)} + \mathcal{Q}^{\zeta} g_{\epsilon\beta} - \tilde{\mathcal{Q}}^{\zeta} g_{\epsilon\beta} - \delta^{\zeta}_{(\epsilon}\mathcal{Q}_{\beta)}\right], \tag{5}$$

where

$$\mathcal{Q}_{\zeta} \equiv \mathcal{Q}_{\zeta}{}^{\epsilon}{}_{\epsilon},\ \ \tilde{\mathcal{Q}}_{\zeta} = \mathcal{Q}^{\epsilon}{}_{\zeta\epsilon}. \tag{6}$$

The above equation provides two different and independent traces, which are very necessary to calculate nonmetricity scalar term as:

$$\mathcal{Q} = -\mathcal{Q}_{\zeta\epsilon\beta}\, P^{\zeta\epsilon\beta}. \tag{7}$$

In order to describe the extended version of field equations for $f(\mathcal{Q})$ theory, one can use the variation approach on action Eq. (1) with respect to the metric tensor $g_{\epsilon\beta}$ and get the following expression

$$\begin{aligned}
&\frac{2}{\sqrt{-g}}\nabla_\gamma\left(\sqrt{-g}\, f_{\mathcal{Q}}\, P^{\gamma}{}_{\epsilon\beta}\right) + \frac{1}{2}g_{\epsilon\beta} f\\
&\quad + f_{\mathcal{Q}}\big(P_{\epsilon\gamma i}\,\mathcal{Q}_{\beta}^{\ \gamma i} - 2\,\mathcal{Q}_{\gamma i\epsilon}\, P^{\gamma i}_{\ \ \beta}\big) = -T_{\epsilon\beta},
\end{aligned} \tag{8}$$

where $f_{\mathcal{Q}} = \frac{df}{d\mathcal{Q}}$, and $T_{\epsilon\,\beta}$ is the energy-momentum tensor, which is further defined as:

$$T_{\epsilon\beta} = -\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\,\mathcal{L}_m\right)}{\delta g^{\epsilon\beta}} \tag{9}$$

Moreover, from the Eq. (1), one can derive the extra constraint over the connection as:

$$\nabla_\epsilon\, \nabla_\beta\left(\sqrt{-g}\, f_{\mathcal{Q}}\, P^{\gamma}{}_{\epsilon\beta}\right) = 0. \tag{10}$$

The constraints through curvature less and torsionless under the affine connection are calculated as follows:

$$\Gamma^{\lambda}{}_{\epsilon\beta} = \left(\frac{\partial x^{\lambda}}{\partial \xi^{\beta}}\right)\partial_\epsilon\partial_\beta\xi^{\beta}. \tag{11}$$

Now, the nonmetricity expression from Eq. (2) under some constrain should be reduced to the following expression:

$$\mathcal{Q}_{\lambda\epsilon\beta} = \partial_\lambda g_{\epsilon\beta}, \tag{12}$$

which, as the metric function is the only fundamental variable, greatly simplifies the computation. Except for general relativity, the action is no longer diffeomorphism invariant in this scenario [24]. To get around this kind of problem, one can use the covariant formulation of $f(\mathcal{Q})$ gravity. One may consider the covariant formulation by first figuring out the affine connection in the absence of gravity, as the affine connection in Eq. (11) is completely inertial [111].

3 System of equations

To this end, we consider the standard static spherically symmetric line element of the form,

$$ds^2 = -H^2(r)dt^2 + N^2(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2, \quad (13)$$

Here, where $H(r)$ and $N(r)$ are metric potentials and depend upon the radial distance r which ensures that the spacetime is static. For the current analysis, we are going to work with an anisotropic matter distribution, then the effective energy-momentum tensor $T_{\epsilon\beta}$ can be expressed as:

$$T_{\epsilon\,\beta} = (\rho + P_t)\, u_\epsilon\, u_\beta + P_t\, g_{\epsilon\,\beta} + (P_r - P_t)\, v_\epsilon\, v_\beta, \quad (14)$$

where ρ and u_ϵ are the effective density and the four-velocity vector, respectively. Besides v_ϵ is the unitary space-like vector in the radial direction, p_r is the effective radial pressure in the direction of u_ϵ, and p_t is an effective tangential pressure orthogonal to v_ϵ. Now, the nonmetricity scalar for the metric (13) is calculated as:

$$\mathcal{Q} = -\frac{2\left(2rH'(r) + H\right)}{HN^2r^2}, \quad (15)$$

For the anisotropic fluid (16), the independent components of the equations of motion (8) in $f(\mathcal{Q})$ gravity are given as,

$$\rho = \frac{f(\mathcal{Q})}{2} - f_{\mathcal{Q}}(\mathcal{Q})\left[\mathcal{Q} + \frac{1}{r^2} + \frac{1}{rN^2}\left(\frac{2H'}{H} + \frac{2N'}{N}\right)\right] + E^2, \quad (16)$$

$$P_r = -\frac{f(\mathcal{Q})}{2} + f_{\mathcal{Q}}(\mathcal{Q})\left[\mathcal{Q} + \frac{1}{r^2}\right] - E^2, \quad (17)$$

$$P_t = -\frac{f(\mathcal{Q})}{2} + f_{\mathcal{Q}}(\mathcal{Q}) \left[\frac{\mathcal{Q}}{2} - \frac{1}{N^2}\left\{\frac{H''}{H} - \frac{H'2}{H^2} + \left(\frac{H'}{2H} + \frac{1}{2r}\right) \times \left(\frac{2H'}{H} - \frac{2N'}{N}\right)\right\} - E^2\right], \quad (18)$$

$$0 = \frac{\cot\theta}{2}\, \mathcal{Q}'\, f_{\mathcal{Q}\mathcal{Q}}, \quad (19)$$

where $f_{\mathcal{Q}}(\mathcal{Q}) = \frac{\partial f(\mathcal{Q})}{\partial \mathcal{Q}}$. In the background of $f(Q)$ theory, it is mentioned that Wang et al. [37] investigated the possible functional forms for $f(\mathcal{Q})$ gravity under the static and spherically symmetric spacetime with an anisotropic fluid. In particular, they have shown that there is no exact Schwarzschild solution for the nontrivial $f(\mathcal{Q})$ function, which can be understand from the following discussions:

The study undertaken by Wang et al. [37] focused on the off-diagonal component seen in Eq. (18). The investigation indicates that the solutions of the $f(Q)$ gravity theory are restricted to only two separate scenarios:

$$f_{\mathcal{Q}\mathcal{Q}} = 0 \quad \Rightarrow \quad f(\mathcal{Q}) = -\chi_1\, \mathcal{Q} - \chi_2, \quad (20)$$

$$\mathcal{Q}' = 0 \quad \Rightarrow \quad \mathcal{Q} = \mathcal{Q}_0. \quad (21)$$

The integration parameters in the formulas above are represented by the symbols χ_1, χ_2, and $\mathcal{Q}_0$. The first solution, referred to as Eq. (20), can be regarded as comparable to general relativity (GR) as it reduces to the STGR. The parameter defining the cosmological constant is indicated as $\chi_2/2\chi_1$. Nevertheless, it is essential to authenticate the accuracy of the Schwarzschild solution under the context of linear $f(\mathcal{Q})$ gravity. Within a vacuum, the values of ρ, P_r, and P_t are all equal to zero. Given the elements indicated above, the equation of motion may be represented as follows:

$$\frac{H'}{H} + \frac{N'}{N} = 0, \quad (22)$$

$$\frac{\chi_2}{\chi_1} - \frac{2}{r^2} - \frac{2q^2}{\chi_1\, r^4} = \mathcal{Q}, \quad (23)$$

$$\frac{\chi_2}{2} - \frac{2\chi_1(N'(H'r + H) - N(H''r + H'))}{HN^3r} = E^2. \quad (24)$$

The equation (22) provides the solution

$$H(r) = \frac{\mathcal{A}_0}{N(r)}, \quad (25)$$

The constant of integration $\mathcal{A}_0$ can be eliminated by adjusting the time coordinate t to a new value of $t/\mathcal{A}_0$. Subsequently, in Eq. (13), it is determined that the rr-component is the inverse of the tt component, specifically

$$H(r) = -N(r), \quad (26)$$

The cosmological constant term, represented by the symbol Λ, may be defined as the quotient of χ_2 divided by χ_1, as seen in the second Eq. (23). The sign of Λ is inverted in relation to the general relativity equivalent owing to the nonmetricity convention specified by Eq. (7). By employing the Eqs. (15), (22), and (23), we may deduce

$$e^{-N} = \left(1 + \frac{\mathcal{C}_1}{r} - \frac{\chi_2}{6\chi_1}r^2 + \frac{q^2}{\chi_1 r^2}\right), \quad (27)$$

The symbol $\mathcal{C}_1$ represents an integration constant.The function $H(r)$ may be obtained from Eqs. (26) and (27) as,

$$e^{H} = \left(1 + \frac{\mathcal{C}_1}{r} - \frac{\chi_2}{6\chi_1}r^2 + \frac{q^2}{\chi_1 r^2}\right). \quad (28)$$

Moreover, the line element in Eq. (13) may be restated as,

$$ds^2 = -\left(1 + \frac{\mathcal{C}_1}{r} - \frac{\chi_2}{6\chi_1}r^2 + \frac{q^2}{\chi_1 r^2}\right)dt^2 + r^2 d\theta^2$$

$$+r^2\sin^2\theta\, d\phi^2 + \left(1 + \frac{\mathcal{C}_1}{r} - \frac{\chi_2}{6\chi_1}r^2 + \frac{q^2}{\chi_1 r^2}\right)^{-1} dr^2, \quad (29)$$

The metric (29) reflects the Reissner–Nordström de Sitter solution when the criteria $\Lambda = \frac{\chi_2}{2\chi_1}$, $\mathcal{C}_1 = 2\mathbf{M}$, and $\mathbb{Q} = \frac{q^2(R)}{R^2}$ are fulfilled, where $\mathbb{M}$ indicates the mass of the object. The Reissner–Nordström de Sitter solution is only found in the linear $f(\mathcal{Q})$ gravity theory, which can accurately reproduce the STGR. Within this particular framework, the Reissner–Nordström de Sitter solution closely aligns with the predictions of General Relativity. Furthermore, they also analyzed the deviation of the metric from the exact Schwarzschild solution by considering the nonmetricity scalar $\mathcal{Q}$ being constant.

It is crucial to acknowledge the Zhao [111] findings about the suitability of a SS spacetime with the coincident gauge. Based on his discoveries, assuming that the affine connection is zero in this coordinate system and further requiring that the $f(Q)$-gravity theory possesses vacuum solutions, i.e.$T_{\epsilon\beta} = 0$, then the off-diagonal component of Eq. (8) can be interpreted as,

$$\frac{\cot\theta}{2}\, Q'\, f_{QQ} = 0, \quad (30)$$

where Q is determined by the Eq. (15). Given the current circumstances, the equations of motion, along with the diagonal components (30), yield the outcome that f_{QQ} is equal to zero. Consequently, it may be inferred that the function $f(Q)$ must exhibit linearity. Put simply, selecting $f(Q)$ as a non-linear function of Q will result in inaccurate equations of motion, especially when $f(Q)$ is equal to Q^2. To summarise, if a theory incorporates $f(Q)$ as a non-linear function of Q, then the metric Eq. (13) with an affine connection $\Gamma^{\mu}{}_{\epsilon\nu} = 0$ will not satisfy the equations of motion. We require a more general version of the SS metric for a consistent coincident gauge in this particular scenario. For a comprehensive understanding of this subject, please refer to the detailed explanation provided in Ref. [111].

For compatibility of affine connection $\Gamma^{\mu}{}_{\epsilon\nu} = 0$ with the spherically symmetric coordinate system, we take f_{QQ} coefficient from the off-diagonal component given in Eq. (19) to be zero for obtaining the solution of $f(\mathcal{Q})$-gravity which restricts functional form of f as,

$$f_{\mathcal{Q}\mathcal{Q}} = 0 \quad \Longrightarrow \quad f(\mathcal{Q}) = -\chi_1\, \mathcal{Q} - \chi_2, \quad (31)$$

where ζ_1 and ζ_2 are constants. By plugging of Eqs. (15) and (31), the Eqs. (16)–(18) provides the following explicit form of equations of motion,

$$\rho = \frac{\chi_1\left(2N'r + N^3 - N\right)}{N^3 r^2} - E^2 - \frac{\chi_2}{2}, \quad (32)$$

$$P_r = \frac{\chi_1}{r^2}\left(\frac{2H'r + H}{HN^2} - 1\right) + E^2 + \frac{\chi_2}{2}, \quad (33)$$

$$P_t = -\frac{2\chi_1(N'(H'r + H) - N(H''r + H'))}{HN^3 r} - E^2 + \frac{\chi_2}{2}, \quad (34)$$

We note that the covariant derivative of effective energy-momentum tensor under the assumption of spherical symmetry (13) vanishes i.e. $\nabla^{\epsilon} T_{\epsilon\,\beta} = 0$, which gives

$$-\frac{2H'}{2H}(\rho + P_r) - (P_r)' + \frac{2}{r}(P_t - P_r) = 0. \quad (35)$$

The above Eq. (35) is known as a Tolman–Oppenheimer–Volkoff (TOV) equation in $f(\mathcal{Q})$-gravity [37] under the linear functional form of $f(\mathcal{Q})$ for the Eq. (8). The coming approach involves seeking a most general exact solution to the field equations (32)–(34) that describe a model of a compact stellar object. To determine the anisotropy condition, we subtract Eq. (33) from Eq. (34) as follows:

$$\chi_1\left[r\{H''Nr - H'(N'r + N)\} + H\left(-N'r + N^3 - N\right)\right] = (2E^2 + \Delta)HN^3r^2. \quad (36)$$

4 New exact solution charged anisotropic star in $f(\mathcal{Q})$-gravity

Since we need to solve the differential equation (36) which contains four unknowns namely: metric functions $H(r)$ and $N(r)$ along with anisotropy Δ and electric field (E). Therefore, we need three extra conditions to solve this equation. In this regard, we consider the ansatz for the metric function $N(r)$ as,

$$N(r) = \sqrt{\frac{1 - 2\lambda r^2}{1 + \lambda r^2}}, \quad (37)$$

here λ denotes a constant with dimension l^{-2}. It is highlighted that the general form of this metric was proposed by Buchdahl [113] which was the form: $N^2 = \frac{1+ar^2}{1+br^2}$. He studied the isotropic fluid model in GR representing a neutron star stellar object for $b = -\frac{a}{2}$. Later on, the same solution was obtained Vaidya–Tikekar [114] for $b = -\frac{a}{2} = \frac{2}{R^2}$. Alternatively, Gupta and Jasim [115] introduced another form of metric, expressed as $e^{\mathcal{S}} = \frac{K(1+ar^2)}{K+ar^2}$, which is considered the most general. This expression yields the same solution as the Vaidya–Tikekar and Buchdahl solutions for $K = -2$. However, Gupta and Jasim derived the most general solutions for all values of K, except for the range $0 < K < 1$. In this connection, Thirukkanesh and Ragel [112] discussed the isotropic solutions utilising same metric: $N^2 = \frac{1+ar^2}{1+br^2}$ for the different cases such as $a + 2b = 0$, $b - 2a = 0$, $7b - 4a = 0$, and $7b + a = 0$. In all the above scenarios, they found that the star's density decreases monotonically while mass increases throughout the stellar configuration.

Recently, the same ansatz (37) was utilized by Baskey et al. [116] to model the anisotropic star in the framework of GR. Given the above observations, the metric $N^2 = \frac{1+ar^2}{1+br^2}$ is much more acceptable for the wide range of parameter values a and b to develop the astrophysical object in the context of GR. Therefore, we considered this particular ansatz of the metric function for modeling astrophysical objects in $f(Q)$-gravity. After plugging the expression $N(r)$ in Eq. (36), we arrive on the following differential equation,

$$Hr(\Delta + 2E^2)\left(1 - 2\lambda r^2\right)^2 = \chi_1\Big[H''r\left(-2\lambda^2 r^4 - \lambda r^2 + 1\right) - H'\left(-2\lambda^2 r^4 - 4\lambda r^2 + 1\right) + 6\lambda^2 H r^3\Big]. \tag{38}$$

As we can see, the master Eq. 38 depends on the two unknowns, namely anisotropy (Δ), and electric charge (E). Therefore, we must choose physically plausible expressions for Δ and E under which this master equation should be solvable. Another important consideration is that both anisotropy and electric field must be zero in the center of the star and positive elsewhere. This is necessary because the forces produced by anisotropy and electric field should work in the outward direction to counteract gravitational collapse. By taking all the above aspects, we consider well-behaved expressions from anisotropy (Δ) and electric field (E) for which the Eq. (38) is integrable,

$$\Delta = \frac{6\lambda^2 r^2 \chi_1}{\left(1 - 2\lambda r^2\right)^2}; \quad \text{and} \quad E^2 = \eta \chi_1 r^2. \tag{39}$$

where, η is a constant with dimension l^{-2} which is called electric charge parameter. It is evident from the expression given in Eq. (39), that there is no singularity in both expressions and they are zero at $r = 0$ and positive everywhere. After plugging Δ and E in Eq. (38), we find the final form,

$$\frac{H''}{H} - \frac{H'\left(-2\lambda^2 r^4 - 4\lambda r^2 + 1\right)}{Hr\left(-2\lambda^2 r^4 - \lambda r^2 + 1\right)} = \frac{\eta\chi_1 r^2\left(2 - 4\lambda r^2\right)}{\chi_1\left(\lambda r^2 + 1\right)}, \tag{40}$$

Now we solve the equation using integrating factor method, we get

$$H(r) = \Upsilon_1 \cosh[\Psi(r)] + \Upsilon_2 \sinh[\Psi(r)] \tag{41}$$

where, Υ_1 and Υ_2 are arbitrary integration constant, and $\Psi(r)$ is denoted as:

$$\begin{aligned}\Psi(r) = {} & \frac{1}{\lambda\sqrt{2\lambda r^2 + 2}\left(\sqrt{6\lambda r^2 + 6} + 3\right)^2}\Bigg[3\sqrt{\eta}\Bigg(\left(\lambda r^2 + 1\right) \\ & \times\sqrt{1 - 2\lambda r^2}\left(2\lambda r^2 + 2\sqrt{6\lambda r^2 + 6} + 5\right) \\ & -3\Big[2\lambda r^2\left(\sqrt{2\lambda r^2 + 2} + 2\sqrt{3}\right) + 5\sqrt{2\lambda r^2 + 2} + 4\sqrt{3}\Big] \\ & \times \tan^{-1}\left(\frac{\sqrt{1 - 2\lambda r^2}}{\sqrt{2\lambda r^2 + 2} + \sqrt{3}}\right)\Bigg)\Bigg].\end{aligned} \tag{42}$$

Now the expressions for density and pressures can be given as,

$$\rho = -\eta\chi_1 r^2 + \frac{3\lambda\chi_1\left(2\lambda r^2 - 3\right)}{\left(1 - 2\lambda r^2\right)^2} - \frac{\chi_2}{2}, \tag{43}$$

$$\begin{aligned}P_r = {} & \eta r^2 \chi_1 + \frac{1}{\sqrt{\lambda r^2 + 1}\left(r - 2\lambda r^3\right)\left(\sqrt{6\lambda r^2 + 6} + 3\right)^4\left(\Upsilon_2 \sinh[\Psi(r)] + \Upsilon_1 \cosh[\Psi(r)]\right)} \\ & \times\Bigg[3\chi_1\Bigg\{6\sqrt{\eta}r\left(4\sqrt{2}\lambda^2 r^4 + 4\lambda r^2\left(4\sqrt{3\lambda r^2 + 3} + 11\sqrt{2}\right) + 40\sqrt{3\lambda r^2 + 3} + 49\sqrt{2}\right) \\ & \times\sqrt{1 - 2\lambda r^2}\Big(\Upsilon_1 \sinh[\Psi(r)] + \Upsilon_2 \cosh[\Psi(r)]\Big) + \lambda r\Bigg(\sqrt{\lambda r^2 + 1} \\ & \times\left(\sqrt{6\lambda r^2 + 6} + 3\right)^4\left(\Upsilon_2 \sinh[\Psi(r)] + \Upsilon_1 \cosh[\Psi(r)]\right) + 6\sqrt{\eta}r^2\Big(4\sqrt{2}\lambda^2 r^4 + 4\lambda r^2\left(4\sqrt{3\lambda r^2 + 3} + 11\sqrt{2}\right) \\ & +40\sqrt{3\lambda r^2 + 3} + 49\sqrt{2}\Big)\sqrt{1 - 2\lambda r^2}\left(\Upsilon_1 \sinh[\Psi(r)] + \Upsilon_2 \cosh[\Psi(r)]\right)\Bigg)\Bigg\}\Bigg] + \frac{\chi_2}{2},\end{aligned} \tag{44}$$

$$
\begin{aligned}
P_t = & \frac{1}{2}\left(\chi_2 - 2\eta r^2 \chi_1\right) + \frac{1}{\left(1-2\lambda r^2\right)^{5/2}\sqrt{\lambda r^2+1}\left(\sqrt{6\lambda r^2+6}+3\right)^8\left(\Upsilon_2 \sinh[\Psi(r)] + \Upsilon_1 \cosh[\Psi(r)]\right)} \\
& \times\Bigg[81\chi_1\Bigg(\Upsilon_1\Bigg\{2\sqrt{\eta}\left(\lambda r^2+1\right)\Big[16\sqrt{2}\lambda^4 r^8 + 8\lambda r^2\left(636\sqrt{3\lambda r^2+3}+1079\sqrt{2}\right) \\
& +3920\sqrt{3\lambda r^2+3} + 32\lambda^3 r^6\left(4\sqrt{3\lambda r^2+3}+23\sqrt{2}\right) + 24\lambda^2 r^4\left(72\sqrt{3\lambda r^2+3}+193\sqrt{2}\right) + 4801\sqrt{2}\Big] \\
& \times\left(1-2\lambda r^2\right)^2 \sinh[\Psi(r)] + \Big(8\lambda r^2\left(1079\sqrt{\lambda r^2+1}+563\sqrt{6}\right) \\
& +4801\sqrt{\lambda r^2+1} + 16\lambda^4 r^8\left(\sqrt{\lambda r^2+1}+4\sqrt{6}\right) + 32\lambda^3 r^6\left(23\sqrt{\lambda r^2+1}+29\sqrt{6}\right) + 24\lambda^2 r^4 \\
& \times\left(193\sqrt{\lambda r^2+1}+142\sqrt{6}\right) + 1960\sqrt{6}\Big)\sqrt{1-2\lambda r^2}\cosh[\Psi(r)]\left(3\lambda + 8\eta\lambda^2 r^6 - 8\eta\lambda r^4 + 2\eta r^2\right)\Bigg\} \\
& +\Upsilon_2\Bigg\{\Big\{8\lambda r^2\left(1079\sqrt{\lambda r^2+1}+563\sqrt{6}\right) + 4801\sqrt{\lambda r^2+1} + 16\lambda^4 r^8\left(\sqrt{\lambda r^2+1}+4\sqrt{6}\right) \\
& +32\lambda^3 r^6\left(23\sqrt{\lambda r^2+1}+29\sqrt{6}\right) + 24\lambda^2 r^4\left(193\sqrt{\lambda r^2+1}+142\sqrt{6}\right) + 1960\sqrt{6}\Big\} \\
& \sqrt{1-2\lambda r^2}\sinh[\Psi(r)]\left(3\lambda + 8\eta\lambda^2 r^6 - 8\eta\lambda r^4 + 2\eta r^2\right) + 2\sqrt{\eta}\left(\lambda r^2+1\right)\Big[16\sqrt{2}\lambda^4 r^8 + 8\lambda r^2 \\
& \times\left(636\sqrt{3\lambda r^2+3}+1079\sqrt{2}\right) + 3920\sqrt{3\lambda r^2+3} + 32\lambda^3 r^6\left(4\sqrt{3\lambda r^2+3}+23\sqrt{2}\right) \\
& +24\lambda^2 r^4\left(72\sqrt{3\lambda r^2+3}+193\sqrt{2}\right) + 4801\sqrt{2}\Big]\left(1-2\lambda r^2\right)^2\cosh[\Psi(r)]\Bigg\}\Bigg)\Bigg].
\end{aligned} \tag{45}
$$

To accurately represent self-gravitating charged compact objects, it is essential to choose an appropriate exterior solution that matches the internal solution at the pressure-free interface, denoted as $r = R$. When examining the linear functional form of the $f(Q)$-gravity theory, the exterior Reissner-Nordstrom de-Sitter solution may provide the most appropriate exterior solution as discussed above, characterized by the following spacetime:

$$
ds^2 = -\left(1 - \frac{2\mathbb{M}}{r} + \frac{\mathbb{Q}^2}{r^2} - \frac{\Lambda}{3}r^2\right)dt^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) + \frac{dr^2}{\left(1 - \frac{2\mathbb{M}}{r} + \frac{\mathbb{Q}^2}{r^2} - \frac{\Lambda}{3}r^2\right)}. \tag{46}
$$

Let $\mathbb{M}$ represent the total mass of the object at the boundary $r = R$. It is given by the equation $\mathbb{M} = m(R)/\alpha$. Additionally, Λ is equal to $\chi_2/2\chi_1$. Furthermore, it is emphasized that the Reissner–Nordström (anti-) di Sitter spacetime will convert into Reissner–Nordström spacetime when $\chi_1 = 1$, and $\chi_2 = 0$ i.e. $\Lambda = \frac{\chi_2}{2\chi_1} = 0$ which is equivalent to GR case.

Moreover, when we use the first and second basic forms, we get the following result:

$$
\left(1 - \frac{2M}{R} + \frac{\mathbb{Q}^2}{R^2} - \frac{\Lambda}{3}R^2\right) = H^2(R), \tag{47}
$$

$$
\left(1 - \frac{2M}{R} + \frac{\mathbb{Q}^2}{R^2} - \frac{\Lambda}{3}R^2\right)^{-1} = N^2(R), \tag{48}
$$

$$
P_r(R) = 0. \tag{49}
$$

Using the following boundary conditions, we find the values of constant Υ_1, Υ_2 and $\mathbb{M}$ as,

$$
\Upsilon_2 = \frac{1}{\Upsilon_3\cosh[\Psi(R)] + \sinh[\Psi(R)]}\sqrt{\frac{1+\lambda R^2}{1-2\lambda R^2}}, \tag{50}
$$

$$
\Upsilon_1 = \frac{\Upsilon_3}{\Upsilon_3\cosh[\Psi(R)] + \sinh[\Psi(R)]}\sqrt{\frac{1+\lambda R^2}{1-2\lambda R^2}}, \tag{51}
$$

$$
\mathbb{M} = \frac{R}{2}\left(-\frac{3\lambda R^2}{(1-2\lambda R^2)} + \eta\chi_1 R^4 - \frac{\Lambda}{3}R^2\right), \tag{52}
$$

where, $\Psi(R)$ is value of $\Psi(r)$ at $r = R$ while Υ_3 is given by

$$
\begin{aligned}
\Upsilon_3 = & \Big[2\chi_1\Big(2\sqrt{\eta}\sqrt{1-2\lambda R^2}\left(\lambda R^2+1\right)\Big(4\sqrt{2}\lambda^2 R^4 + 4\lambda R^2 \\
& \times\left(4\sqrt{3\lambda R^2+3}+11\sqrt{2}\right) + 40\sqrt{3\lambda R^2+3} \\
& +49\sqrt{2}\Big)\cosh[\Psi(R)] \\
& -\Big(4\lambda R^2\left(11\sqrt{\lambda R^2+1}+7\sqrt{6}\right) + 49\sqrt{\lambda R^2+1} \\
& +4\lambda^2 R^4\left(\sqrt{\lambda R^2+1}+2\sqrt{6}\right) + 20\sqrt{6}\Big)\sinh[\Psi(R)] \\
& \times\left\{\lambda\left(2R^4\eta - 3\right) - R^2\eta\right\}\Big) \\
& -\chi_2\left(2\lambda R^2 - 1\right)\Big(4\lambda R^2\Big(11\sqrt{\lambda R^2+1}+7\sqrt{6}\Big)
\end{aligned}
$$

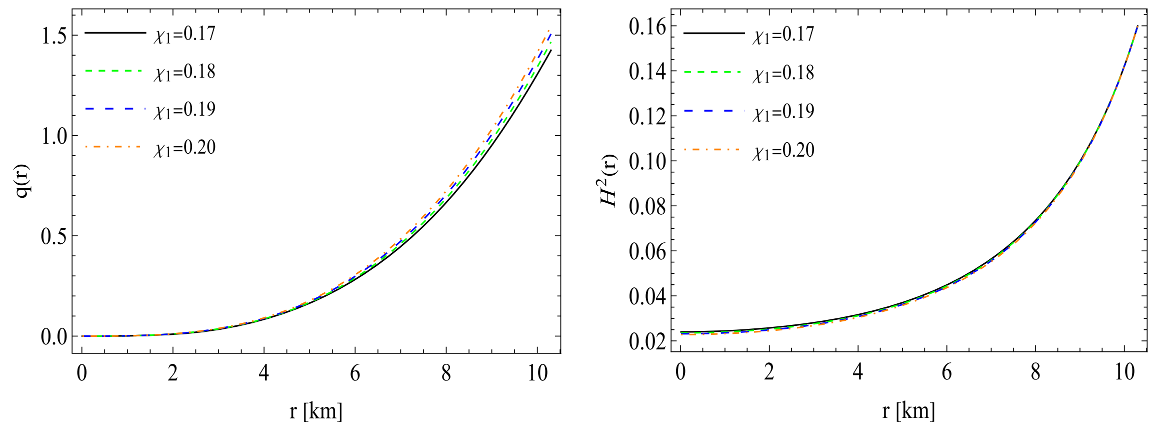

Fig. 1 The variation of electric charge ($q(r)$) versus r for the values of constants $\lambda = -0.006$ km^{-2}, $\chi_2 = 0.0002$ km^2, and $\eta = 0.00001$ km^{-4}

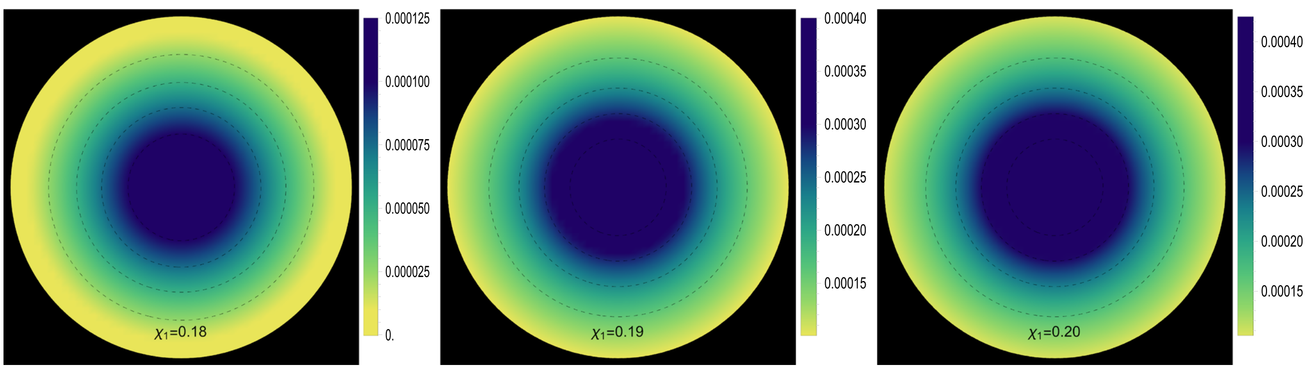

Fig. 2 The variation of the density (ρ) versus r for different value of non-metricity parameter χ_1 with the values of constants $\lambda = -0.006$ km^{-2}, $\chi_2 = 0.0002$ km^2, and $\eta = 0.00001$ km^{-4}

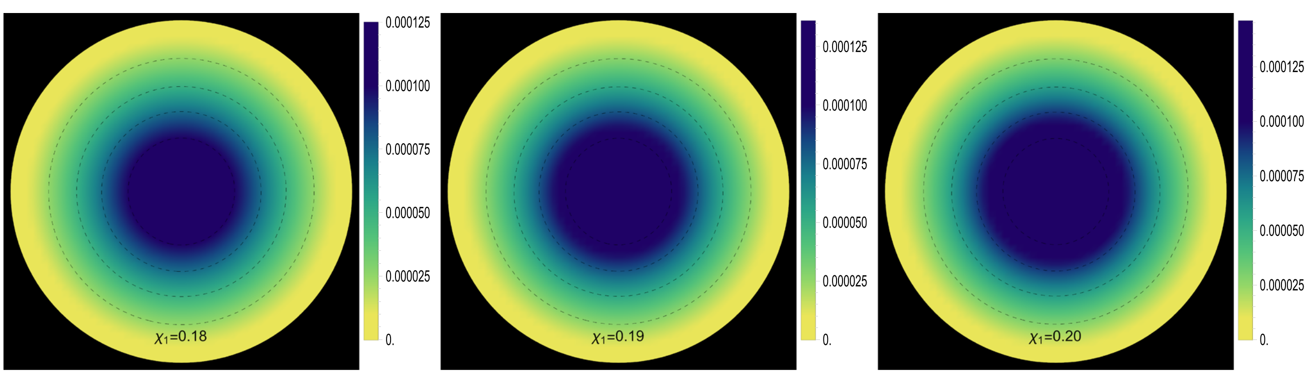

Fig. 3 The variation of the radial pressure (P_r) versus r for different value of non-metricity parameter χ_1 with same values of constants used in Fig. 2

$$
\begin{aligned}
&+49\sqrt{\lambda R^2+1}+4\lambda^2R^4\left(\sqrt{\lambda R^2+1}+2\sqrt{6}\right)+20\sqrt{6}\Big)\\
&\times\sinh[\Psi(R)]\Big]\Big/\Big[2\chi_1\Big\{\Big(4\lambda R^2\left(11\sqrt{\lambda R^2+1}+7\sqrt{6}\right)\\
&+49\sqrt{\lambda R^2+1}+4\lambda^2R^4\left(\sqrt{\lambda R^2+1}+2\sqrt{6}\right)+20\sqrt{6}\Big)\\
&\times\cosh[\Psi(R)]\left(\lambda\left(2R^4\eta-3\right)-R^2\eta\right)-2\sqrt{\eta}\sqrt{1-2\lambda R^2}\\
&\times\left(\lambda R^2+1\right)\Big(4\sqrt{2}\lambda^2R^4+4\lambda R^2\left(4\sqrt{3\lambda R^2+3}+11\sqrt{2}\right)\\
&+40\sqrt{3\lambda R^2+3}+49\sqrt{2}\Big)\sinh[\Psi(R)]\Big\}+\chi_2\left(2\lambda R^2-1\right)\\
&\times\Big(4\lambda R^2\left(11\sqrt{\lambda R^2+1}+7\sqrt{6}\right)+49\sqrt{\lambda R^2+1}\\
&+4\lambda^2R^4\left(\sqrt{\lambda R^2+1}+2\sqrt{6}\right)+20\sqrt{6}\Big)\cosh[\Psi(R)]\Big] \qquad (53)
\end{aligned}
$$

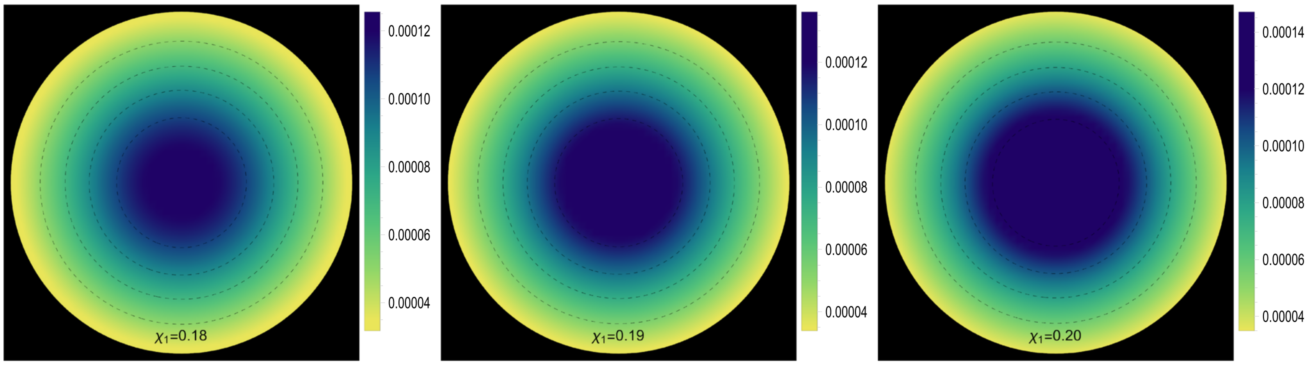

Fig. 4 The variation of the tangential pressure (P_t) versus r for different value of non-metricity parameter χ_1 with same values of constants used in Fig. 2

5 Physical analysis

5.1 Electric charge and metric function

We have assumed a well-behaved and non-singular metric potential $N(r)$ which is equal to unity at the center of the star. Again the solution to Eq. (30) provides another metric potential $H(r)$ which is represented in Fig. 1 and can be seen to be finite at the center. Hence, the metric potentials in the present stellar model governed by $f(\mathcal{Q})$-gravity are regular and non-singular. So, the present solution configuring the stellar system is physically acceptable for being free from singularities. The charge function and metric potential $H(r)$ show increasing behavior with respect to radial distance for different values of χ_1 in Fig. 1. The metric potential $H(r)$ has minimal dependence whereas the charge grows gradually towards the surface of the star for increasing values χ_1. This means that the electric force gradually gets stronger towards the surface of the star. The result is in confirmation with the graphical presentation of electrical force (F_e) in Fig. 6. Again, the electric force acts outward within the configuration and plays a role similar to anisotropic force to attain stability of the gravitational system in the context of $f(\mathcal{Q})$-gravity.

In a study [118] to analyze the impact of including electric charge it is revealed that gravitational spheres having fluid elements of zero net charge contain but with unbounded proper charge density at the fluid-vacuum interface can possess the enormous amount of charge of order 10^{19} in the units of Coulomb. In this regard by numerical calculations in a research work, Ray et al. [119] have shown an intriguing result related to the maximum amount of electric charge of the order 10^{20} C that shall be contained in the star to have considerable balancing effect of the forces present in the charged compact stars. Now, numerical values of the characteristic constants Υ_1 and Υ_2 related to $H(r)$ and values of total charge are listed in Table 1 for different values of χ_1. We can see from Table 1 that the present anisotropic and

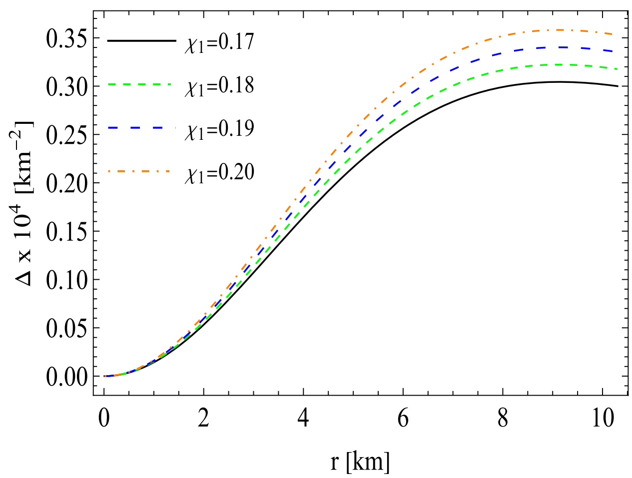

Fig. 5 The variation of anisotropy ($\Delta = P_t - P_r$) versus r

charged configuration in $f(\mathcal{Q})$-gravity is supported by the net charge of the order 10^{20} C for $\chi_1 > 0.17$. This implies that the total amount of charge has an effective role which impacts on balancing effect of forces in the present model.

5.2 Density, pressure, and anisotropy

Energy density, pressure in both radial and tangential direction, anisotropy, and electric charge are the physical quantities that guarantee the physical acceptance of a charged and anisotropic stellar model governed by the linear form of f in $f(\mathcal{Q})$-gravity. The mathematical analysis depicted in Figs. 2, 3, 4 confirms that the quantities fulfill all the physical criteria. It can be seen that ρ, P_r, P_t are non-negative throughout the star which is one of the necessary criteria satisfying energy conditions for the stellar matter. The variation of density, radial pressure, and tangential pressure is decreasing throughout the star. Again we see that only radial pressure vanishes at some value of r which defines the boundary or radius of the star. The numerical values (see Table 1)

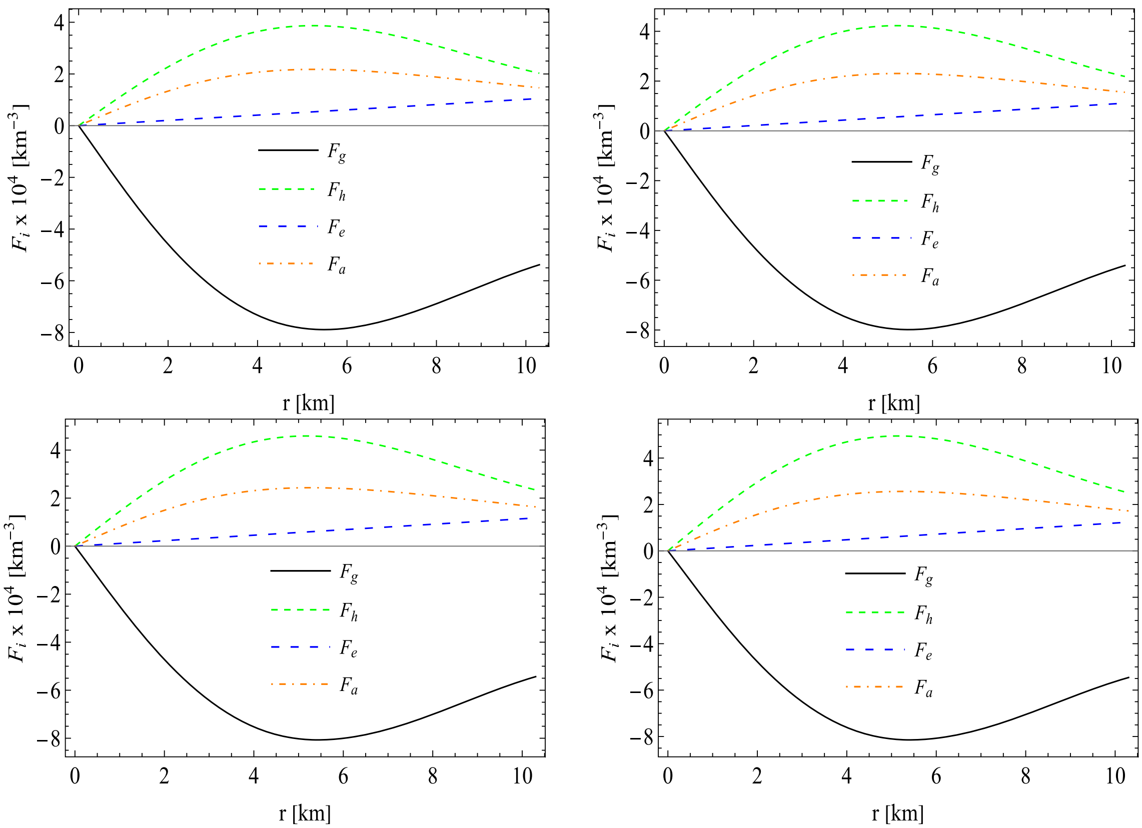

Fig. 6 The equilibrium condition via variation of different forces F_g, F_h, F_e, and F_a for same fixed values of constant as used in Fig. 3. The top-left panel for $\chi_1 = 0.17$, top-right panel for $\chi_1 = 0.18$, bottom-left panel for $\chi_1 = 0.19$, and bottom-right panel for $\chi_1 = 0.20$

of density and pressure are positive and finite at the center and surface of the stellar system.

The numerical values of radial and tangential pressure shall be equal at the center of the star leading to zero central anisotropy which is another important feature satisfied in the present anisotropic stellar model (see Fig. 5). However, the anisotropy is seen to be increasing with respect to the increase in radial distance. This physically indicates that the anisotropic force acts in an outward direction within the configuration of the star. At the surface, the anisotropy is enhanced for increasing values of $f(\mathcal{Q})$-gravity parameter χ_1. Hence, the anisotropic force becomes repulsive in nature and offers stability by countering the gravitational collapse of the system. The central values of ρ, P_r, P_t get larger for increasing values of χ_1.

5.3 Hydrostatic equilibrium

The Tolman–Oppenheimer–Volkoff (TOV) equation defines an equilibrium condition for a charged compact star that depends on gravitational, hydrostatic, anisotropic, and electric forces. The TOV equation is expressed in a generic form as

$$-\frac{H'}{H}(\rho + P_r) - \frac{dP_r}{dr} + \frac{2q}{r^4}\frac{dq}{dr} + \frac{2(P_t - P_r)}{r} = 0. \quad (54)$$

Or, it may be stated as

$$F_g + F_h + F_a + F_e = 0; \quad \text{such that} \quad F_g = -\frac{H^{'}(\rho + p_r)}{H},$$
$$F_h = -\frac{dP_r}{dr}, \quad F_a = \frac{2(P_t - P_r)}{r}, \quad F_e = \frac{q}{r^4}\frac{dq}{dr}.$$

The symbols F_g, F_h, F_e, and F_a, and denote the gravitational, hydrostatic, electric, and anisotropic forces, respectively. By including the energy density, radial pressure, and tangential pressure in the TOV equation, we are to find the variations of these forces within the stellar configuration which are shown in Fig. 6. Individually, these forces are attractive or repulsive, but the sum of these forces is equal which shows that the solution fulfills the TOV equation. Therefore, we may infer that the stellar configuration has achieved the equilibrium.

$$-\frac{H'}{H}(\rho + P_r) - \frac{dP_r}{dr} + \frac{2q}{r^4}\frac{dq}{dr} + \frac{2(P_t - P_r)}{r} = 0. \quad (55)$$

5.4 Energy condition

The following part will analyze the energy conditions. In general relativity, it has been determined that both the trace of the

Table 1 The approximate values of the pressures $P(r)$ & P_t, density $\rho(r)$, the constants γ_2 and γ_1 and central value of adiabatic index Γ_0, electric charge ($\mathbb{Q}$) with $\eta = 1 \times 10^{-5}$ km^{-4}

Parameters	p_0 $(dyne/cm^2)$	ρ_0 (gm/cm^3)	ρ_s (gm/cm^3)	Υ_2	Υ_1	Γ_0	$\mathbb{Q}(C)$
$\chi_1 = 0.17$	1.39823×10^{35}	4.87596×10^{14}	1.20841×10^{14}	0.582386	0.0878481	1.36756	1.66111×10^{19}
$\chi_1 = 0.18$	1.52547×10^{35}	5.16594×10^{14}	1.28265×10^{14}	0.586499	0.0859598	1.39139	1.70926×10^{20}
$\chi_1 = 0.19$	1.65351×10^{35}	5.45592×10^{14}	1.3569×10^{14}	0.590179	0.0842703	1.41305	1.7561×10^{20}
$\beta_1 = 0.20$	1.78225×10^{35}	5.7459×10^{14}	1.43114×10^{14}	0.593491	0.0827498	1.43282	1.80172×10^{20}

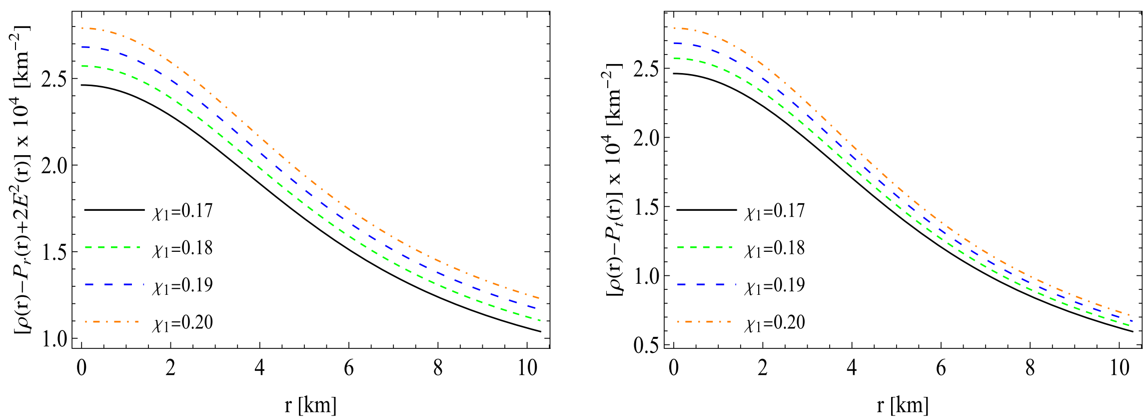

Fig. 7 The behavior of the density (ρ), the radial pressure (P_r), the tangential pressures (P_t) and the anisotropy ($\Delta = P_t - P_r$) against the radial coordinates r for the values as usedd in Fig. 2

tidal tensor $R_{ij}A^iA^j$ and the $R_{ij}B^iB^j$ term in the Raychaudhuri equation are positively oriented. Here, A^i represents any time-like vector and B^i is any null vector directed towards the future. Using this Raychaudhuri equation, we can find four constraints that can be placed on the energy-momentum tensor (T_{ij}). These constraints are known as the energy conditions. In a particular case of $f(Q)$ theory of gravity, the energy conditions for a physical model may be represented as

- WEC: $\rho + \frac{q^2}{r^4} \geq 0$, $\rho + P_r \geq 0$, $\rho + P_t + 2\frac{q^2}{r^4} \geq 0$;
- NEC: $\rho + P_r \geq 0$, $\rho + P_t + 2\frac{q^2}{r^4} \geq 0$;
- SEC: $\rho + P_r + 2P_t + 2\frac{q^2}{r^4} \geq 0$;
- DEC: $\rho - P_r + 2\frac{q^2}{r^4} \geq 0$, $\rho - P_t \geq 0$.

To observe the nature of the energy conditions within the stellar configuration, we need to see the behavior of pressures, density, and electric charge (q). From Figs. 1 and 4, it is clear that the pressures, density, and electric charge are positive throughout the model. Hence, we may infer that our system fulfills the WEC, NEC, and SEC. Now we only focus on the remaining energy condition DEC. For this purpose, we plot the inequalities $\rho - P_r + 2\frac{q^2}{r^4}$ and $\rho - P_t \geq 0$ in Fig. 7. We observe from this figure that both inequalities are also positive within the model. Hence, our charged anisotropic model satisfies all the energy conditions.

6 Stability

6.1 Analysis of the stability of charged models using the Harrison–Zeldovich–Novikov (HZN) criteria

We use our findings to adhere to the Harrison–Zeldovich–Novikov (HZN) stability requirement, as shown in Fig. 8. The HZN stability criterion's validity is determined by the following limits [120,121]:

1. $\frac{d\mathbb{M}}{d\rho_c} < 0$ $\rightarrow$ unstable configuration
2. $\frac{d\mathbb{M}}{d\rho_c} > 0$ $\rightarrow$ stable configuration.

In order to verify this condition for a charged solution, we determine the mass expression as a function of ρ_0 and its derivative with respect to ρ_0 as follows:

$$\mathbb{M} = \frac{2R^7\chi_1\eta\,(2\rho_c + \chi_2) + 3R^3\,(2\rho_c + \chi_2) + 18R^5\chi_1^2\eta}{4\left(R^2\,(2\rho_c + \chi_2) + 9\chi_1\right)}, \quad (56)$$

$$\frac{d\mathbb{M}}{d\rho_0} = \frac{27R^3\chi_1}{2\left(R^2\,(2\rho_c + \chi_2) + 9\chi_1\right)^2} \quad (57)$$

The graph in Fig. 8 shows that the derivative of M with respect to ρ_c is positive i.e. the mass ($M/M_\odot$) is an increasing function of ρ_c, indicating that the resulting charged anisotropic models are stable. On the other hand, decreasing the non-metricity parameter χ_1 improves the stability of the confined

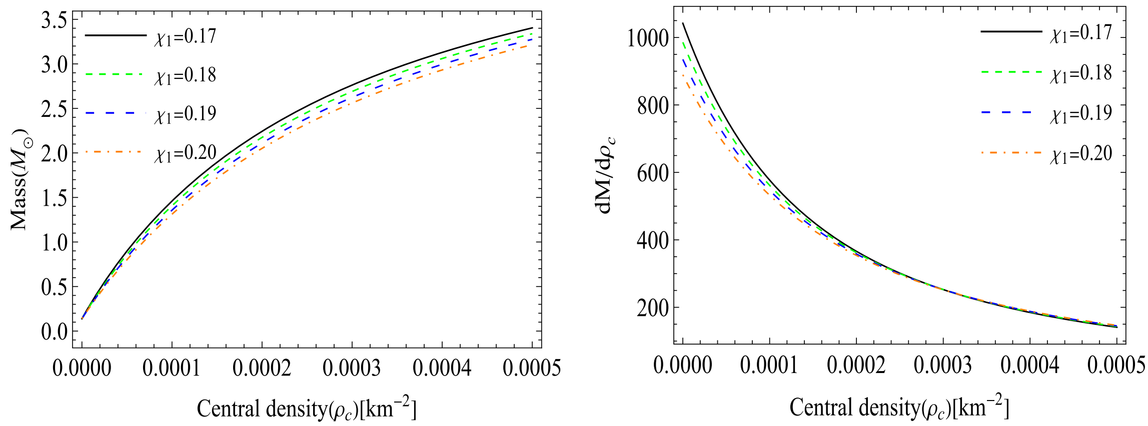

Fig. 8 The behavior of the density (ρ), the radial pressure (P_r), the tangential pressures (P_t) and the anisotropy ($\Delta = P_t - P_r$) against the radial coordinates r for the values as used in Fig. 2

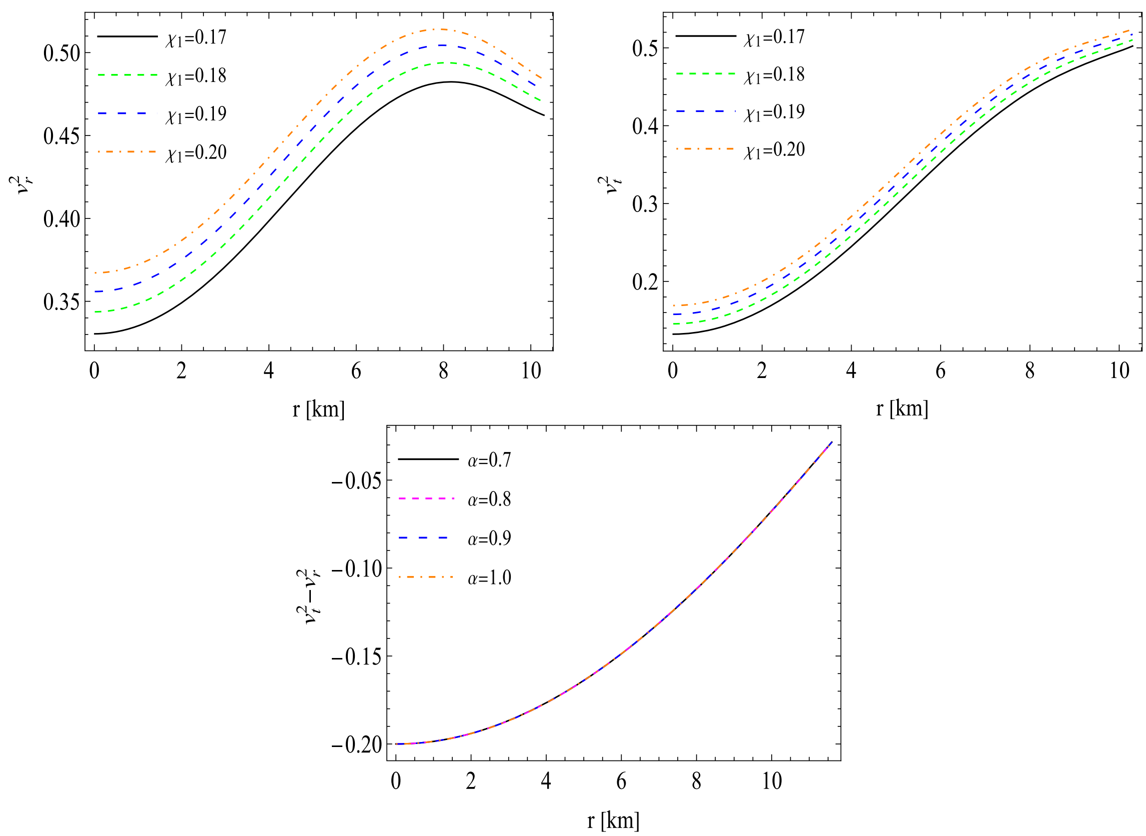

Fig. 9 The variation of radial and tangential speed of sounds (v_r^2 & v_t^2) and stability factor $|v_t^2 - v_r^2|$ versus r for the same values as used in Fig. 1

structures, although the increase in mass relative to the effective central density is minimal.

6.2 Causality and stability via cracking cracking

When studying any physically possible system in astrophysics, the stability of the star configurations is essential. We examine the stability of charged star configurations generated by anisotropic fluid in $f(Q)$-gravity theory, using superluminal speeds derived from Herrera's cracking idea [43]. Physically stable structures in the inside structure of stellar objects must have a squared speed of sound, represented by the formula $v_s^2 = dP/d\rho$, that falls between 0 and 1, meaning $0 \leq v_s^2 = dP/d\rho \leq 1$ according to the causality requirement. Herrera (ref45) introduced the concept of cracking to identify stable and unstable zones inside compact star formations. The formula assesses regions based on the difference between the squared sound speeds in the radial and tangential directions which are denoted by v_r^2 and v_t^2, respectively. Stable regions fall within the range of 0 to 1 i.e. $0 < |v_t^2 - v_r^2| < 1$. We include the sound speeds in Fig. 9 to facilitate stability analysis. The speed of sound is always less than the speed of light, with the radial speed of sound being greater within the object for each χ_1. This indicates that the anisotropic charged stellar solution in $f(Q)$-gravity theory meets causality and stability criteria for all taken values of the non-metricity parameter χ_1.

6.3 Adiabatic index

We are examining the stability of our stellar models by applying the adiabatic stability rule, initially proposed by Chandrasekhar, for isotropic pressure gradients. The adiabatic stability criteria were defined using the following formula:

$$\Gamma = \left(1 + \frac{\rho}{P}\right)\left(\frac{dP}{d\rho}\right)_S. \tag{58}$$

When Γ is greater than 4/3, it represents the limiting scenario for confined structures with isotropic pressure. The velocity of sound is represented by the derivative $\frac{dP}{d\rho}$ with the subscript S indicating a constant specific entropy. This demonstrates that the significance of sound velocity as a crucial parameter related to the adiabatic index. For instance, the adiabatic index for the Schwarzschild solution with constant density is infinite, indicating an incompressible fluid. Glass and Harpaz demonstrated that the adiabatic index at the center must exceed $4/3$ for a stable polytropic star. Furthermore, the recent findings suggest that the range of Γ in large neutron stars should be between 2 and 4, as indicated in Haensel's study. In this connection, Herrera et al. discovered that anisotropy and dissipation alter the condition (58). When pressure anisotropy is present, the stability criteria change to the following form:

$$\Gamma < \frac{4}{3} + \left[\frac{4}{3}\frac{(P_t - P_r)}{|(P_r)'|r}\right]. \tag{59}$$

The expression denotes the absolute value of the derivative of the total pressure with respect to the radial coordinate r. The Newtonian limit, $\Gamma < 4/3$, produces an unstable region when the second component in Eq. (59) becomes zero due to relativistic effects. Figure 8 displays the adiabatic index Γ_r as a function of radial distance r for various values of α. Figure 10 indicates that the adiabatic index Γ is more than 4/3 for all values of χ_1. It is also observed when χ_1 increases the adiabatic index value increases, hence stable configuration is confirmed.

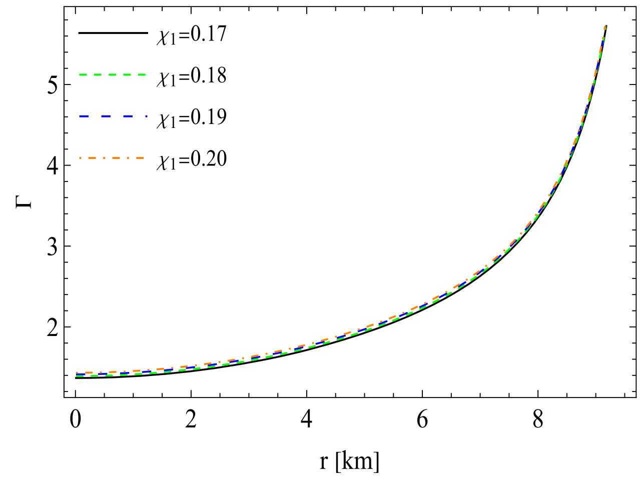

Fig. 10 The behavior of the adiabatic index (Γ) against the radial coordinates r

6.4 Effect of non-metricity parameter (χ_1) and electric charge parameter (η) on mass and radii for different compact objects via $M - R$ curves

The upper and lower bounds on the mass-radius ratio in the presence of a cosmological constant were derived by Andréasson [126] and Böhmer and Harko [127], respectively. In the context of $f(\mathcal{Q})$-gravity, the inequality expressing the bounds on the mass-radius ratio can be formulated as follows,

$$\left(\frac{3\mathbb{Q}^2}{4R^2} + \frac{\chi_2 R^2}{24\chi_1}\right) \leq \frac{M}{R} \leq \left(\frac{2}{9} + \frac{\mathbb{Q}^2}{3R^2} - \frac{\chi_2 R^2}{18\chi_1} + \frac{2}{9}\sqrt{1 + \frac{3\mathbb{Q}^2}{R^2} + \frac{3\chi_2 R^2}{2\chi_1}}\right), \tag{60}$$

The features of mass-radius relation for anisotropic charged compact stars have been explored in the framework of $f(\mathcal{Q})$-gravity with reference to the variations in $f(\mathcal{Q})$-gravity parameter (χ_1) and electric charge parameter (η). In this connection the $M - R$ curves satisfying Eq. 60 are shown in Fig. 11 for different values of parameters χ_1 and η. The values of maximum mass and the corresponding radius obtained from each $M - R$ curve increase gradually for small increments in χ_1. Although one can find minor differences in the values of maximum mass and the radius for increasing values of η. So, χ_1 plays an influential role in developing massive charged compact stars in $f(\mathcal{Q})$-gravity where the effect of η on maximum mass is minimal.

Recently Rawls et al. [122] have applied a refined process for measuring the mass of neutron stars by employing a numerical code related to Roche geometry along with various optimizers that can evaluate the published data for eclipsing X-ray binary systems. This helps to calculate accurate eclipse duration which gives the improved value of mass as $1.29 \pm 0.05 M_\odot$ for LMC X-4. In another study [123], radio timing observations obtained from the Arecibo and Green Bank Observatories are utilized with complete measurement of the relativistic Shapiro delay, analyzed with a very precise determination of the apsidal motion, and linked with the new limitations of the orbital orientation of the system. As a consequence, the mass for PSR J1903+0327 is accurately measured as $1.667 \pm 0.02\ M_\odot$. In accordance with the general relativistic Shapiro delay approach for high precision measurement of masses of a pulsar and its companion in a binary system, Demorest et al. [124] inferred the mass of the millisecond pulsar PSR J1614-2230 to be $1.97 \pm 0.08\ M_\odot$. An astrophysical event GW190814 [125] in connection to the gravitational wave implies that the gravitational wave radiation emerged from a compact binary system involving a black hole of mass 22.2 to 24.3 $M_\odot$ and a compact object of mass 2.50 to 2.67 $M_\odot$.

We have restricted our study to relate observational constraints of the observed stars such as LMC X-4 [122], PSR J1903+327 [123], PSR J1614-2230 [124] and GW190814 [125] to the $M - R$ curves for the practical validity of the present stellar model in $f(\mathcal{Q})$-gravity. Further, we have predicted values of radii for the observed stars with respect to variations in both the parameters χ_1 and η. The predicted radii have been arranged in Tables 2 and 3 for reference. This implies that a star of given total mass and charge can grow physically in size with an increase in values of χ_1. In contrast, a star of given total mass and χ_1 shrinks gradually for increasing values of η. So, the charged anisotropic configuration is more compact in comparison to the neutral configuration (η =0 case) in $f(\mathcal{Q})$-gravity.

7 Conclusion

In the beginning of this paper, we have expressed the gravitational field equations, TOV equation, and anisotropy equation for an anisotropic and charged fluid in $f(\mathcal{Q})$-gravity constrained to the linear form of the function $f(\mathcal{Q})$. We have argued that we need three extra conditions to get a complete solution to the anisotropy equation containing four unknown quantities $(H(r),\ N(r),\ \Delta(r),\ E(r))$. In this regard, we have taken nonlinear but simple functional forms of $N(r),\ \Delta(r),\ E(r)$ regulated by the parameters $\{\lambda,\ \eta,\ \chi_1\}$ into account with complete physical considerations at center and surface of the star. These choices of $N(r),\ \Delta(r),\ E(r)$ are able to produce non-singular and exact solutions to the field equations. Notably, anisotropy and electric field are chosen in such a way that these quantities will vanish for zero value of χ_1. Thus, both the physical features of the stellar matter evolve simultaneously with variation in the $f(\mathcal{Q})$-gravity parameter χ_1 in the present astrophysical model. However, the neutral counterpart of the present model can be retrieved easily by considering η to be zero for non-zero values of χ_1. Since we are interested in the charged case of anisotropic matter, we have assumed non-zero values of η in the present investigation. The properties at the boundary of the $f(\mathcal{Q})$-gravity for anisotropic and charged compact stars have been explored by matching the interior metric solution to the exterior Reissner–Nordstrom de-Sitter solution. The boundary is defined by the standard condition of vanishing of radial pressure.

With the above considerations we have derived the expressions for $H,\ \rho,\ P_r,\ P_t, \Delta,\ q$ and presented graphically in Figs. 1, 2, 3, 4 and 5. By satisfying all the physical criteria, the physical quantities i.e., energy density, pressure in both radial and tangential direction, anisotropy, and electric charge, ensure the physical validity of a charged and anisotropic stellar model governed under $f(\mathcal{Q})$-gravity. Density and pressure show decreasing nature whereas anisotropy and electric charge show increasing nature with respect to the radial distance throughout the star. For increasing values of $f(\mathcal{Q})$-gravity parameter χ_1, the quantities $\{\rho,\ P_r,\ P_t,\ \Delta,\ q\}$ have gradual enhancement inside the star. Central density is found to be of the order of $10^{14}\ gm/cm^3$ listed in Table 1.

On substitution of the physical entities $H,\ \rho,\ P_r,\ P_t,\ q$ in the Tolman–Oppenheimer–Volkoff (TOV) equation, we have shown graphical nature of gravitational (F_g), hydrostatic (F_h), anisotropic (F_a), and electric forces (F_e) in Fig. 6 for four different values of χ_1. It is found that the resultant of three forces $F_h,\ F_a,\ F_e$ individually being repulsive in nature balances the attractive gravitational force. Essentially, this helps to avoid the gravitational collapse and the anisotropic charged stellar system achieve stable equilibrium in the framework of $f(\mathcal{Q})$-gravity.

From the physical behavior of pressures, density, and electric charge (q) shown in Figs. 1, 2, 3, 4, it is evident that the pressures, density, and electric charge are non-negative throughout the model which is the confirmation of the validity of energy conditions such as WEC, NEC, and SEC in the stellar system. Further, Fig. 7 represents the expressions $\rho - P_r + 2\frac{q^2}{r^4}$ and $\rho - P_t$ as non-negative entities confirming the validity of dominant energy condition in the star. Therefore, the present anisotropic charged stellar configuration satisfies all the energy conditions that advocate on behalf of the physical acceptance of the present model in $f(\mathcal{Q})$-gravity.

Further, some additional investigations have been made for stability analysis of the present compact star model. At first, we studied Harrison–Zeldovich–Novikov (HZN) criteria for stability by showing the gradient of total mass with respect to central density in Fig. 5. The gradient $(dM/d\rho_c)$ is found to be positive throughout the star which fulfills the HZN criteria for stable configuration of the present stellar system. The positive values of the gradient are seen to be enhanced by a small amount for decreasing values of the $f(\mathcal{Q})$-gravity parameter χ_1. In another method, we have

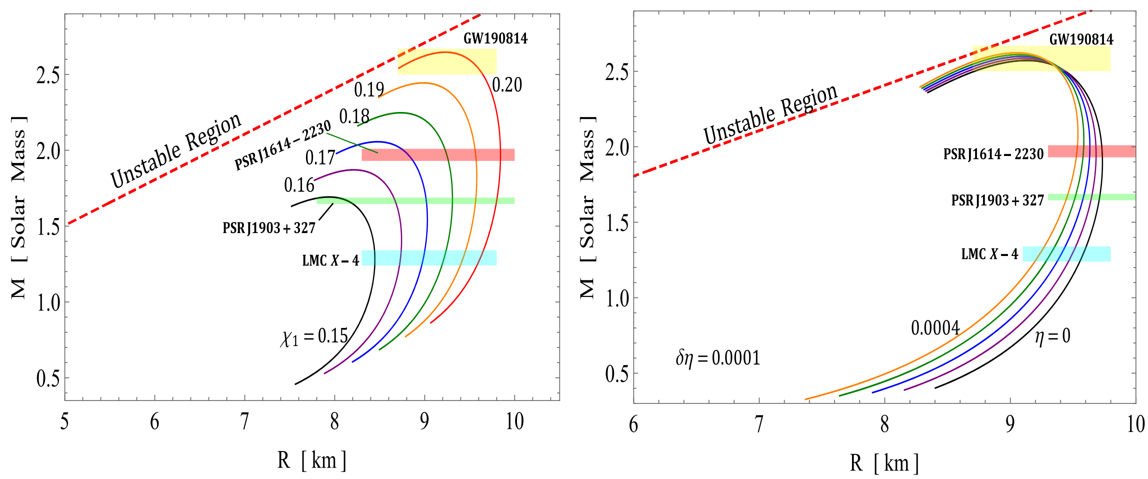

Fig. 11 Mass-radius curves for compact objects under different values of non-metricity and electric charge parameters χ_1 and η, respectively

Table 2 $M - R$ curve and prediction of radii for different values of χ_1

Objects	$M/M_{\odot}$	Predicted R [km]					
		$\chi_1 = 0.15$	$\chi_1 = 0.16$	$\chi_1 = 0.17$	$\chi_1 = 0.18$	$\chi_1 = 0.19$	$\chi_1 = 0.20$
LMC X-4 [122]	1.29 ± 0.05	$8.44^{+0.002}_{-0.002}$	$8.73^{+0.01}_{-0.01}$	$8.98^{+0.02}_{-0.02}$	$9.22^{+0.01}_{-0.03}$	$9.43^{+0.02}_{-0.03}$	$9.61^{+0.04}_{-0.03}$
PSR J1903+327 [123]	1.667 ± 0.021	$8.14^{+0.05}_{-0.05}$	$8.67^{+0.015}_{-0.025}$	$9.01^{+0.01}_{-0.001}$	$9.31^{+0.004}_{-0.001}$	$9.56^{+0.001}_{-0.001}$	$9.81^{+0.01}_{-0.02}$
PSR J1614-2230 [124]	1.97 ± 0.04	–	–	$8.81^{+0.05}_{-0.06}$	$9.42^{+0.02}_{-0.02}$	$9.57^{+0.002}_{-0.009}$	$9.84^{+0.002}_{-0.006}$
GW190814 [125]	$2.5 - 2.67$	–	–	–	–	–	$9.55^{+0.10}_{-}$

Table 3 $M - R$ curve and prediction of radii for different values of η

Objects	$M/M_{\odot}$	Predicted R [km]				
		$\eta = 0$	$\eta = 0.0001$	$\eta = 0.0002$	$\eta = 0.0003$	$\eta = 0.0004$
LMC X-4 [122]	1.29 ± 0.05	$9.58^{+0.01}_{-0.03}$	$9.49^{+0.01}_{-0.03}$	$9.40^{+0.03}_{-0.03}$	$9.32^{+0.03}_{-0.04}$	$9.23^{+0.04}_{-0.03}$
PSR J1903+327 [123]	1.667 ± 0.021	$9.71^{+0.01}_{-0.01}$	$9.65^{+0.01}_{-0.01}$	$9.58^{+0.01}_{-0.01}$	$9.52^{+0.02}_{-0.02}$	$9.45^{+0.01}_{-0.02}$
PSR J1614-2230 [124]	1.97 ± 0.04	$9.73^{+0.01}_{-0.01}$	$9.68^{+0.01}_{-0.01}$	$9.63^{+0.01}_{-0.01}$	$9.58^{+0.01}_{-0.01}$	$9.53^{+0.01}_{-0.01}$
GW190814 [125]	$2.5 - 2.67$	–	–	$9.26^{+0.16}_{-}$	$9.27^{+0.14}_{-}$	$9.29^{+0.09}_{-}$

examined the stability of the present gravitational system by analyzing Herrera's cracking concept based on the idea of causality. Certainly, the system behaves as a causal structure as the square of the sound speeds both in radial and tangential direction is non-negative and less than unity as can be seen from Fig. 8. Even, the inequality $-1 < |v_t^2 - v_r^2| < 0$ is maintained inside the star satisfying Herrera's cracking concept for stable regions with minimal dependence on χ_1. Finally, we have inspected the adiabatic stability rule given by Herrera for anisotropic fluid and confirmed that the adiabatic index shown in Fig. 9 is greater than 4/3 for different values of χ_1. This indicates that the anisotropic charged stellar solution in $f(\mathcal{Q})$-gravity theory meets causality and stability criteria for all taken values of the non-metricity parameter χ_1.

In the present work, with increase in χ_1 the anisotropy (Fig. 5) and the maximum mass (Fig. 11) of the stellar system tend to increase simultaneously. The direct interpretation of the fact may be such that an increase in maximum mass depends on enhancing the anisotropy in the star. This can be justified by a recent study [117] showing that the anisotropy has a direct effect on the maximum mass of a neutron star. It proposes that the maximum mass of an isotropic configuration can be increased up to 15% by including the effect of anisotropy which may be the possible explanation of the maximum masses larger than 2.5 $M_{\odot}$ as in the case of the secondary component of the GW190814. Additionally, the mass-radius curves are related to the observational constraints of the masses of some known neutron stars. With reference to Fig. 11, it can be concluded that an anisotropic and charged configuration in $f(\mathcal{Q})$-gravity has the capability to fulfill the observational constraints. For instance, an charged anisotropic compact star could explain the massive secondary companion of GW190814.

The present investigation reveals that the charged anisotropic stellar model considering a particular ansatz for metric function and linear form of $f(\mathcal{Q})$ in the framework of $f(\mathcal{Q})$-gravity successfully can account for the well-behaved and singularity-free solutions which satisfy all the necessary physical criteria and can be associated with the observational constraints of observed neutron stars.

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Code Availability Statement This manuscript has no associated code/software. [Authors' comment: We use the Mathematica and Python software for numerical computation and graphical analysis of this problem. No other code/software was generated or analysed during the current study.].

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