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On Possible Origin of an Artificial Wormhole

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Abstract: We assume the picture in which a vacuum is filled with virtual wormholes. The presence of external fields changes the distribution of virtual wormholes and, in principle, allows for the creation of a coherent structure that can work as an actual wormhole or the Alcubierre bubble. We establish the explicit functional relationship between the external field and the distribution of virtual wormholes in vacuum. This opens a way for examining the question about the possibility of the artificial creation of such objects in a laboratory. We show that weak and quasihomogeneous fields suppress the density of wormholes and cannot be used. We also discuss the existence of a threshold for the intensity of an external field beyond which the formation of wormhole-type structures becomes possible.

Keywords: Euclidean wormholes; quantum fields; topology changes

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1. Introduction

The physics of wormholes must deal with two important problems. The first is that wormholes violate the averaged null-energy condition (ANEC) [1], i.e., they require the presence of an exotic matter that does not exist in lab experiments. The second problem is that, according to Geroch theorem [2,3], topology changes are rigorously forbidden in classical general relativity. Nevertheless, there exists a widely spread misunderstanding that it is enough to prepare an exotic ANEC-violating matter in order to artificially create a wormhole. This is surely not true because irrespective of what sort of stress-energy tensor we may use as a source in classical general relativity, the space topology remains the same. An exotic energy may produce exotic dynamics but does not change the topology. In particular, the idea of creating a new universe (or baby universe) with an artificial process was first discussed in [4,5]. In connection with ANEC-violating theories in [6], it does not assume any topology change but rather an inflation for a small portion of space [1].

The first rigorous model thay may allow for describing the creation of an artificial wormhole was suggested in [8]. This model is based on quantum gravity effects, namely, on the spacetime foam picture [9,10]. At Planckian scales, space-time is assumed to be filled with a gas of virtual wormholes described by Euclidean wormhole configurations that were first suggested in [11,12]. The virtual wormhole represents a quantum topology fluctuation (tunneling between different topologies) that takes place at Planckian scales and lasts for a very short (Planckian) period of time. It should not obey the Einstein equations; therefore, it can violate ANEC.

Our model is based on the fact that a coherent set of virtual wormholes may work as an actual wormhole; see also [13,14]. In a somewhat different context, an analogous idea was discussed in [15]. Thus, by applying an external classical field, one may change the vacuum distribution of virtual wormholes and try to organize an artificial wormhole or the Alcubierre bubble [7]. Actual topology changes do not occur at all here, so no obstruction emerges from the Geroch theorem standpoint. What actually happens is a specific redistribution of virtual wormholes.

In the present paper, we employ the techniques that we developed in [16] in order to examine how an external field affects the distribution of the number density of virtual wormholes. Naive expectations that, e.g., a homogeneous external electric field may align virtual wormholes along electric lines, do not work here. Indeed, if it were the case, such a phenomenon would be observable in laboratory experiments. Instead, when the characteristic scale of the inhomogeneity of an external field exceeds the characteristic scales of the spacetime foam, the field suppresses the number density of the wormholes. This becomes clear once we recall that the presence of virtual wormholes diminishes the vacuum energy density [17]. The external classical field always carries a positive energy density; therefore, in the presence of any external field, the total energy becomes higher, which means that the number density of virtual wormholes should shrink. This feature is important since it demonstrates that the artificial creation of an actual wormhole requires strongly inhomogeneous external fields with very high energy.

The predicted suppression of the virtual wormhole number density produces an important effect that directly admits the experimental verification, namely, the decrease in the virtual wormhole number density in some spatial region causes the decrease in the speed of light in vacuum; therefore, any sublight particle penetrating such a region should start emitting Cherenkov radiation.

The negative perturbation in the number density of wormholes in the presence of weak external fields may lead to a positive shift in the value of the cosmological constant; see details in [17]. Whether it is possible to use this phenomenon to explain the observed acceleration of the universe requires a further study.

At first glance, the fact that an external classical field suppresses the density of virtual wormholes means that the creation of an actual wormhole meets an obstruction that seems to agree with [6], where an obstruction for creating a universe in the laboratory was reported. However, such a conclusion is valid only for weak fields whose amplitude is less than a certain critical value, and their inhomogeneity scale is sufficiently large in comparison to the characteristic scales of the foam. In fact, the presence of virtual wormholes leads to the emergence of a specific self-interaction between particles, even in free theory [16]. Therefore, the external classical field must also possess the same nonlinearity. In this case, a field of very strong intensity may generate small-scale structures and redistribute virtual wormholes in the required way. Such phenomena should be analogous to the development of turbulence in hydrodynamics and apparently correspond to a specific threshold or critical value of the field intensity. This opens up the possibility to control the distribution of virtual wormholes. As it follows from dimensional estimates, this critical value can be extremely large (the naive estimate gives the Planckian order). One still cannot exclude the possibility that the threshold could be reached experimentally.

2. Effective Action in the Presence of Virtual Wormholes

We now briefly summarize our previous results [16].

Virtual wormhole. A virtual wormhole represents a tunneling event that is described by the Euclidean wormhole configuration as follows [14]. In the integral Euclidean path approach, the dominant contribution comes from metrics with the least Euclidean action. As was first shown by Hawking [11], those are given by conformally flat metrics ($\alpha = 0, 1, 2, 3$):

$$ds^2 = h^2(x)\delta_{\alpha\beta}dx^\alpha dx^\beta, \; h = 1 + \frac{a^2}{(x^\alpha - x_0^\alpha)^2}. \tag{1}$$

Namely, conformal transformation $g_{\alpha\beta} = h^2 g^*_{\alpha\beta}$ transforms the Euclidean action into

$$S_E = -\frac{1}{16\pi}\int R\sqrt{g}d^4x + C = -\frac{3}{8\pi}\int h(-\Delta + \frac{1}{6}R^*)h\sqrt{g^*}d^4x + C, \tag{2}$$

where R is the Ricci scalar, $(\Delta - \frac{1}{6}R^*)$ is the Laplace–Beltrami conformally invariant operator, and C is the surface term that allows for removing second derivatives, e.g., see [18]. Therefore, for the conformally flat metric $ds^2 = h^2 ds_E^2$, the Euclidean vacuum Einstein equations reduce to conformally invariant equation [11] $(\Delta - \frac{1}{6}R^*)h = 0$ with $R^* = 0$. The above conformal factor represents the simplest nontrivial solution of that equation the boundary condition $h \to 1$ as $x \to \infty$ and point x_0^α being removed. The singularity, i.e., the blowing up of the conformal factor at point x_0^α, actually means that there is another asymptotically Euclidean region as $h \gg 1$. This can be directly seen upon the inversion of the coordinate system that transforms $x_0^\alpha \to \infty$.

Metric (1) is invariant under inversion transformation

$$x'^\mu - x_0'^\mu = \frac{a^2 \Lambda_\nu^\mu \left(x^\nu - x_0^\nu\right)}{(x^\alpha - x_0^\alpha)^2},$$

where $\Lambda_\nu^\mu \in O(4)$ and describes a Euclidean wormhole of radius $2a$ that connects two asymptotically Euclidean spaces $\rho \ll a$ and $\rho \gg a$ (where $\rho^2 = (x^\alpha - x_0^\alpha)^2$). Both Euclidean regions $\rho \gg a$ and $\rho \ll a$ can be identified by the gluing, which means that both entrances lead to the same Euclidean space. Let $x_\pm^\alpha$ be coordinates in both spaces, i.e., primed and nonprimed coordinates in the above expression. Then, the identification is the rule

$$x_+^\mu = R_+^\mu + \Lambda_\nu^\mu (x_-^\nu - R_-^\nu), \tag{3}$$

where $R_\pm$ is the position of the center of the wormhole in coordinates $x_\pm^\mu$. If the separation of entrances $X = |R_+ - R_-|$ is a large $X \gg a$, the conformal factor can be approximately written as

$$h \simeq 1 + \frac{a^2}{(x^\alpha - R_+^\alpha)^2} + \frac{a^2}{(x^\alpha - R_-^\alpha)^2}$$

which also gives the least Euclidean action. The influence of entrances on each other and the exact form of h can be obtained with the image method that includes a series of additional images with decreasing values of throats $a_n \sim a\varepsilon^n$, where $\varepsilon = a^n/X^n$, which are placed within spheres of radius a and centers at $x^\alpha = R_\pm^\alpha$. Such a wormhole can also be viewed as a baby universe [11,12] that first branches off at point R_-^α and then at point R_+^α joints onto the mother Euclidean space. The Euclidean action for (1) reduces to surface term C and is $S_E = 3a^2\pi$. Since point x_0^α is removed, there are two such terms, $h \to \infty$ and $h \to 1$.This means that the typical size of a wormhole, weighted by factor e^{-S_E}, does not exceed the Planckian order. The most general distribution of Euclidean wormholes corresponds to the conformal factor of type $h = 1 + \sum \frac{a_N^2}{(x^\alpha - R_N^\alpha)^2}$, where the sum takes over all wormhole entrances and their images. However, to avoid multiple counting in the action, the integration should be performed over the region restricted by spheres $\frac{a_N^2}{(x^\alpha - R_N^\alpha)^2} > 1$, while all additional images lie within those spheres.

The Green function for a scalar field in the presence of virtual wormholes. In the case of a free scalar field (for a more general case and more details, see [19]), wormholes modify the Green function that is the solution of Laplacian equation

$$\left(-\Delta + m^2\right) G\left(x, x'\right) = \delta\left(x - x'\right),$$

where m is the mass of particles. As boundary conditions, we require G and $\partial G/\partial n$ to be continuous at throats. The Green function has the structure of $G_0 + \delta G$, where $G_0(x - x') = \frac{m^2}{4\pi^2} \frac{K_1(m\rho)}{m\rho}$. Source $\delta(x - x')$ generates a set of 4-dimensional multipoles placed in the center of throats that determine the corrections to Green function δG. In the dilute gas approximation, corrections become additive and can be written as

$$\delta G(x, x') = \int \delta G\left(x, x', \xi\right) F(\xi) d\xi, \tag{4}$$

where $F(\xi)$ is the density of wormholes in the configuration space ξ

$$F(\xi) = \sum_i \delta(\xi - \xi_i) \tag{5}$$

and for the lowest monopole term in δG (for the general case, see [19]), we obtain

$$\delta G(x, x') = -2\pi^2 a^2 (G_0(x - R_+) - G_0(x - R_-)) \left(G_0\left(x' - R_+\right) - G_0\left(x' - R_-\right)\right). \tag{6}$$

Background vacuum distribution has an isotropic and homogeneous character, i.e., $\rho(\xi) =< 0|F(\xi)|0 >$ with $\rho(\xi) = \rho(a, X)$, and $X = |R_+ - R_-|$. Then, for the Fourier transform of the Green function, we find

$$G(k) = \frac{1}{k^2 + m^2}\left(1 - \frac{1}{k^2 + m^2}\nu(k)\right) \approx \frac{1}{k^2 + m^2 + \nu(k)}, \tag{7}$$

where $\nu(k)$ is given by [19]

$$\nu(k) = 4\pi^2 \int a^2 (\rho(a, 0) - \rho(a, k)) \frac{J_1(ka)}{ka/2} da, \tag{8}$$

where $J_1(x)$ is the Bessel function that accounts for the finite size of the throat, and $\rho(a, X) = \int \rho(a, k) e^{-ikX} \frac{d^4k}{(2\pi)^4}$. In the analogy to the consideration in [20], we may define the effective radius of virtual wormholes $R^2(k)$ as

$$R^2(k) = \frac{1}{n} \int a^2 \rho(a, 0) \frac{J_1(ka)}{ka/2} da,$$

where $n = \int a^2 \rho(a, 0) da$ has a sense of the vacuum density of wormholes. Then, the polarization function takes the form

$$\nu(k) = 4\pi^2 n R^2(k)(1 - \mu(k)),$$

where $\mu(k)$ is the distribution of wormholes over the separation between entrances into the same wormhole and is determined by

$$\mu(k) = \frac{1}{nR^2(k)} \int a^2 \rho(a, k) \frac{J_1(ka)}{ka/2} da.$$

Generating functional and effective action. The partition function for scalar particles in the presence of an external source is determined in the standard way

$$Z_{total}(J) = \sum_\tau \sum_\varphi e^{-S_E}$$

where the sum is taken over field configurations φ and wormholes τ. We take the Euclidean action in the form

$$S_E = \int \left[-\frac{1}{16\pi} R + \frac{1}{2}\varphi\left(-\Delta + m^2\right)\varphi - J\varphi\right] \sqrt{g} d^4x + C, \tag{9}$$

where C includes all surface terms. We do not account for all gravitational degrees of freedom and take the conformally flat metric of a type (1) with a general conformal factor describing a distribution of virtual or Euclidean wormholes as the background. Gravitons and other gauge fields can be considered to be additional perturbations. For the sake of simplicity, we consider only the scalar field here.

The integration over fields can be performed for any particular distribution of wormholes, which gives

$$Z_{total}(J) = \int [DF] Z_0(G) e^{\frac{1}{2}(JGJ)}, \tag{10}$$

where $G = G_0 + \delta G$ is the Green function for a particular (4) distribution of wormholes, $Z_0(G) = \int D\varphi e^{-S_E(J=0)}$, and $\int [DF]$ defines the sum over the number of wormholes and the integration over their parameters. Expanding this expression by J, we find (see details in [16]):

$$W(J) - W(0) = \ln \frac{Z_{tot}(J)}{Z_{tot}(0)} \approx \frac{1}{2}\overline{(J, GJ)} + \frac{1}{8}\overline{(J, \Delta GJ)^2} + \frac{1}{48}\overline{(J, \Delta GJ)^3} + \dots \tag{11}$$

where we use notion $(J, \varphi) = \int J(x)\varphi(x) d^4x$, and the overbar denotes the vacuum mean value $\overline{G} =< 0|G|0 >$, i.e.,

$$\overline{G} =< 0|G|0 >_{J=0} = \frac{1}{Z_{total}(0)} \int [DF] Z_0(G) G(\xi). \tag{12}$$

These terms are expressed via moments of the number density of wormholes in the configuration space as follows

$$\overline{G} = G_0 + \int \delta G(\xi) \rho(\xi) d\xi,$$

where $\rho(\xi) =< 0|F(\xi)|0 >_{J=0}$ is the mean density (5), and $\delta G(\xi)$ is expressed by (6). Analogously, the next term in (11) that describes topology fluctuations can be expressed as

$$\overline{\Delta G \Delta G} = \int \delta G(\xi) \delta G(\xi) \rho(\xi) d\xi + \int \delta G(\xi) \delta G(\xi') \omega(\xi, \xi') d\xi d\xi'$$

where we denote ($\Delta F(\xi) = F(\xi) - \rho(\xi)$), and for fluctuations in the mean number of wormholes density, we obtain

$$\rho(\xi, \xi') =< 0|\Delta F(\xi) \Delta F(\xi')|0 >_{J=0} = \rho(\xi)\delta(\xi - \xi') + \omega(\xi, \xi').$$

Higher-order mean values in (11), e.g., $\overline{\Delta G \Delta G \Delta G}$, are expressed via the respective momenta $\rho(\xi, \xi', \xi'')$, etc.

Effective action. The effective action for a scalar field can now be obtained by the Legendre transformation

$$\Gamma(\varphi_c) = (\varphi_c J_c) - W(J(\varphi_c)),$$

where $\varphi_c(J) = \frac{\delta W}{\delta J}$ is expressed via source J. In the approximation of a rarefied gas of wormholes, we may neglect the correlations between positions of wormhole, which are $\omega(\xi, \xi') \simeq 0$. Then, this transformation gives (see details in [16]):

$$\Gamma(\varphi) = \frac{1}{2}(\varphi\left(-\Delta + m^2\right)\varphi) + V_1(\varphi) + V_2(\varphi) + \dots$$

where terms $V_1(\varphi)$ and $V_2(\varphi)$ are

$$V_1(\varphi) = -\frac{1}{2}\left(\varphi\left(-\Delta + m^2\right)\overline{\delta G}\left(-\Delta + m^2\right)\varphi\right),$$

$$V_2(\varphi) = -\frac{1}{8}\int \left(\varphi\left(-\Delta + m^2\right)\delta G(\xi)\left(-\Delta + m^2\right)\varphi\right)^2 \rho(\xi) d\xi.$$

Effective action $\Gamma(\varphi)$ produces the self-consistent quantum field theory that is free of divergencies in all orders of perturbations. Indeed, it was previously demonstrated [16] that this action has the Pauli–Villars structure and automatically realizes the invariant Pauli–Villars regularization scheme [21]. In this sense, virtual wormholes cure the basic illness of quantum theory and play the role of a natural regularizator.

The two terms $V_1(\varphi)$ and $V_2(\varphi)$ have a clear interpretation. Term $V_1(\varphi)$ describes the effects of scattering on virtual wormholes and is responsible for the generation of additional massive excitations with masses M_a. The corresponding masses are expressed via moments of wormholes $< 0|a^{2m}X^{2n}|0 >$ (see [16]), and exact values of mass spectrum M_a essentially depend on the typical value of distance X between wormhole entrances. The naive expectations give $M_a \gtrsim M_{Pl}$. Second term $V_2(\varphi)$ corresponds to the back reaction that comes from the quadratic term in generating a functional (11). It describes the effects of a redistribution of virtual wormholes in the external field. Before considering the relation between external field and wormhole number density, we describe possible coherent structures of interest that, as expected, admit formation by external fields.

3. Possible Coherent Structures

As was demonstrated in [13], an actual wormhole-type structure can be modeled with a coherent set of virtual wormholes. We now present the simplest coherent structures. Consider first the distribution function that corresponds to a particular single virtual wormhole. Such a function is given simply by

$$\delta F(\xi, y, y') = \delta(a - a_0)\delta^4(R_+ - y)\delta^4(R_- - y')$$

which corresponds to a single wormhole with entrances at $R_+ = y$, $R_- = y'$, and throat radius $a = a_0$. Let us fix vector $X_0^\alpha = R_+^\alpha - R_-^\alpha = const$ that determines the position of the second entrance via the position of the first as $y' = y - X_0$. This allows for us to define coherent sets of wormholes by integrating the above distribution over some portion of space with an auxiliary density $n(y)$ as

$$\delta\rho(\xi, n) = \int \delta F(\xi, y) n(y) d^4 y. \tag{13}$$

Auxiliary density has a clear physical interpretation. It is the number density of entrances into wormholes that have the same radius a_0, the same separation, and which are directed along the same vector X_0^α. The length of the throat for metric (1) has the value $\ell = 2a$ that tends to zero in limit $a \to 0$. In limit $a_0 \to 0$, the virtual wormhole degenerates into a point; from a topological standpoint, distribution (13) defines the gluing of points of the region of space where $n(y) \neq 0$ with an analogous region shifted on vector X_0. Indeed, let $n(y) = n_0\delta(f(y))$. Then the integration in (13) reduces to the integral over hypersurface $S = \{f(y) = 0\}$. Every point on S, $y \in S$ corresponds to the first entrance into a virtual wormhole, while second entrance y' lies on shifted hypersurface S', i.e., to $y' = y - X_0 \in S'$. Parameter $n_0 \leq 1$ determines the porosity of the gluing for both surfaces. Thus, the above density corresponds to a coherent set of wormholes that roughly glue domains $D = \{n(y) \neq 0\}$ and analogous domain D' shifted in space on vector X_0. Density $n(y)$ here defines the degree of gluing (the transparency coefficient for the resulting wormhole).

If we take different densities $n(y)$, we may obtain different wormhole-type coherent structures that correspond to actual wormholes. An actual wormhole exists for a sufficiently big interval of time t; therefore, density $n(y) = n(r)$ where $y = (t, r)$. The simplest example is the analog of an *astrophysical wormhole* that can be modeled with a spherically symmetric function e.g., $n(y) = n(r) = n_0\delta(r - b)$, where b relates to the radius of the throat, and r is the radial coordinate. Another example can be called the *Star Gate* when density $n(r)$ is concentrated on a thin two-dimensional disk, e.g., $n(\vec{r}) = n(\rho, \varphi, z) = n\theta(b - \rho)\delta(z)$ (where ρ, φ, z are polar coordinates of vector $\vec{r}$). In homogeneous case, $n(y) = n$, and we obtain the example of space with anomalous or anisotropic velocity of light discussed by one of us in [14].

The example of the *Alcubierre bubble*, or the so-called *warp drive*, can also be constructed as follows. From a 4-dimensional standpoint, the bubble is a tube in which the structure of light cones differs from that in the outer region [7]. Such a structure can be obtained with the cut of the tube from the original mother universe and the subsequent gluing onto its place of an analogous tube with a somewhat different direction of the time coordinate. In other words, it can also be considered to be a specific defect of space and can be constructed with a cut and paste technique. Therefore, this can be achieved with function $n(y) = n_0\theta(r_s(x,t) - b)$, where b is the bubble radius, $\theta(x)$ is the step function, and $r_s^2(x,t) = (x - x_s(t))^2 + y^2 + z^2$. It is assumed that the shift on vector X is an element of symmetry of the tube, and (13) defines the gluing of points within the tube only. Within the tube, the causal structure changes due to the change in the speed of light.

By using different densities $n(y)$ in (13) and substituting it into (8), we determine the resulting change in Green function $\delta G(J)$, and may determine the required external source J. In order to create the respective structure in vacuum, we should solve the inverse problem, namely, to determine the external field that produces the respective polarization of the vacuum. We consider the relationship between the external field and the correction to the Green function in the next section.

4. Back Reaction

Since we study corrections to the Green function due to the presence of an external field, and the latter is supposed to be sufficiently small, we may use the approximation of a rarefied gas of additional virtual wormholes. Therefore, to the leading order, we may assume that the correlations between positions of wormholes are absent, and set $\omega(\xi,\xi') \approx 0$. This may give a rather good approximation in the general case. Indeed, as was demonstrated in [17,19], in order to obtain finite quantum field theory, wormholes should degenerate into pointlike objects when the throat radius tends to zero $a \to 0$. In this limit, the vacuum energy density has a finite value, while the portion of the volume in space occupied by wormholes tends to zero. Nevertheless, some corrections appear due to interactions between throats. To account for such correlations, it requires further development of the theory.

Thus, we find the decomposition of the generating functional in the form

$$W(J) - W_0 = \frac{1}{2}(JG_0J) + \frac{1}{2}\int (J\delta G(\xi)J)\rho(\xi)d\xi + \frac{1}{8}\int (J\delta G(\xi)J)^2\rho(\xi)d\xi + \dots \tag{14}$$

First term here corresponds to the standard free scalar field. The second term describes the effects of the scattering of scalar particles on virtual wormholes and corresponds to $V_1(\varphi)$ in the effective action. The third term introduces a nonlinearity due to the back reaction that corresponds to $V_2(\varphi)$. From (14), we find the relation between the external field produced by source J and the additional correction to the Green function as

$$G = \overline{G} + \delta\overline{G}(J) \tag{15}$$

where

$$\delta\overline{G}(J)_{xx'} = \frac{1}{2}\int (J\delta G(\xi)J)\delta G(\xi)_{xx'}\rho(\xi)d\xi + \int \left(\int \delta G(\xi)_{xy}J_y d^4y\right)\left(\int \delta G(\xi)_{x'z}J_z d^4z\right)\rho(\xi)d\xi, \tag{16}$$

$\overline{G}$ is the vacuum mean Green function that is given by (12); see also (7), (8) and $\delta G(\xi)$ is determined by (6).

Equation (15) gives the implicit relation between the distribution of virtual wormholes and the external field. In order to find the necessary field, the corresponding inverse problem must be solved, which is rather serious. In general, inverse problem (15) is incorrect and requires separate study. However, here we present a simple analysis. First, homogeneous and regular quasihomogeneous fields always suppress the density of wormholes. To see this, it is sufficient to look at the sign of the self-interacting term in (14) and (16). For quasihomogeneous fields, typical scales obey inequality $k^2 \ll M_{\min}^2$ where $M_{\min}$ is the minimal mass of additional excitations; then, one may take $\overline{G} \simeq G_0$. Therefore, if the external field does not include very small-scale inhomogeneities, correction $\delta\overline{G}(J)_{xx'}$ to the bare Green function G_0 is always positive, while wormholes give a negative contribution that can be directly seen from (7). This explains the fact why such phenomena were not observed so far.

The possible creation of an artificial wormhole assumes that an external field possesses inhomogeneities with typical scales of order $k^2 \sim M^2_{\min}$. This can be achieved in two ways. The first is direct, e.g., by trying to accelerate particles to almost Planckian energies. This way seems to not work at all. The problem is that the external field should be a classical one that presumes that the number of particles in the given state should be extremely large. Moreover, the possibilities of LHC are still far from such energies, and even cosmic ray particles do not reach Planckian scales. The second way looks to be more promising since it is based on the nonlinearity of vacuum equations for external classical fields, as follows from (14) and the possibility that such fields, e.g., electromagnetic fields, can be produced with very high intensity. For example, strong intensities can be obtained by focusing beams from lasers. By the analogy with the turbulence phenomena, we expect the existence of a specific threshold in the intensity of the field that switches on the generation of inhomogeneities of increasingly smaller scales.

The estimate for the threshold, i.e., the critical value of the field, can be found as follows. Let ℓ be the characteristic scale of the inhomogeneity of the field, φ_{cr} be the critical amplitude, a be the typical scale of throats, X be the typical distance between entrances, and n be the density of virtual wormholes. Then, from (14), we find the estimate

$$\varphi^2_{cr} \sim \frac{\ell^2}{< a^4 >< X^4 > n}.$$

Let us assume that all of the typical scales are $a \sim X \sim n^{-1/4} \sim L_{pl}$; then, we obtain $\varphi_{cr} \sim \ell / L^2_{Pl}$, which was still far from possibilities of modern technologies. The basic hope is that the actual typical scale X is a much larger $X \sim L \gg L_{pl}$, which gives a very modest value $\varphi_{cr} \sim \ell / L^2$. In particular, such a case is realized for the fractal distribution of wormholes [20], which is also predicted by lattice quantum gravity models [22–25]. Even when we reach such intensities, the serious problem remains of creating the desired structure of the field itself.

5. Discussion

Our analyses show that the creation of a classical wormhole or wormhole-type structures requires very large energy densities whose exact value depends on the characteristic scales of spacetime foam a, X, and $n^{-1/4}$. From the formal standpoint, there are no principle obstructions. The naive estimates give $a \sim X \sim n^{-1/4} \sim L_{pl}$, and at the present level of technology, such energies cannot be reached. Even if one of the foam scales is a sufficiently large $X \sim L \gg L_{pl}$, the necessary energies should be enormous. Indeed, the scattering on virtual wormholes produces additional massive particles for all types of fields [16]. Such additional particles were not detected, which means that the typical scale has not yet been reached. There also remains a problem that sufficiently intensive external classical fields produce the effects of particle production that essentially complicate the total picture.

In conclusion, quasihomogeneous fields somewhat reduce the vacuum value of the speed of light. The value of the effect depends on the intensity of the field: $c^2 = 1 - \frac{2b^2}{3!M_2^4}$, where $b^2 = (\nabla\varphi)^2$ and $M_2^{-4} = \frac{\pi^4}{6} < a^4 X^4 > n$, e.g., see [16]. Such an effect can already be verified in laboratory experiments. Moreover, it should produce specific features of radiation from active galactic nuclei and black holes.

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Note

[1] So, it may be used for the Alcubierre warp drive [7], since the latter does not presume a nontrivial topology.

References

1. Visser, M. *Lorentzian Wormholes*; Springer: New York, NY, USA, 1996.
2. Geroch, R. Topology in general relativity. *J. Math. Phys.* **1967**, *8*, 782–786. [CrossRef]
3. Yodzis, P. Lorentz cobordism II. *Gen. Rel. Grav.* **1973**, *4*, 299. [CrossRef]
4. Berezin, V.A.; Kuzmin, V.A.; Tkachev, I.I. Random inflation and global geometry of the universe. *JETP Lett.* **1985**, *41*, 547.
5. Farhi, E.; Guth, A.H. An obstacle to creating a universe in the laboratory. *Phys. Lett. B* **1987** , *183*, 149. [CrossRef]
6. Rubakov, V.A. Towards creating a universe in the laboratory. *Phys. Rev. D* **2013**, *88*, 044015. [CrossRef]
7. Alcubierre, M. The warp drive: Hyper-fast travel within general relativity. *Class. Quantum Grav.* **1994**, *11*, L73–L77. [CrossRef]
8. Kirillov, A.A.; Savelova, E.P. On the Possibility of the Generation of an Artificial Quantum Wormhole. *Yad. Fiz. Eng.* **2013**, *4*, 932–936. (In Russian)
9. Wheeler, J.A. *Relativity, Groups, and Topology*; DeWitt, B.S., DeWitt, C.M., Eds.; Gordan and Breach: New York, NY, USA, 1964.
10. Hawking, S.W. Spacetime foam. *Nucl. Phys.* **1978**, *B144*, 349. [CrossRef]
11. Hawking, S.W. Quantum coherence down the wormhole. *Phys. Lett. B* **1987**, *195*, 337–343. [CrossRef]
12. Lavrelashvili, G.V.; Rubakov, V.A.; Tinyakov, P.G. Disruption of quantum coherence upon a change in spatial topology in quantum gravity. *JETP Lett.* **1987**, *46*, 134.
13. Savelova, E.P. Gas of Wormholes as a Model for Dark Energy. *Gravit. Cosmol.* **2013**, *19*, 101–105. [CrossRef]
14. Savelova, E.P. On possible origin of an anisotropy in the speed of light in vacuum. *Gen. Relativ. Gravit.* **2016**, *48*, 85. [CrossRef]
15. Mielke, E.W. Knot wormholes in geometrodynamics? *Gen. Relativ. Gravit.* **1977**, *8*, 175. [CrossRef]
16. Kirillov, A.A.; Savelova, E.P. Effective action for a free scalar field in the presence of spacetime foam. *Gen. Relativ. Gravit.* **2015**, *47*, 97. [CrossRef]
17. Kirillov, A.A.; Savelova, E.P. On the Value of the Cosmological Constant in a Gas of Virtual Wormholes. *Gravit. Cosmol.* **2013**, *19*, 92–100. [CrossRef]
18. Hawking, S.W. *General Relativity*; Hawking, S.W., Israel, W., Eds.; Cambridge University Press: Cambridge, UK, 1979.
19. Savelova, E.P. Gas of wormholes in Euclidean quantum field theory. *Grav. Cosmol.* **2015**, *21*, 48–56. [CrossRef]
20. Kirillov, A.A.; Savelova, E.P.; Vladykina, P.O. Possible Effects of the Fractal Distribution of Relic Wormholes. *Universe* **2021**, *7*, 178. [CrossRef]
21. Pauli, W.; Villars, F. On the Invariant Regularization in Relativistic Quantum Theory. *Rev. Mod. Phys.* **1949**, *21*, 434. [CrossRef]
22. Knizhnik, V.; Polyakov, A.; Zamolodchikov, A. Fractal structure of 2D quantum gravity. *Mod. Phys. Lett. A* **1988**, *3*, 819. [CrossRef]
23. Kawai, H.; Kawamoto, N.; Mogami, T.; Watabiki, Y. Transfer matrix formalism for twodimensional quantum gravity and fractal structures of space-time. *Phys. Lett. B* **1993**, *306*, 19–26. [CrossRef]
24. Ambjørn, J.; Jurkiewicz, J.; Loll, R. The spectral dimension of the universe is scale dependent. *Phys. Rev. Lett.* **2005**, *95*, 171301. [CrossRef] [PubMed]
25. Ambjørn, J.; Drogosz, Z.; Gizbert-Studnicki, J.; Görlich, A.; Jurkiewicz, J.; Németh, D. Cosmic voids and filaments from quantum gravity. *Eur. Phys. J. C* **2021**, *81*, 708. [CrossRef]